STUDIES ON A MODIFIED ERICSSON CYCLE WITH NEON AS REFRIGERANT

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ABSTRACT

The classical Ericsson cycle consists of two isotherms and two isobars. With an ideal gas as refrigerant a reversible process can be realized. To maintain reversibility with a real fluid as refrigerant, the high pressure isobar is being replaced by a combination of isobaric and isentropic steps.

If the cycle includes the two-phase region, the low temperature isotherm can be obtained by isobaric evaporation without the need for simultaneous work extraction.

This so-called modified Ericsson cycle is still a reversible process. With neon as refrigerant, the heat exchange and work extraction requirements are being studied. In a second step the influence of non-ideal heat exchange and non-ideal work extraction on the total cycle efficiency are being investigated.

INTRODUCTION

The Carnot cycle (Fig. 1) is the "mother" of all reversible refrigeration cycles. It does not contain any internal heat transfer. Therefore it can theoretically be realized with any kind of fluid, even with a real gas with pressure and temperature dependent thermal properties. But the Carnot cycle is impractical in cryogenic applications, because it leads to very high pressure differences.

More practical are the Stirling and the Ericsson cycle, which work with internal heat transfer to bridge the temperature distance between the ambient temperature T_{amb} and the refrigeration temperature T_0. To be reversible, the Stirling and the Ericsson cycle need refrigerants with equal specific heats of the high and the low pressure stream. Therefore one indication of the reversibility is, that the two curves with inner heat exchange are equidistant in the T-s diagram (Fig. 1).
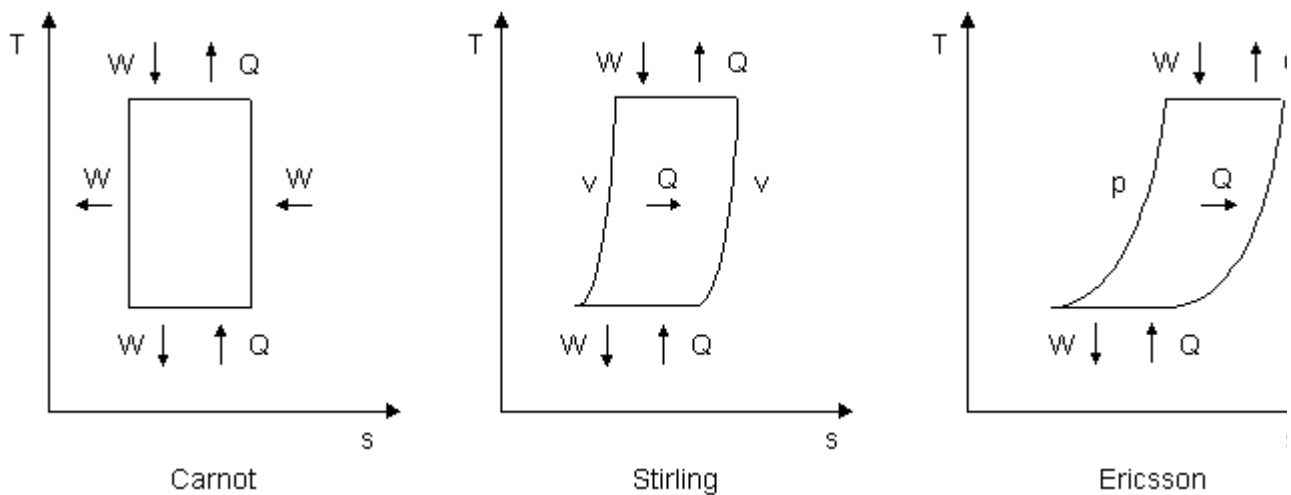
Figure 1. Reversible refrigeration cycles

Real fluids depart from the requirement of pressure independent specific heats, the more the cycle approaches the vapor dome.

Two years ago we reported on the search for the most efficient cycle for refrigerators at the liquid helium temperature level [1]. We found the cycle shown in Fig. 2 to be very promising. It has two isotherms at T_{amb} and T_0 and an isobar at the low pressure side. Because the specific heats of the high and low pressure stream are not equal, heat exchange alone can not secure the equidistance. This can only be reached by a combination of heat transfer with many small isentropic expansions of the high pressure stream. So the high pressure stream is not isobaric. The pressure is decreasing with decreasing temperature.

Because the cycle shown in Fig. 2 has many similarities with the Ericsson cycle, we call it a modified Ericsson cycle. One significant difference is that work extraction occurs only in the high pressure stream and not along the isotherm at T_0, if one chooses to produce refrigeration below the critical temperature of the refrigerant.

In the following we study the change of state on the high pressure side of the cycle.
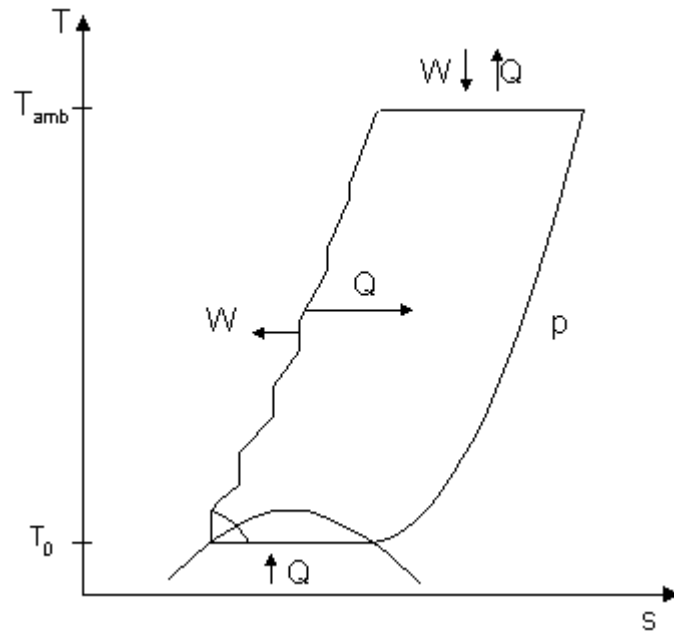
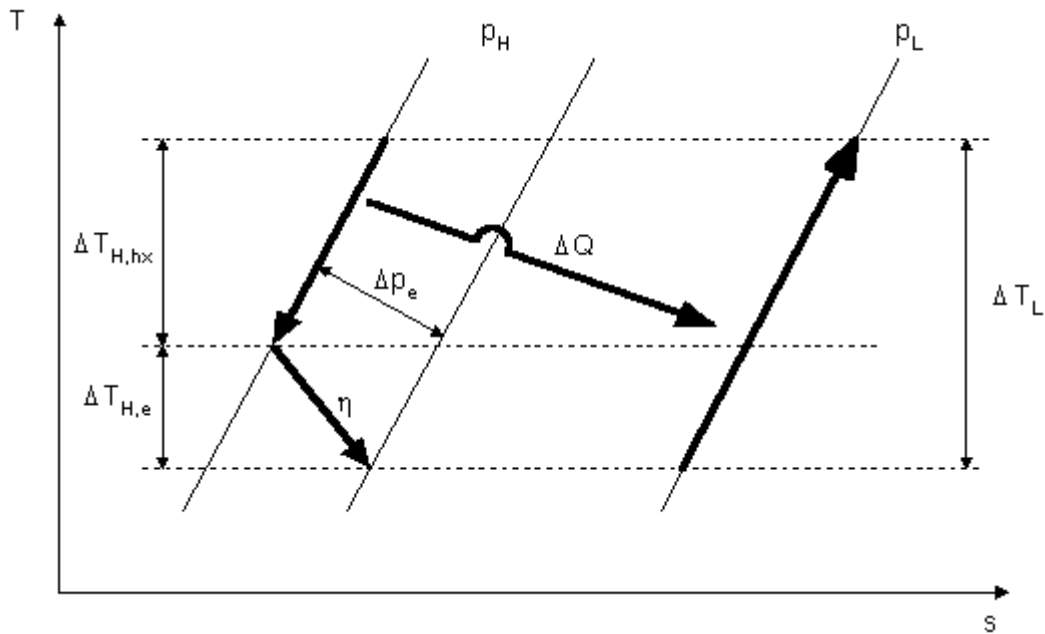Figure 2. Modified Ericsson cycle with a real fluid

Figure 3. Small element of cycle with heat transfer and expansion

REQUIRED TEMPERATURE REDUCTION BY EXPANSION

We investigate a small element of the cycle, where the low pressure stream is warmed up by the temperature step ΔT_L.

Assuming the mass flow rates in the high and low pressure streams to be equal, the transferred heat is able to cool the high pressure stream by

$$\Delta T_{H,hx} = \frac{c_{p,L}}{c_{p,H}} \cdot \Delta T_L \qquad (1)$$

To obtain an overall temperature reduction of the high pressure stream equal to the temperature rise of the low pressure stream, an additional temperature reduction of the high pressure stream must be obtained by work extracting expansion:

$$\Delta T_{H,e} = \Delta T_L - \Delta T_{H,hx} = \left(1 - \frac{c_{p,L}}{c_{p,H}}\right) \cdot \Delta T_L \quad (2)$$

Which pressure drop by expansion is necessary to produce this temperature reduction?

PRESSURE REDUCTION BY WORK EXTRACTING EXPANSION TO OBTAIN THE DESIRED TEMPERATURE REDUCTION

The modified Ericsson cycle is a "real fluid" process, i.e. it has to take into account the deviations from the ideal gas approximation. We make use of the following thermal properties of the fluid, whose values are pressure and temperature dependent:

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \quad \text{(Joule Thomson coefficient)}$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \quad \text{(Specific heat capacity)}$$
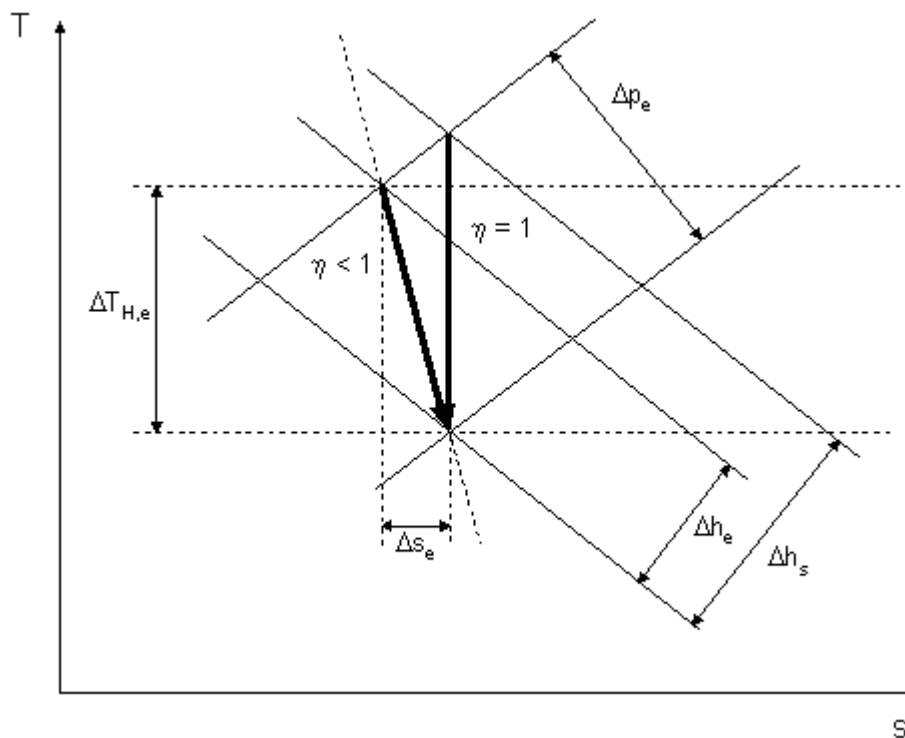
$$v \qquad \text{Specific volume}$$

Figure 4. Isentropic ($\eta = 1$) and non-isentropic ($\eta < 1$) expansion steps shown in a T-s Diagram

We start with the total differential of $T = T(h,P)$:

$$dT = \left(\frac{\partial T}{\partial h}\right)_p dh + \left(\frac{\partial T}{\partial p}\right)_h dp = \frac{1}{c_p} dh + \mu\, dp \quad (3)$$

From the 1st and 2nd law one may write:

$$T ds = dh - v\, dp \quad (4)$$

For an isentropic expansion one obtains:

$$\left(\frac{\partial h}{\partial p}\right)_s = v \quad (5)$$

That means for a small isentropic expansion step (Fig. 4):

$$\Delta h_s = v \cdot \Delta p_e \quad (6)$$

With the help of the definition of the isentropic efficiency

$$\eta = \frac{\Delta h_e}{\Delta h_s} \quad (7)$$

the enthalpy reduction by a non-isentropic expansion step is described by:

$$\Delta h_e = \eta \cdot \Delta h_s = \eta \cdot v \cdot \Delta p_e \quad (8)$$

With change from the differential to a finite, but small difference, Eq. (3) is written:

$$\Delta T_{H,e} = 1/c_{p,H} \cdot \Delta h_e + \mu \cdot \Delta p_e \quad (9)$$

Introducing Eq. (8) into Eq. (9), we get:

$$\Delta T_{H,e} = \frac{\eta \cdot v}{c_{p,H}} \cdot \Delta p_e + \mu \cdot \Delta p_e \quad (10)$$

or

$$\Delta p_e = \frac{1}{\eta \dfrac{v}{c_{p,H}} + \mu} \Delta T_{H,e} \quad (11)$$

Δp_e is the pressure drop of the work extracting expander and $\Delta T_{H,e}$ is the associated temperature drop. This equation is valid for a small expansion step, where the thermal properties v, c_p and μ can be considered constant.

Using the prescribed temperature step of the low pressure stream ΔT_L instead of the temperature drop $\Delta T_{H,e}$ (see Eq. 2 and Fig. 3):

$$\Delta p_e = \frac{1 - \dfrac{c_{p,L}}{c_{p,H}}}{\eta \dfrac{v}{c_{p,H}} + \mu} \Delta T_L \tag{12}$$

While this equation takes into account the inefficiency of the expander, it does not account for a possible inefficiency of the heat exchanger.

NON-IDEAL HEAT TRANSFER

The heat transfer from the high to low pressure gas requires a "driving" temperature difference (δT) between both streams. It can be described by the number of transfer units (NTU) of the counter-current heat exchanger. Thermodynamically it is advantageous, if with a given NTU one chooses a small δT at the cold end, and a larger δT at the warm end of the heat exchanger. The optimum temperature difference distribution for a heat exchanger with a constant specific heat fluids is given by [2]:

$$\delta T = T_H - T_L = \frac{\ln\left(T_{L,w}/T_{L,c}\right)}{NTU} \cdot T_L \tag{13}$$

$T_{L,c}$ and $T_{L,w}$ are the cold inlet and warm outlet temperatures of the low pressure stream. Along a small process element similar to the one shown in Fig. 3, the temperature difference changes by

$$\Delta\left(\delta T\right) = \frac{\ln\left(T_{L,w}/T_{L,c}\right)}{NTU} \cdot \Delta T_L \tag{14}$$

This requires an additional temperature reduction, which has to produced by the expansion of the high pressure gas (see Eq. 2 and 14):

$$\tag{15}$$

$$\Delta T_{H,e} = \Delta T_L - \Delta T_{H,hx} + \Delta(\delta T) = \left(1 - \frac{c_{p,L}}{c_{p,H}} + \frac{\ln\dfrac{T_{L,w}}{T_{L,c}}}{NTU}\right) \cdot \Delta T_L$$

Therefore the associated pressure drop by expansion (see Eq. 11 and 12) has to increase, too.

$$\Delta p_e = \frac{1 - \dfrac{c_{p,L}}{c_{p,H}} + \dfrac{\ln(T_{L,w}/T_{L,c})}{NTU}}{\eta \dfrac{v}{c_{p,H}} + \mu} \, \Delta T_L \qquad (16)$$

This equation is describing the pressure drop over consecutive heat exchange and expansion steps of the high pressure side of the modified Ericsson cycle with both irreversibilities (non-isentropic expansion and non-ideal heat transfer) included.

CALCULATION OF THERMODYNAMIC LOSSES

We study this question again in a process element as shown in Fig. 3, but with an additional temperature difference δT between high and low pressure stream (see Eq. 13).

The entropy production due to the heat flow from the high to the low pressure side is given by [1]:

$$\Delta s_{hx} = \frac{c_{p,L} \cdot \delta T}{T_L (T_L + \delta T)} \cdot \Delta T_L \qquad (17)$$

and with Eq. (13) for δT, one obtains for the total losses connected with the heat transfer, assuming the specific heat of the low pressure gas to be constant over a wide range of temperatures:

$$\sum \Delta s_{hx} \approx c_{p,L} \cdot \frac{\left[\ln(T_{L,w}/T_{L,c})\right]^2}{NTU + \ln(T_{L,w}/T_{L,c})} \qquad (18)$$

The entropy generation Δs_e shown in Fig. 4 due to the non-ideal work extracting expansion with the enthalpy drop Δh_e is determined by [1]:

$$(19)$$

$$\Delta s_e = \frac{\Delta h_e}{T_H} \cdot \frac{1-\eta}{\eta}$$

The total losses by these expansion steps are:

$$\sum \Delta s_e = \frac{1-\eta}{\eta} \cdot \int_{T_c}^{T_w} \frac{\Delta h_e / \Delta T_L}{T_H} \, dT_L \qquad (20)$$

The enthalpy drop Δh_e can be divided into two terms, one due to the pressure dependent specific heat of the refrigerant and a second term due to the non-ideal heat transfer, which produces the optimum temperature difference δT:

$$\Delta h_e = \frac{1 - \dfrac{c_{p,L}}{c_{p,H}}}{\dfrac{1}{c_{p,H}} + \dfrac{1}{\eta} \cdot \dfrac{\mu}{v}} \cdot \Delta T_L + \frac{\dfrac{\ln\left(T_{L,w}/T_{L,c}\right)}{NTU}}{\dfrac{1}{c_{p,H}} + \dfrac{1}{\eta} \cdot \dfrac{\mu}{v}} \cdot \Delta T_L \quad (21)$$

So it is possible to distinguish between the different thermodynamic losses.

NEON AS A REFRIGERANT FOR THE 40 K LEVEL

The 40 K temperature level is of interest for applications of high temperature superconductivity. Neon could be a suitable refrigerant for this purpose. Therefore we try this new cycle with neon. An example for the cycle with four mini-expanders in series and a final throttle valve (Joule-Thomson valve) is given in Fig. 5.

To obtain a compact refrigerator, we are using a low pressure level of 20 bar (2 MPa), which corresponds to a refrigeration temperature of 42.2 K. We choose a rate of refrigeration of 33.6 J/g. In entropy terms this is:

$$\delta s_0 = \frac{33.6 \text{ J/g}}{42.2 \text{ K}} \approx 0.8 \text{ J/g K} \qquad (22)$$

In the "lower left corner" we deviate somewhat from the previously described modified Ericsson cycle and include a small Joule-Thomson stage with isobaric heat transfer followed by a throttle valve. the reason is that below 44 K the specific heat of the high pressure (4 MPa) stream is lower than that of the low pressure stream. The final expansion in the J-T valve from 40 bar (4 Mpa) down to 20 bar and the heat transfer in the J-T heat exchanger are connected with a small entropy generation (0.07...0.08 J/ g K).

We are inserting the real properties of neon into above equations for the thermodynamic losses. The results of the numerical calculation with many small process steps, determined by the temperature step ΔT_L, are summarized in Figure 6.

There is shown the entropy span at ambient temperature δs_{amb} in function of 1/ NTU with the efficiency η as parameter.

The efficiency of the total cycle is obtained by dividing the useful entropy (δs_0) by the value of δs_{amb} shown on Fig. 6. This does not include losses of the room temperature compression.

For e.g. NTU=28 and η =0.6 one obtains a cycle efficiency of

$$\frac{\delta s_0}{\delta s_{amb}} \approx \frac{800 \quad \text{J/kg K}}{1450 \quad \text{J/kg K}} = 0.55 \;.$$

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So this cycle promises to give good performance even with moderately efficient components.
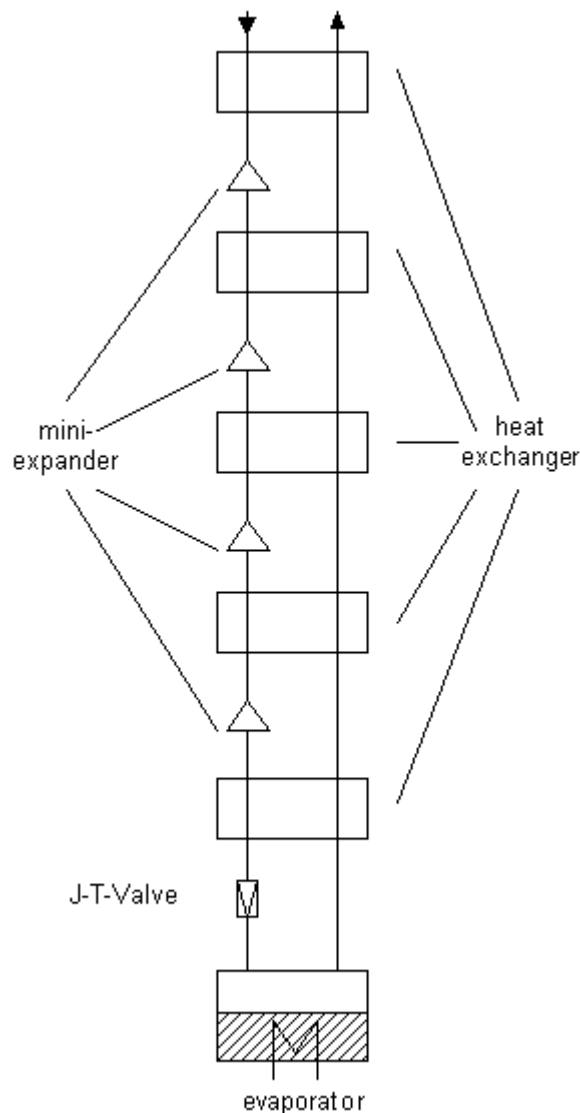

Figure 5. Cycle with 4 mini-expanders in series and a final Joule-Thomson valve
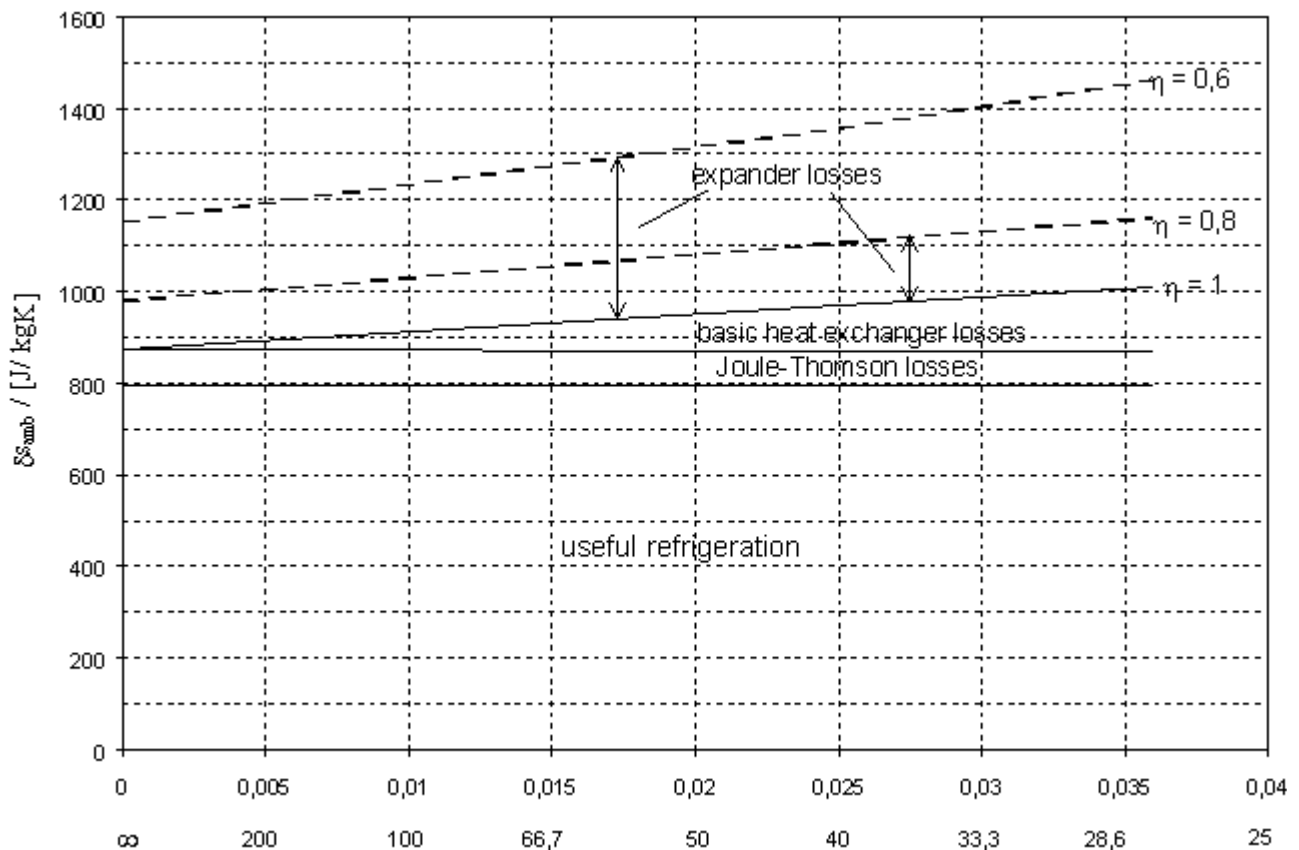
Figure 6. Entropy span δS_{amb} in function of 1/ NTU with the expander efficiency η as parameter

SUMMARY

A new refrigeration cycle, as was mentioned already in [1], with consecutive expansion (work extracting) and heat transfer steps of the high pressure stream, has been identified as a modified Ericsson cycle.

The thermodynamic equations have been derived for an estimation of such a cycle.

With neon as a refrigerant for the 40 K temperature level, the influence of losses (non-ideal heat transfer and non-ideal expansion) on the cycle efficiency is shown. Even with moderately efficient components it is possible to get a good cycle performance.

On the base of the described modified Ericsson cycle a new cryocooler with mini-expanders should be developed.

ACKNOWLEDGMENT

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NOMENCLATURE TABLE

NTU		Number of transfer units
Q	kJ	Heat
T	K	Temperature
W	kJ	Work
c_p	kJ/kg K	Specific heat capacity @ constant pressure
h	kJ/kg	Specific enthalpy
p	Pa	Pressure
s	kJ/kg	Specific entropy
v	m^3/kg	Specific volume
Δ		Small difference for a variable of state of one stream
δ		Difference of a variable of state between two streams
η		Isentropic efficiency of the expansion
μ	K/Pa	Joule-Thomson coefficient

Subscripts

H	high pressure stream
L	low pressure stream
amb	ambient temperature level
c	cold end
e	expansion step
h	isenthalpic change of state
hx	heat exchange step
p	isobaric change of state
s	isentropic step
w	warm end
0	refrigeration temperature level