Short-term Stability of GNSS Satellite Clocks and its Effects on Precise Point Positioning

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BIOGRAPHY

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ABSTRACT

Precise Point Positioning (PPP) requires the availability of precise orbit and clock corrections for all satellites used. Unfortunately, only two Analysis Centers (AC) of the International GNSS Service (IGS) make GLONASS clock corrections available. Both products consist of tabular clock values with 5 minute intervals, which is not dense enough for many kinematic PPP applications. Thus, mathematical interpolation of the satellite clock corrections is required.

We densified the GPS/GLONASS clock corrections produced by the European Space Operation Centre (ESOC) using a carrier-phase based clock interpolation technique to 1 second intervals. Based on these high-rate corrections we analyzed the short-term stability of all GNSS clocks, determined satellite-individual statistical values of the errors due to a mathematical clock correction interpolation, developed an observation function which takes interpolation errors into account, and tested several PPP processing algorithms with kinematic GNSS observations.

Adding GLONASS to GPS considerably improves PPP performance. Further shortening of convergence time and increase of accuracy are obtained if clock corrections at 30 second intervals are used. Performance improvement of combined GPS/GLONASS is achieved by using an observation weighting function which takes the short-term stability of the satellite clocks into account.

INTRODUCTION

The availability of precise satellite orbit and clock products enabled the development of Precise Point Positioning (PPP). This data processing technique is based on single- receiver dual-frequency (code and) carrier-phase observations. An in-depth discussion of the models used in PPP data processing can be found in Kouba and Héroux (2001) and Kouba (2003).

For many years PPP was limited to GPS. With the revival of the GLONASS space segment, the larger number of combined GPS/GLONASS receivers and the availability of combined GPS/GLONASS orbit and satellite clock corrections, PPP can now be extended to GLONASS as well. As a result, a major performance improvement of PPP can be expected even so the GLONASS satellite segment is far from being complete. First results of combined GPS/ GLONASS PPP were presented by Cai and Gao (2007). They could not find, however, a significant impact on PPP results caused by the added two to three GLONASS satellites.

The International GNSS Service (IGS) is the main source of post-mission precise satellite orbit and clock products. IGS combines solutions from several participating Analysis Centers (AC) (Kouba 2003). Besides, the original AC orbit and clock products are available, some of them being more comprehensive than the IGS products.

Tab.1: Precise orbit and clock products for GPS/GLONASS PPP

<table>
<thead>
<tr>
<th>Precise orbits</th>
<th>Precise clock corrections</th>
</tr>
</thead>
<tbody>
<tr>
<td>International GNSS Service (IGS)</td>
<td>GPS, GLONASS</td>
</tr>
<tr>
<td>Information-Analytical Center (IAC), Russia</td>
<td>GPS/GLONASS</td>
</tr>
<tr>
<td>European Space Operation Center (ESOC), Germany</td>
<td>GPS/GLONASS</td>
</tr>
</tbody>
</table>

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Combined GPS/GLONASS orbit products are available from many ACs. GLONASS satellite clock corrections, however, are available from the Information–Analytical Center (IAC) and the European Space Operation Center (ESOC) only (Tab. 1). With just two contributing centers, IGS does not offer precise GLONASS satellite clocks corrections, but only GPS clock corrections (Dach et al., 2008, IGS, 2008). Since May 2008 the Center of Orbit Determination in Europe (CODE) provides clock corrections every 5 seconds, but again for GPS only (Schaer and Dach, 2008).

Biases among clock corrections of different GLONASS satellites or in relation to GPS do not harm PPP positioning as long as they are stable in time, since then they are completely absorbed by the carrier-phase ambiguity parameters. Our analysis of IAC clock products revealed drifts between GPS and GLONASS clock corrections. These drifts may be caused by different reference time scales for GPS and for GLONASS (Oleynik et al., 2006) and their realization. No such drifts were found in the combined GPS/GLONASS clock corrections of ESOC. Consequently, only ESOC precise clock products were used for the research work presented in this paper.

ESOC introduced several improvements to their routine GNSS data processing in early 2008. Now, GPS and GLONASS observations undergo a rigorously combined analysis (Springer et al., 2008). A single receiver clock is selected as reference clock for the processing. Clock biases between the GPS part and the GLONASS part of the receivers are estimated for each receiver - GLONASS satellite pair (Dow and Springer, 2008).

ESOC (and IAC) provide GPS/GLONASS clock corrections at intervals of 5 minutes (Tab. 1). Kinematic PPP applications, however, require satellite clock corrections at a rate as high as the observation rate (e.g. 1 Hz or even higher). In practice, mathematical interpolation algorithms are used to determine high-rate clock corrections. These interpolation algorithms are not able to account for the random behaviour of the satellite clocks. Hence, mathematical interpolation between tabular values at 5 minute intervals causes large observation residuals which can exceed several centimetres (upper panel of Fig. 1).

Better results are achievable by carrier-phase based clock interpolation as described in the next section of this paper. If tabular clock values of closer spacing are produced using this method, the further mathematical interpolation causes much smaller errors (lower panels of Fig. 1).

We analyzed in detail the short-term stability of the different GNSS satellite clocks and recommend a satellite individual weighting in the case that mathematical interpolation of clock corrections is used.

With this background in satellite clock performance, we were then able to process combined GPS/GLONASS observations and look at improvements in convergence time and coordinate accuracies.

![Fig. 1: Examples of PPP observation residuals mainly caused by interpolation errors between tabular clock correction values at intervals of 5 minutes, 30 seconds, and 10 seconds.](image)

**CARRIER-PHASE BASED CLOCK INTERPOLATION**

In order to be able to analyze the short-term stability of GNSS satellite clocks and its effects on PPP, high-rate GNSS satellite clock corrections are required. IGS and its ACs provide 5 minute clock values and carrier-phase based interpolated corrections at 30 second intervals for GPS (Tab.1). Since these sampling intervals were not short enough for our purposes, we implemented a carrier-phase based interpolation algorithm to obtain satellite clock corrections with a sampling interval of 1 second.

The interpolation procedure requires high-rate (1 second) carrier-phase observations of reference stations. Such data are freely available from so call high-rate IGS sites. Starting with 5 minute tabular GNSS clock values of ESOC, PPP residuals were computed for these high-rate observations. This processing step involved mathematical (linear) interpolation to determine high-rate clock corrections at 1 second intervals (Fig. 2).

In a further step, the PPP residuals were used to improve this first set of high-rate clock corrections in such a way that a new set of high-rate clock corrections was obtained with the original 5 minute clock values remaining unchanged. This was achieved by piecewise fitting of the carrier-phase residuals to the 5 minute clock corrections and the linearly interpolated values (Fig. 2).
Time series of PPP residuals of different reference stations have been compared. PPP residuals from identical satellites and epochs but different stations are very similar. This demonstrates that clock interpolation errors have a much larger influence than remaining local tropospheric errors or multi-path effects.

It is very useful to combine residuals of several high-rate reference stations in order to increase the availability of high-rate clock corrections and to improve their reliability by outlier detection and removal.

Our analyses of GNSS satellite clocks and GNSS PPP results were performed with such carrier-phase based interpolated satellite clock corrections. Since our test measurements were observed in the area of Dresden, Germany, it was sufficient to compute high-rate satellite clock corrections for only those satellites being visible above Europe. Hence, three European reference stations with high-rate GNSS observations were used to obtain 1 second clock corrections: Zimmerwald (ZIM2), Herstmonceux (HERT), and Wroclaw (WROC), see Fig. 3.

SHORT-TERM STABILITY OF GNSS SATELLITE CLOCKS

Specialized statistics has been developed for the characterization of clocks. Among these is the Allan variance (or Allan deviation) which is often used to visualize graphically the random characteristics of the clock behavior. The two-sample Allan variance $\sigma^2_2(\tau)$ is a time-domain measure and given by (Allan et al. 1988):

$$\sigma^2_2(\tau) = \frac{1}{2(N-2)} \tau \sum_{i=1}^{N-2} \left( x_{i+2} - 2x_{i+1} + x_i \right)^2$$

with

- $N$ - number of original clock corrections,
- $\tau$ - interval of clock corrections,
- $x_i$ - clock correction at epoch $i$.

The Allan deviation is defined as the square root of the Allan variance.

Allan deviations of all available GNSS clocks were calculated based on the 1 second satellite clock file of day-of-year 17608 obtained by carrier-phase based interpolation as described above. They are graphically shown for intervals from 1 second to 10 minutes (Fig. 4). This figure can be considered as an extension of Fig. 1 in Senior et al. (2008) where (modified) Allan deviations of all GNSS satellite clocks are published for intervals from 30 seconds to several days.

In order to increase the readability of Fig. 4, we have subdivided the satellite clocks into three distinct groups (USNO, 2008): GPS cesium (Cs) clocks (PRN 3, 8, 9, 10, 24, 27, and 30), GPS rubidium (Rb) clocks and GLONASS Cs clocks. Actually, the group of GPS Rb clocks is formed by two subgroups: Rb clocks of the older Block IIA satellites (PRN 4, 5, 6, 25, 26, and 32) have higher stability for intervals greater 10 seconds than Block IIR and Block IIR-M satellite clocks.

Only satellites of the newer GLONASS-M generation were usable in June 2008. All their clocks are of type Cs. The clock characteristics of GLONASS satellites vary considerably (Fig. 4). A clear trend can be identified: the older a satel
the larger is its short-term clock instability. R06 and R24 were launched in 2003 and 2005, respectively. R13 and R19 participated in the very last launch of December 2007 (IAC, 2008). The short-term clock stabilities of newer GLONASS-M satellites clocks are on the level of those of GPS satellite clocks.

In order to get even more valuable information from the 1 second clock corrections we computed interpolation errors in the form of root mean square (RMS) values at midpoints between tabular clock values with sampling intervals of 30 seconds and 5 minutes. We tested two low-order polynomial interpolation algorithms: linear and cubic interpolation based on two and four neighboring data points, respectively. Linear interpolation always performed slightly better. Hence, our mathematical interpolation algorithm of choice is linear interpolation between adjacent tabular clock correction values.

The RMS values \( s_M \) of interpolation errors at midpoints are shown for each GNSS satellite clock in Fig. 5. The RMS variations correspond very well to the Allan deviations of

![Short-term Allan deviations of GNSS satellite clocks](image)

**Fig. 4: Short-term Allan deviations of GNSS satellite clocks (day of year 176/2008)**

Interpolation RMS values at midpoints vary by a factor of 3.5 and 6.9 among GPS satellites for clock sampling intervals of 30 seconds and 5 minutes, respectively. Taking GLONASS-M satellites into account as well, these factors increase to 7.3 and 12.9. It can thus be expected that an observation weighting according to these values improves PPP positioning performance.

![GNSS satellite clock interpolation errors](image)

**Fig. 5: GNSS satellite clock interpolation errors \( s_M \) (RMS) at midpoints between tabular clock values, day of year 176/2008.**

**WEIGHTING OF CARRIER-PHASE OBSERVATIONS**

A PPP positioning solution is computed from dual-frequency GNSS carrier-phase (and code) observations. The proper modeling of this positioning problem requires stochastic information for all observations, i.e. accuracy estimates and correlations among observations. In this section the proper weighting of the carrier-phase observations is discussed. Correlations among carrier-phase observations are not taken into account.

The amount of observation errors like remaining tropospheric delays, multipath, and observation noise is elevation...
dependent. The lower the GNSS satellite elevation, the larger can be the effects of these errors on the observations.

In order to determine an elevation-dependent observation weighting function, we examined the relationship of satellite elevation and PPP observation residuals. Several 24 h data sets of various IGS reference station were processed using ESOC precise orbits and clock corrections (Fig. 6). As expected, GPS and GLONASS residuals decrease with increasing elevation angle. No significant differences can be found comparing GPS and GLONASS residuals. This fact confirms that the ESOC orbit and clock determination and the carrier-phase observations at the GNSS reference sites perform equally well for both satellite systems.

RMS values of elevation bins of size 2° were calculated from the PPP observation residuals shown in Fig. 6. Finally, an elevation dependent function was fitted to these RMS values:

\[ s_E = \frac{1}{\sin E} \cdot 0.0063 \text{ m} \quad E \leq 25° \]
\[ s_E = 0.0150 \text{ m} \quad E > 25° \] (2)

When using tabular clock values with a spacing of 30 seconds or 5 minutes and a mathematical interpolation algorithm, we have to deal with interpolation errors. Their effects have already been shown in Fig. 1 and their satellite dependence was discussed in the previous section. Since a clock interpolation error affects the positioning model like an observation error, we can take its stochastic properties into account when setting up the stochastic model for the carrier-phase observations.

Two aspects have to be considered concerning clock interpolation errors:

- the satellite clock dependent interpolation error at midpoints between tabular values as discussed in the previous section, and
- the behavior of the interpolation errors between the reference time of the nearest tabular clock value and the time of the observation epoch.

In order to model the second effect, interpolation errors have been determined based on the carrier-phase interpolated 1 second clock corrections and mathematically interpolated clock corrections for tabular values spaced 30 seconds and 5 minutes. Fig. 7 presents the clock interpolation errors as a function of the time difference to the nearest tabular value. The series of RMS data points were normalized with respect to the spacing of tabular clock values and with respect to the midpoint RMS values. A function was fitted to the data sets of various stations, satellites (GPS and GLONASS), and two spacing intervals \( \tau \) (30 seconds and 5 minutes):

\[ s_\Delta t = 1 - e^{-\left|\Delta t\right|/\tau} \] (3)

with \( \left|\Delta t\right| \) - time difference to the nearest tabular value
\( \tau \) - spacing of tabular clock values (30 seconds or 5 minutes)

Finally, the complete weighting function results from the combination of the three contributing factors which depend on elevation angle, time difference to nearest tabular clock value, and satellite clock interpolation error at midpoints:
\[
\omega = \frac{1}{s_E^2 + s_M^2} \cdot \Delta t. \tag{4}
\]

If the clock correction values have an interval of less than 30 seconds, a simplified weighting function maybe used:

\[
w = \frac{1}{s_E^2}. \tag{5}
\]

In the following examples, PPP positions were computed either by applying the complete weighting function (Eq. 4) or by using the simplified weighting function (Eq. 5).

**GPS/GLONASS PPP positioning results**

In June of 2008 a kinematic measurement campaign was performed using Leica GRX 1200 GG Pro equipment in a testing area in Dresden, Germany. The following test results refer to almost 4 hours of continuous dual-frequency kinematic GPS/GLONASS observations.

GNSS satellite orbit and clock corrections were copied from the ESOC server. Since ESOC GNSS clock corrections are produced with a time interval of 5 minutes only, clock corrections of 1, 5, 10, and 30 seconds were computed by carrier-phase based interpolation as described in an earlier section. The further densification to the observation epoch rate of 1 Hz was performed by mathematical interpolation.

The carrier-phase based interpolation procedure requires high-rate observation data from reference stations. We selected the 3 IGS sites shown in Fig. 3. PPP residuals of these stations were averaged. Unfortunately, a data gap of 150 seconds occurred at two of the stations (and many more IGS/EPN high-rate stations) simultaneously. Hence, for this time span only observations of the reference station ZIM2 were available. This, however, had no adverse effect on the positioning results.

Unfortunately, the GNSS Leica receiver had difficulties to produce continuous GLONASS L2 phase measurements in kinematic mode. Loss-of-lock and short data gaps occurred frequently. We were forced to fix these cycle-slips in a pre-processing step involving local reference station observations. Hence, our results do not reflect the present GPS/GLONASS PPP performance of this receiver type but rather the potential of GNSS PPP assuming a better receiver performance.

For the calculation of PPP positioning results we used our own PPP software modules (WaP and TripleP). A reference solution was obtained from the differential kinematic carrier-phase processing of a short (2.8 km) baseline to a permanent reference station using the baseline processor Wa1.

PPP data processing was performed with an elevation mask of 15° in order to mitigate multipath effects and to reduce residual tropospheric errors. Observations to 7.5 GPS satellites and to 3.5 GLONASS satellites were available on average at time intervals of 1 second.

Antenna phase center corrections for the Leica AX 1202 GG antenna were applied in a pre-processing step.

In our analysis we compare several variations of PPP data processing:

- GPS only, and combined GPS/GLONASS,
- time intervals between tabular satellite clock values of 10 seconds, 30 seconds, and 5 minutes
- standard weighting according to Eq. 5, and application of the complete weighting function according to Eq. 4

Two criteria were selected for the comparison of the different PPP solutions:

- convergence time and
- accuracy of the coordinates.

The convergence time of 10 different solution types was determined by computing position errors at 1 second intervals for different lengths of observation periods (5, 10, 15 min etc.). Therefore, the complete kinematic data set was subdivided into data blocks of appropriate length. Thus, we were able to determine the 3D position RMS values which are shown in Fig. 8.

Comparing the upper and lower panel of Fig. 8, it can clearly be seen that adding GLONASS observations to GPS reduces the convergence times by factors of 1.5 to 2.5. It can be expected that this effect would even be larger if the GLONASS satellite constellation would be complete.

Large differences in convergence times exist among the various solutions with different clock correction intervals and weighting functions. Taking the clock interpolation errors into account according to the weighting function of Eq. 4 improves the solutions based on clock corrections at 5 min intervals. For shorter clock correction intervals, however, no further reduction of position errors can be achieved by applying the improved weighting function.

Hardly any differences in convergence times can be found comparing solutions based on clock correction intervals of 30 seconds or 10 seconds. Our conclusion is that 30 second clock corrections for GPS or GPS/GLONASS are appropriate for this kind of application.
Fig. 8: 3D position errors (RMS) as a function of the observation period for various PPP processing approaches.

We also tested positioning results based on 1 second and 5 second intervals of tabular clock values, but their performance did not improve as compared to 10 second interval corrections. Hence, no positioning results based on 1 second or 5 second clock corrections need to be presented here. Similarly, no results are shown for clock corrections at 10 second intervals and the complete weighting function since they do not differ from those obtained with elevation-dependent weighting.

In a second step we analyzed coordinate accuracies of the various PPP processing approaches based on the complete kinematic data set. RMS-values of all 3 coordinate components are listed in Tab. 2a+b. All solutions reach a high level of accuracy due to the long time of continuous observations (almost 4 hours). The accuracy of the height component is worse by a factor of about 2 as compared to the horizontal components. RMS values of the north component are mostly smaller than those of the east component. This effect is expected for float (ambiguities unfixed) solutions in the mid-latitudes (cp. Santerre and Lavoie, 1991).

Accuracy differences between corresponding GPS and GPS/GLONASS results are mostly small. When the complete weighting function according to Eq. 4 is applied, combined GPS/GLONASS results are more accurate as GPS only results.

Accuracy increases with smaller intervals of satellite clock corrections. The 5 minute clock corrections made available by ESOC do not allow exploiting the full potential of kinematic PPP. 30 second clock corrections for both satellite systems GPS and GLONASS should be made available.

In order to give a more detailed insight into time-series of coordinate errors, a 20 minute example of the kinematic data set is presented in Fig. 9. Two solutions are compared: one solution based on satellite clock corrections at 10 second intervals and the other based on clock corrections at 5 minute intervals. The 5 minute solution uses the complete weighting approach (Eq. 4).

It is interesting to notice that residuals are smallest at those observation epochs which coincide with the reference time of a tabular clock value. This example confirms again (like Fig. 1) that in the case of tabular clock corrections at 5 minute intervals the errors due short-term instabilities of the satellite clocks in connection with mathematical interpolation are the dominant error source in PPP.
SUMMARY AND RECOMMENDATIONS

Short-term stability of GNSS satellite clocks varies among the different kinds of clocks. Lowest short-term stability occurs for older GLONASS-M cesium clocks, whereas the newer GLONASS-M clocks are as stable as GPS satellite clocks. Highest short-term stability was found for the rubidium clocks of GPS Block IIA satellites.

This short-term stability is of much importance for Precise Point Positioning (PPP) since precise clock corrections are usually made available for time intervals of 30 seconds or 5 minutes only. Thus, mathematical interpolation of these corrections is required and the interpolation errors depend on the short-term stability of the satellite clocks.

We developed an observation weighting function which takes these interpolation errors into account. Furthermore, precise clock corrections at various intervals were computed using a carrier-phase based interpolation technique.

Several different PPP processing strategies were tested with kinematic GPS/GLONASS observation data sets. In summary, the following conclusion can be drawn:

- Adding GLONASS satellites to GPS reduces PPP convergence times by a factor of about 2, although the GLONASS constellation is still incomplete. When the GLONASS space segment will be complete, a further reduction of convergence times can be expected.
- Precise satellite clock corrections at 5 minute intervals do not allow exploiting the full potential of kinematic PPP.
- If clock corrections at 5 minute intervals are used (presently no other precise GLONASS clock corrections are available), observation weighting has to take clock interpolation errors and thus the short-term stability of the satellite clocks into account.
- Better results were obtained with clock intervals of 30 seconds. Observation weighting should take the clock interpolation errors into account, especially for GLONASS. Shorter intervals of clock corrections seem not to be necessary for kinematic PPP applications like the ones presented in this paper.

We would very much welcome an IGS product of combined GPS/GLONASS clock corrections referring to a common reference time system, i.e. a product similar to the one ESOC is offering right now. In contrast to the present ESOC product, clock corrections at intervals of 30 seconds (are even closer) would be preferred, since they considerably improve PPP performance, especially reduce the convergence time.

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REFERENCES


