

Towards sub-microarcsecond models for relativistic astrometry

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ABSTRACT. Astrometric space missions like Gaia have stimulated a rapid advance in the field of relativistic astrometry. Present investigations in that field aim at accuracies significantly less than a microarcsecond. We review the present status of relativistic astrometry. As far as the problem of light propagation is concerned we face two problems: the form of the BCRS metric and solutions to the light-ray equation. Finally, work in progress in that field is briefly mentioned.

1. INTRODUCTION

The Hipparcos astrometric space mission has determined positions (proper motions) of some 120000 stars with a precision of a milliarcsecond (mas/y). The forthcoming mission Gaia is expected to reach a level of up to some μas for one billion stars depending on stellar brightness. Proposed missions like the 'Nearby Earth Astrometric Telescope' (NEAT) envisage an accuracy of 50 nanoarcseconds (nas). This stunning progress in astrometry implies the necessity to formulate appropriate relativistic astrometric models with an intrinsic accuracy of 1 nas. One is still far from that goal but there has been a lot of work in that direction.

Any relativistic astrometric model based on Einstein's theory of gravity employs one or several different reference systems (4-dimensional coordinate systems) to describe the location and motion of gravitating bodies and the light trajectory from the emitter to the observer. In one of these well related coordinate systems one has to formulate the astrometric observable as a coordinate independent quantity (i.e., as a scalar). A model for a concrete astrometric mission will contain a certain set of coordinate-dependent parameters that have to be fitted from observational data. The reference system then becomes the corresponding reference frame, materialized e.g., by a stellar (or quasar) catalog.

If physically relevant local coordinates, co-moving with the observer are introduced, then it might be possible to derive observables from coordinate quantities as it is the case in the Gaia Relativistic Model (GREM) developed by Klioner (2003a). An astrometric model thus involves the following constructions: i) one or several space-time reference systems, i.e., space-time coordinates and the corresponding metric tensor, ii) the trajectories of light-rays, iii) the trajectories of the observer and gravitating bodies, and iv) the calculation of astrometric observables. This contribution focuses on the first two aspects i) and ii).

2. APPROXIMATION METHODS FOR REFERENCE SYSTEMS

To construct a space-time reference system with a metric tensor as solution of Einstein's field equations for real high precision astrometric observations one resorts to approximation schemes, either to a post-Newtonian hierarchy (weak field, slow motion) or to the post-Minkowskian approximation (weak field). For light rays the post-Newtonian (PN) metric is of the form

$$g_{00} = -1 + \frac{2w}{c^2}, \quad g_{0i} = 0, \quad g_{ij} = \left(1 + \frac{2w}{c^2}\right) \delta_{ij}, \quad (1)$$

where w is the gravitational potential. The corresponding post-post Newtonian (2PN) metric for light rays can be written in the form

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4}, \quad g_{0i} = -\frac{4w^i}{c^3}, \quad g_{ij} = \left(1 + \frac{2w}{c^2} + \frac{2w^2}{c^4}\right) \delta_{ij} + \frac{4}{c^4} q_{ij}, \quad (2)$$

where w^i is the gravito-magnetic potential induced by moving or rotating masses. For one body at rest (rotating, vibrating, with arbitrary shape and decomposition) the exterior metric is known for both the post-Newtonian (Blanchet & Damour, 1989) and the post-Minkowskian case (Damour & Iyer, 1991), and it has been demonstrated that the metric in both cases is determined by only two families of multipole moments, M_L (mass-moments) and S_L (spin-moments).

In what follows we will give an overview of the present status of the theory of light propagation.

3. LIGHT PROPAGATION IN THE FIELD OF MASS MONOPOLES

Light propagation in the field of mass monopoles with constant velocity: Explicit post-Newtonian solutions for the light propagation in the case of uniformly moving bodies, where the position of the body is given by $\mathbf{x}_A(t) = \mathbf{x}_A^{\text{eph}}(t_A^0) + \dot{\mathbf{x}}_A^{\text{eph}}(t_A^0)(t - t_A^0)$, were derived by Klioner (1989); here $\mathbf{x}_A^{\text{eph}}$ and $\dot{\mathbf{x}}_A^{\text{eph}}$ are the actual position and velocity of body A taken from an ephemeris for some instant of time t_A^0 . Following a suggestion by Hellings (1986), Klioner & Kopeikin (1992) have argued that in order to minimize the errors in the light propagation the free parameter t_A^0 should be chosen to coincide with the moment of closest approach between the body and the light ray. Furthermore, Klioner (2003b) has suggested a straightforward way to calculate the effect of uniform translational motion of a body on the light propagation by a Lorentz transformation of the light trajectory in a reference system where the body is at rest. In this way Klioner (2003b) has derived a post-Minkowskian solution for the light propagation in the field of a mass monopole moving with constant velocity. It has been demonstrated that the more general solution of Kopeikin & Schäfer (1999) can be reproduced in the limiting case of uniform motion.

Light propagation in the field of arbitrarily moving mass monopoles: A rigorous solution of the problem in the first post-Minkowskian approximation has been found by Kopeikin & Schäfer (1999), where the geodetic equations for photons are integrated using retarded potentials. The numerical accuracy of various approaches has been investigated by Klioner & Peip (2003). Especially, Klioner & Peip (2003) have numerically compared various available solutions for the light propagation for observations made within the Solar system. The authors used both artificial orbits for deflecting bodies as well as planetary trajectories taken from JPL solar system ephemerides. It has been demonstrated that the simple solution obtained in (Klioner, 1991) and (Klioner & Kopeikin, 1992) is sufficient for an accuracy of about 2 nas, provided that $\mathbf{x}_A^{\text{eph}}$ and $\dot{\mathbf{x}}_A^{\text{eph}}$ are taken in the optimal way.

2-PN Light propagation in the field of mass monopoles at rest: The light trajectories in the Schwarzschild field, that means in the field of a single mass monopole at rest, can be found in an analytically closed form as it has been demonstrated at the first time by Hagihara (1931); for a re-derivation we refer to Chandrasekhar (1983). However, this exact analytical solution is not convenient for data reduction of astrometric observations, since the light curve is not given by an explicit time dependence of the coordinates of the photon $x(t), y(t)$ but only implicitly in terms of $y(x)$.

From a practical point of view, post-post-Newtonian effects in the light propagation in the Schwarzschild field have been considered by many authors. An important progress has been made by Brumberg (1991) who has found an explicit post-post-Newtonian solution for light trajectories in the Schwarzschild field as function of coordinate time in a number of coordinate gauges. Generalizations of that solution for the case of the parametrized post-post-Newtonian metric have been given by Klioner & Zschocke (2010). The latter authors have investigated in great detail the numerical magnitudes of various post-post-Newtonian terms and formulated practical algorithms for highly-effective computation of the post-post-Newtonian effects. It has been demonstrated that the so-called enhanced post-post-Newtonian terms are due to a physically inadequate choice of the parametrization of the light rays; see also Bodenner & Will (2003).

Two alternative approaches to the calculation of propagation times and directions of light rays have been formulated recently. Both approaches allow one to avoid explicit integration of the geodetic equations for light rays. The first approach (Le Poncin-Lafitte, Linet & Teyssandier, 2004; Teyssandier & Le Poncin-Lafitte, 2008) is based on the use of Synge's world function. Several applications of this approach have been published: higher post-Newtonian approximations in spherically symmetric gravitational fields and post-Newtonian effects in the gravitational field with multipole moments. Another approach based on the eikonal concept has been developed by Ashby & Bertotti (2010) to investigate the light propagation in the field of a spherically symmetric body. All the results of these authors confirm the conclusions and formulas obtained in Klioner & Zschocke (2010).

2-PN Light propagation in the field of moving mass monopoles: There are only very limited results dealing with moving deflecting bodies in the post-post-Newtonian approximation. Especially, Brüggmann (2005) has investigated some effects of the light propagation in the post-post-Newtonian gravitational field of a system of two bodies, where two important approximations are used. First, both the light source and the observer are assumed to be located at infinity in an asymptotically flat space. Second, some of the results were obtained in form of an expansion in powers of the ratio between the distance between two bodies and the impact parameter of the light ray with respect to the center of mass of the two-body system. These assumptions imply, however, that the results are not applicable to observations in the solar system.

4. LIGHT PROPAGATION IN THE FIELD OF MASS QUADRUPOLES

Light propagation in the quadrupole field of bodies at rest: Analytical solutions of light deflection in a quadrupole gravitational field have previously been investigated by many authors. However, for the first time the full analytical solution for the light trajectory in a quadrupole field has been obtained by Klioner (1991), where an explicit time dependence of the coordinates of a photon and the solution of the boundary value problem for the geodesic equation has been obtained. These results were confirmed by a different approach in Le Poncin-Lafitte & Teyssandier (2008), while a simplified expression with μas accuracies has been derived in Zschocke & Klioner (2011).

Light propagation in the quadrupole field of arbitrarily moving bodies: The light-deflection at moving massive bodies, having monopole and quadrupole structure, has been investigated by Kopeikin & Makarov (2007), where the quadrupole term is taken into account in local coordinates of the body in Newtonian approximation. Using the harmonic gauge, the linearized Einstein equations are inhomogeneous wave equations and a general solution is given in terms of a multipole expansion (Thorne, 1980; Blanchet & Damour, 1986). In Kopeikin & Makarov (2007) the geodesic equation is rewritten into a considerably simpler form. Using a special integration method, they succeeded to integrate analytically the geodesic equation by neglecting all terms that contribute by less than $1 \mu\text{as}$.

5. LIGHT PROPAGATION IN THE FIELD OF BODIES WITH SPIN

Light propagation in the field of bodies at rest with a spin-dipole: The first explicit post-Newtonian solution of the light trajectory in the gravitational field of massive bodies at rest possessing a spin dipole has been obtained by Klioner (1991). This solution provides all the details of light propagation, especially the explicit time dependence of the coordinates of the photon and the solution of the corresponding boundary value problem. Kopeikin (1997) has generalized the solution for the case of motionless bodies possessing any set of time-independent spin (and mass) moments, and it has been shown that the expression in Klioner (1991) and Kopeikin (1997) agree with each other.

Light propagation in the field of arbitrarily moving bodies with spin-dipole: Kopeikin & Mashhoon (2002) have derived formulas for the case of light propagation in the field of arbitrarily moving bodies possessing mass monopole and spin dipole.

6. LIGHT PROPAGATION IN THE FIELD OF HIGHER MASS AND SPIN MULTIPOLE MOMENTS

Mass and spin multipole moments at rest: A systematic approach to the integration of light geodesic equation in the stationary post-Newtonian gravitational field of an isolated system of N bodies having a complex but time-independent multipole structure has been worked out in Kopeikin (1997) and Kopeikin et al. (1999). Especially, the work of Kopeikin (1997) represents a generalized solution for the case of motionless bodies possessing any set of time-independent mass and spin moments, that is M_L and S_L , respectively. Later, in Kopeikin, Korobkov & Polnarev (2006) and Kopeikin & Korobkov (2005), the propagation of light rays in the field of localized sources which are completely characterized by time-dependent mass and spin multipoles, $M_L(t)$ and $S_L(t)$, respectively, has been investigated. Kopeikin, Korobkov & Polnarev (2006) and Kopeikin & Korobkov (2005) have found an analytical solution for the light propagation in such gravitating systems.

7. WORK IN PROGRESS

Presently several groups try to extend relativistic astrometry to still higher accuracies. Our group presently concentrates on the 2PN field of arbitrarily moving bodies endowed with arbitrary mass- and spin multipole moments where the metric in harmonic gauge is given by (2). There have been first attempts to tackle this problem (e.g., Xu & Wu 2003; Minazzoli & Chauvineau 2009) but they are far from being complete.

Problems, that have been ignored in these preliminary papers, are related with the internal structure of the bodies. For a single body at rest these problems are well understood for both the post-Newtonian and the post-Minkowskian case (Blanchet & Damour, 1989; Damour & Iyer, 1991) where many structure dependent terms appear in intermediate calculations that cancel exactly in virtue of the local equations of motion or can be eliminated by corresponding gauge transformations. However, for the post-linear case the situation is still unclear. In course of our studies for the general problem just mentioned we found that even for the spherically symmetric case of a single body the complete derivation of the exterior metric (the Schwarzschild metric) is interesting. In a forthcoming paper (Klioner & Soffel, 2014) we will show how such structure-dependent terms cancel and one ends up with the Schwarzschild solution in harmonic gauge.

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