Abstract:
Airborne lasercanning has proven to be a powerful technique for the detection and modeling of man-made objects such as buildings, which form a substantial part of 3-D city models. As an active technique, lasercanning delivers reliable 3-D data points without requirements to surface reflectance variations, thus evading a number of problems occurring with techniques based on 2-D imagery.

The publication discusses the use of invariant moments applied to lasercanning data for the determination of roof parameters of simple building types. The technique works on the original, irregularly-distributed lasercanner data points, thus avoiding effects caused by an interpolation to a regular grid. Using only first and second order invariant moments, a number of basic parameters of a building (position, orientation, length, width, height, roof type and roof steepness) can be determined as closed solutions from ratios of binary and height-weighted moments of segmented point clouds. Using higher order moments, more complex roof shapes can be modeled as well. By analyzing differences between point cloud and building model in a second processing step outliers can be detected and systematic deviations from the assumed model such as dorms on a roof can be modeled.

The technique was applied to a section of a FLI-MAP lasercanner dataset with an average point density of five points per square meter. No a priori information, such as 2-D GIS data, was used. Instead, the dataset was segmented by a the analysis of height texture measures, followed by morphological filtering and connected component labeling. All detected buildings complying with the assumed simple building types could be detected and modeled successfully. Moreover, most dorms with an extension of greater than two square meters could be modeled in the step of model fit analysis. The precision potential for the building parameters is in the order of 0.1-0.2m for the dimensions and 1-2 degrees for orientation and roof steepness.

Keywords:
Airborne lasercanning, building extraction, invariant moments

1. Introduction
The detection and modeling of man-made objects has become a major issue of photogrammetric research in the past few years, with the primary focus on the automatic (e.g. Henricsson et al., 1996) or semi-automatic (e.g. Lang/Förstner, 1996) modeling of buildings. Starting with two-dimensional image processing techniques, researchers soon turned towards 3-D approaches such as the grouping of features matched in multiple images and the use of dense digital elevation models as an additional source of information. Data fusion of photogrammetric image data with available 2-D ground maps and digital elevation models generated by airborne lasercanning has also been investigated by several research groups. Delivering reliable 3-D point clouds without requirements to surface reflectance variations and without the necessity of time consuming and potentially erroneous image matching techniques, lasercanner data may provide a perfect supplement to photogrammetrically determined boundary representations. Provided that data is sampled with sufficient spatial resolution, building models may also be derived from lasercanning data exclusively. Although the point densities delivered by most lasercanning systems in standard operation mode are still too small (often in the order of one point per 10m²), some sensors do deliver a spatial resolution of more than one point per square meter already today.

The potential of airborne lasercanner data for building model generation has been examined by several authors
in the past few years. Haala/Brenner (1997) extract planar roof primitives from dense laserscanner data (TopoSys system, four points per m²) by a planar segmentation algorithm and use additional ground plan information to gain knowledge about topological relations between roof planes. Brenner/Haala (1998) derive parameters for 3-D CAD models of basic building primitives by least-squares adjustment, minimizing the distance between a digital surface model generated by laserscanning and corresponding points on a building primitive. The boundaries of buildings are derived from available ground plans. The implementation is limited to four standard building primitives; further refinement has to be performed interactively. Hug/Wehr (1997) show the detection and segmentation of houses from ScalARS (Hug, 1994) height and reflectivity data based on morphological filtering with successive progressive local histogram analysis; in addition, they use the laser reflectivity measure for separating buildings from vegetation.

Lemmens et al. (1997) show the fusion of laser-altimeter data with a topographical database to derive heights for roof-less cube type building primitives. Brunn/Weidner (1997) show the detection of buildings in digital surface model data in general, their approach driven extraction of building structures. While referring to differential geometric quantities and attempts to data-driven extraction of building structures. While referring to digital surface model data in general, their approach shows good results when applied to laserscanner data with three points per square meter, but fails when applied to a surface model derived from stereo imagery.

In the following, the determination of simple house models exclusively from airborne laserscanning data processed with fast moment-based techniques will be discussed. The model contains the center coordinates, length, width and height of a building as well as its orientation, roof type and roof steepness (Figure 1). These parameters are derived as closed solutions from ratios of 0th, 1st and 2nd order moments of point clouds generated by laserscanning. In addition, asymmetries like dorms on roofs are detected from a model fit analysis and subsequently modeled using the same approaches (Figure 2).

Figure 1: Standard gable roof building

2. Invariant moments

The analysis of moments has been used in image processing for a long time. Early publications go back to the sixties (e.g. Hu, 1962). A major application field of invariant moments is shape recognition of 2-D objects from segmented images.

In the continuous domain, moments are defined as:

$$M_{ij} = \int \int x^i y^j f(x,y) \, dx \, dy$$  \hspace{1cm}  (eq. 1)

In the discrete domain of image raster data the integrals have to be replaced by sums:

$$M_{ij} = \sum_{P=P_1}^{P_n} x_P^i y_P^j$$  \hspace{1cm}  (eq. 2)

For non-regularly distributed 2 1/2-D data such as airborne laserscanner points, summation has to be performed over a segmented group of data points ($P_1, \ldots, P_n$), and the height $H_p$ can be used as a weight function:

$$M_{ij} = \frac{\sum_{P=P_1}^{P_n} X_P^i \cdot Y_P^j \cdot H_P}{P=P_1}$$  \hspace{1cm}  (eq. 3)

In most image processing applications the invariance of moments towards shift, scale and rotation is required; in some applications, also mirror or affine invariance is required.

- Shift invariance is obtained by relating coordinates to the center of gravity:

$$\bar{X} = \frac{M_{00}}{M_{00}}, \quad \bar{Y} = \frac{M_{01}}{M_{00}}.$$  \hspace{1cm}  (eq. 4)

- Scale invariance, if desired, can be obtained by setting $M_{00}$ to 1:

$$\tilde{M}_{ij} = \left(\frac{1}{\sqrt{M_{00}}}\right)^{i+j+2} \cdot M_{ij}$$  \hspace{1cm}  (eq. 5)

- Rotation invariance is obtained by principle axis transformation:

$$\Theta = \frac{1}{2} \arctan \frac{2M_{10}}{M_{02}-M_{20}}.$$  \hspace{1cm}  (eq. 6)

$$M_{pq}' = \sum_{r=0}^{p} \sum_{s=0}^{q} (-1)^{q-s} \binom{p}{r} \binom{q}{s} \Psi \cdot \tilde{M}_{(p+q-r-s)r+s(x0,y0)}$$

with $\Psi = (\cos \Theta)^{r-r+s} \cdot (\sin \Theta)^{q+r-s}$

See e.g. (Teh/Chin, 1988) for a further discussion on invariant moments.

Many implementations of moments in shape recognition are based on the computation of measures like the Mahalanobis distance between sets of higher order moments of a segmented object in a binarized image towards the moments of a number of model objects in a database. A general disadvantage of the use of moments is the noise sensitivity of higher order moments, making object recognition rather sensitive to segmentation errors. In the case of an insecure segmentation or if object data such as image greyvalues or point heights are used as
weights in the computation of moments, the results will show a dependency on noise which increases with the distance to the center or gravity. This noise sensitivity limits the use of higher order moments and will restrict many applications to the analysis of lower order moments and ratios of these. Therefore the following considerations are restricted to the analysis of 1st and 2nd order moments. Moreover, the comparison with moments of known objects is replaced by closed solutions for a parametric building model.

3. Computing and interpreting invariant moments of lasercanner data

Airborne lasercanning delivers $2^{1/2}$-D point data with the height $H$ as a function of planimetry coordinates $X$ and $Y$. Although the distribution of the data depends on the scanning process and topography, the data provider will often deliver a dataset which has been interpolated to a regular grid. To avoid the effects of imperfections in the interpolation, the following investigations will be based on the original irregularly distributed data.

The dataset used in the work described in the following was segmented by the analysis of height texture measures in a classification-like approach, followed by morphological filtering and connected component labeling. See (Maas, 1999b) for a detailed discussion of this technique and (Maas/Vosselman, 1999) for a general discussion of some techniques for the segmentation of lasercanner data.

The basic idea of the moment-based procedure can be described as follows:

1. First and second order height-weighted shift- and rotation-invariant moments $M_{ij}$ (eq. 1) as well as binary moments $m_{ij}$ (with $f(x,y)=1$, used for the determination of position, ground size and orientation of a building) are expressed as a function of the parameters of an assumed building model.

2. The equation system formed by these moments is solved for the building model parameters, i.e. the building model parameters are expressed as functions of 1st and 2nd order moments.

3. Shift- and rotation-invariant moments of all segmented point clouds are computed, and the building parameters are derived from these moments. A model fit analysis procedure delivers quality measures for the reconstructed building and allows for the rejection of gross errors. Moreover, systematic deviations from roofs such as dorms can be modeled.

Note that building parameters can only be derived from ratios of moments, since in the case of irregularly distributed discrete data points the absolute values of moments depend on the number of data points in the segmented region. See chapter 5.3.1 for a discussion of some problems caused by this fact.

For the analysis shown in the following chapters, the order of moments can be limited to two. Higher order moments may be used to model more complex roof types or to indicate asymmetries of a roof.

3.1 Building orientation and extension from 2nd order moments of binarized height data

Basically, the two-dimensional extension of a building is already available from the segmentation process. Usually, however, only the extension in the X- and Y-direction of the original data coordinate system will be determined in the segmentation process. The determination of rotation invariant moments does provide the principal axis angle, which describes the orientation of the building. To avoid distortions caused by irregularities of the roof shape, moments of binarized height data are used in this step. The coordinates of the centroid of the building can be obtained from [eq. 4], the principle axis angle $\Theta$ from [eq. 5]. If the ratio of the rotation invariant moments of inertia $q_2 = m'_{20}/m'_{00}$ (eq. 7) is smaller than 1, 90E has to be added to the principle axis angle $\Theta$ in order to describe the orientation of the longer axis.

A principal axis transformation of the segmented point cloud will deliver the extensions of the building in its local coordinate system. Alternatively, assuming a rectangular shape of the building and an unbiased distribution of lasercanning data points, the dimensions can be obtained directly from the formulation of 2nd order moments of a building with a rectangular ground plan (Figure 1) in the continuous domain (eq. 1):

$$m'_{00} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 y^2 \, dx \, dy$$

$$m'_{02} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 y^4 \, dx \, dy$$

$$m'_{20} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^6 y^2 \, dx \, dy$$

$$m'_{20} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^4 y^4 \, dx \, dy$$

$$m'_{12} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^2 y^6 \, dx \, dy$$

$$m'_{12} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^6 y^4 \, dx \, dy$$

Given the moments of the segmented point cloud, the dimensions $X$ and $Y$ of the building can be computed from the ratios $(m'_{20}/m'_{00}), (m'_{02}/m'_{00})$ (eq. 8):

$$X = \sqrt{\frac{12 m'_{20}}{m'_{00}}}$$

$$Y = \sqrt{\frac{12 m'_{02}}{m'_{00}}}$$

3.2 Roof parameters from 2nd order height-weighted moments

For the computation of roof type and shape parameters, the height of the lasercanner data points is used as a weight function in [eq. 1] and [eq. 3]. The center of gravity and principle axis used for the computation of invariant height-weighted moments are fixed to the values determined from the binarized height data. This is in order to avoid effects of the roof shape on these parameters in case of asymmetries.
Information on the roof type can be obtained from a comparison of the ratios of 2nd order moments of the binarized \[ Q_2 = M_{20}^{r} / M_{02}^{r} \] and the height-weighted data:

\[
M_{20}^{r} = \frac{1}{12} \int_{-Y/2}^{Y/2} \int_{-X/2}^{X/2} x^2 H(y) dxdy
\]

If the roof is a flat roof, the ratio \[ r_q = (Q_2 / q_2) \]

will be equal to 1. If \( r_q \) is larger than 1, a roof oriented parallel with the principle axis of the building can be assumed; if \( r_q \) is smaller than one, the roof will be oriented perpendicular to the principle axis of the building.

Assuming a standard gable roof oriented parallel with the principle axis of the building, the height of the building and the inclination of the roof can be derived from 2nd order moments. The height is expressed as a function of the y-coordinate in the local coordinate system after principal axis transformation

\[
H = H_{avg} + \left( \frac{Y}{4} - \frac{Y}{2} \right) \cdot \tan \alpha
\]

with the average building height

\[
H_{avg} = M_{00}^{r}
\]

and used as weight in the computation of the 2nd order moments

\[
M_{20}^{r} = \int_{-X/2}^{X/2} \int_{-Y/2}^{Y/2} x^2 H(y) dxdy
\]

Solving e.g. the ratio \( r_q \) for \( \alpha \), the roof inclination angle becomes

\[
\alpha = \arctan \left( 8 \cdot \frac{H_{avg} \cdot (r_q - 1)}{r_q \cdot Y} \right)
\]

Introducing this inclination into the building model, the height of the building can be computed. Obviously, the height of the building might also be taken from the height histogram of the segmented region. Using the approach as outlined before, however, all measurements contribute to the determination of the height and the calculated height fits to an assumed roof model. Thus the effects of outliers such as data points on chimneys or antennae can be avoided.

### 3.3 Use of higher order moments

The above considerations apply for the parametrization of rectangular houses with a flat roof or a gable roof. Higher order moments may be used for the diagnosis of other roof types. In addition, higher order moments can also be used to detect asymmetries on gable roofs such as dorms.

A rather simple indication is the analysis of the existence of significant values in the odd rows and columns in the matrix \( M[i][j] = M_{ij} \):

- Symmetry to the principle axes of the building leads to zeros in the odd rows and columns of the rotation invariant moments: Symmetry to the x-axis means that \( M_{ij} = 0 \) if \( j \) is odd; symmetry to the y-axis means that \( M_{ij} = 0 \) if \( i \) is odd.

- In the case of an asymmetry about \( x \), an object above the axis leads to \( M_{ij} > 0 \) for odd \( j \); in case of asymmetry about \( y \), an object left of the axis leads to \( M_{ij} > 0 \) for odd \( i \).

This analysis can only detect asymmetries, i.e. deviations from a chosen primitive standard roof form. It does not detect the location and type of distortion. Also, the reverse conclusion is not valid: The absence of zeros in odd rows and columns does not mean that there are no deviations from the standard roof form - there might e.g. be two distortions compensating each other, like dorms towards the northern and southern side. For modeling more complex building types, higher order moments have to be analyzed in more detail. A consequent extension of the model shown in eq. 12 is e.g. a parametrization of hip roofs by defining a second roof inclination and solving an equation system consisting of 2nd and 4th order moments for the two inclination parameters. In practice, this approach is of limited applicability: Already for a building model including a dorm modeled by four parameters the solution of the equation systems becomes very complicated. Moreover, this approach does not take into consideration that there may be more than one object on a roof.

As an alternative, higher order moments for a number of roof types might be derived from synthetic house models and stored in a database. Roof types may then be determined by the comparison of sets of moments of detected houses to those of the database. With a realistically large number of roof types including varying inclination values, however, this database would become rather large, and the reliability of the method would suffer from the noise sensitivity of higher order moments. For these reasons, the analysis was limited to 0th, 1st and 2nd order moments, and asymmetries were modeled during a subsequent model fit analysis procedure.

### 3.4 Model fit analysis and modeling of dorms

After the determination of building model parameters, a goodness of fit can be determined by projecting the model into the point cloud and computing residuals for every data point. This allows for a rejection of the computed house model in case of bad fit and for the detection and
elimination of outliers in the data points or a refinement of the segmentation.

In addition, the model fit analysis procedure allows the detection of systematic deviations from the roof, such as dorms (Figure 2). This can be performed by a recursive use of the procedures described above: As a first step, a standard gable roof house model is determined. Then the differences between the points of the original point cloud and the model are calculated. Points above the ridge height are discarded as potentially lying on chimneys or antennae. Points below the ridge but significantly above the roof are assumed to belong to dorms. This new point cloud can be segmented into parts belonging to multiple dorms by binning and connectivity analysis, assuming a certain minimum size of dorms, uniform data point coverage and a minimum distance between dorms. This way, groups of outliers are segmented. For each of the subsets, binarized and height-weighted moments are computed. As the number of points on the dorms is rather small (mostly less than 20), the height is assumed to be constant. Moreover, the orientation of the dorms is assumed to be perpendicular to the principle axis of the building. Thus a dorm is described by four parameters (a coordinate and a length along the roof axis, a distance from the roof edge and a height), which can be determined as closed solutions from moments $m_{20}$, $m_{02}$, $m_{10}$, $m_{01}$, $m_{00}$ of the dorms point cloud.

Figure 2: Successfully reconstructed building with two dorms

Optionally, a 5-parameter dorm with an extra gable may be modelled in the same manner.

### 3.5 Correction of biases

Using irregularly distributed laserscanner data points, the results obtained from the computation of moments will be biased if the point distribution is inhomogeneous. While the pattern of dense laserscanner points in normal operation mode over flat surfaces with ample reflectivity can usually be considered sufficiently homogeneous, problems may occur under the following conditions:

- Neighbouring laserscanning strips will usually show a certain overlap. If a part of a building is covered by two strips and data of both strips is being merged, the results will show a bias. This effect can easily be avoided by using data from only one strip. However, special solutions have to be found for buildings that are not completely covered by one strip.
- An inhomogeneous point distribution may also occur on roofs with varying reflectivity, where parts of the roof do not return sufficient laser signal strength, or in the case of mirror reflectance away from the sensor caused by water on horizontal parts.
- The scanning principle of lasercanner systems will lead to a higher point density on the roof side that is oriented towards the scanner. The size of this effect depends on the scan angle, the roof inclination and the relative position and orientation of the building with respect to the flight path.

Taking these parameters into account, a numerical correction scheme can be formulated. For this correction, the number of points actually falling onto both roof faces has to be counted. This can be performed iteratively by analyzing flight path and building geometry, or by recursively counting the points that actually fall on both roof halves. Both procedures will converge quickly. Assuming that there are $n_1$ and $n_2$ points on the two faces of the roof and defining a weight factor $P = 2n_1/(n_1 + n_2)$, the correction terms for the centroid of the building derived from the data points with binarized heights depend on the width $Y$ and orientation $\Theta$ of the building:

$$\Delta \bar{X} = \frac{Y (P-1)}{4} \cdot \sin \Theta$$

$$\Delta \bar{Y} = \frac{Y (P-1)}{4} \cdot \cos \Theta$$

(eq. 16)

After applying this correction, the other model parameters can be computed as shown before.

Obviously, these problems of inhomogeneously distributed data will not occur when data interpolated to a regular grid is being used, or if the planimetry information of a building is derived from transformed boundaries of the bounding box as discussed before.

### 4. Application example

The analysis of moments as discussed in chapter 3 was applied to laserscanning data acquired by the FLI-MAP system. FLI-MAP (Fugro N.V., see e.g. Pottle, 1998) is a helicopter-based laserscanning system with 8000Hz sampling rate, which is mainly used for corridor mapping. It acquires 40 profiles per second with 200 points per profile. Range measurement is limited to first-pulse capture at 20-200 meter distance, thus providing a maximum strip width of 200m at a scan width of 60°. Due to these system parameters, the point density is usually rather large (more than one point per square meter). Orientation parameters are determined by a set of four GPS receivers and a vertical reference unit. In addition to the laser range measurements, the FLI-MAP system is capable of delivering 6-bit intensity data.
Figure 3 shows a group of 10 buildings, which form part of a settlement in the Netherlands, to which the technique was applied. The average point density of the dataset is 5.3 points per m².

Segmentation of this data could be performed by the analysis of height texture followed by morphological filtering and connectivity analysis. All buildings could be modeled successfully, including most dorms (Figure 4). Two dorms could not be modeled due to a lack of data points, possibly caused by mirror reflection on water present on the horizontal surfaces.

The computation time per building was in the order of 0.8 seconds in a non-optimised implementation on a HP-9000 workstation.

As no ground truth was available in the study, precision figures can only be derived from the variation of the parameters, assuming identical width, height, orientation and roof inclination for the houses which belong to a settlement of equally designed buildings with varying length. Within this group of houses, the following parameters and standard deviations were determined from a laserscanner dataset covering roofs with areas between 100 and 240 m²:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>width (m)</td>
<td>8.09</td>
<td>0.16</td>
</tr>
<tr>
<td>height (m)</td>
<td>11.33</td>
<td>0.11</td>
</tr>
<tr>
<td>principle axis</td>
<td>6.9E</td>
<td>0.8E</td>
</tr>
<tr>
<td>inclination</td>
<td>37.4E</td>
<td>1.6E</td>
</tr>
</tbody>
</table>

Table 1: Average and RMS of identical model parameters of the buildings shown in Figure 4

The model fit analysis delivered an RMS deviation of 10 cm between point clouds and models of buildings complying with the assumed models. The reader is referred to the web page [http://www.geo.tudelft.nl/hrs/laserscan/laser_mom.html](http://www.geo.tudelft.nl/hrs/laserscan/laser_mom.html) for a VRML visualization of a building model overlaid with a segmented point cloud.

An analysis of the effect of the reduction of the spatial resolution is shown in (Maas, 1999a): While the high point density of approximately 5.3 points per m² is crucial for the reconstruction of dorms, buildings could still be reconstructed at a reduced point density in the order of one point per m² at a loss of accuracy, which may be acceptable for a number of applications.

5. Conclusion

The analysis of ratios of 2nd order invariant moments does provide a fast and efficient tool for the derivation of simple building descriptions. From datasets with densities of about five points per square meter, the parameters centroid, orientation, length, width, height and roof steepness of gable roof houses can be determined. Analyzing deviations between model and point cloud, dorms on roofs can also be modeled.

More complex building types may be modeled after splitting non-rectangular ground plans into primitives, as e.g. shown by (Weidner/Förstner, 1995) and (Haala/Brenner, 1997). More work has to be performed on aspects of error propagation, blunder detection and the analysis of the quality of results.

Advantages of the technique are the fact that the parameters of standard gable roof type buildings can be formulated as a closed solution. The technique can be applied to the original, irregularly distributed laserscanner data points without the requirement of interpolation to a regular grid. Segmentation may be performed based solely on the laserscanner points without the requirement of additional information such as 2-D GIS data. A practical test indicated a precision of 0.1-0.2 meter for the building dimensions and 1-2° for the orientation and the steepness of the roof.

Obviously, the resolution requirements for detailed building reconstruction are higher than the point density flown in standard airborne laserscanning applications nowadays. However, a resolution of significantly more than one point per square meter can be obtained by several laserscanning systems already today, and a clear trend towards higher spatial resolution is recognizable in current sensor development.
Acknowledgement

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