DESIGN AND TESTING OF MATHEMATICAL MODELS FOR A FULL-SPHERICAL CAMERA ON THE BASIS OF A ROTATING LINEAR ARRAY SENSOR AND A FISHEYE LENS

Danilo SCHNEIDER, Ellen SCHWALBE
Institute of Photogrammetry and Remote Sensing
Dresden University of Technology
(danilo.schneider, ellen.schwalbe)@mailbox.tu-dresden.de

Abstract: The paper describes two different geometric projection models for a full-spherical camera and presents results of the calibration of full spherical camera systems based on them. Full spherical images can be generated by a rotating linear array camera equipped with a fisheye lens. Their projection (Fig. 1) differs considerably from the central perspective geometry. It can be modelled by introducing spherical coordinates or by applying a sensor related coordinate system for each recording position of the linear array sensor extended by fisheye lens projection equations. For a first test of the system, a calibration was performed by spatial resection from a single image of a calibration room suitable for full-spherical records. Full-spherical camera systems were calibrated with an accuracy better than 1/5 pixels.

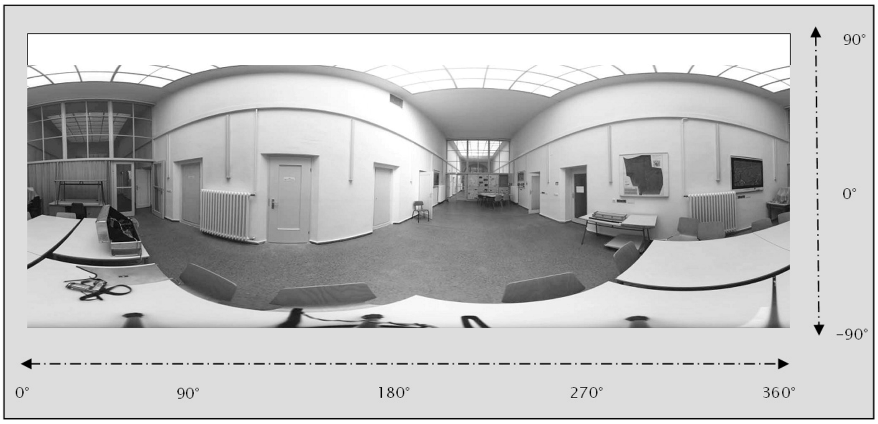


Figure 1: Full spherical image (360° horizontal, 180° vertical) of a room

1. Introduction

Panoramic images have become very popular in the last years mainly for photographic purposes as for example presenting landscapes, rooms of a museum or hotel for advertising and web presentations such as virtual round tours [6]. Beyond these applications, a number of photogrammetric applications, such as cultural heritage documentation or measuring of rooms or city squares, benefit from panoramic recording methods because panoramic cameras offer a very high resolution and the possibility of recording 360° 3D object geometries by taking only a few images [7].

There are different panoramic imaging methods. Besides the 'amateur-like' stitching of conventional central perspective images [4], [10], it is possible to use a rotating linear array sensor which is able to achieve a field of view up to 360° in the horizontal image coordinate direction. The field of view in the vertical image coordinate direction depends on the used lens and is usually far less than 180°. Therefore panoramic cameras do not create full-spherical images. There are different approaches for the calibration of this kind of panoramic cameras as described for example in [5], [8], [1], [2]. The angular limitation in the vertical image direction can be solved by the use of a fisheye lens, which is able to cover 180° in the second image coordinate direction. In this way a full-spherical image can be captured, which may be useful for various photogrammetric applications (e.g. silvicultural applications [11], measurement of interiors, facility management, site of crime documentation).

The photogrammetric use of such a combined full-spherical sensor requires a geometrical model for both, rotating line camera and fisheye lens, which do both not comply with the central perspective principle. A precise model of rotating line cameras has been developed and extended successfully using additional parameters in order to compensate systematic effects, resulting in an accuracy potential of ¼ pixels [8]. In addition, a geometric model for fisheye lenses has been developed. It is based on a linear relation between the angle of incidence of an object point's beam and the distance from the corresponding image point to the principle point (equi-angular model), refined with additional parameters. The paper will present two different approaches for an integrated mathematical model which allows describing and calibrating the geometry of full-spherical panoramic images. The combined geometrical model was implemented in a spatial resection and tested using a reference field.

Two full-spherical camera systems were used for the development and assessment of the geometric models (Fig. 2). The first is the SpheroCam HDR from Spheron (www.spheron.de) with a 5300 pixel linear array sensor. In combination with a 14 mm fisheye lens it captures 60 megapixel full-spherical images. The second camera system is a Nikkor 8 mm fisheye mounted on an EyeScan MM1 panoramic camera from Kamera & System Technik (www.kst-dresden.de) with a 3600 pixel linear sensor.

SpheroCam HDR:

- Image format (360° × 180°): 10600 × 5300
- Pixel size: 8 µm × 8 µm
- Radiometric resolution: 3 × 32 bit RGB

EyeScan MM1:

- Image format (360° × 180°): 6700 × 3600
- Pixel size: 9 µm × 9 µm
- Radiometric resolution: 3 × 8 bit RGB

Figure 2: Panoramic cameras with fisheye lenses from Spheron and KST

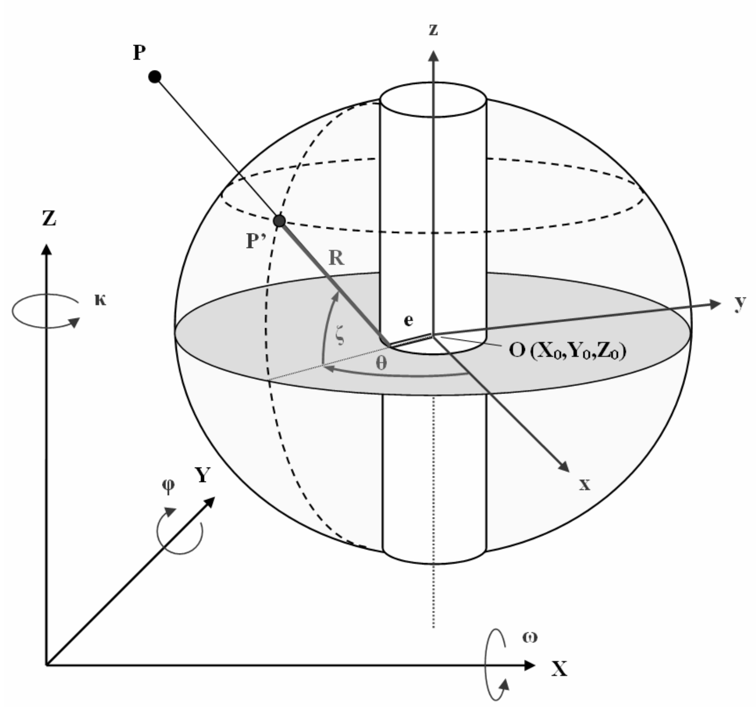
2. Mathematical models

The following chapter describes two mathematical models for a full spherical camera based on two different considerations. One uses spherical coordinates which in the first instance are perspicuous and illustrative for the mathematical description of a full-spherical projection but are less appropriate to strictly model systematical effects. The other model is based on transformations of the object point coordinates into the individual coordinate system of the linear array sensor for the record of each image column. While reconstructing, the position and the orientation of the linear array sensor in this manner the second model is more suitable concerning the description of plausible additional parameters.

2.1. Model based on spherical coordinates

2.1.1. Coordinate systems

In order to develop the mathematical model, suitable coordinate systems have to be introduced. According to [8] four consecutive systems are applied as illustrated in Fig. 3. The geometric model of a panoramic camera equipped with a conventional lens can be described by central perspective geometry in one image direction and a cylindrical geometry in the other image direction. Since the combination of a panoramic camera and a fisheye lens does not comply with the central perspective geometry in any image direction, the cylindrical coordinate system is substituted by a spherical system. The spherical coordinate system allows describing full spherical images in a quite simple way. While the z-axis of the camera coordinate system is equal to the rotation axis of the camera, the x-axis defines the direction of the first column in the image. In addition, a numerical eccentricity parameter e has to be considered, since the projection centre is not located within the rotation axis.



- Object coordinate system (X, Y, Z)
- Cartesian camera coordinate system (x, y, z)
- Spherical camera coordinate system (θ, ζ, R)
- Image coordinate system (x', y')

Figure 3: Geometrical model based on spherical coordinates

2.1.2. Derivation of the mathematical model

The purpose of a mathematical model is to describe the projection of an object point into the image mathematically. This model is usually presented by equations which transform object coordinates (X, Y, Z) of an arbitrary object point into image coordinates (x', y'). The image coordinates also depend on the exterior and interior camera orientation. The development of the model is based on transformation equations between the coordinate systems. The transformation (translation and rotation) between object coordinates (X, Y, Z) and Cartesian camera coordinates (x, y, z) occurs by the following formula:

$$\underline{x} = \underline{X}_0 + \underline{R} \cdot \underline{X} \tag{1}$$

$\underline{X}_0$ … Translation between object and camera coordinates
$\underline{R}$ … Rotation matrix between object and camera coordinates

Spherical angles (θ, ζ) are calculated from Cartesian camera coordinates (x, y, z):

$$\theta = \arctan\frac{y}{x} \qquad \zeta = \arctan\frac{z}{\sqrt{x^2 + y^2} - e} \tag{2}$$

e … Numerical eccentricity of the projection centre w.r.t. the rotation axis
R … Spherical radius (cp. Figure 3)

Finally the spherical coordinates (θ, ζ, R) have to be transformed into the image coordinate system (x', y') according to Figure 4. The principle point in y'-direction is also considered in the following formula:

$$x' = (R + e) \cdot \theta \qquad y' = R \cdot \zeta + y_P \tag{3}$$

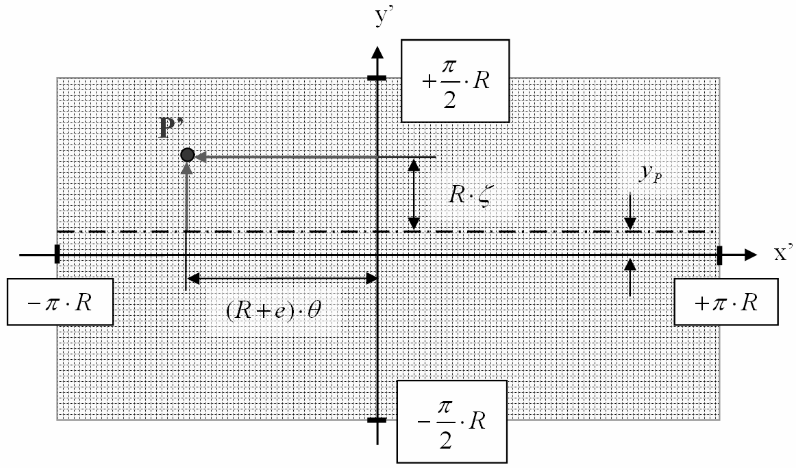


Figure 4: Definition of the image coordinate system

Finally, by inserting equation (1)-(3) into each other we achieve the final equations, which describe the transformation of an arbitrary object point onto the image:

$$x' = (R + e) \cdot \arctan\frac{r_{12} \cdot (X - X_0) + r_{22} \cdot (Y - Y_0) + r_{32} \cdot (Z - Z_0)}{r_{11} \cdot (X - X_0) + r_{21} \cdot (Y - Y_0) + r_{31} \cdot (Z - Z_0)} + dx' \tag{4}$$

$$y' = R \cdot \arctan\frac{r_{13} \cdot (X - X_0) + r_{23} \cdot (Y - Y_0) + r_{33} \cdot (Z - Z_0)}{\sqrt{(r_{11} \cdot (X - X_0) + r_{21} \cdot (Y - Y_0) + r_{31} \cdot (Z - Z_0))^2 + (r_{12} \cdot (X - X_0) + r_{22} \cdot (Y - Y_0) + r_{32} \cdot (Z - Z_0))^2} - e} + y_P + dy' \tag{5}$$

r_{ij} … Elements of the rotation matrix $\underline{R}$

In analogy to the collinearity equations, which describe the central perspective geometry, the formulas (4) and (5) express the image coordinates of a full spherical image depending on the exterior orientation (rotation and translation), the interior orientation (spherical radius and principle point in y'-direction) and the object coordinates of an object point (X, Y, Z). Hence, they are the basis for diverse adjustment calculations, such as spatial resection for the calibration and accuracy assessment of panoramic cameras combined with a fisheye lens as specified later in this paper. Furthermore, the formulas are extended by the correction terms dx' and dy', which contain additional parameters in order to compensate remaining systematic effects as itemised in the following chapter.

2.1.3. Additional parameters

Since the geometrical model described above complies only approximately with the actual physical imaging process, additional parameters have to be considered in order to achieve a high precision. Deviations from the basic model may be caused by limitations of the mechanical and optical properties of the physical camera system. These deviations produce systematical variations of the image coordinate residuals. By analysing the residuals resulting from a spatial resection (cp. chapter 3), such systematics were detected and appropriate parameters were developed. One of these parameters is the numerical eccentricity of the projection centre with respect to the rotation axis, which is already introduced (Fig. 3). Further parameters are summarized in the following table:

Parameter	Description
e	Numerical eccentricity with respect to the rotation axis
A_1, A_2, A_3	Radial-symmetric lens distortion [3], considered primarily in y'-direction
B_1, B_2	Asymmetric and tangential lens distortion, according to [3]
N	Non-parallelism of the CCD-line with respect to the rotation axis
C_1	Affinity (different scales of the image coordinate axes due to the panoramic recording principle)
C_2	Shear of the image coordinate axes (due to a infinitesimal rotation of the CCD-line in recording direction)
S_i	Polynomial coefficients for compensation of fisheye-caused deviations in x'-direction (since the CCD-line in not perfectly located within the optical axis)

Table 1: Additional parameters

2.2. Model based on a rotating image coordinate system

Another mathematical model of a full-spherical camera is based on the assumption that each image column can be treated as an individual image with its own centre of projection and its own orientation. In this way the fisheye projection equations can be applied to every single image column.

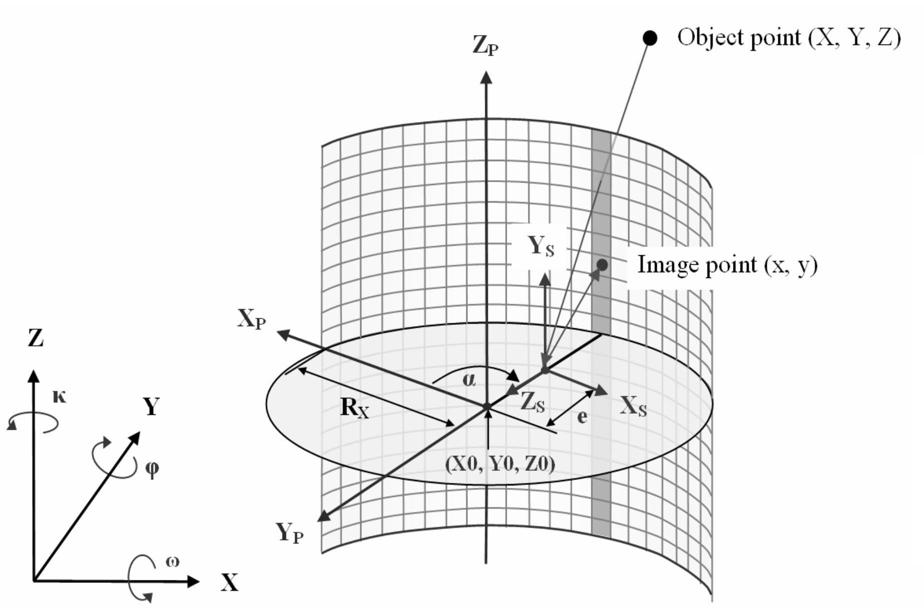
2.2.1. Coordinate systems

In order to describe the projection of a full-spherical camera several coordinate systems have to be defined (see Figure 5). The coordinates of the object points are given in a world coordinate system (X, Y, Z). A second coordinate system is the local camera coordinate system of the panoramic camera (X_P, Y_P, Z_P), where the z-axis coincides with the rotation axis of the camera. The determination of the image coordinates in x-direction is based on this coordinate system. Therefore, in a first step the object coordinates have to be transformed into the panoramic camera coordinate system by the according transformation equation (1).

The image coordinates in y-direction are calculated using the mathematical model for the fisheye projection (see chapter 2.2.2). For this purpose a coordinate system for the record of each image column (X_S, Y_S, Z_S) is required where the z-axis coincides with the optical axis. This coordinate system exists individually for each position of the sensor and depends on the numerical eccentricity e. The latter is caused by the fact that the projection centre is not the rotation centre of the camera. Thus the origin of the sensor coordinate system is the projection centre for the record of an image column. The object coordinates given in the panoramic camera coordinate system are transformed into the sensor coordinate system by the following equations:

$$X_S = 0 \qquad Y_S = Z_P \qquad Z_S = \sqrt{(X_p)^2 + (Y_p)^2} + e \tag{6}$$

X_S, Y_S, Z_S … Object point coordinates in the sensor coordinate system
X_P, Y_P, Z_P … Object point coordinates in the panoramic camera coordinate system
e … Eccentricity (cp. Figure 5)



- Object coordinate system (X, Y, Z)
- Panoramic camera coordinate system (X_P, Y_P, Z_P)
- Rotating sensor coordinate system (X_S, Y_S, Z_S)
- Image coordinate

Figure 5: Full-spherical projection principle based on a rotated sensor coordinate system

2.2.2. Basic full-spherical model

The basic mathematical model for a full-spherical projection is based on the assumption of an ideal projection. This means that it is first supposed that the linear sensor line coincides with the vertical plane through the optical axis of the fisheye lens. Thus, only the y-coordinate is influenced by the geometry of the fisheye lens. The x-coordinate can be calculated as a product of the radius R_X, which can be seen as a scale factor for the x-direction, and the azimuth α of the corresponding object point.

$$\tilde{x} = R_X \cdot \alpha \tag{7}$$

R_X … Radius (distance between sensor and rotation axis)
α … Azimuth (cp. Figure 5)

Since the linear sensor array does not coincide with the optical axis in physical reality, the x-coordinate of the image point is also slightly influenced by the fisheye lens. This effect can be compensated by an additional rotation of the sensor coordinate system around the Y_S-axis by an angle $\Delta\alpha$. The latter is depending on the angle of incidence of the ray from the particular object point. This dependency can preliminary be modelled using a fifth order polynomial. In future work a strict solution will be searched for. The object point coordinates are now given in the individual rotated sensor coordinate system (X_{Sr}, Y_{Sr}, Z_{Sr}). For the calculation of the y-coordinate ($\tilde{y}$) of the image and the varying offset for the x-coordinate (dx) the equations for the fisheye projection described in [9] are used:

$$\tilde{y} = \frac{\frac{2 \cdot R_y}{\pi} \cdot \arctan\left(\frac{\sqrt{(X_{Sr})^2 + (Y_{Sr})^2}}{Z_{Sr}}\right)}{\sqrt{\left(\frac{X_{Sr}}{Y_{Sr}}\right)^2 + 1}} \qquad dx = \frac{\frac{2 \cdot R_y}{\pi} \cdot \arctan\left(\frac{\sqrt{(X_{Sr})^2 + (Y_{Sr})^2}}{Z_{Sr}}\right)}{\sqrt{\left(\frac{Y_{Sr}}{X_{Sr}}\right)^2 + 1}} \tag{8}$$

R_y … Radius of the fisheye image
X_{Sr}, Y_{Sr}, Z_{Sr} … Object point coordinates in the rotated sensor coordinate system

They were developed based on the assumption of a constant ratio between the angle of incidence of an object points ray and the distance between the corresponding image point and the principle point. The input values are the coordinates of the object point given in the according sensor coordinate system (X_{Sr}, Y_{Sr}, Z_{Sr}).

This basic model still has to be refined by adding some additional parameters (cp. table 2) as the coordinate of the principle point in y-direction (y_P) and parameters that compensate the distortions caused by the fisheye lens. The final equations describing the projection of a full-spherical camera can be set up as follows:

$$\begin{aligned} x &= \tilde{x}(\underline{X_P}) + dx(\underline{X_{Sr}}) + dx_{rot}(\underline{X_{Sr}}) + dx_{\tan}(\underline{X_{Sr}}) \\ y &= \tilde{y}(\underline{X_{Sr}}) + y_P \qquad\quad + dy_{rot}(\underline{X_{Sr}}) + dy_{\tan}(\underline{X_{Sr}}) \end{aligned} \tag{9}$$

x, y … Image coordinates
$\tilde{x}$, $\tilde{y}$ … Image coordinates of the basic projection model
dx … Varying offset caused by the discrepancy between optical axis and the linear array sensor
y_P … Coordinate of the principle point
dx_{rot}, dy_{rot} … Components of symmetric radial distortion including A1, A2, and A3
dx_{tan}, dy_{tan} … Components of asymmetric radial and tangential distortion including B1 and B2

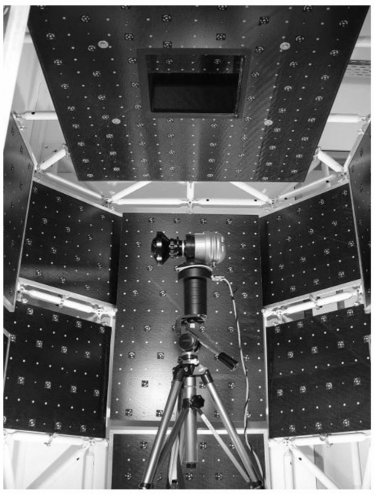
3. Calibration by spatial resection

To determine the exterior and interior orientation as well as the additional parameters of the full spherical camera systems, software tools for spatial resection based on the two mathematical models described above were implemented. The unknown parameters were calculated iteratively in a least squares adjustment. Besides the parameters themselves, statistical parameters allowing the evaluation of the accuracy and the significance of the estimated parameters are determined. At the same time, the resection delivers information on the precision potential of the camera and the mathematical model.

3.1. Calibration room

The cameras and the geometric models were extensively tested by using images taken in a test field room. Due to the full spherical imaging geometry, the test field should consist of object points all around the camera. The test field used for the investigations was a calibration room of AICON 3D Systems GmbH (Fig. 6). The dimensions of this room are ca. $2.5 \times 1.8 \times 2.0$ m³, and it consists of ca. 430 coded retro-reflecting targets with an average coordinate precision of 0.01 - 0.07 mm. To use the retro-reflecting effect of the targets in combination with a fisheye lens, 3 spotlights were placed close to the camera position.

The object points are distributed at the surrounding walls as well as on the ceiling of the calibration room. Due to the fact that there are no points on the floor, there is a lack of points in the lower part of the image. This is disadvantageous for the modelling of the fisheye geometry, since the fisheye distortions mainly affect the border areas of the image.



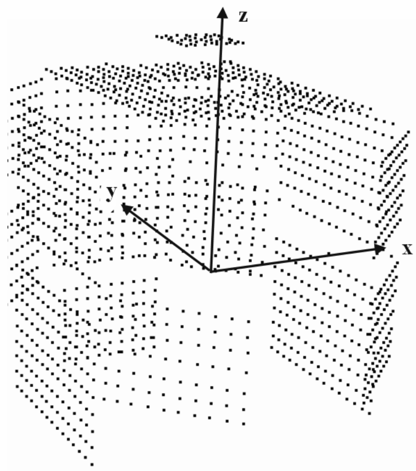


Figure 6: Calibration room (courtesy of Acion 3D Systems GmbH)

3.2. Calibration results

Applying the mathematical models as developed above, a standard deviation of unit weight of 0.15 pixels and 0.20 pixels was obtained for the SpheroCam and the EyeScan, respectively. Translated into the object space, this corresponds to a lateral accuracy of 0.225 mm at a distance of 1 m (dimension of the calibration room) or 2.250 mm at a distance of 10 m for the EyeScan camera. Due to the higher resolution of the SpheroCam the accuracy in object space amounts 0.086 mm at a distance of 1 m and 0.857 mm at a distance of 10 m.

To show the effect of additional parameters on the standard deviation of unit weight of the resection, the parameters were calculated using the second model and the data recorded by the SpheroCam. Table 2 shows that the standard deviation of unit weight was increased by a factor 200 from more than 31 pixels to 0.15 pixels by successively extending the model with additional parameters. It is recognizable that the principle point and the radial symmetric-distortion have the strongest influence on the accuracy.

Estimated Parameters		σ_0 [pixel]
basic model with exterior orientation and scale factors only	$X_0, Y_0, Z_0, \omega, \varphi, \kappa, R_X, R_Y$	31,131
with the y-component of the principle point additionally	y_P	10,177
with the offset caused by the discrepancy between optical axis and the linear array sensor additionally	dx	9.766
with numerical eccentricity additionally	e	9.464
with radial-symmetric distortion additionally	A1, A2, A3	0.260
with radial-asymmetric and tangential distortion additionally	B1, B2	0.151

Table 2: Example for the influence of additional parameters on σ_0

As mentioned above the resulting residuals for the image points are almost free of systematical influences. Some remaining systematical effects (see Figure 7) after adding the additional parameters may be attributed back to mechanical instabilities and external effects such as vibrations during to the recording time.

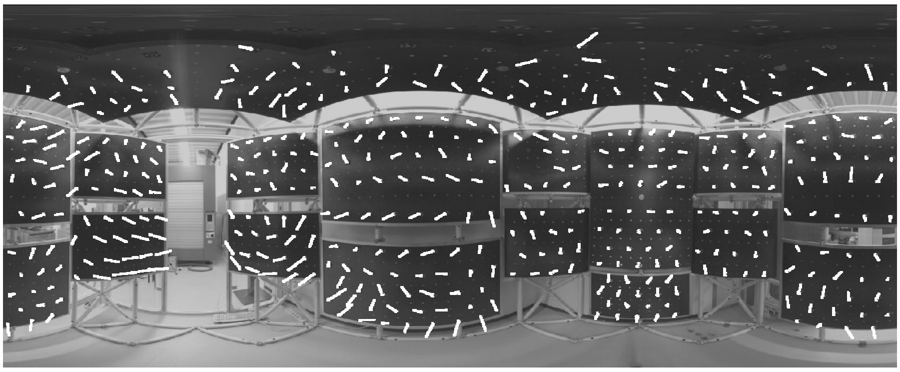


Figure 7: Calibration room and residuals from spatial resection

4. Summary and future work

Two mathematical models for full-spherical camera systems based on different approaches were developed. They were implemented in a spatial resection and tested with datasets recorded by an EyeScan and a SpheroCam. Applying the geometric model, a standard deviation of unit weight of better than 0.2 pixels could be achieved. The use of additional parameters improved the result by a factor of 200.

In the future the mathematical models will be refined. The temporal stability of the camera model parameters has to be verified by recording and processing sequences of spherical images. The geometric spherical camera models, which have so far been implemented in a spatial resection, should be ported to a self-calibrating bundle adjustment, allowing for reference field independent camera calibration. The models will also be tested on new full-spherical cameras specially designed for high-precision measurement purposes, which are under development.

Acknowledgement

The authors would like to thank SpheronVR AG (www.spheron.de) and Kamera & System Technik (www.kst-dresden.de) for providing their cameras and AICON 3D-Systems GmbH (www.aicon.de) for providing the calibration room.

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