DRYING CREEP OF CONCRETE IN TERMS OF AGE-ADJUSTED EFFECTIVE MODULUS METHOD

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The problem of drying creep of concrete is considered. It is emphasized that there is no single explanation of the paradoxical concrete performance called Pickett effect. The existing experimental data cannot clearly distinguish between the different mechanisms of drying creep, namely, microcracking and stress-induced shrinkage. The phenomenological approach describing the drying creep under tension as a sum of shrinkage-induced creep and creep-induced shrinkage is corrected by taking into account the swelling deformation of sealed concrete in basic creep tests. In such a way the drying creep strain represents only the extra creep component, therefore the mechanism of drying creep based on the stress-induced shrinkage is called into question. A new phenomenological approach to the problem of drying creep using the terms of age-adjusted effective modulus method is demonstrated. The work of concrete element under simultaneous loading and drying is represented as restrained shrinkage in addition basic creep under gradually developed stresses. It is shown that the drying creep strain depends on the values of aging coefficients at basic and drying creep and on the individual components of concrete deformation: instantaneous strain, basic creep and free shrinkage. It is found that the instantaneous strain plays the minor role. The experimental data of the creep tests under tension are used to calculate the aging coefficient of concrete at basic creep.

Notation

\( E \) elasticity modulus of concrete
\( E_e \) effective elasticity modulus of concrete
\( E_\text{age} \) age-adjusted effective elasticity modulus of concrete
\( t \) time
\( \varepsilon \) total strain in concrete
\( \varepsilon_{bc} \) basic creep strain
\( \varepsilon_{cr} \) total creep strain
\( \varepsilon_{dc} \) drying creep strain
\( \varepsilon_{el} \) elastic (instantaneous) strain
\( \varepsilon_{fs} \) free shrinkage strain
\( \sigma \) stress in concrete
\( \sigma_{rs} \) stress due to restrained shrinkage
\( \sigma_0 \) stress at the time of first loading
\( \tau_0 \) age at first loading
\( \varphi \) creep coefficient
\( \varphi_{bc} \) basic creep coefficient
\( \varphi_{cr} \) total creep coefficient
\( \varphi_{dc} \) drying creep coefficient
\( \chi \) aging coefficient
\( \chi_{bc} \) aging coefficient at basic creep
\( \chi_{cr} \) aging coefficient at total creep
\( \chi_{dc} \) aging coefficient at drying creep

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**Introduction**

Usually, shrinkage of concrete occurs simultaneously with creep. The common practice over many years has been to consider these two phenomena to be additive, that is suitable for the many practical applications. In fact, we well know that they are not independent and the principle of superposition cannot be applied. The problem is either in the definitions, which are not so correct, or in assuming that these two phenomena are additive.

Shrinkage is defined as a change of strain in time due to moisture withdrawal in the absence of an applied load, creep - as a change of strain in time due to stress. These definitions were stated a long time ago, much before assuming that these two phenomena are independent, and now they are widely used in various fields of mechanical science. That is why it is more logical to keep the definitions. Indeed, why must shrinkage and creep be additive? If they are not, the well-known problem of drying creep, which has not yet been explained and discussions on this still continue, would not exist at all.

We have to distinguish between the component of creep of concrete under conditions of no moisture movement to or from the ambient medium (so called "basic creep") and the component caused by the concurrent drying process ("drying creep"). The idea of drying creep was introduced by Pickett\(^1\) to explain the observed excess of total creep at drying over basic creep. In other words, drying creep is a deviation from the idea of additivity of shrinkage and creep. As it was shown in the work\(^2\), this deviation should inevitably arise if we will express the deformation of a drying specimen through that of a sealed one, because these two specimens represent different materials, in spite of the same structure and geometry, age and load conditions, and even in spite of the same total moisture content (in the initial moment of sealing).

Concrete creep (\(\varepsilon_{cr}\)) is commonly modeled in terms of a basic creep component (\(\varepsilon_{bc}\)) which is independent of concrete drying, and a drying creep component (\(\varepsilon_{dc}\)):

\[
\varepsilon_{cr} = \varepsilon_{bc} + \varepsilon_{dc}
\]

However, the internal physical mechanisms resulting in drying creep are not completely understood yet. The review of the main physical models explaining drying creep has been recently published in the work\(^3\). It is accepted that there are two main possible physical mechanisms resulting in drying creep: microcracking effect in concrete skin and so-called stress-induced shrinkage. Both of them have the limitations, because cannot explain some evident experimental facts. For, example, the microcracking effect fails to explain more than 50% of the difference in strain, obtained by Thelandersson et al\(^4\). The microcracking was observed neither directly, nor by the analysis of the mechanical characteristics of dried concrete under axial tension (elasticity modulus, tensile strength and hysteresis loops at loading/unloading cycles)\(^3\). As far as the second mechanism, the stress-induced shrinkage, is concerned, it is also called into question, as it will be shown in the present paper.

The goal of the present study is to use the known age-adjusted effective modulus method for the explanation of the extra drying creep deformation of concrete. Such an approach is phenomenological and cannot be applied for the analysis of the internal physical causes resulting in drying creep. Let us first recall the main principles of this method.

**The principles of the age-adjusted effective modulus method**

As is known, the total strain at any time \(t\) at a point in a uniaxially loaded element by constant stress \(\sigma\) at constant temperature can be expressed as the sum of the instantaneous (elastic), creep and shrinkage components:
\[ \varepsilon(t, \tau) = \varepsilon_{el}(t, \tau) + \varepsilon_{cr}(t, \tau) + \varepsilon_{fs}(t) = \frac{\sigma}{E(\tau)} + \frac{\sigma}{E(\tau)} \varphi(t, \tau) + \varepsilon_{fs}(t) \]  \hfill (2)

where \( \tau \) is a time of load application, \( E(\tau) \) is the elasticity modulus of concrete, \( \varphi(t, \tau) \) is the creep coefficient defined as the ratio of creep strain at time \( t \) to instantaneous elastic strain.

The simplest technique for including creep in structural analysis is effective modulus method, where the elastic and creep components of strain are combined:

\[ \varepsilon(t, \tau) = \frac{\sigma}{E(\tau)}[1 + \varphi(t, \tau)] + \varepsilon_{fs}(t) = \frac{\sigma}{E_e(t, \tau)} + \varepsilon_{fs}(t) \]  \hfill (3)

where the effective (reduced) modulus is defined as

\[ E_e(t, \tau) = \frac{E(\tau)}{1 + \varphi(t, \tau)} \]  \hfill (4)

In order to take the previous stress history into account, Trost and Bazant developed so-called age-adjusted effective modulus method. The principles of this method are shown in Fig. 1.

\[ \sigma(t) = \frac{\sigma_0 \varphi(t, \tau_0)}{E(\tau_0)} + \frac{\sigma_0 \chi(t, \tau_0) \varphi(t, \tau_0)}{E(\tau_0)} \]

\[ \varepsilon(t) = \frac{\sigma_0}{E(\tau_0)}[1 + \varphi(t, \tau_0)] + \frac{\sigma(t) - \sigma_0}{E(\tau_0)}[1 + \chi(t, \tau_0) \varphi(t, \tau_0)] + \varepsilon_{fs}(t) = \]

\[ = \frac{\sigma_0}{E_e(t, \tau_0)} + \frac{\sigma(t) - \sigma_0}{E_e(t, \tau_0)} + \varepsilon_{fs}(t) \]  \hfill (5)

where the \( E_e(t, \tau_0) \) is the age-adjusted effective modulus, defined as

*Fig. 1. Creep due to both constant and variable stress histories*
Like the creep coefficient, \( \chi(t, \tau_0) \) depends on the age at first loading, the duration of load, geometry of the loaded element, drying conditions and other factors. As was shown by Gilbert, the experimental magnitude of \( \chi(t, \tau_0) \) falls between 0.6 and 0.9. As far as the theoretical value of the aging coefficient is concerned, there are contradictory opinions on this issue. For example, in the period immediately after initial loading the aging coefficient should be theoretically equal to 0.5. However, according to the norms CEB-FIP and others, the aging coefficient is close to 1.0 after the loading. For many practical implications, the aging coefficient can be accepted constant with an average value of 0.8.

The introduction of the aging coefficient helps to overcome the numerous inadequacies of the effective modulus method and to solve structural tasks of time analysis using the methods of elastic analysis. The age-adjusted effective modulus method can be successfully applied, for example, for the evaluation of shrinkage-induced stresses in restrained concrete. If the restraint is absolute, the sum of the individual elastic, creep and shrinkage components of concrete strain should be zero:

\[
\varepsilon(t) = \varepsilon_{el}(t) + \varepsilon_{cr}(t) + \varepsilon_{fs}(t) = 0
\]

Because the tensile shrinkage-induced stress is developed gradually, as shown in Fig. 1, the sum of instantaneous and creep strains, which is equal to \(-\varepsilon_0(t)\), should be related to stress by the age-adjusted effective modulus. Hence, the stress induced by restrained shrinkage can be calculated as

\[
\sigma_{rs}(t, \tau_0) = -\varepsilon_{fs}(t)\overline{E}_e(t, \tau_0)
\]

**Creep under drying as restrained shrinkage plus basic creep at variable stresses**

Let us consider the concrete element exposed simultaneously to drying and loading with the constant stress \( \sigma_0 \). Instead of the principle of additivity of shrinkage and creep strain under simultaneous drying and loading, we will apply the principle of additivity of stresses. The idea is to get the total time-dependent deformation, including the unknown drying creep component, as a function of simultaneous action of two stresses: the first is shrinkage-induced stress due to restraint of drying concrete element, and the second is the stress developed in the same element, when there is no moisture exchange with the ambient medium. In other words, the work of the concrete member is represented as restrained shrinkage plus basic creep. The only condition is to have the sum of these stresses equaled to the constant stress \( \sigma \). To provide the resulting stress constant, each of these two stresses has to be variable (Fig. 2).

The main assumptions and simplifications are the following:
1. The drying commencement coincides with the commencement of loading.
2. To simplify the problem, let us consider that the elasticity modulus of concrete does not change with time: \( E(\tau_0) = E \). This fact, for example, was found even for the early age concrete exposed to drying in hot dry climate conditions.
3. Generally, the aging coefficients at basic creep and at creep under drying are not the same: \( \chi_{bc}(t, \tau_0) \neq \chi_{cr}(t, \tau_0) \).

The total time-dependent strain under simultaneous drying and loading with the stress \( \sigma_0 \), according to (3), is
\[ \varepsilon(t, \tau_0) = \frac{\sigma_0}{E} [1 + \varphi_{cr}(t, \tau_0)] + \varepsilon_{fs}(t) = \frac{\sigma_0}{E} (1 + \varphi_{bc} + \varphi_{dc}) + \varepsilon_{fs} \]  \hspace{1cm} (9)

<table>
<thead>
<tr>
<th>Creep under constant compressive stress ( (\sigma_0) ) and drying</th>
<th>Restraint under drying and gradually increased tensile stress ( \sigma_{rs} ) (restrained shrinkage)</th>
<th>Basic creep under gradually increased compressive stress ( (\sigma_0 - \sigma_{rs}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_0 = \text{const} )</td>
<td>( \sigma_{rs} = \text{var} )</td>
<td>( \sigma_0 - \sigma_{rs} = \text{var} )</td>
</tr>
<tr>
<td>Stress ( \sigma_0 = \text{const} ) (compression)</td>
<td>Stress ( \sigma_{rs} ) ( = \text{var} ) (tension)</td>
<td>Stress ( \sigma_0 - \sigma_{rs} = \text{var} ) (compression)</td>
</tr>
</tbody>
</table>

Fig. 2. Simplified scheme of the work of concrete element under simultaneous loading and drying as restrained shrinkage plus basic creep under gradually increased stresses

On the other hand, the total strain is equal to the strain of the element under conditions of basic creep (see Fig. 2):

\[ \varepsilon(t, \tau_0) = \frac{\sigma_0}{E} [1 + \varphi_{bc}(t, \tau_0)] - \frac{\sigma_{rs}(t, \tau_0)}{E} [1 + \chi_{bc}(t, \tau_0)\varphi_{bc}(t, \tau_0)] = \]

\[ = \frac{\sigma_0}{E} (1 + \varphi_{bc}) - \frac{\sigma_{rs}}{E} (1 + \chi_{bc}\varphi_{bc}) \]  \hspace{1cm} (10)

Equating (9) and (10) to each other and taking into account that
\[ \sigma_{rs}(t, \tau_0) = - \frac{\varepsilon_{fs}(t)}{E} \left[ 1 + \varphi_{cr}(t, \tau_0) \right] = - \frac{\varepsilon_{fs}}{E} \left( 1 + \frac{\varepsilon_{be} + \varepsilon_{dc}}{\varepsilon_{el}} \right) \]  

(11)

the quadratic equation for the drying creep strain is obtained:

\[ \varepsilon_{dc}^2 + Z \varepsilon_{dc} - \left( \frac{\chi_{bc}}{\chi_{cr}} - 1 \right) \varepsilon_{be} \varepsilon_{fs} = 0 \]  

(12)

where \( Z = \frac{1}{\chi_{cr}} \varepsilon_{el} + \varepsilon_{be} + \varepsilon_{fs} \).

The general solution of the equation (12) is

\[ \varepsilon_{dc} = - \frac{Z}{2} \pm \sqrt{\left( \frac{Z}{2} \right)^2 + \left( \frac{\chi_{bc}}{\chi_{cr}} - 1 \right) \varepsilon_{be} \varepsilon_{fs}} \]  

(13)

At the very small values of basic creep and/or free shrinkage the solution (13) is reduced to the simple form \( \varepsilon_{dc} = - \frac{Z}{2} \pm \sqrt{\left( \frac{Z}{2} \right)^2} \). In this case, according to the common sense, the drying creep strain should also achieve zero. This is correct also when the values of the aging coefficients at basic creep and at creep under drying are very close. That is why the sign before the square root has to be negative, if the function \( Z \) is negative too, and vice versa. In the opposite case \( \varepsilon_{dc} = - Z = - \left( \frac{1}{\chi_{cr}} \varepsilon_{el} + \varepsilon_{be} + \varepsilon_{fs} \right) \), that at \( \chi_{cr}=1 \) corresponds to the absolute restraint of deformations, that is not a case of our consideration.

**Drying creep under compression**

From the expression (13) it follows, that for the case of compression, when all of the components of strain are negative, the drying creep strain is

\[ \varepsilon_{dc} = - \frac{1}{\chi_{cr}} \varepsilon_{el} + \varepsilon_{be} + \varepsilon_{fs} \]  

(14)

The dependence of drying creep under compression on basic creep and free shrinkage is shown in Figs 3-5 for the certain levels of elastic strain (\( \varepsilon_{el}=-150 \) microstrain in Figs 3 and 4 and \( \varepsilon_{el}=-300 \) microstrain in Fig. 5) and for the fixed constant values of aging coefficients at total and basic creep (\( \chi_{cr}=0.7 \) and \( \chi_{bc}=0.8 \) in Fig. 3; \( \chi_{cr}=0.6 \) and \( \chi_{bc}=0.9 \) in Figs 4 and 5).
Fig. 3. Dependence of drying creep on basic creep and free shrinkage (elastic strain = -150 microstrain, aging coefficients at total and basic creep = 0.7 and 0.8, respectively)

Fig. 4. Dependence of drying creep on basic creep and free shrinkage (elastic strain = -150 microstrain, aging coefficients at total and basic creep = 0.6 and 0.9, respectively)
The small difference between the aging coefficients, when the values of aging coefficients were accepted 0.7 and 0.8 at total and basic creep, respectively, is considered in Fig. 3. The relatively big difference, when the above aging coefficients were accepted 0.6 and 0.9, is the case of Fig. 4. From the comparison of Figs 3 and 4 it follows that the greater is the difference between the aging coefficients at total and basic creep, the more is the absolute value of the drying creep strain. In other words, the difference in aging coefficients is the decisive factor influencing drying creep. At the same time both of the strain components, basic creep and free shrinkage, influence the drying creep directly, and by the same extent, i.e. the drying creep in Figs 3-5 is symmetrical in relation to both basic creep and free shrinkage.

The most of the models dealing with drying creep of concrete state the linear relation between drying creep strain and free shrinkage. For example, Gamble and Parrott\textsuperscript{8} found the following dependence for the total creep strain $\varepsilon_{cr}$ of loaded concrete specimens under drying:

$$\varepsilon_{cr} = \varepsilon_{bc} + k_1\sigma \varepsilon_{fs}$$  \hspace{1cm} (15)

where $\varepsilon_{bc}$ is basic creep strain, $k_1$ is a constant for the particular concrete used, $\sigma$ is stress and $\varepsilon_{fs}$ is free shrinkage strain.

According to the approach of L’Hermite\textsuperscript{9} and Neville\textsuperscript{10}, drying creep strain depends both on basic creep and free shrinkage, and is proportional to their product:

$$\varepsilon_{cr} = \varepsilon_{bc} + k_2 \varepsilon_{bc} \varepsilon_{fs}$$  \hspace{1cm} (16)

where $k_2$ is a constant depending on the properties of material.

From the analysis of equation (14) and Figs 3-5 it follows that the dependence of drying creep on free shrinkage is not exactly linear, as was stated in the previous models. Secondly, the level of compressive load (or the value of elastic strain) should evidently influence the drying creep. However, it is difficult to see both these effects from Figs 3-5.
The character of the dependence of drying creep on free shrinkage at different levels of elastic strain is shown in Fig. 6 for the given values of basic creep strain. It can be seen both the degrees of linearity of the drying creep strain curve versus free shrinkage and the range of the influence of elastic strain achieved. As it follows from Fig. 6, the influence of the elastic strain is minor, because the increase of \( \varepsilon_{el} \) by twice resulted in the decrease of the drying creep strain only by 13%.

**Drying creep under tension**

The dependence of drying creep under tension is

\[
\varepsilon_{dc} = -\frac{1}{\chi_{cr}} \varepsilon_{el} + \varepsilon_{bc} + \varepsilon_{fs} \pm \sqrt{\left(\frac{1}{\chi_{cr}} \varepsilon_{el} + \varepsilon_{bc} + \varepsilon_{fs}\right)^2 + \left(\frac{\chi_{bc}}{\chi_{cr}} - 1\right) \varepsilon_{bc} \varepsilon_{fs}}
\]

where the sign before the square root is negative, when \( \frac{1}{\chi_{cr}} \varepsilon_{el} + \varepsilon_{bc} + \varepsilon_{fs} \) is also negative (like in the case of compression, discussed earlier), and vice versa. Therefore, we cannot assume that there are the constant values of aging coefficients to demonstrate the results of the present approach, otherwise or the expression under the square root can be negative, either the rule of signs stated above would not work correctly. For the calculations we will assume, that one of the aging coefficients,

![Figure 6. Dependencies of drying creep on free shrinkage (aging coefficients at total and basic creep = 0.6 and 0.9, respectively)](image)

*Figure 6. Dependencies of drying creep on free shrinkage (aging coefficients at total and basic creep = 0.6 and 0.9, respectively)*
namely that at creep under drying (at total creep) is given, for example, by the norms of CEB-FIP. So, the value of the second aging coefficient (at basic creep) will be found as a function by means of the same equation (17). For this purpose we will use the experimental data obtained in the tests with concrete of the following characteristic.

The concrete was a microconcrete, with maximum size aggregate of 7 mm), specially graded for this purpose. Concrete composition (cement:sand:gravel by weight) was 1:2:2 at a water/cement ratio of 0.7. The materials used were: ordinary Portland cement having standard compressive strength of 30 MPa and specific gravity of 3,100 kg/m$^3$, coarse aggregate - crushed gravel with a maximum particle size of 7 mm, specific gravity of 2,750 kg/m$^3$, fine aggregate - quartz sand from natural source having fineness modulus of 1.76, specific gravity of 2,630 kg/m$^3$. Shrinkage tests were carried out in special horizontal beam moulds with net cross-section of 40x40 mm. There were 2 twin specimens, one - for drying creep (or basic creep) and another - for free shrinkage, with a working length of 1,000 mm. The experimental set-up was described in detail in the work$^{11}$.

The specimens were cast and exposed to drying in a special environmental room having a temperature of (30.8±0.2)$^\circ$C, and air relative humidity of (35±3)%, which simulated hot-dry climate conditions.

Immediately after casting, the upper surface of each specimen was sealed by a synthetic film, being tightly applied onto the fresh concrete surface with no visibly trapped air. The specimens were completely demoulded after 24 hours and then exposed to drying environment for 24 hours more, to exclude the phenomenon of so-called “shock of evaporative cooling”, observed and described in the work$^{12}$. So, the creep tests were started 48 hours after casting. For the basic creep test, specimens were sealed by means of a multilayer synthetic film.

The experimental curves obtained for the total creep (under drying and loading with the tensile stress of 1.0 MPa), free shrinkage, and basic creep (with the same stress) are presented in Fig. 7. It can be seen that the drying creep as the difference between the curves of total and basic creep is not characterized by so-called “abnormal” behavior, which was revealed in the work$^{3}$ and analyzed in detail in the work$^{2}$. Such “abnormal” behavior means that the drying creep deformation changes the sign with time. Here the value of the basic creep strain is corrected by the subtraction of the swelling component from the measured basic creep. The swelling phenomenon was found in the work$^{2}$ in sealed concrete specimens exposed to some drying before. After the correction the drying creep deformation is positive within the whole range of the duration of loading and drying. It can be also concluded, that the drying creep of concrete (at least under tension) is not caused by so-called “stress-induced shrinkage”, because it represents extra creep component, as it clearly follows from Fig. 7.

In order to satisfy to equation (17) and to the given value of the aging coefficient at total creep, the aging coefficient at basic creep under axial tension should follow the data shown in Fig. 8. It can be seen, that in some initial duration of loading it is somewhat smaller than the value of $\chi_{cr}$, however after ~1 month of the testing $\chi_{bc}$ starts to exceed $\chi_{cr}$.
Conclusions

The phenomenological approach describing the drying creep under tension as a sum of shrinkage-induced creep and creep-induced shrinkage is corrected by taking into account the swelling deformation of sealed concrete in basic creep tests. In such a way the drying creep strain represents only the extra creep component, therefore the mechanism of drying creep based on stress-induced shrinkage is called into question.

A new phenomenological approach to the problem of drying creep in terms of age-adjusted effective modulus method is demonstrated. The work of concrete element under simultaneous loading and drying is represented as restrained shrinkage in addition basic creep under gradually developed stresses. It is shown that the drying creep strain depends on the values of aging coefficients at basic and drying creep and on the individual components of concrete strain. It is found that the instantaneous strain plays the minor role. The experimental data of the creep tests under tension can be used to calculate the aging coefficient of concrete at basic creep.
The main conclusion of the present work is the explanation of drying creep by the difference in aging coefficients. This difference is the necessary condition for developing drying creep in concrete. Knowing the ratio between aging coefficients and the instantaneous, basic creep and free shrinkage components of concrete strain, the drying creep can be easily evaluated. However, the present approach is only phenomenological and therefore cannot resolve the internal physical causes resulting in drying creep. The object of the future study is to find these physical mechanisms.

Acknowledgment
The valuable advice and support of Prof. A. Bentur are gratefully acknowledged.

References