TESTING LOCAL LORENTZ INVARIANCE WITH HIGH-ACCURACY ASTROMETRIC OBSERVATIONS

S. A. Klioner*, S. Zschocke, M. Soffel, A. V. Butkevich

Lohrmann-Observatory, Dresden Technical University, 01062 Dresden, Germany
**E-mail: Sergei.Klioner@tu-dresden.de*

This paper summarizes the analysis of the consequences of the violation of the Local Lorentz Invariance (LLI) on astrometric observations. We demonstrate that from the point of view of the LLI astrometric observations represent an experiment of Michelson-Morley type. The future high-accuracy astrometric projects (e.g., Gaia) will be used to test the LLI.

Keywords: Local Lorentz Invariance, aberration, astrometry

1. Introduction

Motivated by ideas about quantum gravity, a tremendous amount of efforts over the past decade has gone into testing Local Lorentz Invariance (LLI) in various regimes [5]. This paper summarizes the framework allowing one to test LLI using high-accuracy astrometric observations. The basic idea is that the usual special-relativistic aberrational formulas used in the corresponding relativistic models is a direct consequence of the Lorentz transformations [1]. A generalization of that aberrational formula obtained with generalized Lorentz transformation contains parameters (similar to the Mansouri-Sexl ones) and can be directly used to test LLI [2]. Especially the future ESA mission Gaia [6] will provide a lot of high-accuracy astrometric data that will be used to make independent tests of LLI.

2. Parametrized coordinate transformations

The transformation between preferred coordinates (T, X^a) and non-preferred ones (t, x^i) read [4]:

$$c\,t = \Lambda^0_0\, c\,T + \Lambda^0_a\, X^a, \tag{1}$$

$$x^i = \Lambda^i_0\, c\,T + \Lambda^i_a\, X^a, \tag{2}$$

where

$$\Lambda^0_0 = a - b\,(\boldsymbol{\epsilon}\cdot\boldsymbol{K})\,, \tag{3}$$

$$\Lambda^0_a = d\,\epsilon^a + (b-d)\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{K}}{K^2}\,K^a\,, \tag{4}$$

$$\Lambda^i_0 = -b\,K^i\,, \tag{5}$$

$$\Lambda^i_a = d\delta^{ia} + (b-d)\frac{K^a\,K^i}{K^2}\,. \tag{6}$$

Here, $\boldsymbol{K} = \boldsymbol{V}/c$, $\boldsymbol{V}$ is the velocity of the origin of the system (t, x^i) with respect to (T, X^a), and a, b, d, and $\boldsymbol{\epsilon}$ are arbitrary functions of $\boldsymbol{K}$.

3. Basic aberrational formulas

Let us consider the relation between directions of light propagation of a given light ray in the preferred frame S^a and that in the non-preferred one s^i. Here we consider the same light ray as seen by an observer at rest relative to (T, X^i) and another observer (co-located with the first one) at rest relative to (t, x^i). Taking the differentials along the light ray we have

$$S^a = \frac{1}{c}\,\frac{dX^a}{dT}, \tag{7}$$

for the preferred frame ($\boldsymbol{S}\cdot\boldsymbol{S} = 1$) and

$$p^i = \frac{1}{c}\,\frac{dx^i}{dt}\,, \tag{8}$$

$$s^i = p^i/|\boldsymbol{p}| \tag{9}$$

for the non-preferred frame. The last normalization is needed since the light velocity is not equal to c in the non-preferred frames and therefore vector p^i is not an Euclidean unit vector. Using the coordinate transformations between (T, X^a) and (t, x^i) given above we get the transformations between $\boldsymbol{S}$ and $\boldsymbol{s}$ in closed form:

$$\boldsymbol{s} = \frac{f\,\boldsymbol{S} + (1-f)\,\dfrac{\boldsymbol{K}\,(\boldsymbol{K}\cdot\boldsymbol{S})}{K^2} - \boldsymbol{K}}{\left(f^2 + K^2 - 2\,\boldsymbol{K}\cdot\boldsymbol{S} + (1-f^2)\,\dfrac{(\boldsymbol{K}\cdot\boldsymbol{S})^2}{K^2}\right)^{1/2}}\,, \tag{10}$$

$$\begin{aligned}\boldsymbol{S} = \boldsymbol{K} &+ \left(\left(f^2\,(\boldsymbol{K}\cdot\boldsymbol{s})^2 + (1-K^2)\,\left(K^2 - (\boldsymbol{K}\cdot\boldsymbol{s})^2\right)\right)^{1/2} - f\,K\,(\boldsymbol{K}\cdot\boldsymbol{s})\right)\\ &\times \frac{K}{K^2 - (1-f^2)\,(\boldsymbol{K}\cdot\boldsymbol{s})^2}\,\left(\boldsymbol{s} - (1-f)\,\frac{\boldsymbol{K}\,(\boldsymbol{K}\cdot\boldsymbol{s})}{K^2}\right)\,,\end{aligned} \tag{11}$$

where $f = d/b$. In the limit of special relativity one gets the normal special-relativistic aberrational formulas. Note that the transformation between $\boldsymbol{S}$ and $\boldsymbol{s}$ depends only on f and does not depend on a and ϵ. This demonstrates that the aberrational formula tests the same properties of the Lorentz transformation as the Michelson-Morley experiment, that is, the isotropy of light velocity.

4. Realistic aberrational formula

In practice the aberrational formula entering relativistic models [1] corrects for aberration due to the velocity of the observer relative to the barycenter of the solar system. The solar system barycentric reference system is not usually assumed to be the preferred system in the sense of the LLI. Therefore, we have to consider three reference systems: one preferred system (T, X^a), two non-preferred ones – system (t, x^i) attached to the barycenter of the solar system, and one more system $(t', x^{i\prime})$ attached to the observer. The transformation between the preferred and non-preferred coordinates are given above. The only parameter of the transformations is the velocity of the origin of the non-preferred coordinates in the preferred ones. The velocity of the origin of (t, x^i) relative to (T, X^a) is $\boldsymbol{V}$. The velocity of the origin of $(t', x^{i\prime})$ relative to (T, X^a) is $\boldsymbol{V}'$ and relative to (t, x^i) is $\boldsymbol{v}$. The relation between these three velocities follows from the coordinate transformations and reads ($\boldsymbol{K}' = \boldsymbol{V}'/c$, $\boldsymbol{k} = \boldsymbol{v}/c$)

$$\boldsymbol{K}' = \boldsymbol{K} + \frac{a}{d}\,(1 - \boldsymbol{\epsilon}\cdot\boldsymbol{k})^{-1}\,\left(\boldsymbol{k} - (1-f)\,\frac{\boldsymbol{k}\cdot\boldsymbol{K}}{K^2}\,\boldsymbol{K}\right)\,. \tag{12}$$

Finally, denoting $\boldsymbol{s}'$ the direction of light relative to $(t', x^{i\prime})$, combining (10)–(11) written for two non-preferred coordinate systems and using (12) one gets

$$\boldsymbol{s}' = \mathbf{P}\,\boldsymbol{s}'', \tag{13}$$

$$\begin{aligned}\boldsymbol{s}'' = {} & \boldsymbol{s} + (\boldsymbol{s}\cdot\boldsymbol{k})\,\boldsymbol{s} - \boldsymbol{k} - \frac{1}{2}\,(\boldsymbol{s}\cdot\boldsymbol{k})\,\boldsymbol{k} - \frac{1}{2}\,k^2\,\boldsymbol{s} + (\boldsymbol{s}\cdot\boldsymbol{k})^2\;\boldsymbol{s}\\ & -\eta\,(\boldsymbol{s}\cdot\boldsymbol{K})\,\boldsymbol{k} - \eta\,(\boldsymbol{s}\cdot\boldsymbol{k})\,(\boldsymbol{k}+\boldsymbol{K}) + \eta\;(\boldsymbol{s}\cdot\boldsymbol{k})^2\;\boldsymbol{s} + 2\eta\,(\boldsymbol{s}\cdot\boldsymbol{k})\,(\boldsymbol{s}\cdot\boldsymbol{K})\,\boldsymbol{s}\\ & +\mathcal{O}(c^{-3})\,,\end{aligned} \tag{14}$$

where $\mathbf{P}$ is an orthogonal matrix of the Thomas-like precession, and $\eta = \frac{1}{2} - \beta + \delta$, where β and δ are the usual Mansouri-Sexl parameters ($f = d/b = 1 + (\eta - \frac{1}{2})\,K^2 + \mathcal{O}(c^{-4})$, $\eta = 0$ in special relativity). The Thomas precession plays no role here since pure rotation cannot be observed in astrometry (cannot be distinguish from the local rotation of the observer) and the operational reference system attached to the observer is chosen not to rotate with respect to the barycentric reference system. Eq. (14) gives the generalized aberrational formula. In addition to the barycentric velocity of the observer $\boldsymbol{v} = c\,\boldsymbol{k}$, this formula contains the parameter η and the velocity of the solar system barycenter relative to the preferred frame $\boldsymbol{V} = c\,\boldsymbol{K}$. Taking the value of $\boldsymbol{V}$ from the dipole of the cosmic microwave background, one can determine η. Alternatively, both η and $\boldsymbol{V}$ can be determined from astrometric observations. More details on the derivation and interpretation of this formula will be given elsewhere [3].

Acknowledgments

This work was partially supported by the BMWi grants 50 QG 0601 and 50 QG 0901 awarded by the Deutsche Zentrum für Luft- und Raumfahrt e.V. (DLR).

References

1. S.A. Klioner, Astron.J., 125, 1580 (2003)
2. S.A. Klioner, In: Lasers, Clocks and Drag-Free: Exploration of Relativistic Gravity in Space, H. Dittus, C. Lämmerzahl, S.G. Turyshev (eds.), 399 (Springer, Berlin, 2007)
3. S.A. Klioner et al., Gen.Rel.Grav., in preparation (2010)
4. R. Mansouri, R.U. Sexl, Gen.Rel.Grav., 8, 497 (1977)
5. D. Mattingly, Living Rev. Relativity, 8, 5 (2005)
6. M.A.C. Perryman et al., Astron.Astrophys. 369, 339 (2001)