



# Speedup for Multi-Level Parallel Computing

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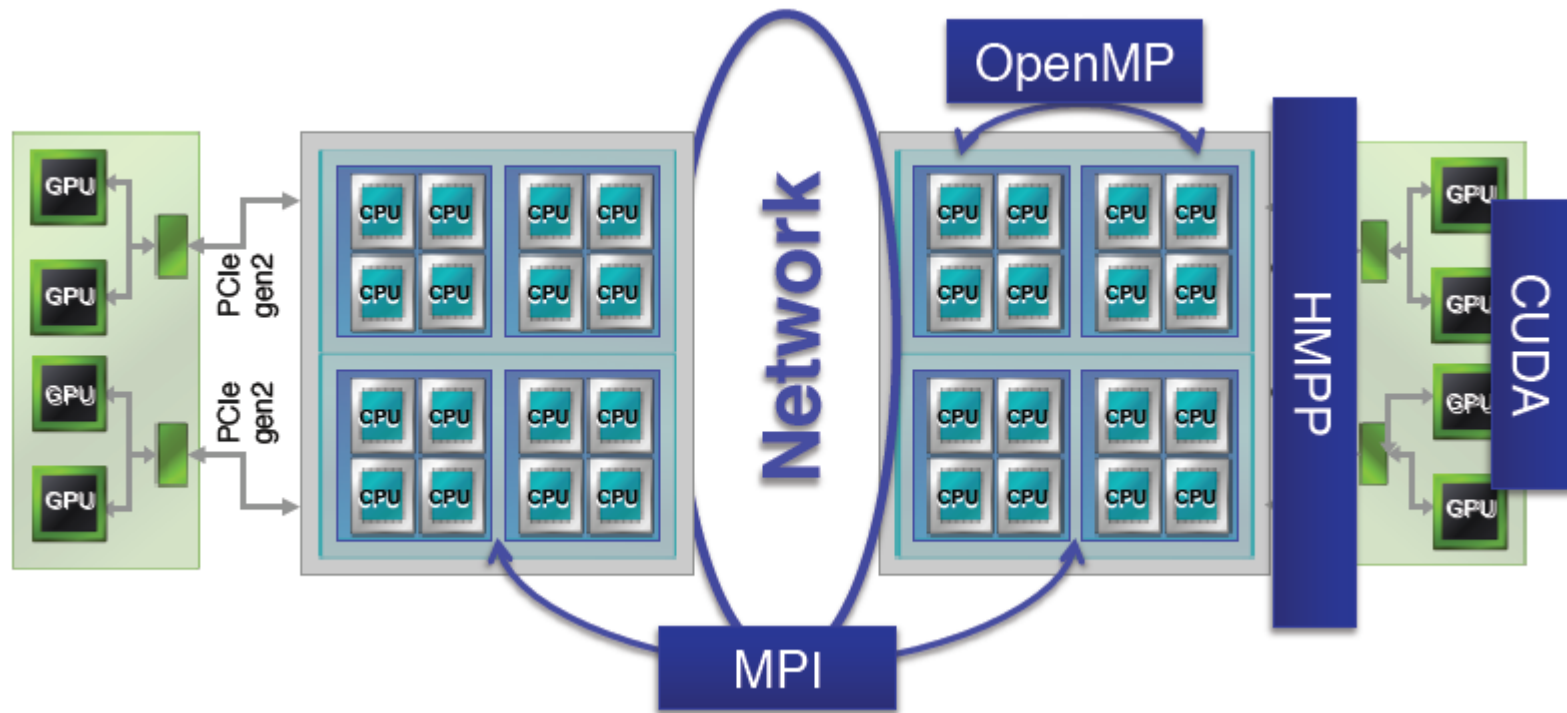


# OutLine

- Background & Motivation
- Multi-Level Parallel Speedup
- Evaluation
- Conclusion



# Multi-Level Computing Architecture and Paradigm

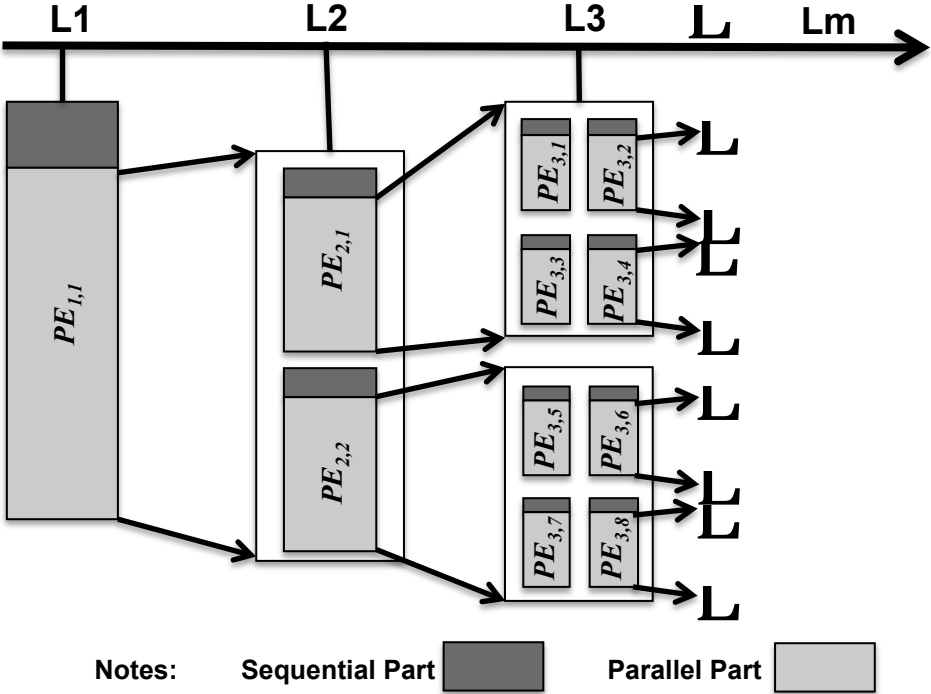


# Multi-Level Computing Architecture and Paradigm

- MPI+OpenMP
- MPI+CUDA
- MPI+OpenMP+CUDA
- .....



# Multi-Level Parallel Computing Model



# Parallel Speedup

- Definition

$$\text{Speedup} = \frac{\text{SequentialExecutionTime}}{\text{ParallelExecutionTime}}$$

- Classification

- Absolute Speedup

$$\text{Speedup} = \frac{\text{BestSequentialALGExecutionTime}}{\text{ParallelALGExecutionTime}}$$

- Relative Speedup

$$\text{Speedup} = \frac{\text{ParallelALGSequentialExecutionTime}}{\text{ParallelALGExecutionTime}}$$



# Relative Speedup Model

- Fixed-size Speedup

- Amdahl's Law

$$Speedup = \frac{sequentialTime}{parallelTime} = \frac{1}{1 - \alpha + \frac{\alpha}{p}}$$

where  $\alpha$  is parallel fraction workload of the program,  $p$  is the number of processors.

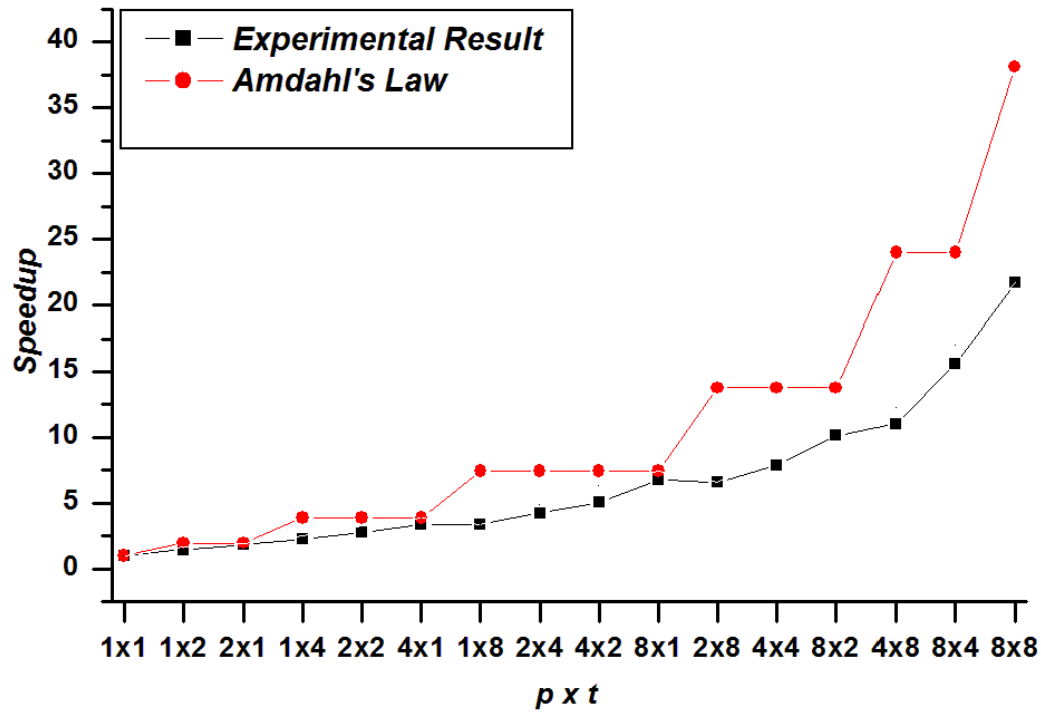
- Fixed-time Speedup

- Gustafson's Law

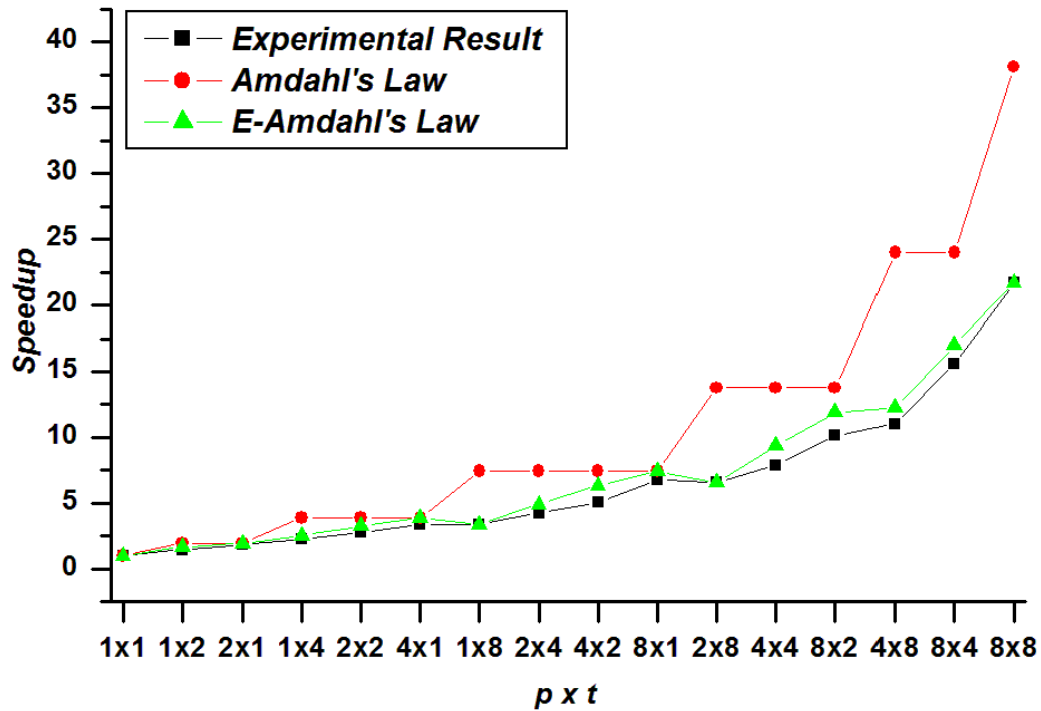
$$Speedup = \frac{sequentialTime}{parallelTime} = \frac{1 - \alpha + \alpha p}{1 - \alpha + \alpha} = 1 - \alpha + \alpha p$$



# Motivation Example—NAS Benchmark (MPI+OpenMP)



# Motivation Example—NAS Benchmark (MPI+OpenMP)



**Amdahl's Law is UNSUITABLE for Multi-Level Parallel Computing**

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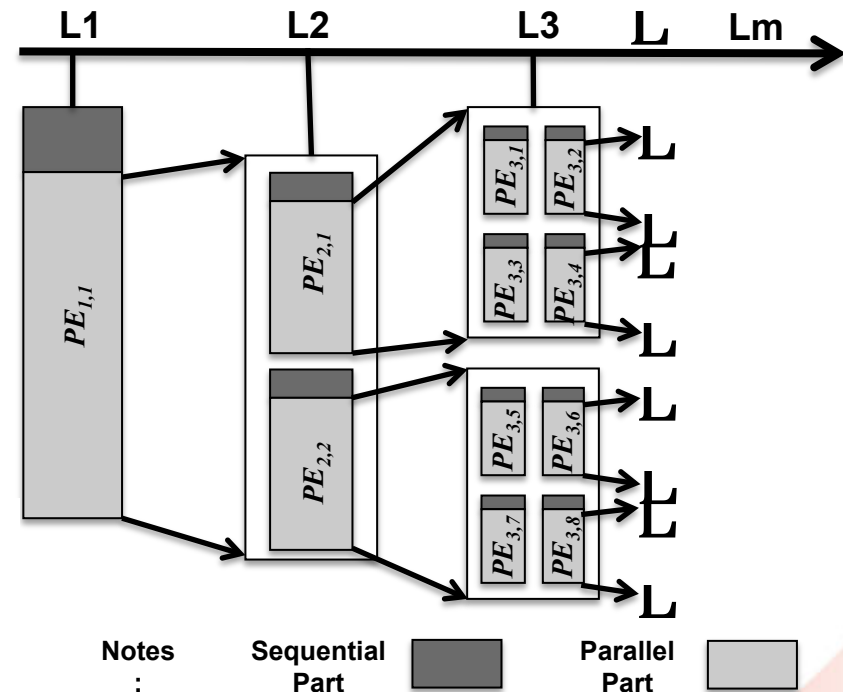


# E-Amdahl's Law

- Awareness of Different Grained-Level Parallelism

$$sp(i) = \begin{cases} \frac{1}{1 - f(m) + \frac{f(m)}{p(m)}} & (i = m) \\ \frac{1}{1 - f(i) + \frac{f(i)}{p(i)sp(i+1)}} & (1 \leq i < m) \end{cases}$$

Symbol	Definition
$m$	The number of nested parallelism levels. ( $m \geq 1$ )
$p(i)$	The number of parallel processing elements in the $i^{th}$ level. ( $p(i) \geq 1$ )
$f(i)$	The portion of workload in the $i^{th}$ level that can be parallelized. ( $0 \leq f(i) \leq 1$ ).
$sp(i)$	The multi-level speedup for the $i^{th}$ level.



# E-Amdahl's Law

- Two-Level Parallelism Speedup Model (MPI+OpenMP)

$$sp(\alpha, \beta, p, t) = \frac{1}{1 - \alpha + \frac{\alpha(1 - \beta + \frac{\beta}{t})}{p}}$$

where

$\alpha$  is the parallel fraction of coarse-grained (MPI-level) parallelism.

$\beta$  is the parallel fraction of fine-grained (OpenMP-level) parallelism.

$p$  is the number of processes spawned.

$t$  is the number of threads spawned per process.

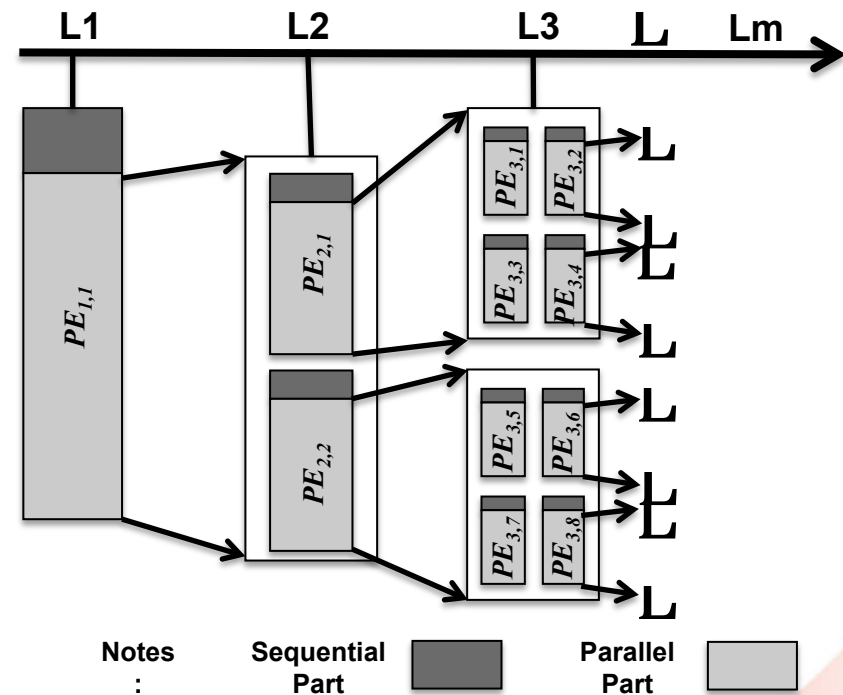


# E-Gustafson's Law

- Awareness of Different Grained-Level Parallelism

$$sp(i) = \begin{cases} 1 - f(m) + f(m)p(m) & (i = m) \\ 1 - f(i) + f(i)p(i)sp(i+1) & (1 \leq i < m) \end{cases}$$

Symbol	Definition
$m$	The number of nested parallelism levels. ( $m \geq 1$ )
$p(i)$	The number of parallel processing elements in the $i^{th}$ level. ( $p(i) \geq 1$ )
$f(i)$	The portion of workload in the $i^{th}$ level that can be parallelized. ( $0 \leq f(i) \leq 1$ ).
$sp(i)$	The multi-level speedup for the $i^{th}$ level.



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# Experiment Setup

- Platform and Configuration

- A linux cluster consisting of eight computing nodes each with two quad-core chips
- Configuration: One thread per CPU core

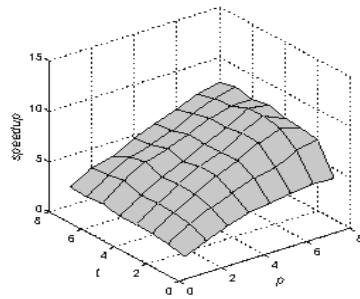
- Benchmarks

NAS Parallel Benchmark (NPB) Multi-Zone (MZ) Version:

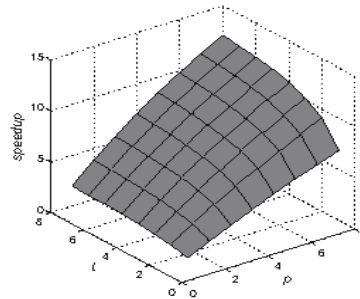
- BT-MZ (Unbalanced Workload Partitioning)
- SP-MZ (balanced Workload Partitioning)
- LU-MZ (balanced Workload Partitioning)



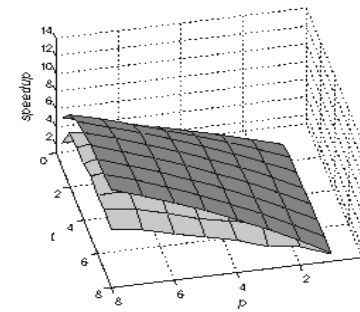
# Performance Prediction



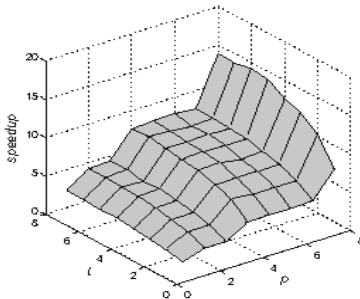
(a) Experimental result of BT-MZ (class W).



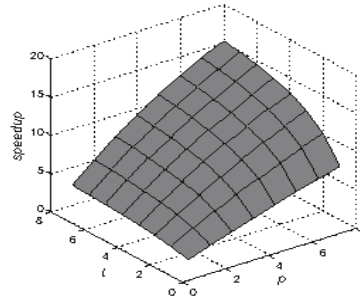
(b) Estimated result of BT-MZ.  $\alpha = 0.9771, \beta = 0.5822$



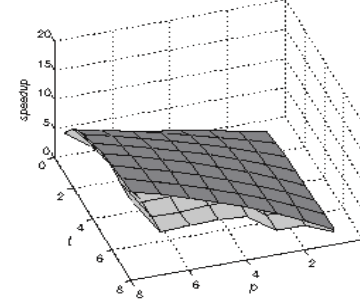
(c) Comparison result of (a) and (b).



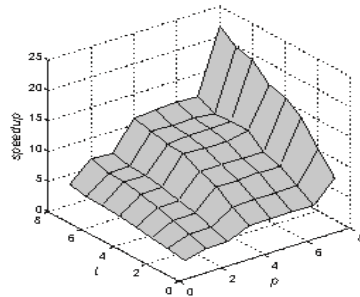
(d) Experimental result of SP-MZ (class A).



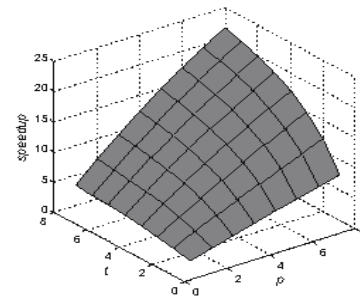
(e) Estimated result of SP-MZ.  $\alpha = 0.9790, \beta = 0.7263$



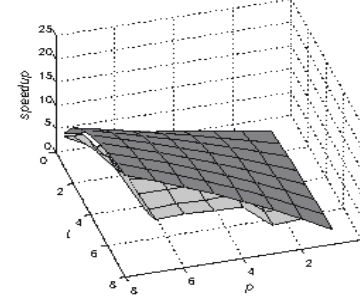
(f) Comparison result of (d) and (e).



(g) Experimental result of LU-MZ (class A).



(h) Estimated result of LU-MZ.  $\alpha = 0.9892, \beta = 0.8161$



(i) Comparison result of (g) and (h)

# Prediction Result Comparison

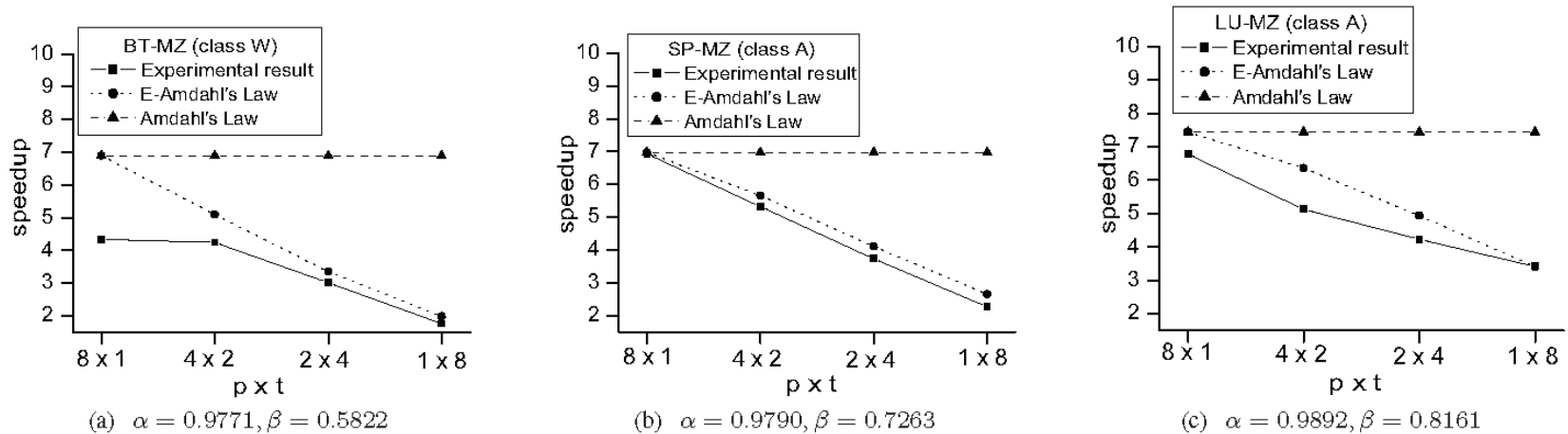


Fig. 8: Experimental and estimated speedups of NPB-MZ for different combinations of  $p \times t$  under a given total number of 8 processors. The speedup based on *Amdahl's Law* is estimated with the formula  $\frac{1}{1-\alpha+\frac{\alpha}{p \times t}}$ . The speedup based on *E-Amdahl's Law* is estimated by using Formula (17).

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# Conclusion

- Traditional speedup models are unsuitable for multi-level parallelism
  - Unable to be awareness of different granularities of parallelism for multi-level parallel computing.
- Multi-level Parallelism Model
  - A guidance model for multi-level optimization.
  - A prediction model for multi-level parallelism.



# Thank You !

## Question?



# Argument Estimation

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**Algorithm 1** *Argument Estimation for  $\alpha, \beta$ .*

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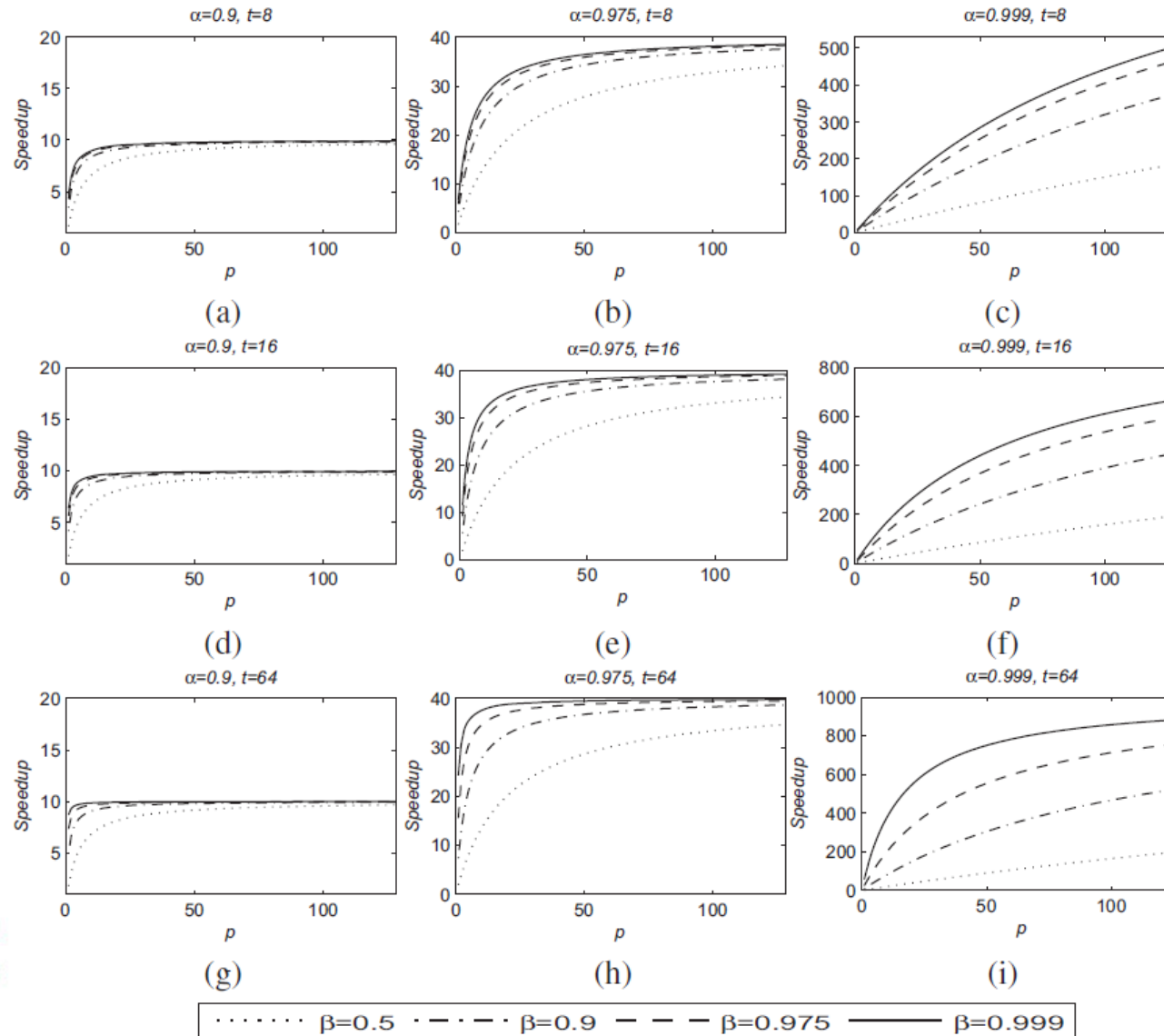
- 1: Execute the program  $k$  times in the multi-level parallel execution mode with the chosen parameters  $(p_1, t_1), (p_2, t_2), \dots, (p_k, t_k)$  respectively. Then it gets  $(p_1, t_1, sp_1), (p_2, t_2, sp_2), \dots, (p_k, t_k, sp_k)$ .
- 2: Choose all possible combinations of two arrays  $(p_i, t_i, sp_i)$  and  $(p_j, t_j, sp_j)$  to figure out the value  $(\alpha_s, \beta_s)$  based on Equation (17), where  $s$  denotes the  $s^{th}$  combination and  $1 \leq i, j \leq k$  &  $i \neq j$ .
- 3: Check all possible pairs of value  $(\alpha_s, \beta_s)$  to guarantee that the pair of estimated values is valid (i.e.  $0 \leq \alpha_s \leq 1, 0 \leq \beta_s \leq 1$ ). Otherwise, discards it.
- 4: Collect all valid pairs of  $(\alpha_i, \beta_i), (i = 1, 2, \dots, k')$ , where  $k'$  represents the number of valid pairs. Remove the noise pairs by clustering with the guard condition:  $|\alpha_i - \alpha_j| < \varepsilon$  &  $|\beta_i - \beta_j| < \varepsilon$ .
- 5: The exact value of  $\alpha, \beta$  can thereby be estimated with the formula:

$$\begin{cases} \hat{\alpha} = \frac{1}{\hat{k}} \sum_{i=1}^{\hat{k}} \alpha_i \\ \hat{\beta} = \frac{1}{\hat{k}} \sum_{i=1}^{\hat{k}} \beta_i, \end{cases}$$

where  $\hat{k}$  is the number of clustered pairs.

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# Speedup Under E-Amdahl's Law



# Speedup Under E-Gustafson's Law

