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Triangulation methods for height profile measurements on instationary water surfaces

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Abstract. The precise measurement of water surface models or profiles plays an important role in experimental hydromechanics. Conventional gauge-based techniques often come with a large instrumental effort and a limited spatial resolution. The paper shows an efficient non-contact photogrammetric technique for the measurement of water-surface profiles, which is based on an extension of the well-known laser light sheet projection technique. While the original laser lightsheet triangulation technique is limited to surfaces with diffuse reflection properties, the developed technique is capable of measuring on reflecting instationary surfaces.

This article presents the basic principle, potential and limitations of the method. Several evolution steps of the system with different applicability and different complexity are shown. A double projection plane system capable of simultaneously measuring water surface height and tilt profiles marks the ceiling of the development. Besides the geometrical models of different levels of complexity, system calibration procedures are described. The applicability of the techniques and their accuracy potential are shown in several practical tests.

Keywords. Water surface measurement, triangulation sensor, hydromechanics.

1. Motivation

The application of scaled physical models is an often used method to solve complex problems in connection with project planning in river engineering (ATV-DVWK 2004, Block 1936). Due to improved computational techniques, hydromechanics phenomena can be simulated and analysed theoretically in mathematical models nowadays. To verify and improve these theoretical approaches, practical experiments on modelled systems like water channels (see Figure 1) are still necessary (Godding et al. 2003). One of the most important parameters in hydromechanical phenomena is a height model of the water level. Water surface models are often determined by pointwise water gauge measurements, monitoring the vertical motion of a floater or by ultrasonic height measurements in cylinders, which are connected with the channel bed via conduits.

These methods are limited in their temporal and spatial resolution, and they may affect the behaviour of the water surface. To overcome these limitations, a non-contact profile-wise photogrammetric water surface measurement technique has been developed in a cooperation of the Institute of Photogrammetry and Remote Sensing at TU Dresden and the Federal Waterways Engineering and Research Institute (BAW) in Karlsruhe.

The requirement of simultaneous area- or profile-wise measurement is fulfilled by most optical measurement techniques. However, the application of optical measurement techniques to water surface measurements necessitates methods to map the water level in images of a camera. Basically, two methods for water surface visualization can be distinguished: Physical visualization, for instance by markers floating on the water surface, or optical visualization realized by suitable illumination or projected light patterns.

Small particles on a water surface may be imaged by a stereo camera system and used to determine an instantaneous water surface model over a given observation area via their 3D coordinates interpolated to a regular grid. If stereoscopic image sequences are acquired and processed, the technique may be extended to a 4-D method by tracking particles in time and simultaneously determining 3-D surface models and

Figure 1: Test bed of a river model, pointwise gauge measurement, generated 3D model (source BAW).

surface velocity fields (particle tracking velocimetry, Maas et al. 1993). A drawback of physical marking techniques based on adding markers is often formed by the fact that markers pollute the experimental facility. In this sense, the technique can not really be considered a non-contact measurement technique.

Marker-free optical methods can be differentiated into triangulation based methods and photometric methods. Triangulation methods are based on the projection of a light pattern onto a surface and imaging it by a camera under a parallax. Photometric methods use image intensities rather than image coordinates as measurements. The photometric stereo approach (Woodham 1978), an extension of shape-from-shading, determines surface albedo and surface normal from image intensity measurements under three illumination directions, assuming diffuse surface reflection. The technique has been extended to be utilized on surfaces with specular reflection properties by using a large number of spatially distributed light sources (Nayar et al. 1990). A general disadvantage of photometric methods is the fact that they determine the surface normal; a surface height model can only be determined by an integration approach, leading to unfavorable error propagation.

Optical triangulation methods based on the projection of a laser light sheet onto a surface and imaging it by a camera under a parallax form a widely used technique for object surface height profile measurements (e.g. Jarvis 1983). The technique is based on the projection of a line onto a diffuse reflecting surface, which is imaged by a camera under a parallax ideally with an angle close to 90° between the optical axis of the camera and the laser. The physical properties of water surfaces are, however, in conflict with the requirement of diffuse surface reflection. The interaction of a laser beam with a clear water surface is characterized by specular reflection and refraction. A part of the energy of the beam will penetrate into the water, with the direction of the beam determined by Snell's law. The rest of the energy is reflected according to the angle-of-incidence law. This specular reflection on clean water surfaces can not be used for measurements in a conventional optical triangulation setup.

In the following, the fundamental technique of laser lightsheet triangulation is extended in four steps to adapt it to different needs in experimental hydromechanics. The introduction of a vertical projection plane allows measurements on still water surfaces (Section 2). Further, the analysis of accumulator images enables measurements on regular one-dimensional wave patterns (Section 3). The introduction of a second projection plane renders the detection of valid measurements on wave peaks or valleys (Section 4.1). The full-grade system marks the peak of development and allows simultaneous determination of local water level height and tilt (Section 4.2).

2. Optical triangulation on reflecting surfaces

As mentioned above, the estimation of water surfaces cannot be realized with traditional photogrammetric methods for surface measurement, because these methods assume diffuse-reflecting textured surfaces. At first glance, the mirror-like reflecting properties of water surfaces prohibit surface determination by optical triangulation methods. However, an extension of the technique may allow for measuring on reflecting surfaces (Maas et al. 2003): Rather than observing the laser line projection on the water surface directly, the reflected laser light sheet can be projected onto a plane, which is orientated vertically to the water surface (see Figure 2). The resulting laser line can be observed by a camera and translated into a water level height profile (equation). With a band-pass filter on the camera lens adapted to the laser diode wavelength, the images are quasi binary and the laser line projection vertical position can easily be detected by scanning the image columns for peaks.

$$\Delta H_W = \lambda \cdot \Delta H_{\text{Laser}} \tag{1}$$

with

ΔH_W water level change,
λ conversion factor, and
ΔH_{Laser} change of laser line height.

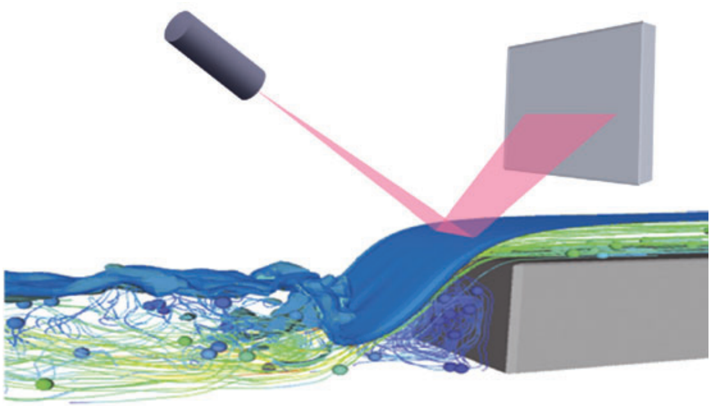

Figure 2: Basic measurement principle.

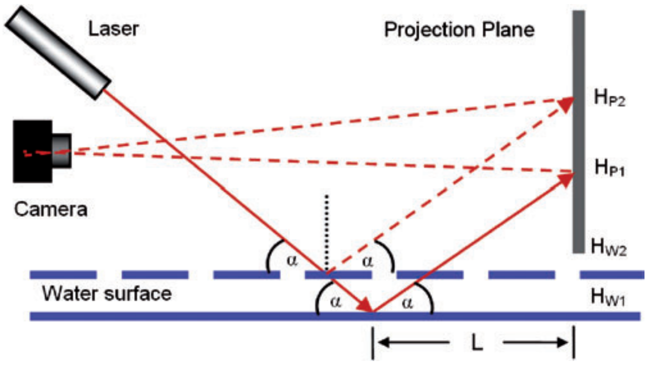

Figure 3: Calibration principle

The system calibration is rather simple and can be performed by measuring the laser line at minimum and maximum water level height (Figure 3). The required conversion factor λ can be calculated from equation (2), and the absolute height of a certain water level j is obtained from equation (3). In the simplest case with a vertical projection plane we obtain $\lambda = 2$. Tilting the plane backward may help to further increase the sensitivity of the method.

$$\lambda = \frac{H_{P2} - H_{P1}}{H_{W2} - H_{PW1}} \tag{1}$$

$$H_{W_j} = H_{W1} + \lambda(H_{P1} - H_{P_j}) \tag{2}$$

Maas et al. (2003) confirmed the applicability of the basic principle and the high accuracy potential of the technique on still water surfaces. Using a 1024×768 pixel camera to record a 70 cm wide profile, an accuracy of 0.03 mm was achieved for the determination of water level variations.

3. Accumulator image technique

A limitation of the practical application of the method as described before is the requirement of a still water surface. Under the presence of waves, water-level induced effects cannot be separated from slope-induced effects (see Figure 4a), leading to a severe degradation of the accuracy potential if neglected. This limitation to static phenomena can be overcome by further modifications of the technique, which are shown in the following. Under the assumption of a pre-dominant wave pattern in flow direction (i.e. perpendicular to the projection plane), a solution to this surface normal dependency of the results can be provided by the analysis of short image sequences covering at least one complete wave cycle in time: As an effect of waves, the projected line will move up and down in the image, with the minima and maxima reached at the extreme values of the surface tilts in the direction perpendicular to the line (Figure 4). Assuming a symmetric and primarily one-directional wave pattern, a wave-independent position of the line representing the average water level can be found by searching for minima and maxima of the line position and analysing those.

A very efficient method to determine the average line position is the use of a max-store algorithm storing the highest greyvalue of each pixel over an image sequence in an accumulator image. In this accumulator image, the original thin laser line will be widened to a band, with the width of the band depending on the maximum surface tilts. Again assuming spatial and temporal evenness of the wave pattern, the vertical position of the band will be an average over a wave cycle, and the vertical position of the center of the band will be a wave-independent measure for the water level.

From Figure 3 and Figure 4, one can derive that symmetric waves with a given maximum slope β will introduce an asymmetry into equation (3), with a

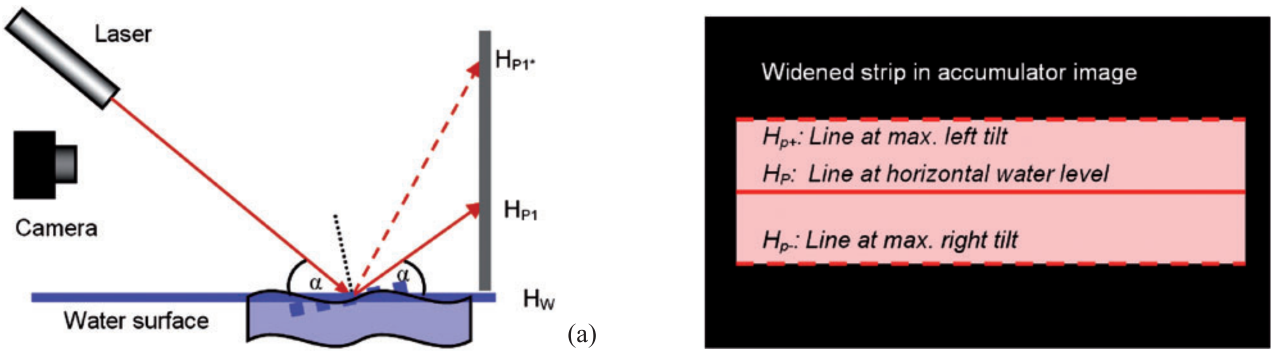

Figure 4: (a) Effect of waves on height measurement. (b) Principle of max-store accumulator image.

positive projection height deviation caused by a surface tilt towards the laser source being larger than a negative deviation caused by a surface tilt to the opposite side.

If the vertical line position on the projection plane at still water level is

$$H_P = H_{P_{\min}} + 2 \cdot (H_W - H_{W_{\min}}), \quad (3)$$

the highest and lowest position of the line on the projection plane (H_{p^+}, H_{p^-}) become

$$H_{P_+} = H_{P_{\min}} + 2 \cdot (H_W - H_{W_{\min}}) + L \cdot (\tan(\alpha + \beta) - \tan \alpha), \quad (4)$$

$$H_{P_-} = H_{P_{\min}} + 2 \cdot (H_W - H_{W_{\min}}) + L \cdot (\tan(\alpha - \beta) - \tan \alpha) \quad (5)$$

with an incidence angle α and a lever arm L between the laser-water intersection point at $H_{\min}$ and the position of the projection plane (see Figure 3). This deviation introduced by the water surface normal is not symmetric.

The vertical center H_{P_m} of the widened line in the max-store accumulator image becomes

$$\begin{aligned} H_{P_m} &= \tfrac{1}{2}(H_{P_+} + H_{P_-}) \\ &= H_{P_{\min}} + 2 \cdot (H_W - H_{W_{\min}}) + L \cdot \left(\tfrac{1}{2}\tan(\alpha + \beta) + \tfrac{1}{2}\tan(\alpha - \beta) - \tan \alpha\right). \end{aligned} \quad (6)$$

As a consequence of the asymmetry in equations (4) and (5), there will be a positive bias with respect to the still water surface solution equation (3):

$$\begin{aligned} \Delta H_P &= H_{P_m} - H_P \\ &= L \cdot \left(\tfrac{1}{2}\tan(\alpha + \beta) + \tfrac{1}{2}\tan(\alpha - \beta) - \tan \alpha\right) \\ &\geq 0. \end{aligned} \quad (7)$$

Assuming a lever arm of 200 mm and an incidence angle $a = 45°$, the bias correction is approximately 0.25 mm at a wave slope of 2° and 1.5 mm at a wave slope of 5°.

Computing the width of the widened line in the accumulator image

$$\begin{aligned} W_P &= H_{P_+} - H_{P_-} \\ &= L \cdot (\tan(\alpha + \beta) - \tan(\alpha - \beta)) \end{aligned} \quad (8)$$

and resolving it for the maximum wave slope β

$$\beta = \arctan\left(\frac{L \cdot \tan^2\alpha + L - (L^2 \cdot \tan^4\alpha + 2 \cdot L^2 \cdot \tan^2\alpha + L^2 + H^2 \cdot \tan^2\alpha)^{0.5}}{H \cdot \tan^2\alpha}\right) \quad (9)$$

the height bias can be computed equation (7) and applied to the result. Introducing equations (2), (8) and (9) into equation (3) and assuming symmetric one-dimensional wave effects, the corrected water level profile height for a vertical line position measurement h_j in column j of the accumulator image becomes

Table 1: Accuracies achieved in practical experiments (Maas et al. 2003).

	Verified water level height precision
Still water level (single image)	0.03 mm
Moderate waves (single image)	0.56 mm
Moderate waves (max-store image)	0.10 mm

$$H_{W_j} = H_{\min} + (h'_j - h'_{\min}) \cdot \frac{H_{\max} - H_{\min}}{h'_{\max} - h'_{\min}} - \frac{L}{2} \cdot \left(\frac{1}{2}\tan(\alpha + \beta) + \frac{1}{2}\tan(\alpha - \beta) - \tan \alpha\right). \quad (10)$$

The technique has the advantage of simplicity in image processing, geometric modelling and calibration. It is, however, limited to simple wave patterns with moderate surface tilts and a pre-dominant formation in flow direction.

In a practical verification on measuring a 70 cm wide water level height profile (Table 1), the method based on max-store images obtained from short image sequences could reduce the errors resulting from water surface tilts by a factor of six, compared to applying the single image method for still water surfaces as shown in Section 2. However, the precision level achieved in these experiments is three times worse as the results achieved on still surfaces (Maas et al. 2003).

4. Double projection plane system

A more general solution to the discussed wave problem can be achieved by the integration of a second projection plane into the design of the system. The basic idea of the double-plane technique is the detection of the direction of the reflected laser lightsheet by analysing its projection on two planes (Figure 5). This allows for the elimination of wave affected measurements and for a rigorous geometric solution for surface determination.

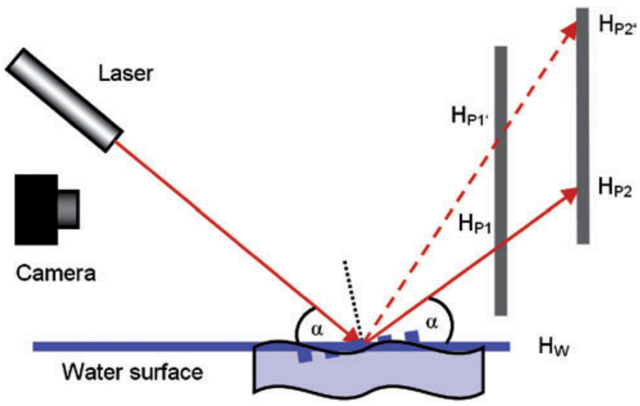

Figure 5: System with two projection planes.

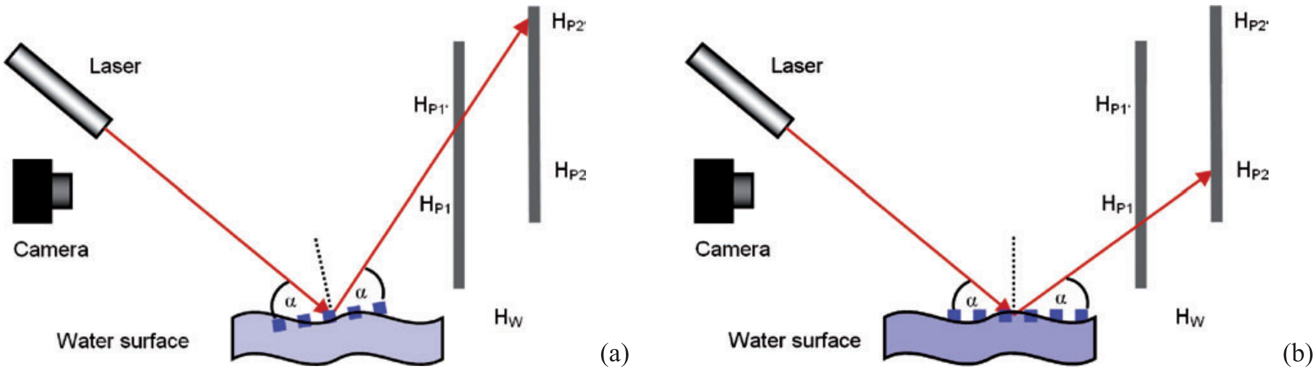

Figure 6: (a) Invalid measurement. (b) Valid measurement.

The double-plane system can be realized in two ways: The method presented in Section 4.1 is a straight-forward extension of the single plane system, where the height measurement itself is still carried out on the first projection plane only. Observations on the second plane are solely used as a detector to separate water-level induced projection variations from slope induced ones. In Section 4.2, a method based on a complete geometric reconstruction of the reflected laser light sheet is shown, wherein the deformed reflected laser sheet is used for the calculation of the water surface profile by intersecting it with the projected laser sheet. This technique allows for the determination of water height profiles and surface normal vectors simultaneously.

Figure 7: A test setup for the wave detection system with two projection planes.

4.1. System with wave-detection

As mentioned before, the main drawback of the single plane system is its limitation to measuring on still water surfaces or on regular one-dimensional wave patterns. A straight-forward solution to this problem can be achieved by the introduction of a second projection plane into the system (see Figure 5). The height measurement itself is performed in the same manner as the single plane system. For this task observations of the laserline on the front projection plane are taken as a basis. Additionally, measurements of the laserline on the rear plane are acquired, which are only used to recognize height measurements on locally and temporarily horizontal surface patches (Figure 7 and 6b) thus allowing to avoid wave affected measurements (Figure 6a).

The separation between valid and invalid measurements can be performed by analysing the proportion between the measured vertical line shifts on the front and rear plane. A measurement is valid if equation (12) is fulfilled.

$$\frac{H_{P1\,\max} - H_{P1\,\min}}{H_{P2\,\max} - H_{P2\,\min}} = \frac{H_{P1} - H_{P1\,\min}}{H_{P2} - H_{P2\,\min}} \qquad (11)$$

To allow a proper projection on both planes and a discretization along the profile, the front plane was constructed as a vertical grating (see Figure 7). Some sections of the laser lightsheet are projected onto the grating, while the remaining sections pass through the front plane and are projected onto the rear plane. This system design turned out to be superior to using semi-transparent material for the front projection plane, as this is accompanied by an undesired widening of the laser light sheet. Instead, the reflected profile is represented by the intersection points of the vertical grating edges and the detected laser line segments, with the number of measurements per profile determined by the front plane grating.

In addition to the end points of the projected laser line on the front grating, segments on the rear plane are used to allow for the separation of water-level induced and slope-induced projection height variations. As one can see in the schematic front view in Figure 8, wave-induced lateral water surface tilts will lead to a shift of the projection on the rear plane.

Using the system calibration parameters, the position of the projection on the rear plane can be predicted from the projection of a line segment onto the front grating, if the water surface is horizontal. A projection on the rear plane, with deviates from the pre-

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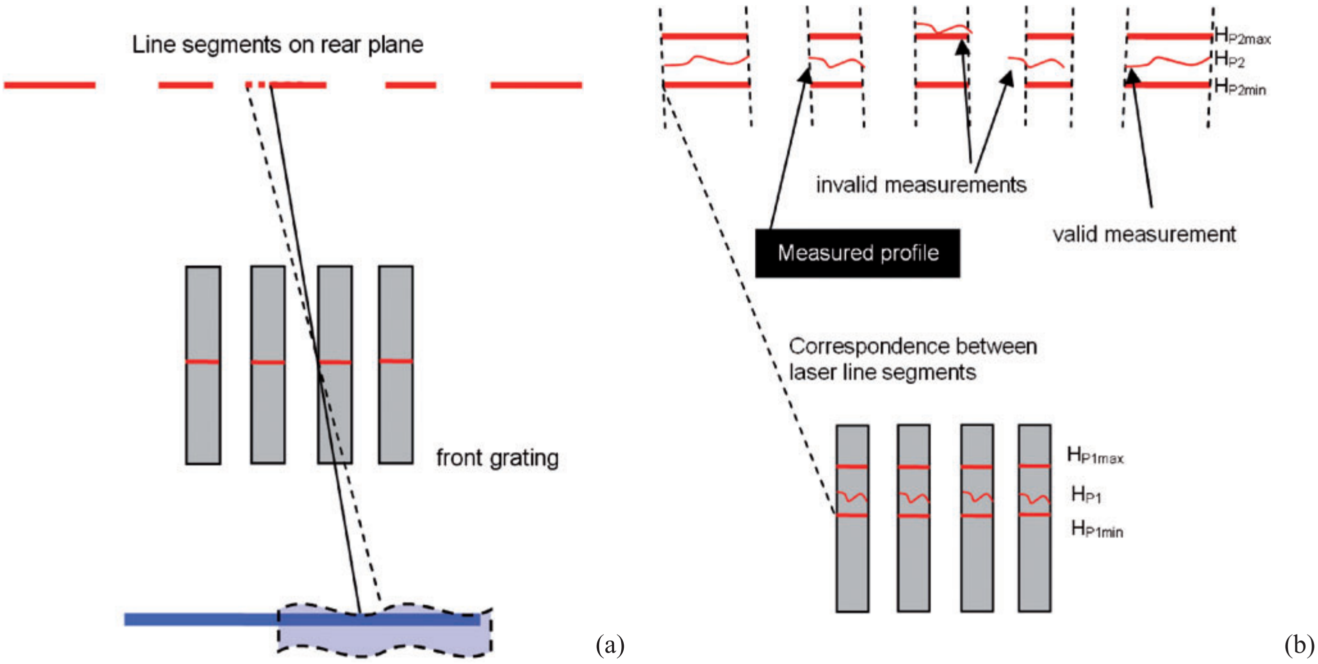


Figure 8: (a) Wave-induced shifts on rear plane. (b) Validation of measurements on rear plane.

dicted position, indicates an invalid measurement, which is affected by waves. In this case, equation (12) is not fulfilled.

In practical use, the system will permanently measure the position of the projected line segments on the front plane. Line segments can be extracted by a simple connectivity analysis. Subpixel-accuracy measurement is performed by a centroid operator in vertical and by the moment-preservation algorithm (Tabatabai and Mitchell 1984) in horizontal image coordinate direction at a precision of 0.1–0.2 pixel. The measured line end points coordinates are corrected by effects of lens distortion before passing to further analysis.

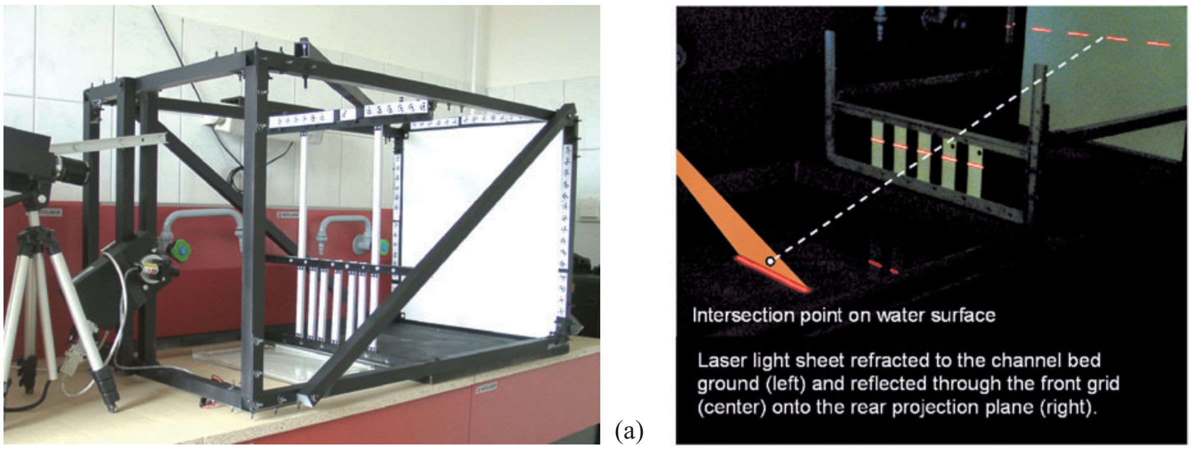
Measurements are indexed valid if equation (12) is fulfilled within a pre-set tolerance and invalid if not. The water level height for valid measurements (i.e. measurements negligibly affected by waves) is computed in the same way as outlined in Section 2. The system calibration procedure is similar to the procedure of the single plane system (Section 2). The method works well if the lateral wave effects are by one order of magnitude smaller than those from longitudinal waves (Mulsow 2007). The accuracy is similar to the accuracy obtained for the single plane system.

4.2. Full-grade system

A major limitation of the system with wave detection as described before is in the fact that it delivers measurements only at time instances with a horizontal surface. An essential step forward can be achieved by the full geometric reconstruction of the reflected laser lightsheet segments and their intersection with the projected laser lightsheet. This full-grade system is shown in Figure 9a. The main parts are a Sony XC700 Firewire camera equipped with an optical band pass filter, a laser line projector on a rotating stepper motor and the two projection planes. All parts are integrated in a rugged aluminium frame. A homogeneous laser light sheet with an almost rectangular intensity profile is generated by a 35mW diode with a Powell lens.

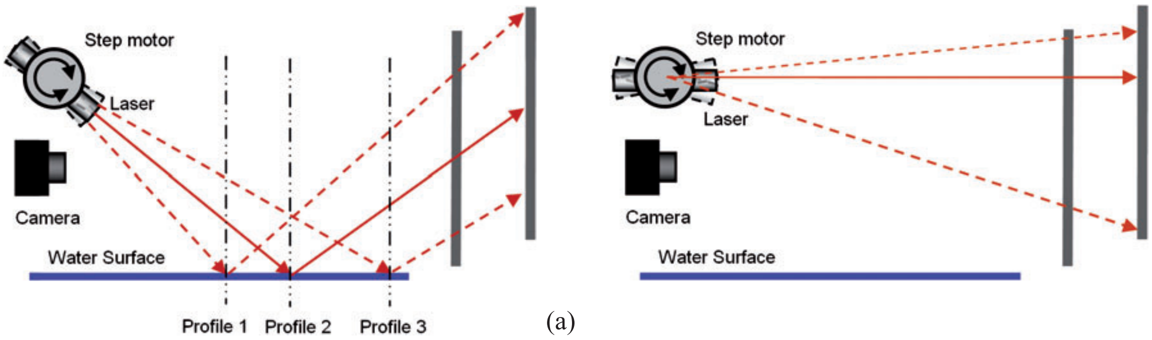
A discretized water surface profile is represented by the intersection of the vectors between the corresponding end points of the reflected laser lightsheet on both planes and the projected laser lightsheet (see Figure 9b). Reference targets on the projection planes allow for the transformation of the line observations from image space to 3D object space. The number of profile points is two times the number of gaps in the front plane grating. It should be chosen in a way that an overtake of projections on the rear projection plane resulting from lateral water level changes is avoided (i.e. that the maximum expected movement is smaller than the line spacing).

As an additional component, a stepper motor rotating the projection unit was integrated into the system (see Figure 10). This addition allows for variable settings of the incidence angle of the laser light sheet and a sequential measurement of multiple parallel profiles (see Figure 10a). The angular rotation sensor of the stepper motor can determine the motor position with a precision of 0.001°. The detected laser line end points can be transformed into the 3D frame system with a precision of 0.2 mm on the vertical grating and 0.3 mm on the rear plane.



(b)

Figure 9: (a) Experimental system layout. (b) Measurements principle.



(b)

Figure 10: (a) System with step motor. (b) Data acquisition for calibration – direct projection at different rotation angles.

4.2.1. *Calibration*

The calibration of the full-grade configuration becomes rather complex and requires a large number of parameters, including the determination of the motor rotation axis, the orientation of the laser plane and the system orientation with respect to a still water surface. To allow for an efficient implementation, the mathematical model is based on a system of constraint equations rather than a set of observation equations. The model describes the variable projection of the laser light sheet on to the planes with a set of 14 parameters (see Table 2).

Based on this model a calibration procedure was implemented. Image sequences of different incidence angles (at least two) of the laser sheet on the projection planes are taken as a basis for the calibration. The resulting line segments on the vertical grating and the rear plane are first projected directly, i.e. without reflection from the waters surface. The mathematical estimation of the calibration parameters is implemented as least-squares adjustment in the Gauss–Markov model. The complete calibration procedure takes place fully automatically in an integrated software package. A detailed description can be found in Mulsow et al. (2006).

Table 2: Full-grade system calibration parameters.

Parameter	Description
A_i, B_i, C_i, D_i	Parameters (of which 3 are independent) of plane i ($i = 1, \dots, n$)
φ_i	Rotation angle of the motor
V_X, V_Y, V_Z	Direction of the motor rotating axis
R_X, R_Y, R_Z	Origin of the motor rotating axis
N_X, N_Y, N_Z	Normal direction of the reference plane for the rotation angle ($\varphi = 0$ in the stepper motor system)
A_W, B_W, C_W, D_W	Parameters of still water surface

4.2.2. *Measurement*

The actual water surface height profile measurement can be performed in realtime after the calibration of the system. The laser line segments can be detected in the images by a modified Hough transformation.

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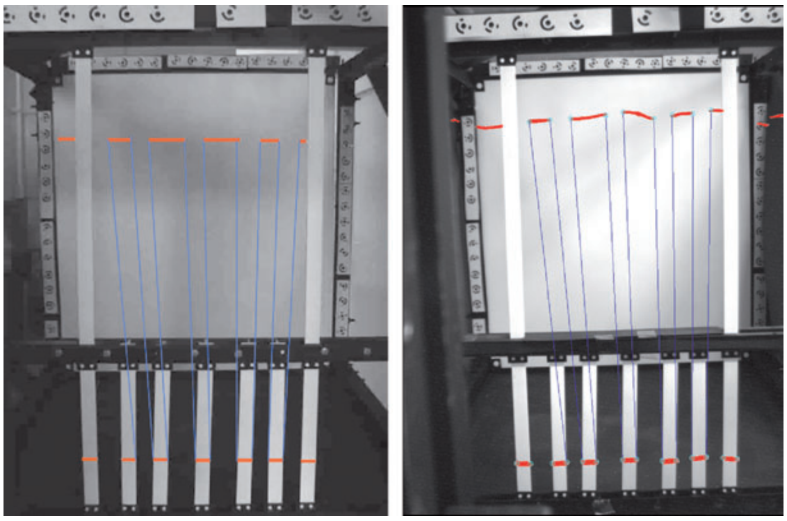

Figure 11: Corresponding laser ends (left: still surface; right: dynamic surface).

In the setup the laser lines will just appear nearly horizontal in the image. Improving the detection reliability and processing time of the Hough transformation the parameter space is limited for angles around −20 to +20 degrees. Subsequently, the line segment end points are determined with subpixel accuracy, and corresponding laser ends of the front and the rear plane are connected for every image. Initial correspondences between front and rear plane points are established during the calibration procedure and then maintained by tracking. The image coordinates are corrected for lens distortion and transformed into 3D object space by a rectification using the known 3D coordinates of reference targets on each plane. 3D vectors are generated by connecting corresponding points on the front and rear plane. These vectors represent the reflected laser lightsheet. An intersection of these vectors with the original laser lightsheet delivers 3D coordinates on the water surface. Simultaneously, the water surface normal vector can be reconstructed from the components of the reflected vectors and the lightsheet. Figure 11 shows two screenshots from the processing software package.

4.2.3. Practical tests

To determine the practical accuracy of the system, the water level in a test basin was sequentially raised by adding exactly 100 ml per step resulting in a level change of 3.20 mm. The measurement was carried out on still water level. An RMS of water level difference determination of about 0.20 mm could be achieved. The result shows, that the precision of the water level measurement is worse than the precision obtained from the single plane system shown in Maas et al. (2003). The reason for this can be seen in the different calculation principles for water level height determination. In the single plane system, the height variations of the laser lines correspond directly with the water level. In contrast, in the full-grade system the determination of the water level is based on the intersection of vector and laser sheet. This indirect method is subject to more disturbing effects, for instance the laser endpoint measurements on both projection planes in contrast to the observation of the laser line on a single plane.

However, while delivering worse results in the case of still water surfaces, the full-grade system is superior in its actual domain, the determination of water surfaces under the presence of wave patterns. Because of the nature of the measurement (intersection of vector and laser sheet), the accuracy figures for still water can be transferred to the measurement on moving water surfaces, at the most with a slight decrease due to temporary widening of the projected laser lines. A thorough evaluation of the measurement accuracy on moving surfaces with external reference measurements could not be performed so far due to the lack of a suitable reference measurement technique.

5. Conclusion

Several extensions of the principle of optical triangulation have been shown to adapt the technique to measurements on reflecting surfaces. Stepwise modifications of the system configuration and the data processing procedure were realized to meet different requirements on applicability, accuracy, reliability and system complexity. As a result, a chain of four different optical triangulation system configurations for measurements on water surfaces has been developed and presented:

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System	Pro's	Con's
Single Plane system		
Single image technique (Section 2)	+ simple system setup + very simple data processing + high accuracy on still surfaces	– only applicable to still water surfaces without any waves
Accumulator image technique (Section 3)	+ simple system setup + simple data processing + high accuracy on still surfaces	– only applicable when measuring on moving surfaces with small and regular one-dimensional wave patterns
Double Plane system		
System with wave detection (Section 4.1)	+ moderate data processing effort + high accuracy on still and moving surfaces	– more complex system configuration – measurement rate reduced by acceptance of measurements at horizontal water surface (wave peaks and valleys) only
Full-grade system (Section 4.2)	+ delivers measurements under presence of waves without limitations to certain wave patterns	– more complex system configuration with high requirements to geometric stability – elaborate calibration procedure – complex data processing – accuracy affected by stability of system component alignment

All four system configurations have their application fields in experimental hydromechanics and related fields. The water level height determination accuracy potential achieved in practical tests varies between 0.03 mm and 0.2 mm, depending on the system configuration and the complexity of the water surface and the system design. A further optimization of the system design, the data processing chain and the calibration procedures is envisaged. Another future goal is taking the step from laboratory vessels to real nature measurements, which will require several modifications in the system configuration.

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