A new approach for nonlinear BWR stability analysis

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Introduction

A new approach for nonlinear stability analysis is presented. In the framework of this approach, integrated BWR (system) codes and simplified BWR models (reduced order model, ROM) are used as complementary tools to examine the stability characteristic of fixed points and periodic solutions of the nonlinear differential equations describing the stability behaviour of a BWR loop [1-4]. This work is a continuation of the previous work at Paul Scherrer Institute (PSI, Switzerland) and University of Illinois (USA) on this field. The current ROM was extended by adding the recirculation loop model. The necessity of consideration of the effect of subcooled boiling in an approximate manner was discussed. Furthermore, a new calculation methodology for the feedback reactivity was implemented. The modified ROM is coupled with the code BIFDD [5] which performs semi-analytical bifurcation analysis. In addition to the ROM extensions, a new procedure for calculation of the ROM input data was developed [4]. The results of the new approach for nonlinear BWR stability analysis are presented for NPP Leibstadt. This investigation is carried out for an operational point, where an out-of-phase power oscillation was observed during a stability test at the beginning of cycle 7 (KKL cycle 7 record #4) [6]. The new procedure for the calculation of the ROM input data qualify the modified ROM for BWR stability analysis in the framework of the approach demonstrated in this paper.

New approach for nonlinear BWR stability analysis

In the framework of the new approach, integrated BWR (system) codes (RAMONA5, Studsvik/Scanpower) and simplified BWR models are used as complementary tools to examine the stability characteristic of fixed points and periodic solutions of the nonlinear differential equations describing the stability behaviour of a BWR loop (RAM-ROM methodology, RAM is used as synonym for system codes). The intention is, firstly, to identify the stability properties of certain operational points by performing ROM analysis and, secondly, to apply the system code (here RAMONA5) for a detailed stability investigation in the neighbourhood of these operational points.

The ROM is characterized by a minimum number of system equations which is mainly realized by the reduction of the geometrical complexity. One demand on the TUD ROM is, because the ROM sub-models should be as close as possible to the sub-models used in RAMONA, that the solution manifold of the RAMONA model should be as close as possible to the solution manifold of the ROM. E.g., regarding model resemblance, both neutron kinetic models (ROM and RAMONA) are based on the two neutron energy group diffusion equations

and both thermal-hydraulic two-phase flow models are represented by models which consider the mechanical non-equilibrium (different velocities of the phases of the fluid) [1,4].
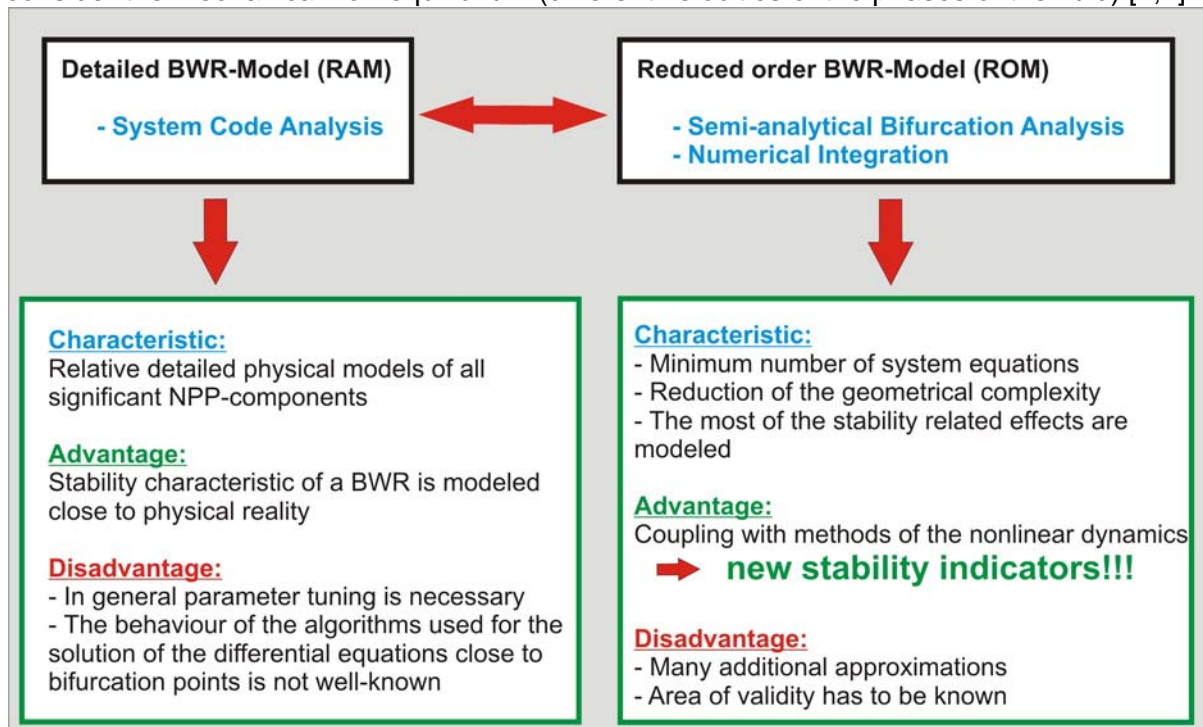

Figure 1:   Overview over the methodology applied for the nonlinear BWR stability analyses where RAMONA5 and ROM are used as complementary tools [4].

The main advantage of employing ROM's is the possible coupling with codes using methods of nonlinear dynamics like **bifurcation analysis** (see explanation box at the end of this paper). In the framework of application of such techniques, the scope of BWR stability analyses can be expanded significantly. The existence of **stable and unstable periodical solutions** (correspond to limit cycles) can be examined reliably. Further, the **stability behaviour of global and regional power oscillation** states can be investigated separately [4].

In the framework of the present ROM analyses two independent techniques are employed (Figure 1). These are the semi-analytical bifurcation analysis with the bifurcation code BIFDD [1,4,5] and the numerical integration [1] of the ROM differential equation system. Bifurcation analysis with BIFDD determines the stability properties of fixed points and periodical solutions. For independent confirmation of these results, the ROM system will be solved directly by numerical integration for selected parameters.

The procedure of the nonlinear BWR stability analysis is depicted in Figure 2. The goal is to simulate the stability behaviour of the power plant with the ROM as close as possible to that one calculated by RAMONA5 in the neighbourhood of a selected operational point. Hence, at first, the reference OP has to be selected for which the nonlinear BWR stability analysis will be performed. Secondly, the new procedure for the ROM input data calculation is applied. Thereby, all ROM input data are calculated from the specific RAMONA5 model and its steady state solution corresponding to the reference point. We demand: the ROM should provide the correct steady state values in the reference operational point, wherein the most essential values (for the BWR stability behaviour) are the mode feedback reactivity coefficients, the core inlet mass flow, the axial void profile and the channel pressure drops over the reactor vessel components along the closed flow path. E.g. the subcooling number and the pressure loss coefficients cannot be calculated directly from the RAMONA5 model

(and its steady state output) because the models describing the axial power profile and the pressure drop along the closed flow path are different in both codes. Hence a special calculation procedure for the pressure loss coefficients and the core inlet subcooling of the ROM is developed and applied. Finally, after the calculation of the ROM input data, nonlinear BWR analysis is performed by using ROM and RAMONA5 as **complementary** tools.
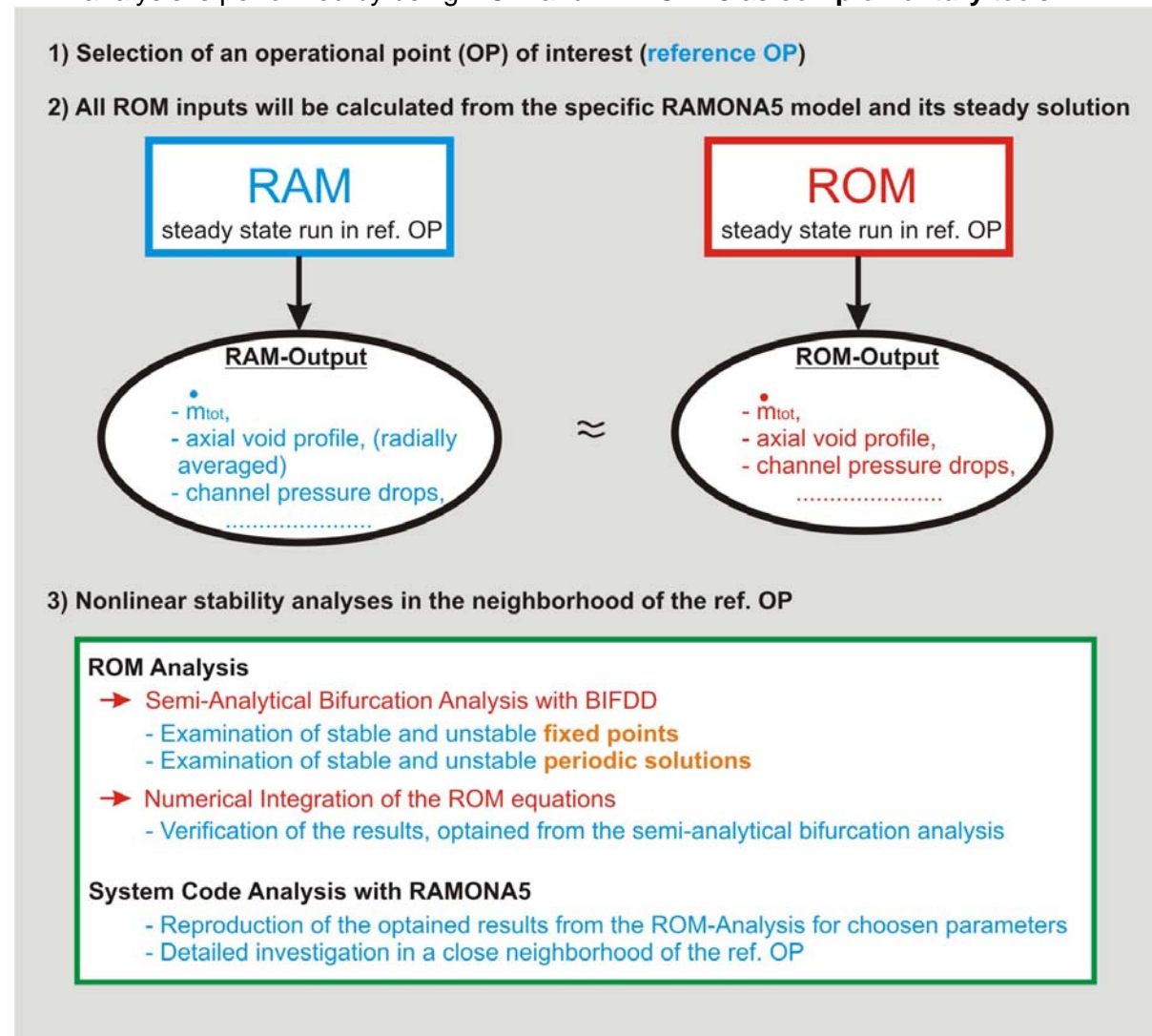

Figure 2: This figure depicts the new approach for nonlinear BWR stability analyses using RAMONA5 and ROM as complementary tools.

Results for NPP Leibstadt (KKL cycle 7 record #4)

The nonlinear analysis has been carried out for an operational point for which an out-of-phase power oscillation was observed during a stability test at the beginning of cycle 7 (KKL cycle 7 record #4, KKLc7_rec4) [6]. This OP is defined as reference OP (but with slight modified core inlet subcooling). In this paper, only representative results of the ROM analyses are presented.

Bifurcation analysis using BIFDD

The results of the semi-analytical bifurcation analysis of the ROM equation system are presented in the N_{sub} - DP_{ext} -operating plane (N_{sub} and DP_{ext} are defined in **Nomenclature** at the end of this paper**)**. Notice, a variation of DP_{ext} (steady state external pressure drop)

corresponds to a movement on the rod-line which crosses the reference OP while the 3D-distributions will not be affected. Thus the stability properties of operational points along a fixed rod-line (fixed control rod configuration) and its close neighbourhood are analysed. The stability boundary (SB) and the bifurcation characteristic (BCH, see explanation box) are shown in Figure 3. The SB is defined as the set of fixed points for which the Hopf conditions are fulfilled (see explanation box). Roughly speaking, this means, in each of these fixed points a limit cycle is "born" and exists either in the (linear) stable or (linear) unstable region. The stability characteristic of the limit cycle is determined by the Floquet parameter β_2 [5]. If $\beta_2 < 0$, the limit cycle is stable (supercritical bifurcation) while if $\beta_2 > 0$, the limit cycle is unstable (subcritical bifurcation) [1,2,4,5] (see explanation box, Figure 7a,c).
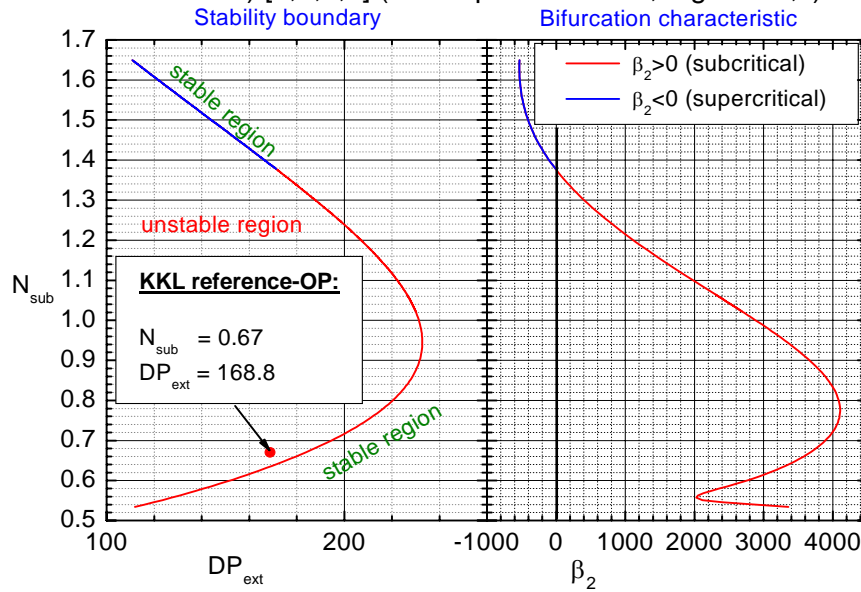

Figure 3: Stability boundary and the bifurcation characteristic (nature of the Poincarè-Andronov-Hopf bifurcation) for the reference OP.
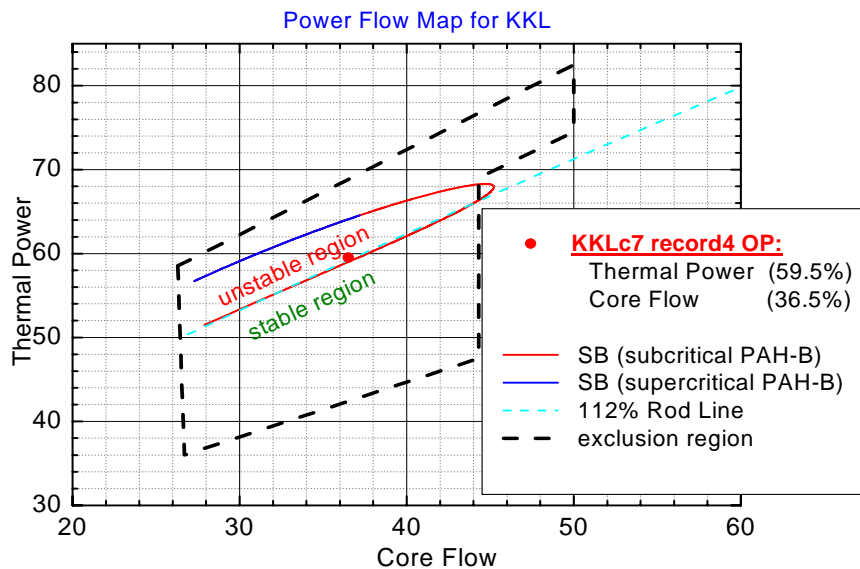

Figure 4: Stability boundary transformed into the power flow map.

Unstable periodical solutions (unstable limit cycle) close to the KKLc7_rec4-OP are predicted by the semi-analytical bifurcation analysis (for $0.53 < N_{sub} < 1.38$, see Figure 3). These solutions are located in the linear stable region close to the stability boundary. This means, in

this region <u>coexist</u> **stable fixed points** <u>and</u> **unstable limit cycles** (see Figure 7a and Figure 8a). Notice, the asymptotic decay ratio (linear stability indicator) is less than 1 ($DR < 1$) in this region. A linear stability analysis is not capable to examine the stability properties of limit cycles.

Numerical integration

For independent confirmation of the results which are predicted by the bifurcation analyses, numerical integration of the ROM equation system (in the time domain) has been carried out for selected parameters. The ROM equations are solved in an operational point that is located in the (linear) stable region (see Figure 5). In particular, the selected OP is located in a region in which unstable limit cycles are predicted by the bifurcation analysis (remark: as shown in Figure 3, the reference OP is located in the (linear) unstable region).
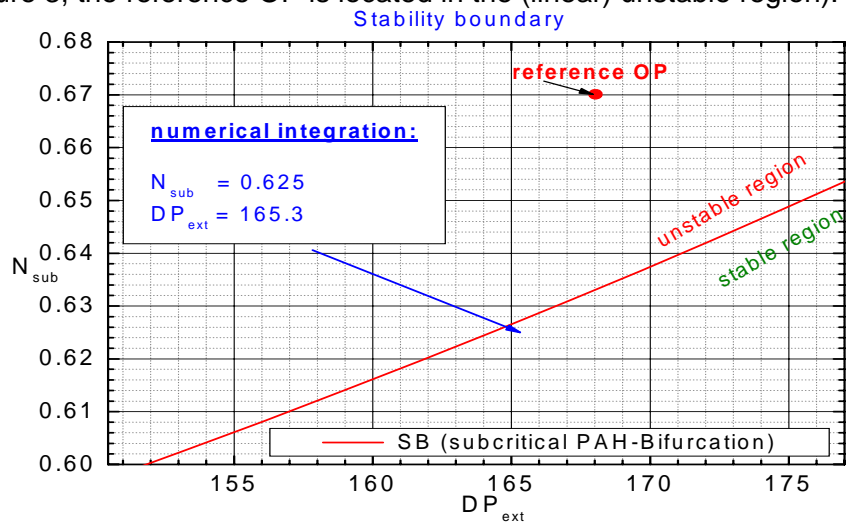
Figure 5: SB and the point for which the unstable limit cycle will be verified by numerical integration (see Figure 6).
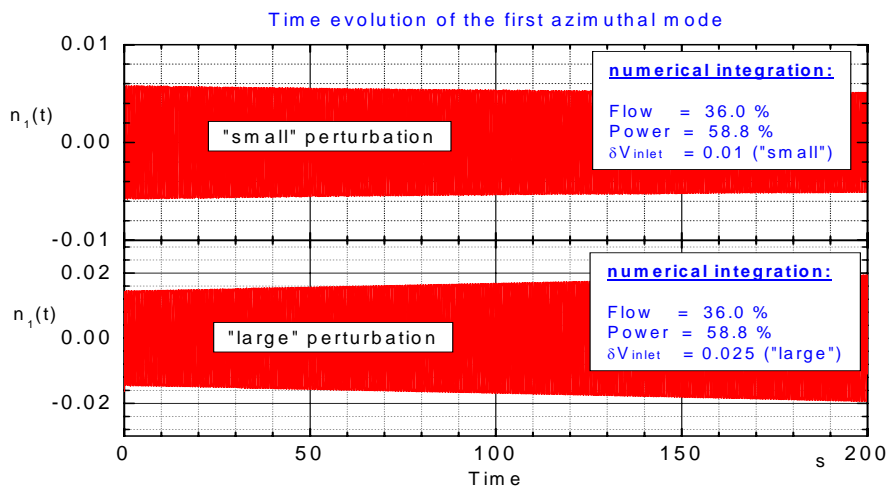
Figure 6: Numerical integration is carried out in an operational point in which an unstable limit cycle is predicted by the bifurcation analysis. The transient was initiated by imposing perturbations of the core inlet mass flow with (small $\delta v_{inlet} = 0.01$, large $\delta v_{inlet} = 0.025$).

In order to verify the existence of the unstable limit cycle, perturbations of different amplitudes are imposed on the system. This means, according to $\vec{X}(t) = \vec{X}_0 + \delta\vec{X}(t)$, the steady state solution $\vec{X}_0$ is perturbed by different perturbation amplitudes $\delta\vec{X}(t)$ and the

transient behaviour of the system state $\vec{X}(t)$ is calculated by numerical integration of the ROM equation system. If a sufficient small perturbation is imposed on the system, the state variables will return to the steady state solution. The terminus "sufficient small perturbation" means that the trajectory starts within the basin of attraction of the fixed point. Roughly speaking, the perturbation amplitude is less than the repellor amplitude (see phase space portrait depicted in Figure 8a). But if a sufficient large perturbation is imposed on the system, the state variables will diverge in an oscillatory manner. The terminus "sufficient large perturbation" means that the perturbation amplitude is larger than the repellor amplitude. In this case the trajectory will start out of the basin of attraction of the fixed point (see explanation box).

As shown in Figure 6, the results of the numerical integration method confirm the prediction of the bifurcation analysis. This example show that conceivably unstable conditions (from the nonlinear point of view) are not recognized and the operational safety limits could be violated.

Summary and conclusions

The analysis has shown that the stability behaviour of the reference OP and its close neighbourhood can be simulated reliably by the new ROM. In this OP, the results of RAMONA5 and ROM are consistent. Under stability related parameter variations the stability behaviour calculated by both ROM and RAM are consistent. Hence, the new ROM and the new procedure for the calculation of the ROM input data are qualified for BWR stability analysis in the framework of the new approach (RAM-ROM methodology): **The ROM analysis provides an overview about types of instability which we have to expect** (in a selected BWR power-flow map region)**, by RAM (system code) we analyse these phenomena in detail (but with pre-informations received by ROM analysis). Hence the reliability of the stability analysis is improved.** Methods and results of this work will be the basis for further nonlinear BWR stability analyses. An in-depth understanding of the BWR stability behaviour should definitely contribute to an efficient design of "detection and suppression" systems.

Nomenclature: The subcooling number N_{sub} represents the core inlet subcooling and appears as a boundary condition in the single phase energy equation.

$$N_{sub} = \frac{(h_{sat}^* - h_{inlet}^*)}{\Delta h_{fg}^*} \cdot \frac{\Delta \rho^*}{\rho_g^*} \qquad DP_{ext} = \frac{DP_{ext}^*}{\rho_f^* v_0^{*2}}$$

where h_{sat}^* is the saturation enthalpy, h_{inlet}^* inlet enthalpy, $\Delta \rho^* = \rho_f^* - \rho_g^*$ liquid-vapor density, $\Delta h_{fg}^* = h_g^* - h_f^*$ liquid-vapor enthalpy, v_0^* reference velocity and DP_{ext}^* is the steady state pressure drop over the vessel high. $\vec{X}(t)$ with $\vec{X}(t) \in \mathbb{R}^n$ is the state vector of the dynamical system.

Explanation box: *Bifurcation analysis* (see e.g. [5,7])

The dynamics of a BWR is described by a system of coupled nonlinear partial differential equations. From the nonlinear dynamics point of view, it is well known that such systems show, under specific conditions, a very complex behaviour (e.g. spontaneous limit cycle

oscillations) which is reflected in the solution manifold of the corresponding equation system. Hence, in order to understand the nonlinear stability behaviour of a BWR, the solution manifold of the differential equation system which describes the BWR stability behaviour must be examined. In particular, examination means to perform stability analysis in nonlinear dynamic terms. Bifurcation analysis is an especial methodology in the scope of nonlinear stability analysis.

The nonlinear dynamical system (BWR loop) encountered a **bifurcation** when the qualitative solution structure (the so-called phase portrait) in the space of state variables (also called phase space) is changed under system parameter variations. Supposing the system parameter γ_k (with $\gamma_k \in \mathbb{R}^m$) is varied in its domain of definition and at the critical parameter value $\gamma_k = \gamma_{k,c}$ the dynamical system experienced a bifurcation. At $\gamma_{k,c}$ the dynamical system will have at least one pair of eigenvalues of the Jacobian matrix, λ_i, with a zero real part ($\mathrm{Re}(\lambda_i) = 0$ with $\lambda_i(\gamma_{k,c})$). As a consequence, at $\gamma_{k,c}$ the dynamical system lost its hyperbolicity. Hyperbolicity is an important property of nonlinear dynamic systems: If only hyperbolic fixed points (operational points) exist (the real part of the complex eigenvalues of the (system) Jacobian matrix is different from zero), the application of linear stability indicators like Decay Ratio is allowed (there exist only stable or unstable fixed points, decreasing or increasing oscillations). Hence in many cases linear stability analysis is sufficient (e.g. frequency domain codes use the transfer function technique). But if the system experienced a so-called Hopf bifurcation (stable or unstable) limit cycles are born (if a critical parameter is reached). **Hence the existence of a Hopf bifurcation is the (mathematical) reason for the sudden appearance of periodic oscillations (limit cycles).** These periodic solutions will be observed in the BWR reactor dynamics as global (in-phase) or regional (e.g. out-of-phase or azimuthal mode) power oscillations. Please note, a Hopf bifurcation is a dynamical one; there are so-called static bifurcations (appearance of 2 solution branches like pitchfork bifurcations) which are not interesting in the scope of this investigations.

Bifurcation analysis (using the BIFDD code [5]) does mean: Set up a differential equation system describing the dynamics of a nonlinear system (in our case the ROM or RAM equations). Define a control parameter (like the subcooling number (N_{sub}), or the steady state external pressure drop (DP_{ext}^*)) and change it until a "critical" value is reached (for which a pair of complex conjugated eigenvalues of the Jacobian matrix of the linearized system with zero real part exists). Check the compliance of the so-called Hopf conditions in the "critical" point (at $\gamma_{k,c}$). If the Hopf conditions are fulfilled, the system can be reduced (in a formal mathematical sense) to a second order system (Poincare normal form). In this case the system is converted into a subspace of the phase space, the so-called center space (where the decaying parameters are eliminated). The second order system in the Poincare normal form provides directly a nonlinear stability indicator, the so-called **Floquet exponent** (β_2) which **identifies the stability of the limit cycles** [5]. The result of a bifurcation analysis is to get an indication on the existence of stable or unstable limit cycles (which we have to search with the system code in the next step, **hence the system code analysis is made with more pre-information's, is made more reliable**).

In Figure 7b is shown a selected stability map (incremental parameter, e.g. N_{sub}, vs. control or bifurcation parameter, γ_k, left hand side) and the corresponding bifurcation characteristic (β_2 vs. incremental parameter, right hand side). The bifurcation characteristic predicts unstable limit cycles coexisting with stable fixed points (Figure 7a) and stable limit cycles coexisting with unstable fixed points (Figure 7c) in the environment of the stability line as depicted in Figure 7b, left picture.
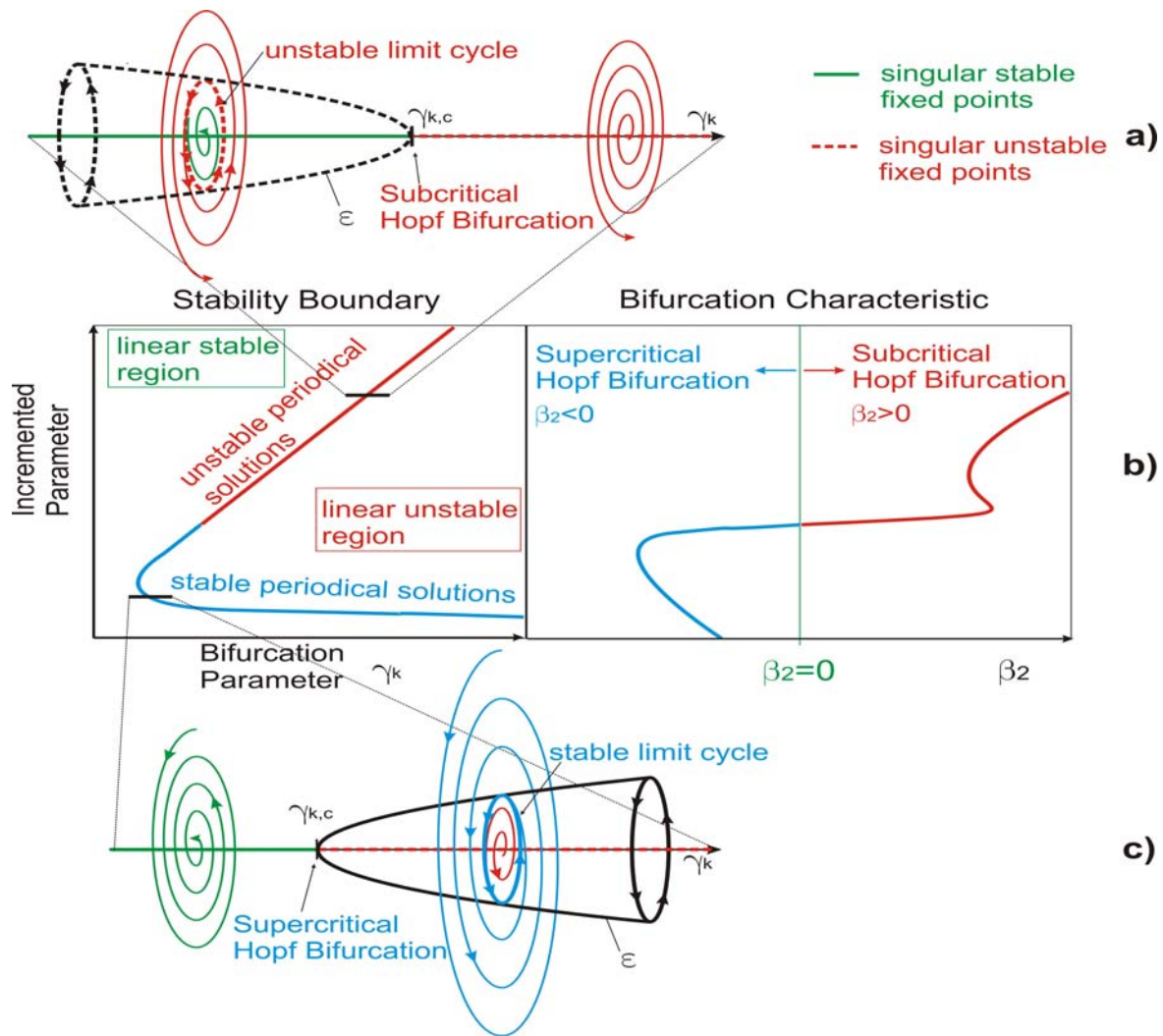
Figure 7: If the nonlinear dynamical system encountered a Hopf bifurcation point (during control parameter variation) the bifurcation analysis is **used to define a (nonlinear) limit cycle stability indicator** β_2 **.**

To understand the existence of unstable limit cycles (subcritical bifurcations) is a crucial point in the scope of nonlinear stability analysis (and a very strange phenomenon from the conventional thinking point of view). Loosely spoken, an unstable limit cycle is a set of operational points (on a circle) which separates the phase space in nonlinear stable and unstable regions (like a separatrix, see Figure 8a,b). Note the nonlinear unstable region is located at the linear stable side of the (linear) stability boundary (stability boundary, SB, is reached if the eigenvalues of the Jacobian matrix are zero under bifurcation parameter variation). **This is the reason that unstable limit cycles exist at the stable side of the SB** (Figure 7b). An unstable limit cycle "born" in a subcritical Hopf bifurcation separates a set of trajectories (in phase space) which spiral into the steady state solution (fixed point) from a set of trajectories which spiral away ad infinitum of the phase space (the state variables oscillations diverges; the system behaves unstable, Figure 8a). Or we say: Because of the coexistence of unstable limit cycles and stable fixed points (left of the SB) the stability behaviour of the system is dependent on the **parameter** perturbation amplitude: For parameter perturbations leading to **phase space variable elongations** less than the separatrix "amplitude" the stable fixed point acts as an attractor. For **large system**

parameter perturbations which lead to **phase space variables passing the separatrix** the unstable limit cycle acts as a repellor which means the state variables diverge in an oscillatory manner (see Figure 8a) or in technical terms: the system is stable for sufficient small but unstable for large system parameter perturbations (in system codes we use in many cases reactivity perturbations by control rod up and down moving for exiting power oscillations, hence we should find an unstable limit cycle by different control rod elongations).
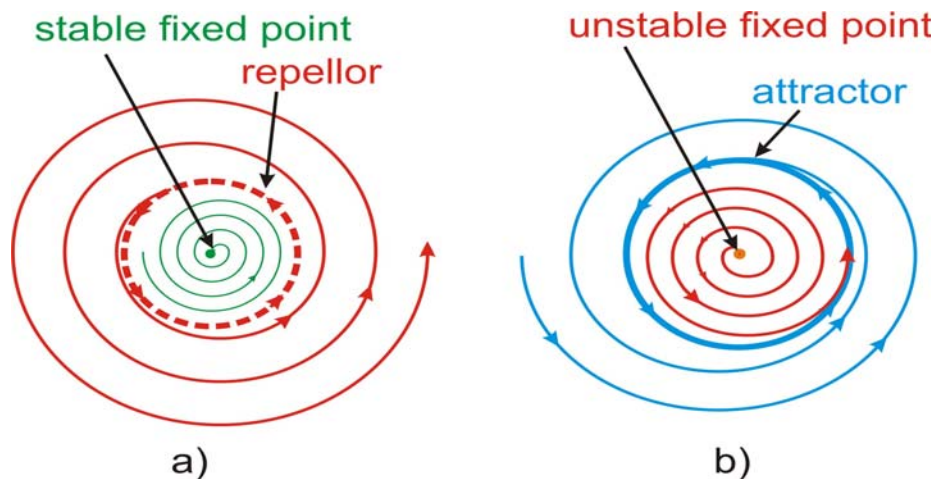

Figure 8: Phase space portrait of an unstable limit cycle (unstable periodical solution, repellor) close to a subcritical Hopf bifurcation (a) and phase space portrait of a stable limit cycle (stable periodical solution, attractor) close to a supercritical Hopf bifurcation (b).

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