GEOMETRIC MODELLING AND CALIBRATION
OF FISHEYE LENS CAMERA SYSTEMS

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ABSTRACT:

The paper describes a geometric projection model for fisheye lenses and presents results of the calibration of fisheye lens camera systems based on it. For fisheye images the real projection model does not comply with the central perspective. In addition, the lenses are subject to considerable distortions. The fisheye lens projection follows an approximately linear relation between the incidence angle of an object point's beam and the distance from the corresponding image point to the principle point. The developed projection model is extended by several distortion parameters. For the calibration of fisheye lens camera systems, a calibration room was created and equipped with number targets. They were arranged to have a good distribution all over the image. The reference object coordinates of the targets were determined with high precision. The calibration itself is done by spatial resection from a single image of the calibration room. The calibration parameters include interior orientation, exterior orientation and lens distortion. Besides the geometric model for fisheye lenses, the paper shows practical results from the calibration of a high quality fisheye lens on a 14 mega pixel camera as well as a consumer camera equipped with a fisheye lens.

1. INTRODUCTION

Fisheye lenses allow imaging a large sector of the surrounding space by a single photo. Therefore they are useful for several applications. Besides the pure visualisation of landscapes or interiors (e.g. ceiling frescos in historical buildings) for advertisement or internet presentations, fisheye images can also be used for measuring tasks. These tasks include e.g. the position determination of celestial bodies or the documentation and analysis of the distribution of clouds.

The background of the work presented here is the recording of forest stand hemispheres for radiation-ecological analysis. Hemispherical photography using 180° fisheye lenses has been established as an instrument for the measurement of solar radiation in ecosystems since many years [Evans & Coombe, 1959; Dohrenbusch, 1989; Wagner, 1998]. It is used for the evaluation of the light situation of young growing trees in order to determine site-related factors within forest stands. Hemispherical images offer the advantage of providing spatially resolved information about the canopy coverage above a certain position using one record only (see Figure 1). This means that the sectors of the hemisphere which provide the most or the least photosynthetically utilizable radiation can be located. Alternative radiation sensors such as PAR sensors (photosynthetically active radiation) on the other hand acquire radiation only as an integral value for the whole hemisphere. To overcome this deficit, these sensors require recording time series, while a spatially resolved hemispherical image, combined with the daily and annual path of the sun, allows for integral statements over the illumination conditions from a single shot.

For the purpose of deriving photosynthetic relevant radiation values from hemispherical images two tasks have to be solved. First it is necessary to segment and classify the images into radiation relevant and radiation non-relevant areas [Schwalbe et al., 2004]. The second task is the evaluation of the classified hemispherical images using radiation models and a sun path model. Hence, it is necessary to know the geometrical relation between image space and object space. This means that the fisheye lens camera system used has to be calibrated in order to derive the inner orientation of the camera as well as several lens distortion parameters.

Figure 1. Hemispherical image of a pure spruce forest stand

The goal was to develop a mathematical model that describes the projection for fisheye lens camera systems. Furthermore the task was to create a calibration field that is suitable for camera fisheye lens systems. Finally the mathematical projection model has to be implemented in a software tool for a spatial resection in order to allow the calibration of fisheye camera systems using one image only.

2. PROJECTION MODEL

2.1 Central perspective projection vs. fisheye projection

Similar to panoramic images [Schneider & Maas, 2004; Parian & Grün, 2004], the geometry of images taken with a fisheye lens does not comply with the central perspective projection. Therefore the collinearity equation can not be used to mathematically describe the imaging process. The mathematical model of the central perspective projection is based on the assumption that the angle of incidence of the ray from an object point is equal to the angle between the ray and the optical axis within the image space (see Figure 2a).

Figure 2b. Fisheye projection model

2.2 Fisheye projection model

To reconstruct the projection of an object point into the hemispherical image, the object coordinates and image coordinates have to refer to the same coordinate system. Therefore, first the object coordinates which are known in the superior coordinate system have to be transformed into the camera coordinate system with the following transformation equations:

$$X_C = r_{11} \cdot (X - X_0) + r_{21} \cdot (Y - Y_0) + r_{31} \cdot (Z - Z_0)$$

$$Y_C = r_{12} \cdot (X - X_0) + r_{22} \cdot (Y - Y_0) + r_{32} \cdot (Z - Z_0) \qquad (1)$$

$$Z_C = r_{13} \cdot (X - X_0) + r_{23} \cdot (Y - Y_0) + r_{33} \cdot (Z - Z_0)$$

where X_C, Y_C, Z_C = object point coordinates in the camera coordinate system

X, Y, Z = object point coordinates in the object coordinate system

X_0, Y_0, Z_0 = coordinates of the projection centre
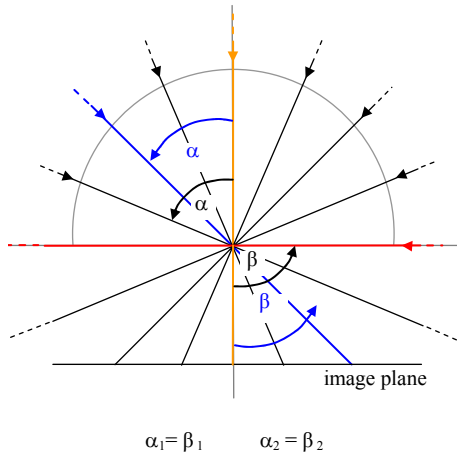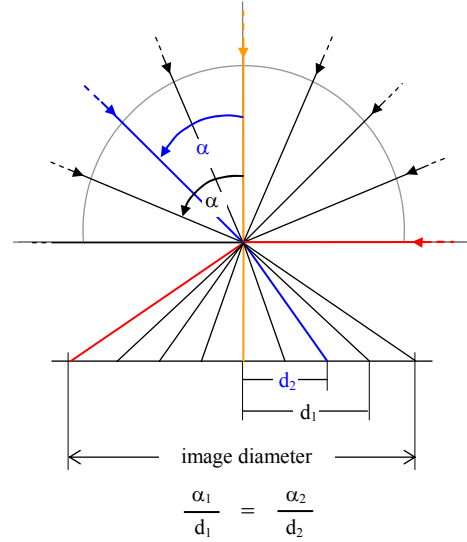
r_{ij} = elements of the rotation matrix

Figure 2a. Central perspective projection model

To realise a wider opening angle for a lens, the principle distance has to be shortened. This can only be done to a certain extent in the central perspective projection. At an opening angle of 180 degrees, an object point's ray with an incidence angle of 90 degree would be projected onto the image plane at infinite distance to the principle point, independent of how short the principle distance is. To enable a complete projection of the hemisphere onto the image plane with a defined image format, a different projection model becomes necessary.

The fisheye projection is based on the principle that in the ideal case the distance between an image point and the principle point is linearly dependent on the angle of incidence of the ray from the corresponding object point (see Figure 2b). Thus the incoming object ray is refracted in direction of the optical axis. This is realised by the types and the configuration of the lenses of a fisheye lens system.

All the following considerations refer to the Cartesian camera coordinate system. Figure 3 illustrates the principle of the fisheye projection graphically. For a 180 degree fisheye lens an object point's ray with an angle of incidence of 90 degree is projected onto the outer border of the circular fisheye image. This means that the resulting image point has the maximal distance to the optical axis. The relation between the angle of incidence and the resulting distance of the image point from the principle point is constant for the whole image. Consequently the following ratio equation can be set up as basic equation for the fisheye projection:

$$\frac{\alpha}{r} = \frac{90°}{R} \qquad \text{where} \qquad r = \sqrt{x'^2 + y'^2} \qquad (2)$$

where α   = angle of incidence
      r   = distance between image point and optical axis
      R   = image radius
      x', y' = image coordinates

The angle of incidence α is defined by the coordinates of its corresponding object point X, Y, Z and the exterior orientation parameters. For the fisheye projection the image radius R replaces the principle distance of the central perspective projection as a scale factor.

In equation (2) the image coordinates x' and y' are still included in r. To obtain separate equations for the single image coordinates another ratio equation becomes necessary. Since an object point, its corresponding image point and the z-axis of the camera coordinate system belong to the same plane, the following equation based on the intercept theorem can be set up:

$$\frac{x'}{y'} = \frac{X_C}{Y_C} \qquad (3)$$

where X_C, Y_C = object point coordinates in the camera
                         coordinate system
      x', y'   = image coordinates

After some transformations of the equations described above, the final fisheye projection equations for the image coordinates can be derived. They describe the ideal projection of an object point onto the image plane when using fisheye lenses. The equations are then extended by the conventional distortion polynomials and by the coordinates of the principle point to reduce the remaining systematic effects.

$$x' = \frac{\dfrac{2 \cdot R}{\pi} \cdot \operatorname{atan}\left[\dfrac{\sqrt{(X_C)^2 + (Y_C)^2}}{Z_C}\right]}{\sqrt{\left(\dfrac{Y_C}{X_C}\right)^2 + 1}} + dx + x_h$$

$$(4)$$

$$y' = \frac{\dfrac{2 \cdot R}{\pi} \cdot \operatorname{atan}\left[\dfrac{\sqrt{(X_C)^2 + (Y_C)^2}}{Z_C}\right]}{\sqrt{\left(\dfrac{X_C}{Y_C}\right)^2 + 1}} + dy + y_h$$

where x', y'   = image coordinates
      X_C, Y_C, Z_C = object point coordinates in the camera
                         coordinate system
      R   = image radius
      dx, dy   = distortion polynomials
      x_H, y_H = coordinates of the principle point

The object point coordinates in the camera coordinate system X_C, Y_C, Z_C have to be replaced by the according transformation equations (1), so that the image coordinates now depend on the

object coordinates given in the superior coordinate system and on the parameters of the exterior orientation of the camera. The inner orientation is described by the image radius R and the coordinates of the principle point x_H, y_H. The additional parameter set (dx, dy) modelling lens distortion is equivalent to the parameter set introduced for central perspective cameras by [Brown, 1971], extended by two parameters of an affine transformation (x'-scale and shear) [El-Hakim, 1986]. Based on the developed mathematical projection model, the camera parameters of fisheye lens camera systems can now be derived.
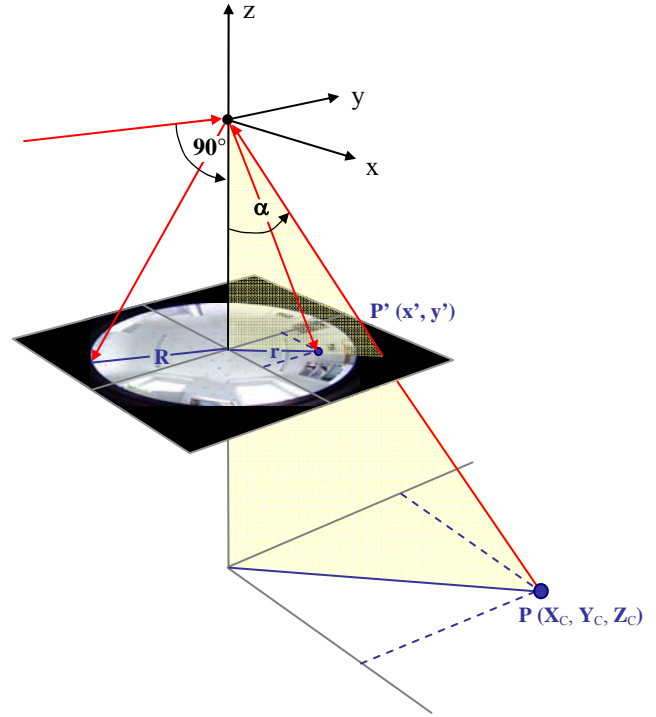

Figure 3. Geometric relation between object point and image
point within the camera coordinate system

3. CALIBRATION BY SPATIAL RESECTION

3.1 Calibration room

The mathematical model for the calibration of a fisheye lens camera system was implemented in a spatial resection using one image only. For that purpose a calibration room was established and equipped with a number of object points with known 3D coordinates (see Figure 4). The points are distributed in a way that they form concentric circles on the image plane. The distances between neighbouring points on the respective circles are equal. Thus, a frame-filling and consistent distribution of the points all over the image is achieved. To signalise the object point numbers, coded targets were used for points which are projected into the inner part of the image. Points that are projected into the outer parts of the fisheye image were signalised with uncoded targets because of the higher distortion within this image areas, which sometimes disabled the proper measurement of coded targets. A sufficient number of targets that cover the outer part of the image is necessary in order to obtain representative results in the calculation of the camera parameters.

Since it is not possible to apply a ring flash while taking fisheye images retro reflecting targets could not be used. The illumination deficits had to be compensated by longer exposure times.

The reference coordinates of the 140 object points of the calibration room were determined using an off-the-shelf industrial photogrammetry system. The standard deviation of the object coordinates is between 0.02 mm and 0.23 mm (see Table 1).

	X [mm]	Y [mm]	Z [mm]
Mean	0.05	0.04	0.04
Max	0.23	0.11	0.08
Min	0.02	0.02	0.02

Table 1. Object point standard deviations

3.2 Spatial resection

To determine the exterior and interior orientation as well as the distortion parameters of fisheye lens camera systems, a software tool for a spatial resection based on equation (4) and (5) was implemented. The unknown parameters were calculated iteratively in a least squares adjustment. Besides the parameters themselves, statistical parameters allowing the evaluation of the accuracy and the significance of the estimated parameters are determined.

The additional parameters describing the distortion of the hemispherical images include the radial-symmetric distortion, the radial-asymmetric and tangential distortion as well as an affinity and a shear parameter.
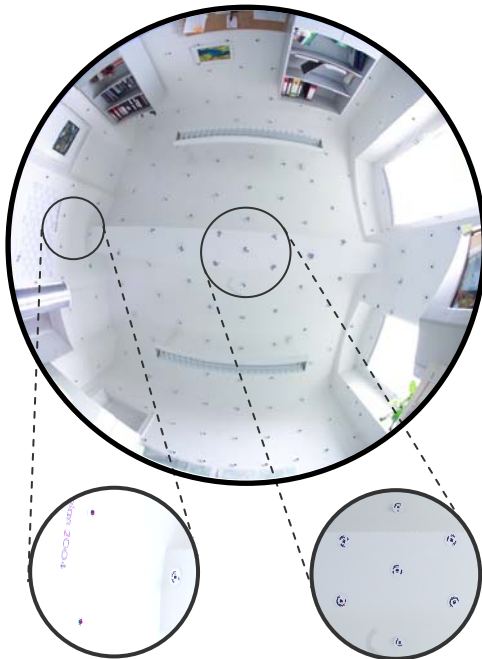

Figure 4. Calibration room

4. RESULTS

4.1 Used fisheye lens camera systems

The fisheye lens camera systems which have been tested are a Kodak DCS 14N Pro 14 mega pixel digital stillvideo camera with an 8 mm Nikkor fisheye lens and a Nikon Coolpix 990 amateur camera with the Nikon fisheye converter FC-E8 (see figure 5). Both of the used fisheye lenses are circular fisheye lenses with a field of view of 180 degrees (Nikkor) and 183 degrees (Coolpix). For the current research project the high resolution DCS camera is used. The camera was chosen to evaluate the technical possible accuracy potential of the developed method for the radiation determination. However, these kinds of camera fisheye lens systems are still too expensive for the practical use in forestry. Therefore the method should also be tested for low cost cameras to determine the influence of the lower resolution on the accuracy of the calculated radiation values. For that purpose the Coolpix camera was calibrated as well.

Figure 5. Camera fisheye lens systems:
a)   DCS 14N PRO & 8mm Nikkor fisheye lens
b)   Coolpix 990 & Nikon fisheye converter FC-E8

4.2 Calibration results

As mentioned above the focus is mainly set on the DCS camera with the Nikkor fisheye lens. Table 2 shows an example for calibration results of this fisheye lens camera system:

Parameter		Value
interior orientation	R	11.506 $\pm$ 0.0012 mm
	x_H	0.142 $\pm$ 0.0006 mm
	y_H	0.018 $\pm$ 0.0007 mm
radial-symmetric distortion	A1	$-7.9\cdot10^{-4}$ $\pm$ $6.2\cdot10^{-6}$
	A2	$2.0\cdot10^{-6}$ $\pm$ $9.7\cdot10^{-8}$
	A3	$-7.7\cdot10^{-9}$ $\pm$ $4.8\cdot10^{-10}$
radial-asymmetric and tangential distortion	B1	$1.5\cdot10^{-5}$ $\pm$ $1.6\cdot10^{-6}$
	B2	$1.0\cdot10^{-5}$ $\pm$ $1.8\cdot10^{-6}$
affinity and shear	C1	$6.4\cdot10^{-5}$ $\pm$ $2.1\cdot10^{-5}$
	C2	$2.8\cdot10^{-5}$ $\pm$ $2.0\cdot10^{-5}$

Table 2. Example for calibration results of the DCS 14N Pro with the Nikkor fisheye lens

The standard deviation of unit weight obtained from the adjustment was 0.1 pixels. Referring to the object space this corresponds to a lateral accuracy of 0.3 mm in a distance of 3 m (dimension of the calibration room) or 1.5 mm in a distance of 15 m (tree height within a typical forest stand). This means that the remaining residuals for the image points (see Figure 6) al-

ready reflect the influence of the accuracy of the object points themselves. It also means that the accuracy of the calibrated fisheye lens camera system is much better than it is demanded for the application of the determination of radiation values in forest stand.

For the DCS camera it was evaluated how much the additional parameters influence the standard deviation of unit weight of the resection. Table 3 shows that the standard deviation of unit weight was increased by a factor 100 from more than 9.5 pixels to less than 0.1 pixels by extending the model with additional parameters.

Estimated Parameters	σ_0 [pixel]
with exterior orientation only $(X_0, Y_0, Z_0, \omega, \varphi, \kappa)$	9,540
with interior orientation additionally (R, x_H, y_H)	8,781
with radial-symmetric distortion additionally (A1, A2, A3)	0.117
with radial-asymmetric and tangential distortion additionally (B1, B2)	0,098
with affinity and shear additionally (C1, C2)	0,096

Table 3. Influence of additional parameters on σ_0

As expected for fisheye lenses, the radial symmetric-distortion has the strongest influence on the accuracy. The other distortion parameters give only minor improvements to the standard deviation of unit weight. The parameters for affinity and shear are hardly significant.

The radial-symmetric distortion of the Nikkor fisheye lens causes residuals up to 35 pixels at the image border. The maximal residuals caused by the radial asymmetric and tangential distortion as well as by affinity and shear amount up to 0.5 pixels. The coordinates of the principle point have a special influence. If they are not estimated, the magnitude of the residuals is depending on the distance of the corresponding object point. In a calibration room with object point distances of 1.5 m up to 4 m the coordinates of the principle point cause residuals up to 5 pixels.

The standard deviation of unit weight for the calibration of the Coolpix camera was 0.3 pixels. This may also be attributed to the calibration room target size, which was optimized for the larger DCS camera. Within the object space this corresponds to a lateral accuracy of 2 mm in a distance of 3 m (dimension of the calibration room) or 10 mm in a distance of 15 m (tree height within the forest stand). This means that the accuracy (referring to the object space) which can be obtained with the DCS is 7 times better then the accuracy obtained with the Coolpix. Nevertheless the accuracy of the Coolpix is acceptable for the forestry application.
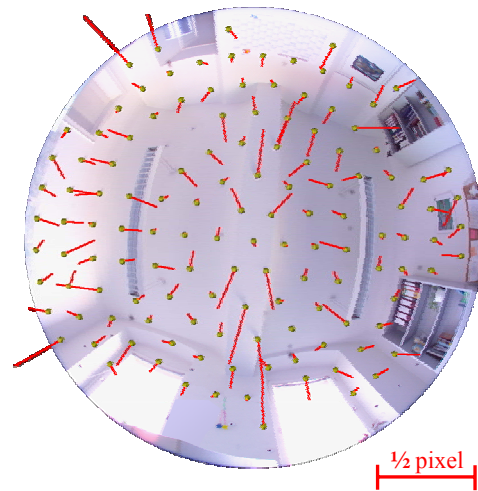

Figure 6. Remaining residuals after the calibration of the DCS 14N Pro with the Nikkor fisheye lens

5. SUMMARY AND OUTLOOK

A mathematical model for fisheye lens camera systems was developed, implemented and tested. The model deviates from the conventional central perspective camera model by introducing spherical coordinates. This means that a ray in object space is described by its azimuth and its inclination angle. In order to adapt to the physical reality, the model was extended by additional parameters modelling lens distortion.

The model was implemented in a spatial resection and tested with a 180° Nikkor fisheye lens on a 14 mega pixel professional digital stillvideo camera. Applying the geometric model, a standard deviation of unit weight of better than 0.1 pixels could be achieved. The use of additional parameters removed residuals of up to 35 pixels and improved the result by a factor of 100.

Future work will extend the implementation of the model to a self-calibrating bundle adjustment for hemispherical images, allowing to efficiently solve general photogrammetric measurement tasks by fisheye cameras.

6. ACKNOWLEGMENTS

7. REFERENCES

Amiri Parian, J., Grün, A., 2004: An advanced sensor model for panoramic cameras. International archives of Photogrammetry, Remote Sensing and Spatial Information Sciences. Vol. XXXV, Part B

Brown, D., 1971: Close-Range Camera Calibration. Photogrammetric Engineering, Vol. 37, No. 8.

Dohrenbusch, A., 1989: Die Anwendung fotografischer Verfahren zur Erfassung des Kronenschlußgrades. Forstarchiv, Jg. 60, S. 151-155.

El-Hakim, S.F., 1986: Real-Time Image Meteorology with CCD Cameras. Photogrammetric Engineering and Remote Sensing, Vol 52, No. 11, pp. 1757 - 1766.

Evans, G.C., Coombe, D.E., 1959: Hemispherical and woodland canopy photography and the light climate. Journal of Ecology, Jg. 47, S. 103-113.

Schneider, D., Maas, H.-G., 2004: Development and application of an extended geometrical model for high resolution panoramic cameras. International archives of Photogrammetry, Remote Sensing and Spatial Information Sciences. Vol. XXXV, Part B

Schwalbe, E., Maas, H.-G., Wagner, S., Roscher, M., 2004: Akquisition und Auswertung digitaler Hemisphärenbilder für waldökologische Untersuchungen. Publikationen der Deutschen Gesellschaft für Photogrammetrie, Fernerkundung und Geoinformation, Band 13, S. 113-120

Wagner, S., 1998: Calibration of grey values of hemispherical photographs for image analysis. Agricultural and Forest Meteorology, Jg. 90, Nr. 1/2, S. 103-117.