Carrier phase multipath mitigation based on GNSS signal quality measurements

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Abstract. Carrier phase multipath caused by signal reflections in the vicinity of GNSS stations is a major error source of precise differential positioning. Detection and mitigation of carrier phase multipath errors can be based on signal quality values (e.g. signal-to-noise ratio values) as provided by GNSS receivers. These values are influenced by various factors as e.g.the actual strength of the transmitted signal, space loss, atmospheric effects, and antenna gain pattern. After their removal, deviations from nominal signal quality values mainly contain multipath information which can be used to calculate carrier phase multipath corrections. The described technique is valid only for observations collected in static mode and single dominant reflectors. Practical experiences show that corrections based on signal quality observations are able to remove a large portion of the carrier phase multipath errors.

Keywords. GNSS, carrier phase multipath, signal-to-noise ratio, signal quality

1. Introduction

Multipath caused by diffuse or specular reflections (Braasch 1996, Hannah 2001) in the vicinity of GNSS (*Global Navigation Satellite Systems*) stations is a major error source of cm-level positioning. Multipath effects on code and phase measurements differ considerably. The maximum carrier phase error is frequency dependent and amounts to 4.8 cm and 6.1 cm for GPS frequencies L1 and L2, respectively (Georgiadou and Kleusberg 1988). Code multipath errors, however, can reach up to several meters.

There are various methods to mitigate carrier phase multipath influences. Basically, they can be divided into two categories: multi-antenna systems and solutions which exploit the information content of the signal quality measurements. Multi-antenna systems couple a number of small and closely spaced antennas to a single antenna. It is thus possible to detect signal disturbances, e.g. caused by multipath, and mitigate their effects on the observations (Brown 2001, Brown and Mathews 2005). Multi-antenna systems are capable for working in real-time. Ray (2000) uses the well-known geometry of several closely spaced (not coupled) antennas to determine the multipath influence based on signal quality measurements. The majority of solutions employing signal quality measurements are found in post-processing applications. For this purpose, the signal quality measurements (signal-to-noise ratios or carrier-to-noise ratios) are used to estimate carrier phase multipath errors. Early research results were published by Axelrad et al. (1996), Comp and Axelrad (1998), Reichert and Axelrad (1999). A detailed discourse and newer results on multipath error estimating based on signal quality measurements, are given in Bilich (2006), Bilich and Larson (2007), Bilich et al. (2008) and Lau and Cross (2005, 2006). This paper describes a new efficient processing strategy to determine carrier phase multipath corrections based on signal quality measurements.

The following section discusses multipath effects on carrier phase observations and on signal quality observations, and how these two effects are connected. The third section describes our processing strategy in more detail. Section 4 presents test results and examines the limitations of the presented technique. The paper does not deal with multipath errors of the code observable.

2. Carrier phase multipath and GNSS signal quality theory

2.1. Phase Multipath Model

Ideally, a signal transmitted by a satellite arrives at a receiving antenna on its direct path only. In actual fact however, the direct signal is superimposed by indirect signals which are either reflected in the antenna surroundings (Figure 1) or at the transmitting satellite. This causes the received signal to become a composite signal whose phase is shifted when compared to the direct signal.

Let us assume that there is just a single reflected signal involved. In this case the direct signal s_D , the one reflected signal s_R and the composite signal s_C can be described after Bishop et al. (1985) and Georgiadou and Kleusberg (1988) by

$$s_{\rm D} = A_{\rm D} \cdot \cos \phi_{\rm D},\tag{1a}$$

$$s_{\mathbf{R}} = \alpha \cdot A_{\mathbf{D}} \cdot \cos\left(\phi_{\mathbf{D}} + \Delta\phi_{\mathbf{R}}\right),$$
 (1b)



Figure 1: Multipath due to signal reflections in the surroundings of a GNSS station. Direct signal and indirect signals form the composite signal received at the antenna.

$$s_{\rm C} = s_{\rm D} + s_{\rm R},$$

= $A_{\rm D} \cdot \cos \phi_{\rm D} + \alpha \cdot A_{\rm D} \cdot \cos (\phi_{\rm D} + \Delta \phi_{\rm R}).$ (1c)

where $A_{\rm D}$ denotes the amplitude (in volts) and $\phi_{\rm D}$ the signal phase of the direct signal. The amplitude of the reflected signal is attenuated by α ($0 \le \alpha \le 1$) and its phase is shifted with respect to the direct signal by $\Delta\phi_{\rm R}$.

The amplitude $A_{\rm C}$ of the composed signal can be written as

$$A_{\rm C} = A_{\rm D} \cdot \sqrt{1 + 2 \cdot \alpha \cdot \cos \Delta \phi_{\rm R} + \alpha^2}, \qquad (2)$$

and the carrier phase error due to multipath interference as

$$\delta \phi = \arctan\left(\frac{\alpha \cdot \sin \Delta \phi_{\rm R}}{1 + \alpha \cdot \cos \Delta \phi_{\rm R}}\right). \tag{3}$$

The theoratical maximum carrier phase error occurs for $\alpha = 1$ and $\Delta \phi_{\rm R} = \pi$. This yields maximum phase range errors of 4.8 cm and 6.1 cm for GPS L1 and L2, respectively.

2.2. GNSS signal quality

Signal quality is a generic term which describes the ratio of signal power to noise power. The signal strength can be defined either by amplitude A (in volts) or power S (in watts). Signal-to-noise ratio can usually be found in the context of signals at baseband (using power S_{Corr} of the modulated signal at correlator output) (Langley 1997, Butsch 2002). As written in Langley (1997) and Ward et al. (2006b), the quality of a received GNSS signal is commonly described by its carrier-to-noise ratio (using power C_{Ant} of the unmodulated carrier at the receiving antenna). Due to the filter process of the modulated signal, S is negligibly smaller than C (assuming the same reference point). Nevertheless, signal quality is given either as signal-to-noise ratio (S/N) or as carrier-to-noise ratio (C/N).

The system noise N affecting the signal quality originates from various sources. Langley (1997) examined the origins of noise and assessed their effects on GNSS observations. Pseudorange code and carrier phase measurement errors depend on the ratio of signal power (C or S) and system noise (N). Due to the fact that noise and signal are amplified in the same way (between antenna and correlator output), these ratios are almost identical:

$$\frac{C_{\text{Ant}}}{N_{\text{Ant}}} \approx \frac{S_{\text{Corr}}}{N_{\text{Corr}}} \approx S_{\text{Q}}.$$
(4)

The system noise N is several magnitudes smaller than the signal strength S. Therefore, the values are usually converted to decibel (dB):

$$S_{\rm Q} ({\rm dB}) = 10 \cdot \log_{10} (S_{\rm Q}).$$
 (5)

In order to ensure comparability among various GNSS receivers, S/N is often normalised to a specific bandwidth, e.g. 1 Hz. Hence, system noise N is substituted by the product of noise power density N_0 and loop bandwidth $B_{\rm L}$:

$$\mathbf{N} = N_0 \cdot B_{\mathrm{L}}.\tag{6}$$

Equations (5) and (6) yield the normalised signal quality (Butsch 2002):

$$S_{\mathsf{Q}_0} \left(\mathsf{dBHz} \right) = 10 \cdot \log_{10} \left(S_{\mathsf{Q}} \right) \, \left(\mathsf{dB} \right) + B_{\mathsf{L}} \left(\mathsf{dBHz} \right). \tag{7}$$

 S_{Q_0} is an important parameter for characterising or comparing the performance of GNSS receivers. The user should however be careful when comparing different GNSS receivers. Some manufacturers, e.g. Trimble, have provided signal quality in arbitrary manufacturer units (AMU) for older receiver models. These AMU are receiver dependent and need to be converted to S_Q or S_{Q_0} by a conversion formula. The converted values can differ up to 3 dB from the original value (Butsch 2002). ICD-GPS-200D (2006) defines the minimum received signal strength with –160 dBW. If the signal strength falls below this specified minimum it is usually not possible to correlate the received signal with the locally generated replica. The new generation of GPS satellites (Block IIR-M) includes a modernised antenna panel that provides increased signal strength (Lockheed Martin 2007). As a consequence, each satellite must be analysed separately in respect to its signal quality measurements.

The transmitted signal strength is influenced by the gain of the transmitting satellite and thus by the satellite type (Czopek and Shollenberger 1993, Misra and Enge 2006, Ward et al. 2006a), as well as by polarisation errors (Misra and Enge 2006, Ward et al. 2006a) and by the position of the solar panels and moon turn manoeuvres (Czopek and Shollenberger 1993). The signal is moreover affected by free space attenuation and atmospheric loss. Both effects greatly depend on the elevation angle of the satellite (Spilker 1996). The gain of the receiving antenna, cable attenuation, and correlation errors also decrease signal power.

Figure 2 shows normalised carrier-to-noise ratios $(B_{\rm L} = 1 \text{ dBHz})$ for L1 of a full-arc pass of a GPS satellite. The difference of the signal strengths between lowest and highest elevation angle reaches 10 dBHz, mainly caused by the elevation dependence of the receiving antenna's gain. It is necessary to remove this elevation dependence in order to extract carrier phase multipath information from these signal quality observations.

2.3. Link between signal quality and carrier phase multipath

The connection of signal quality and carrier phase multipath parameters can be shown in the phasor diagram (Figure 3). In-phase (I) and quadrature phase (Q) components are used to determine the carrier phase. The relations shown in figure 3 can be used to express the signal-to-noise ratio in terms of multipath parame-



Figure 2: Normalised carrier-to-noise ratio C/N_0 for L1 of a full-arc pass of a GPS satellite.



Figure 3: Phasor diagram: in-phase (I) and quadrature phase (Q) components of direct signal (amplitude A_D , phase angle ϕ_D), **one** reflected signal (selected attenuation factor $\alpha = 0.3$, amplitude A_R , relative phase $\Delta \phi_R$), and the resulting composite signal (amplitude A_C , phase angle ϕ_C). The phase error due to multipath, is labelled $\delta \phi$.

ters:

$$(A_{\rm SQ})^2 \equiv A_{\rm C}^2 = A_{\rm D}^2 + A_{\rm R}^2 + 2A_{\rm D}A_{\rm R}\cos\Delta\phi_{\rm R}.$$
 (8)

Substituting $A_{\rm R}$ by the ratio of indirect and direct signal amplitudes (α) yields equation (2).

Neglecting the influences on the signal strength described above, the amplitude of the indirect/reflected signal and the multipath relative phase will be the dominant factors in the equation (8). In the presence of multipath, $\Delta\phi_{\rm R}$ changes with time, thus causing fluctuations of the signal-to-noise ratio measurements.

Equations (2) and (8) require signal quality measurements in units of volts (Bilich 2006). Our experience has shown that a resolution of at least 0.25 dB of signal quality values is required.

3. Carrier phase multipath estimation based on measured signal quality

The processing algorithms are based on equation (2) and assume specular reflection from one horizontal reflector (which is of larger dimension than the signal wavelength). Furthermore, knowledge of the magnitude of the direct signal amplitude (A_D) is required. In absence of an antenna gain pattern of the receiving antenna, A_D is estimated by a polynomial fit to the signal quality measurements (Figure 4).

The ratio of the amplitude of the composite signal $A_{\rm C}$ and the amplitude of the direct signal $A_{\rm D}$ is obtained from (2):

$$Q = \frac{A_{\rm C}}{A_{\rm D}} = \sqrt{1 + 2 \cdot \alpha \cdot \cos \Delta \phi_{\rm R} + \alpha^2}.$$
 (9)

The attenuation factor α is assumed to be constant over limited periods of time. Thus, fluctuations in this ratio Q of signal amplitudes are related to variations of $\Delta\phi_{\rm R}$. The key idea of our approach is to differentiate Q with respect to $\Delta\phi_{\rm R}$:

$$\frac{\mathrm{d}Q}{\mathrm{d}\Delta\phi_{\mathrm{R}}} = -\frac{\alpha\sin\Delta\phi_{\mathrm{R}}}{\sqrt{1 + 2\alpha\cos\Delta\phi_{\mathrm{R}} + \alpha^{2}}}.$$
 (10)

Rearranging equation (3) to gain an expression for $\alpha \sin \Delta \phi_{\rm R}$ and substituting the numerator of the right hand side of equation (10) yields equation (11). Further rearrangements and the application of trigonometric relationships finally gives equation (12).

$$\frac{\mathrm{d}Q}{\mathrm{d}\Delta\phi_{\mathrm{R}}} = -\frac{\tan\delta\phi\cdot(1+\alpha\cos\Delta\phi_{\mathrm{R}})}{\sqrt{1+2\alpha\cos\Delta\phi_{\mathrm{R}}+\alpha^{2}}},\qquad(11)$$
$$= -\frac{\tan\delta\phi}{\sqrt{1+\left(\frac{\alpha\sin\Delta\phi_{\mathrm{R}}}{1+\alpha\cos\Delta\phi_{\mathrm{R}}}\right)^{2}}},$$
$$= -\frac{\tan\delta\phi}{\sqrt{1+\tan^{2}\delta\phi}},$$
$$\Rightarrow \frac{\mathrm{d}Q}{\mathrm{d}\Delta\phi_{\mathrm{R}}} = -\sin\delta\phi.\qquad(12)$$

Consequently, the carrier phase multipath error is obtained by differentiating Q with respect to $\Delta \phi_{\rm R}$:



 $\delta\phi = -\arcsin\left(\frac{\mathrm{d}Q}{\mathrm{d}\Delta\phi_{\mathrm{R}}}\right).\tag{13}$

Figure 4: Amplitude separation for a full-arc pass of a GPS satellite. (a) Amplitude $A_{\rm C}$ of the composite signal (solid) and amplitude $A_{\rm D}$ (polynomial fit to the raw data) for the direct signal (dashed). (b) Residuals as a result of the ratio $Q = A_{\rm C}/A_{\rm D}$ (shifted by -1).

Unfortunately, we do not have any information of changes of Q with $\Delta\phi_{\rm R}$ in the first place. What we can estimate from the raw signal quality values, however, are changes of Q over time (compare Figure 4). The connection comes from the geometry of satellite orbits, reflector, and antenna. In this case we have restricted ourselves to the simple geometric situation of one horizontal reflector below the antenna horizon (Figure 5). The incidence angle $\zeta_{\rm in}$ at the reflector equals the angle of reflection $\zeta_{\rm out}$, and furthermore: $(90 - \zeta_{\rm in}) = (90 - \zeta_{\rm out}) = el$. This results in a model of the changes of $\Delta\phi_{\rm R}$ with time:

$$l_{\rm R} = 2h\sin el,\tag{14}$$

$$\Delta\phi_{\rm R} = \frac{2\pi}{\lambda} l_{\rm R},\tag{15}$$

$$\frac{\mathrm{d}\Delta\phi_{\mathrm{R}}}{\mathrm{d}t} = \frac{2\pi}{\lambda} 2h\cos el\frac{\mathrm{d}el}{\mathrm{d}t}.$$
 (16)

Changes of the satellites elevation angle with time (del/dt) can easily be computed from broadcast ephemeris. Figure 6 shows examples of del/dt-values as a function of the elevation angle. The vertical distance between antenna and horizontal reflector must be known.

When applying this process to real observation data,



Figure 5: Multipath caused by a horizontal reflector. The distance between antenna phase centre and reflector is labelled with *h*. In this case the incidence angle ζ_{in} at the surface is equal to the angle of reflection ζ_{out} . The additional path length $l_{\rm R}$ (in meter) which is due to surface reflection is shown as a bold (red) line.

⇐



Figure 6: Change of elevation per 10 seconds (del/dt) as a function of the elevation el itself, calculated from broadcast ephemeris.

the differentiation shown in equation 13 is achieved by using the differentiation property of Fourier transforms (Misra and Enge 2006, sec. 8.5.3).

4. Practical experiences

Data sets were collected at the car park of the Dresden Exhibition Centre in order to verify the theory described above. Two Leica GRX1200GG Pro receivers connected to light-weight Septentrio antennas (PolaNt) recorded observations for more than 10 hours on day 260/08. The antennas were set-up at different heights: station HIGH and station LOW (Figure 7, 8).

The car park surface is a strong reflector. Multipath caused by this large and horizontal reflector should affect all satellite signals independent of their incidence azimuth, but influenced by antenna height and incidence elevation. That is why a set-up with very different antenna heights was chosen to ensure different multipath effects.

Observation data of all satellites above an elevation angle of 10° were selected for detailed analysis. This elevation mask angle was chosen to ensure that all re-



Figure 7: Observing stations at Dresden Exhibition Centre car park.



Figure 8: Station set-up with different tripod heights.

flected signals originate from the car park surface and to avoid signal diffraction by trees in the surroundings. Carrier phase multipath corrections were estimated from signal quality measurements using equations (10) and (16). The direct signal amplitudes were determined by polynomial fits to the raw signal quality measurements. Figure 9 gives an overview of all corrections as a function of the signal incidence angle. It confirms that no azimuth dependencies exist. Due to the large horizontal reflector, the corrections are a function of antenna height and elevation angle only. Furthermore, the two kinds of surface materials seem to reflect the GPS signals with similar attenuation factors. The most noticeable corrections were obtained for elevation angles below 30.

In order to enable a validation of the calculated carrier phase multipath corrections $\delta \phi$ the short (10 m) baseline HIGH – LOW was processed with the baseline processor Wa1. Antenna phase centre corrections were taken into account. In such a short baseline satellite clock and orbit errors as well as ionospheric and tropospheric effects cancel out by forming single difference observations, so that multipath remains as the dominant error source. Validation is performed on the observation level, i.e. by analysing the double difference phase residuals (Hofmann-Wellenhof et al. 2008), as well as on the coordinate level.

In order to validate the potential of the presented algorithm, the estimated carrier phase multipath corrections $\delta \phi$ were applied to the original observations of both stations. Double difference residuals were computed for three kinds of solutions: L1, L2, and the ionosphere-free linear combination L0. The comparison of original with corrected double-difference residuals reveals that a large portion of the carrier phase multipath effects can be removed (see sample in Figure 10). This is especially true for multipath at lower elevation angles because of its shorter periods. On



Figure 9: Estimated carrier phase multipath corrections $\delta \phi$ for stations HIGH (left) and LOW (right). Satellite tracks of PRN 09 and 12 are added to the polar plot.

the other hand, no improvements could be achieved for multipath periods longer than two hours which occurred at higher elevations.

RMS values across all double difference residuals experienced a reduction of approximately 25% whenever multipath corrections were being applied (Table 1).

The effects of multipath corrections were also tested



Figure 10: Comparison of double differences for L1 (top), L2 (middle) and L0 (bottom) calculated from phase observations (dashed) and estimated carrier phase multipath error (solid). The curves are low pass filtered for clarity.

Table 1: L1, L2 and ionosphere-free L0 double of	difference
residuals from original and corrected data.	

	RMS, in mm		
data set	L1	L2	L0
original corrected	8.1 6.3	8.6 6.5	26.6 19.8

on the coordinate level. Single epoch baseline coordinate solutions were computed from the original as well as from corrected data set. The distribution of L1 coordinate residuals is shown in Figure 11. The number of residuals being smaller than a few millimetres is greater for the corrected observations in comparison to the original data. The number of residuals being greater than a few millimetres decreases after applying corrections.

Table 2 summarises RMS values for L1, L2 and ionosphere-free L0 solutions separately for the horizontal components and height. Carrier phase multipath



Figure 11: L1 single epoch coordinate residuals from original and corrected data.

Table 2: RMS-values of L1, L2 and ionosphere-free L0 single epoch coordinate residuals from original and corrected data.

	RMS, in mm	
sional	original	corrected
L1	3.9/2.0/ 6.0	2.8/1.6/ 4.7
L2	4.0/2.3/ 7.0	2.6/2.0/ 5.4
L0	12.2/5.9/20.0	7.9/4.8/15.0

corrections improved the positioning results by 20 to 30%.

The described practical experiences refer to observations recorded by Leica GRX1200GG Pro receivers. With this receiver type, L2 multipath corrections from signal quality observations perform as well as L1 corrections. We also tested the algorithms with L2 observations of other receivers but could not obtain useful L2 multipath corrections. These receivers obviously use different code-less measurement techniques on the second frequency.

A successful multipath mitigation using the aforementioned technique is restricted to certain applications and multipath environments. These restrictions include: observations collected in static mode, multipath caused by geometrically well-defined reflectors and reflections with fairly constant attenuation factors α . Furthermore, if the composite signal contains more than one strong reflected signal, estimating the carrier phase multipath corrections as shown in this paper will fail.

5. Conclusions

Carrier phase multipath corrections can be computed from signal quality measurements (signal-to-noise ratio, or carrier-to-noise ratio). The developed algorithms make use of the typical multipath variations in time which are found in observations collected in static mode. GNSS satellite motion induces oscillations of the same frequency in both observables, the carrier phase and the signal quality.

Observation data sets were collected on a car park with the car park surface being a strong signal reflector. Baseline processing was performed without and with application of multipath corrections based on the signal quality values. Double difference observation residuals could be reduced by almost 25%. Positioning results improved by about the same amount. Due to several restrictions, the described technique can not be used for multipath mitigation in general. It is, however, a promising approach for carrier phase multipath reduction of GNSS reference station observations.

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