Line Optimization in Public Transport Systems

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1 Introduction

Line planning is one of the strategic tasks a transport company is faced with. The aim is to create a line plan with line routes and service frequencies. Line optimization means to determine a line plan that is optimal regarding to a defined objective like the number of direct travelers [3], the total ride time, number of changes [5], the total cost [2] or the total traveling time. The literature offers approaches with choosing lines from a given set as well as construct line routes from the scratch [1], [4].

All of these approaches presume a given origin-destination-matrix. At least for urban areas this is not realistic. The most important questions of a traffic planner of a transportation company are: "How much does the new line plan cost?" and "How many passengers will go by public transport under the new circumstances?". Obviously it is necessary to consider the movement in demand for public transport within line optimization.

In this paper we include frequency depending changing times. In urban public transport systems often more than one line connects two points in a direct way. The expected traveling time is therefore lower than riding time plus half of the frequency time of the used line(s). The waiting times will decrease if there are e.g. two lines that connect two points by parallel line routes.

In practice, transport companies take advantage of lines that are parallel in the city and separate in the periphery to give a good service in the area with a great demand and connect the suburbs more efficient with the city. By experience (i.e. tested with data of Dresden) minimizing traveling times without regarding parallel line routes yields unrealistic results for the waiting times.

2 Model

In this section we present a model that can cope with (partially) parallel lines and traveling time dependent passenger demand.

2.1 Assumptions

Let $G[V, A]$ be a directed graph with a set of nodes V and a set of node connecting arcs A. The nodes represent stops for public transport. The arcs symbolize connections between nodes that can be passed by public transport vehicles. For each arc a ride time t_{ij} is defined. Furthermore, we know a set of line routes L. The arcs (i, j) which are part of the route of line l are given by set $\hat{A}_{lij}$. F is a set of possible frequencies a line can be operated with.

Every node pair that is connected by at least one potential line route yields for each combination of potential line routes l and frequencies f one arc. The larger the pool of lines L is, the more such arcs are required. For practical reasons we generate combinations with no more than five parallel line routes. Thus for each combination the expected traveling time can be estimated. Furthermore, we are able to calculate the proportion of passengers for each line frequency combination within a subset of lines. We assume that a path $p \in P$ is a connection between one pair of nodes u, v i.e. possibility for passengers to get from node u to node v. While the arcs are direct connections by one or more lines a path can be a combination of more than one arc. So necessary changes on the way from u to v can be modeled.

Example We show the computation of traveling times for one path. In Figure 1 you can see the connection between node u and node v with three lines. Line 2 connects the origin and the destination directly with a detour of four minutes and a frequency of 2 vehicles per hour while line 1 and line 3 offer only a part of this connection. We are now able to calculate traveling times (including the expected waiting times) of the given connections. Exemplary the traveling times of two generated arcs are shown in Figure 2.

The traveling times of the arcs are calculated as follows:

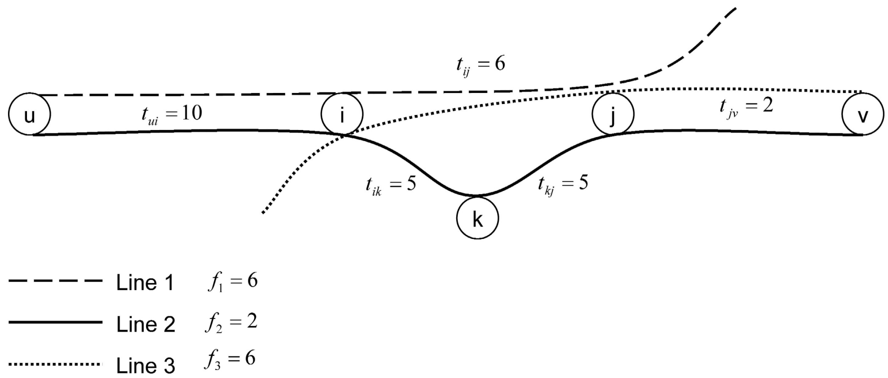

Fig. 1. Generated path with three involved lines

$$t^*_{puj} = \underbrace{\frac{60}{2 \cdot (2+6)}}_{\text{waiting time}} + \underbrace{\frac{2 \cdot (10+10) + 6 \cdot (10+6)}{(2+6)}}_{\text{riding time}} = 20.75 \quad (1)$$

$$t^*_{pjv} = \frac{60}{2 \cdot (2+6)} + \frac{2 \cdot 2 + 6 \cdot 2}{(2+6)} = 5.75 \quad (2)$$

The expected traveling time of the shown path is $t^*_{puv} = 20.75 + 5.75 = 26.5$ minutes. If only line 2 is available for the path, the expected traveling time is $15 + 22 = 37$ minutes. Similar to this we can calculate the proportions of the demand of each original arc and each line frequency combination as follows:

$$\beta_{p,1,6,i,j} = \frac{6}{6+2} = 0.75 \quad (3)$$

This means that 75% of the demand of passengers from u terminating in v will go by line 1 with the frequency of 6 vehicles per hour on arc

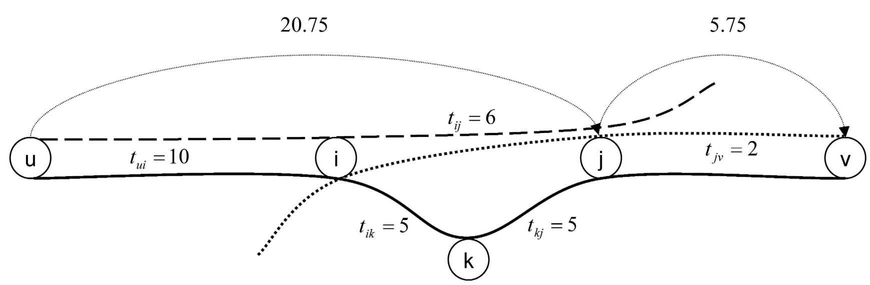

Fig. 2. Traveling times under the condition of parallel line routes

(i, j) when the line plan contains path p. We assume that 75% of the passengers will go by line one because six out of eight vehicles per hour belong to line 1.

On the basis of generating a large set of paths before optimization (e.g. by a n-shortest-path-algorithm) we get the following model:

$$\max F = \sum_p d_p \cdot z_p \tag{4}$$

$$\sum_{p,u,v \in \bar{p}_{puv}} z_p \leq 1 \qquad \forall\,(u, v) \in V^2 | u \neq v \tag{5}$$

$$\sum_f y_{lf} \leq 1 \qquad \forall l \in L \tag{6}$$

$$\sum_{p \in \hat{P}_{plfij}} d_p \cdot \beta_{plfij} \cdot z_p \leq K_{lf} \cdot y_{lf} \qquad \forall\,(l, i, j) \in \hat{A}_{lij}, \forall f \in F \tag{7}$$

$$\sum_{l,f} c_{lf} \cdot y_{lf} \leq C \tag{8}$$

$$z_p \in \{0, 1\} \qquad \forall p \in P \tag{9}$$

$$y_{lf} \in \{0, 1\} \qquad \forall l \in L, \forall f \in F \tag{10}$$

The objective function (4) maximizes the expected total number of passengers. To every path p an expected traveling time is assigned, that defines the expected number of passengers d_p. The binary variables z_p decide whether path p is selected or not. Obviously one pair (u, v) of nodes can be connected by many different paths. The constraints (5) ensure that the demand of one node pair (u, v) can be met by maximum one path p. The set $\bar{p}_{puv}$ gives for each path p the corresponding origin and destination. It is allowed to choose at most one frequency f for every line path l (6). The binary variables y_{lf} take the value 1, when line route l with the frequency f is selected. The capacity constraints (7) for all arcs (i, j), belonging to the line route l and the frequency f, give at least the capacity to handle the number of passengers moving along it. The demand of a path d_p multiplied by β_{plfij} represents the expected number of passengers. Set $\hat{P}_{plfij}$ denotes line routes l frequencies f and arcs (i, j) which correspond to path p. The parameter K_{lf} gives the capacity per vehicle of line l and frequency f. Let c_{lf} be the (proportional to the riding time) operating cost of line route l and frequency f. So the constraint (8) bounds the total operating cost to a given maximum of total cost C.

2.2 Discussion

One obvious problem is the large amount of possible and reasonable paths. When generating them before the optimization process we enlarge the model unnecessarily because most of the paths will not be part of a solution. So it seems to be appropriate to generate only those paths which will be probably part of a solution. A decomposition method for problems with many possible but only a few reasonable alternatives could be helpful.

3 Example

To clarify the above statements we present a small example. Starting with a directed graph with 10 nodes and 36 arcs. Before the optimization process we defined 16 possible line routes. Based on it 4320 arcs have to be created to model all (parallel) line frequency combinations. After that 24728 possible paths are generated by a modified n-shortest-path-algorithm. These paths contain the line routes, the frequencies and the passenger demand for each arc of the original graph.

For each pair of nodes we assume linear demand functions $d_{puv} = a_{uv} - b_{uv} \cdot t^*_{puv}$ with random parameters a_{uv} and b_{uv}.

Now we are in a position to solve the problem and vary the maximum total cost. The results are shown in table 1. One obvious result is that the increase of the maximum total cost yields no fundamental increase of the total number of expected passengers at a certain point.

No	max. Cost	objective	computing time [s]	gap
1	100	1016	1.92	-
2	200	1852	33.04	-
3	400	2840	1000.00	0.038
4	500	3228	1000.00	0.033
5	700	3621	31.28	-
6	800	3631	14.26	-
7	1600	3656	7.62	-

Table 1. Results of the example

For all scenarios, which took more than 1000 seconds the gap between the best possible and the actual integer solution is denoted. For example in Figure 3 we show the solution of scenario 5 with the maximum total cost of 700 minutes of total vehicle operating time and the objective of 3621 passengers.

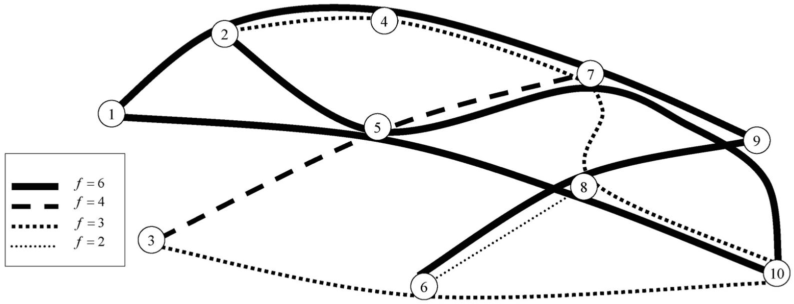

Fig. 3. Solution of scenario 5

4 Conclusions

In this article we have presented an approach on line optimization in urban public transport systems. It is shown that it is possible to take into account parallel line routes (with decreasing waiting times) and changing demand. There is still a lot of work to be done in the field of estimation of relation specific demand regarding to the expected traveling time and e.g. the number of changes needed or socioeconomic structures of corresponding districts. Moreover, the solution process should be made more efficient to get the ability for solving real world instances. An advantage of our approach is that it is possible to include non-linear demand functions in the data but nevertheless the model will stay linear.

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