Algebraic approach to bare nucleon matrix elements of quark operators

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An algebraic method for evaluating bare nucleon matrix elements of quark operators is proposed. Thereby, bare nucleon matrix elements are traced back to vacuum matrix elements. The method is similar to the soft pion theorem. Matrix elements of two-quark, four-quark and six-quark operators inside the bare nucleon are considered.

I. INTRODUCTION

One ultimate goal of contemporary strong interaction physics is to find a comprehension of the physical properties of hadrons by means of the underlying theory of quantum chromodynamics (QCD). Among several methods which provide a link between QCD (quark and gluon) degrees of freedom and the hadronic spectrum are the QCD sum rules which have to be considered as important nonperturbative approach in understanding the physical observables of hadrons. The sum rule method, first developed for the vacuum [1], has later been extended to finite density [2–4], finite temperature [5,6], and mixed finite density and finite temperature [7]. Within the QCD sum rule approach, and more generally in hadron physics, pions and nucleons have to be considered as important degrees of freedom because the pion is the lightest (Goldstone) meson, while the nucleon is the lightest baryon. In-medium QCD sum rules provide a direct way to relate changes of hadronic properties to changes of the various condensates, i.e. nucleon and pion expectation values of quark and gluon fields. Therefore, expectation values of a local operator $\hat{\mathcal{O}}$ taken between these states, $\langle \pi_{\text{phys}} | \hat{\mathcal{O}} | \pi_{\text{phys}} \rangle$ and $\langle N_{\text{phys}} | \hat{\mathcal{O}} | N_{\text{phys}} \rangle$, need to be known. However, the predictive power of the QCD sum rule method in matter meets uncertainties when evaluating condensates, especially higher mass dimension condensates inside the nucleon. Accordingly, the exploration of nucleon matrix elements is presently an active field of hadron physics, cf. [8,9].

If the operator $\hat{\mathcal{O}}$ consists of hadronic fields, then in principle one needs an effective hadronic theory which describes the interaction between pions and nucleons, respectively, and the hadrons from which the operator $\hat{\mathcal{O}}$ is made of for evaluating these matrix elements. However, if one is concerned with pion matrix elements then the use of

soft pion theorems [10–14] gives in general good estimates for such expressions, which are related to several so called low-energy theorems like Goldberger-Treiman relation [15], Adler-Weisberger sum rule [16] or Cabibbo-Radicati sum rule [17]. These soft pion theorems as algebraic tools are based on the hypothesis of partially conserved axial vector current (PCAC) [18–20] and postulated current algebra commutation relations [12,13], and allow in general to trace the pion matrix elements of operators made of effective hadronic fields back to vacuum matrix elements. A feature of the soft pion theorems is that they can also be deduced within quark degrees of freedom. Accordingly, pion matrix elements of quark field operators have also been evaluated by means of the soft pion theorem (if we speak about the soft pion theorem then we mean the special theorem considered in the Appendix A which is the relevant one in our context) expressing the pion field and axial vector current, respectively, by interpolating fields made of quark degrees of freedom [21,22].

After discovering the powerful method of current algebra for mesons several attempts have been made to investigate the possibilities for extending this algebra to the case of baryons. Especially, the analog hypothesis of a partially conserved baryon current (PCBC) and the related (and postulated) baryon current algebra has been investigated long time ago [23–28]. These attempts focussed on the construction of baryon currents by products of nucleon fields. Furthermore, in [29] this procedure has been studied by considering baryon currents made of quark degrees of freedom where several relations between form factors, e.g. baryon-meson vertex form factors, have been obtained. However, it turned out that, while the PCAC directly leads to the mentioned soft pion theorems for evaluating pion matrix elements, the PCBC does not provide a comprehensive algebraic theorem for evaluating nucleon matrix elements. Therefore, up to now for evaluating nucleon matrix ele-

elements of an operator $\hat{\mathcal{O}}$ consisting of quark fields more involved tools are needed like chiral quark model [8], lattice evaluations [9], or Nambu-Jona-Lasinio model [30]. From this point of view it seems very tempting to look for an algebraic approach for evaluating nucleon matrix elements in analogy to the soft pion theorem. Here, by using directly the nucleon field instead the nucleon current, we propose such an algebraic approach for evaluating matrix elements of quark operators taken between a bare nucleon, i.e. the valence quark contribution.

To clarify what the terminology ''bare nucleon'' means we recall the basic QCD structure of nucleons. From deep inelastic lepton-nucleon scattering (DIS) experiments we know that nucleons are composite color-singlet systems made of partons. In the language of QCD these are three valence quarks with a current quark mass, accompanied by virtual sea quarks and gluons. Accordingly, the physical nucleon state $|N_{\mathrm{phys}}\rangle$ is a highly complicated object consisting of many configurations in the Fock space. For instance, in the case of the proton, the Fock expansion begins with the color-singlet state $|uud\rangle$ consisting of three valence quarks which is the so called bare proton state, and continues with $|uudg\rangle$, $|uud\bar{q}q\rangle$ and further sea quark and gluon states that span the degrees of freedom of the proton in QCD.

In the low-energy region, many properties of the nucleon can rather successfully be described by approximating the virtual sea quarks and gluons by a cloud of mesons, especially pions, surrounding the bare valence quark core. Accordingly, in the pion cloud model, which resembles the Tamm-Dancoff method [31–36] the physical nucleon is viewed as a bare nucleon, which accounts for the three valence quarks, accompanied by the pion cloud which accounts for the virtual sea quarks and gluons. Then the Fock representation for the physical nucleon reads [33,34,37–40]

$$|N_{\mathrm{phys}}\rangle = Z_N^{1/2}(|N\rangle + \phi_1|N\pi\rangle + \phi_2|N\pi\pi\rangle + ...), \quad (1)$$

where the Fock state $|N\rangle$ represents a bare nucleon state, $|N\pi\rangle$ and $|N\pi\pi\rangle$ represent a bare nucleon with one pion and two pions, respectively, and the dots stand for all of the Fock states consisting of one bare nucleon with more than one pion or heavier mesons. The probability amplitudes ϕ_n to find the nucleon in the state $|Nn\pi\rangle$ can be evaluated by using a Hamiltonian which describes the pion-nucleon interaction [31–34,37]. Then the bare nucleon probability can also be determined and turns out to be $Z_N \simeq 0.9$ [32,38]. Since the deviation of Z_N from 1 comes from pion-nucleon interaction one has to put $Z_N = 1$ if the pion cloud is not taken into account. By using the Fock expansion (1) the expectation value of an observable $\hat{\mathcal{O}}$ taken between the physical nucleon states is given by [32,33,41],

$$\langle N_{\mathrm{phys}}|\hat{\mathcal{O}}|N_{\mathrm{phys}}\rangle = Z_N(\langle N|\hat{\mathcal{O}}|N\rangle + \phi_1^2\langle N\pi|\hat{\mathcal{O}}|N\pi\rangle$$
$$+ \phi_2^2\langle N\pi\pi|\hat{\mathcal{O}}|N\pi\pi\rangle + \ldots). \quad (2)$$

The first term on the right side of (2), i.e. the contribution of the bare nucleon without pions, plays an important role for two reasons. First, the bare nucleon is expected to give the main contribution in many cases [42]. And second, for the leading chiral correction one needs only the contributions of the lowest-momentum pions in the cloud allowing an application of the soft pion theorem (see Appendix A), which then reduces the pion cloud terms in (2) also to bare nucleon matrix elements [43,44]. Accordingly, in this paper we focus on bare nucleon matrix elements and propose an algebraic method for evaluating them. This approach seems capable to estimate nucleon matrix elements of quark operators in a straightforward way. We also note that within the algebraic approach new parameters are not necessary since the bare nucleon matrix elements are traced back to vacuum matrix elements, like in the soft pion theorem. We apply the method on two-quark, four-quark and, finally, on six-quark operators inside the nucleon which so far have not been evaluated.

The paper is organized as follows. In Sec. II we derive an algebraic formula for evaluating matrix elements taken between the state of a bare nucleon. In Sec. III a valence quark field operator with the quantum numbers of a bare nucleon is introduced. A few tests of the nucleon formula on well known bare nucleon matrix elements of two-quark operators are given in Sec. IV A (currents) and IV B (chiral condensate). In Sec. IV C we explore the valence quark contribution of four-quark condensates within the algebraic method developed and assert an interesting agreement with the results of groundstate saturation approximation when taking properly the valence quark contribution. We also compare our findings for the valence quark contribution of four-quark condensates with recently obtained results within a chiral quark model. In Sec. V we evaluate six-quark condensates inside the bare nucleon. A summary of the results and an outlook can be found in Sec. VI. In Appendix A a derivation of the soft pion theorem is given which shows the similarity of it with our algebraic approach. Details of some evaluations are relegated to the Appendix B.

II. NUCLEON FORMULA

Let $\hat{\mathcal{O}}(x)$ be a local operator which may depend on space and time, $x = (\mathbf{r}, t)$. We are interested in matrix elements taken between two bare nucleon states $|N(k, \sigma)\rangle$ with four-momentum k and spin σ (i.e. $|N\rangle$ is either a bare proton $|p\rangle$ or a bare neutron $|n\rangle$ state, which are considered as QCD eigenstates). To derive a formula for such matrix elements between bare nucleons with finite nucleon masses and momenta we first apply the Lehmann-Symanzik-Zimmermann (LSZ) reduction [45,46] on one nucleon

state,

$$\langle N(k_2, \sigma_2)|\hat{\mathcal{O}}(x)|N(k_1, \sigma_1)\rangle = iZ_\Psi^{-1/2} \int d^4x_1 \langle N(k_2, \sigma_2)|T_W(\hat{\mathcal{O}}(x)\overline{\hat{\Psi}}_N^\alpha(x_1))|0\rangle (i\gamma_\mu \overleftarrow{\partial}_{x_1}^\mu + M_N)_{\alpha\beta} u_N^\beta(k_1, \sigma_1)e^{-ik_1x_1}, \quad (3)$$

where the greek letters α, β are Dirac indices. The normalization of the nonperturbative QCD vacuum is $\langle 0|0\rangle = 1$, and the normalization for the nucleon state reads $\langle N(k_2, \sigma_2)|N(k_1, \sigma_1)\rangle = 2E_{k_1}(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)\delta_{\sigma_1\sigma_2}$, where $E_{k_1} = \sqrt{\mathbf{k}_1^2 + M_N^2}$. Throughout the paper we take the sum convention: If two Dirac (or later color) indices are equal or not given explicitly, then a sum over them is implied. The four-momenta are on-shell, $k_1^2 = k_2^2 = M_N^2$; for noninteracting nucleons the bare nucleon mass equals the physical nucleon mass, $M_N = 938$ MeV. The field $\overline{\Psi}_N(x_1)$ is the interacting (adjoint) nucleon field operator. i.e. off-shell. The equal-time anticommutator for the interacting nucleon field operator is the same as for the free fields and reads

$$[\hat{\Psi}_N^\alpha(\mathbf{r}_1, t), \hat{\Psi}_N^{\beta\dagger}(\mathbf{r}_2, t)]_+ = \delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)\delta^{\alpha\beta}. \quad (4)$$

For the wave function renormalization constant we have $0 \leq Z_\Psi^{-1/2} \leq 1$. The free nucleon spinor satisfies $(\gamma^\mu k_\mu - M_N)u_N(k, \sigma) = 0$, with normalization $\overline{u}_N(k, \sigma_2) \times u_N(k, \sigma_1) = 2M_N\delta_{\sigma_1\sigma_2}$. The operator $\hat{\mathcal{O}}$ is, for physical reasons, assumed to consist of an even number of fermionic fields, i.e. a bosonic operator, according to which the Wick

time-ordering, $T_W\hat{A}(x_1)\hat{B}(x_2) = \hat{A}(x_1)\hat{B}(x_2)\Theta(t_1 - t_2) + \hat{B}(x_2)\hat{A}(x_1)\Theta(t_2 - t_1)$, has been taken in Eq. (3).

We approximate Eq. (3) by introducing a noninteracting nucleon field operator given by

$$\hat{\Psi}_N^\alpha(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2E_k} \sum_{\sigma=1}^2 (\hat{a}_N(k, \sigma)u_N^\alpha(k, \sigma)e^{-ikx} + \hat{b}_N^\dagger(k, \sigma)v_N^\alpha(k, \sigma)e^{ikx}), \quad (5)$$

with the corresponding anticommutator relations in momentum space

$$[\hat{a}_N(k_1, \sigma_1), \hat{a}_N^\dagger(k_2, \sigma_2)]_+ = [\hat{b}_N(k_1, \sigma_1), \hat{b}_N^\dagger(k_2, \sigma_2)]_+$$
$$= 2E_{k_1}(2\pi)^3\delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2)$$
$$\times \delta_{\sigma_1\sigma_2}. \quad (6)$$

Accordingly, $|N(k, \sigma)\rangle = \hat{a}_N^\dagger(k, \sigma)|0\rangle$. For the noninteracting nucleon field operator $Z_\Psi^{-1/2} = 1$, and the equation of motion follows from (5), $(i\gamma_\mu \vec{\partial}_x^\mu - M_N)\hat{\Psi}_N(x) = 0$, and for the adjoint noninteracting nucleon field operator it reads $\overline{\hat{\Psi}}_N(x)(i\overleftarrow{\partial}_x^\mu\gamma_\mu + M_N) = 0$, respectively. Then one arrives at

$$\langle N(k_2, \sigma_2)|\hat{\mathcal{O}}(x)|N(k_1, \sigma_1)\rangle = \int d^4x_1 e^{-ik_1x_1}\delta(t - t_1)\langle N(k_2, \sigma_2)|[\hat{\mathcal{O}}(x), \overline{\hat{\Psi}}_N^\alpha(x_1)]_-|0\rangle(\gamma_0)_{\alpha\beta}u_N^\beta(k_1, \sigma_1). \quad (7)$$

Applying this procedure on the left nucleon state yields

$$\langle N(k_2, \sigma_2)|\hat{\mathcal{O}}(x)|N(k_1, \sigma_1)\rangle = \int d^4x_1 \int d^4x_2 e^{-ik_1x_1}e^{ik_2x_2}\delta(t - t_1)\delta(t - t_2)\overline{u}_N^{\beta_2}(k_2, \sigma_2)(\gamma_0)_{\beta_2\alpha_2}$$
$$\times \langle 0|[\hat{\Psi}_N^{\alpha_2}(x_2), [\hat{\mathcal{O}}(x), \overline{\hat{\Psi}}_N^{\alpha_1}(x_1)]_-]_+|0\rangle(\gamma_0)_{\alpha_1\beta_1}u_N^{\beta_1}(k_1, \sigma_1), \quad (8)$$

which is symmetric under the replacement $\langle 0|[\hat{\Psi}, [\hat{\mathcal{O}}, \overline{\hat{\Psi}}]_-]_+|0\rangle \rightarrow \langle 0|[[\hat{\Psi}, \hat{\mathcal{O}}]_-, \overline{\hat{\Psi}}]_+|0\rangle$. Equation (8) is the central point of our investigation and we call it nucleon formula. This formula resembles the soft pion theorem given in Appendix A. It is worth to underline that, due to the δ-functions in (8), only the equal-time commutator and anticommutator occur. The anticommutator comes into due to the fact that the commutator in (7) between the operator $\hat{\mathcal{O}}$ (consisting of an even number of fermionic operators) and the (adjoint) fermionic field operator $\overline{\Psi}_N$ yields an operator consisting of an odd number of fermionic field operators. Therefore, when applying LSZ [cf. Equation (3)] on the other nucleon state a Dirac time-ordering, $T_D\hat{A}(x_1)\hat{B}(x_2) = \hat{A}(x_1)\hat{B}(x_2)\Theta(t_1 - t_2) - \hat{B}(x_2)\hat{A}(x_1)\Theta(t_2 - t_1)$, is needed.

The nucleon formula is valid for a noninteracting nucleon with finite mass M_N and finite three momentum $\mathbf{k}$,

and in this respect it goes beyond the soft pion theorem, which is valid for pions with vanishing four-momentum only. In the next section we supplement the nucleon formula with a nucleon field operator expressed by quark fields, which allows then the algebraic evaluation of bare nucleon matrix elements of quark operators.

We note a remarkable advantage of the algebraic approach. The operator $\hat{\mathcal{O}}$ is a composite operator, i.e. a product of field operators taken at the same space-time point. As it stands, such a composite operator needs to be renormalized. Therefore, a renormalization $\hat{\mathcal{O}}^{\text{ren}} = \hat{\mathcal{O}} - \langle\hat{\mathcal{O}}\rangle_0$ (we abbreviate $\langle\hat{\mathcal{O}}\rangle_0 \equiv \langle 0|\hat{\mathcal{O}}|0\rangle$), which applies for products of noninteracting field operators, has to be implemented [46]. However, the term $\langle\hat{\mathcal{O}}\rangle_0$ is a c-number and, according to Eq. (8), does not contribute because of $[\langle\hat{\mathcal{O}}\rangle_0, \overline{\hat{\Psi}}_N]_- = 0$. Another kind of renormalization for

products of interacting field operators is also based on subtracting of c-numbers (so called renormalization constants) which vanish when applying the commutator in the nucleon formula. Therefore, one may consider the composite operator $\hat{\mathcal{O}}(x)$ in Eq. (8) as a renormalized operator. This feature is also known within PCAC and PCBC algebra, and, in particular, within the soft pion theorem.

III. CHOICE OF NUCLEON FIELD OPERATOR

The nucleon formula (8) can directly be applied on a local operator $\hat{\mathcal{O}}$ which consists of bare nucleonic degrees of freedom, e.g. $\hat{\mathcal{O}} = \hat{\bar{\Psi}}_N \hat{\Psi}_N, \hat{\bar{\Psi}}_N \gamma_\mu \hat{\Psi}_N$, etc. However, we are interested in operators basing on quark degrees of freedom, e.g. $\hat{\mathcal{O}} = \hat{\bar{q}} \hat{q}, \hat{\bar{q}} \gamma_\mu \hat{q}$, etc. To make the relation (8) applicable for such cases one needs to decompose the bare nucleon field operator $\hat{\Psi}_N$ into the three valence quarks, yielding a composite operator $\hat{\psi}_N$ to be specified by now [47]. Although there has been considerable success in understanding the properties of nucleons on the basis of their quark substructure as derived within QCD, a rigorous use of QCD for the nucleons is not yet in reach. Therefore, in order to gain a nucleon field operator which shows up the main features (quantum numbers) of the bare nucleon a more phenomenological approach on the basis of the quark-diquark picture of baryons [48] is used.

To be specific we consider the proton. The bare proton state $|uud\rangle$ is defined by the SU(2) flavor, SU(2) spin wavefunction of three valence quarks. In the quark-diquark model of the bare proton two of these valence quarks are regarded as a composite colored particle (diquark) which obeys the Bose statistic and which has a mass of the corresponding meson (e.g. for QCD with $N_c = 2$ Pauli-Gürsey symmetry [49]), i.e. a mass which is significantly larger than the current quark mass. Within quark degrees of freedom the general expression for such a diquark can be written as [29,50]

$$\hat{\Phi}_{q_1 q_2}^{ab}(x) = \hat{q}_1^{aT}(x) C \Gamma \hat{q}_2^b(x). \qquad (9)$$

Here, $\hat{q}_1^a$ and $\hat{q}_2^b$ are quark field operators of flavor u or d with color index a and b, respectively. Throughout the paper all quark field operators are solutions of the full Dirac equation, $(i\gamma^\mu \hat{D}_\mu - m_q)\hat{q}(x) = 0$ with $\hat{D}_\mu = \partial_\mu - ig_s \hat{A}_\mu^a \lambda^a/2$, where $\hat{A}_\mu^a$ are the gluon fields and $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ ($a, b = 1, \ldots, 8$ are Gell-Mann indices, which should not be confused with the color indices (in roman style) a, b, c (later also i..., n) of quark fields). The equal-time anticommutator for these interacting quark fields is the same as for free quark fields and reads

$$[\hat{q}_\alpha^a(\mathbf{r}_1, t), \hat{q}_\beta^{b\dagger}(\mathbf{r}_2, t)]_+ = \delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)\delta_{\alpha\beta}\delta^{ab}. \qquad (10)$$

The charge conjugation matrix is $C = i\gamma_0 \gamma_2$, and $\Gamma = \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}\}$ is an element of the Clifford algebra. C changes the parity of Γ, e.g. $C\gamma_5$ has positive parity.

The diquark (9), considered as a composite operator made of quark fields, does generally not commute with quark field operators. On the other side, if the diquark is regarded as an effective boson, it commutes with the fermionic quark fields. This feature of the diquark, considered as a bosonic quasiparticle, can be retained on quark level when neglecting the quantum corrections for the quark fields which are participants of the diquark. Accordingly, the diquark is separated into a classical part and a quantum correction

$$\hat{\Phi}_{q_1 q_2}^{ab}(x) = q_1^{aT}(x) C \Gamma q_2^b(x) + \Delta\hat{\Phi}_{q_1 q_2}^{ab}(x). \qquad (11)$$

The classical Dirac spinors q_1^a, q_2^b are solutions of the full Dirac equation $(i\gamma^\mu D_\mu - m_q)q(x) = 0$. The classical part in (11), $\Phi_{q_1 q_2}^{ab} = q_1^{aT} C \Gamma q_2^b$, commutes with quark field operators. To specify the diquark relevant for a proton we note that there are only two structures, $\Gamma = \gamma_5$ and $\Gamma = \gamma_5 \gamma_0$, which have positive parity and vanishing total spin, $J^P = 0^+$ [50]. This is in line with [51], where it was found that the proton has indeed a large overlap with the interpolating field $\hat{\eta}_p = \epsilon^{abc}(\hat{u}^{aT} C \gamma_5 \hat{d}^b)\hat{u}^c$, where ϵ^{abc} is the total antisymmetric tensor. We also remark that in lattice calculations the field $\hat{\eta}_p$ is usually used [52,53] since this interpolating field has an appropriate nonrelativistic limit. In addition, the field $\hat{\eta}_p$ is also a part of the so called Ioffe interpolating field, which for the proton is given by $\hat{\eta}_{\text{Ioffe}} = 2\epsilon^{abc}((\hat{u}^{aT} C \hat{d}^b)\gamma_5 \hat{u}^c - (\hat{u}^{aT} C \gamma_5 \hat{d}^b)\hat{u}^c)$ [54]. The Ioffe interpolating field is usually used in QCD sum rule evaluations; for a more detailed motivation see also [55]. These properties in mind, we take $\hat{\eta}_p$ as a guide for constructing a proton field operator and obtain a semiclassical interpolating proton field by neglecting the quantum correction of the diquark. Further, we assume that any quark of the nucleon can either be a participant of the diquark or can be located outside the diquark. In this line only two different structures for a semiclassical interpolating proton field may occur and, according to this, the general semiclassical field operator for a bare proton is a linear combination of both of them:

$$\hat{\psi}_p^\alpha(x) = \epsilon^{abc}[A_p(u^{aT}(x) C \gamma_5 d^b(x))\hat{u}^{c\alpha}(x)$$
$$+ B_p(u^{aT}(x) C \gamma_5 u^b(x))\hat{d}^{c\alpha}(x)]. \qquad (12)$$

The colorless operator (12) leads to the quantum numbers of a proton (charge, parity, spin, isospin). In the following we evaluate proton matrix elements on quark level by means of the nucleon formula (8) where the field operator $\hat{\Psi}_p(x)$ is replaced by $\hat{\psi}_p(x)$ given in Eq. (12).

Before going further we have to comment on the normalization of the field operator (12), i.e. on the determination of the coefficients A_p and B_p which, in general, are complex quantities. With the relativistic normalization for the nucleon state and taking into account that there are two u quarks inside the proton we demand [56]

$\langle p(k, \sigma_1)|\hat{u}_\alpha^{i\dagger}(0)\hat{u}_\alpha^i(0)|p(k, \sigma_2)\rangle = 4E_k\delta_{\sigma_1\sigma_2}$. Evaluating this term using the nucleon formula (8) with (12) we find the normalization, at $x = 0$, to be [57]

$$|A_p|^2 \epsilon^{abc}\epsilon^{a'b'c'}\delta^{cc'}\overline{u}^{a'T}C\gamma_5\overline{d}^{b'}u^{aT}C\gamma_5 d^b = 2. \quad (13)$$

From $\langle p(k, \sigma_1)|\hat{d}_\alpha^{i\dagger}(0)\hat{d}_\alpha^i(0)|p(k, \sigma_2)\rangle = 2E_k\delta_{\sigma_1\sigma_2}$ we deduce

$$|B_p|^2 \epsilon^{abc}\epsilon^{a'b'c'}\delta^{cc'}\overline{u}^{a'T}C\gamma_5\overline{u}^{b'}u^{aT}C\gamma_5 u^b = 1. \quad (14)$$

Formula (8) in combination with the field operator (12) and the normalizations (13) and (14) summarizes our propositions made for obtaining bare proton matrix elements for quark operators.

For neutron matrix elements in Eq. (8) we have to insert the semiclassical field operator for the bare neutron, $\hat{\psi}_n(x)$, which is achieved from (12) by interchanging $u \leftrightarrow d$, $\hat{u} \leftrightarrow \hat{d}$ and by the replacements $A_p \rightarrow A_n$, $B_p \rightarrow B_n$. The corresponding normalizations for the bare neutron field operator, i.e. the determination of A_n and B_n, are obtained from Eqs. (13) and (14) by interchanging the up and down quarks, and $A_p \rightarrow A_n$, $B_p \rightarrow B_n$.

IV. TESTING THE NUCLEON FORMULA

In the following we will test the outlined formula, Eq. (8) with field operator Eq. (12), and compare with known bare nucleon matrix elements. Throughout the paper we evaluate matrix elements of a composite operator $\hat{O}(x)$ at $x = 0$ and therefore omit the argument x in matrix elements.

A. Electromagnetic and axial vector current

The electromagnetic current for the noninteracting pointlike neutron on hadronic level is zero, due to the vanishing electric charge of the neutron. For the noninteracting pointlike proton it is given by $\hat{J}_\mu^{\mathrm{em}}(x) = e_p\overline{\hat{\Psi}}_p(x)\gamma_\mu\hat{\Psi}_p(x)$, where the electric charge of proton equals the elementary electric charge, $e_p = e$. Now there are two possibilities to evaluate such a matrix element on hadronic level: either by means of the algebraic approach Eq. (8) and the anticommutator relation (4), or the usual way by means of the field operator Eq. (5) and the anticommutator relations (6). In both cases it is straightforward to show that $\langle p(k_2, \sigma_2)|\hat{J}_\mu^{\mathrm{em}}|p(k_1, \sigma_1)\rangle = e_p\overline{u}_p(k_2, \sigma_2)\times \gamma_\mu u_p(k_1, \sigma_1)$ on effective hadronic level. As a first test of the nucleon formula we verify this relation on quark level where the electromagnetic current is given by $\hat{J}_\mu^{\mathrm{em}}(x) = \frac{2}{3}e\overline{\hat{u}}(x)\gamma_\mu\hat{u}(x) - \frac{1}{3}e\overline{\hat{d}}(x)\gamma_\mu\hat{d}(x)$. Indeed, by using Eqs. (B5) and (B6) from Appendix B for the bare proton we get on quark level

$$\langle p(k_2, \sigma_2)|\hat{J}_\mu^{\mathrm{em}}|p(k_1, \sigma_1)\rangle = e_p\overline{u}_p(k_2, \sigma_2)\gamma_\mu u_p(k_1, \sigma_1). \quad (15)$$

Similarly, for the bare neutron, with Eqs. (B7) and (B8) from Appendix B, we obtain on quark level $\langle n(k_2, \sigma_2)|\hat{J}_\mu^{\mathrm{em}}|n(k_1, \sigma_1)\rangle = 0$. Both of these findings are in agreement with the results on effective hadronic level.

Now we look at the axial vector current which on hadronic level for a noninteracting pointlike nucleon is given by $\hat{A}_\mu^a(x) = g_A^v\overline{\hat{\Psi}}_N(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\hat{\Psi}_N(x)$ [58], where g_A^v is the axial charge of a bare nucleon. For the moment being in this paragraph up to Eq. (16) $a = 1, 2, 3$ are isospin indices, and $\hat{\Psi}_N$, $|N\rangle$ and u_N are isoduplets. The isospin matrices τ^a coincide with Pauli's spin matrices with normalization $\mathrm{Tr}(\tau^a\tau^b) = 2\delta^{ab}$.

Similar to the case of electromagnetic current, there are two possibilities to evaluate this matrix element on hadronic level: by means of the algebraic approach Eq. (8) and the anticommutator relation (4), or directly by means of the field operator Eq. (5) and the anticommutator relations (6). In both cases one obtains on effective hadronic level the well known result for pointlike nucleons, $\langle N(k_2, \sigma_2)|\hat{A}_\mu^a|N(k_1, \sigma_1)\rangle = g_A^v\overline{u}_N(k_2, \sigma_2)\gamma_\mu\gamma_5\frac{\tau^a}{2}u_N(k_1, \sigma_1)$. We will verify this relation on quark level, where the axial vector current is defined as $\hat{A}_\mu^a(x) = (\overline{\hat{u}}(x)\overline{\hat{d}}(x))\gamma_\mu\gamma_5\frac{\tau^a}{2} \times (\hat{u}(x)\hat{d}(x))^{\mathrm{T}}$. To operate with matrix elements between either bare proton states or bare neutron states we use for the nondiagonal cases ($a = 1, 2$) the assumed isospin symmetry relations, cf. [59], $\langle p|\overline{\hat{u}}\gamma_\mu\gamma_5\hat{d}|n\rangle = \langle p|\overline{\hat{u}}\gamma_\mu\gamma_5\hat{u} - \overline{\hat{d}}\gamma_\mu\gamma_5\hat{d}|p\rangle$ and $\langle n|\overline{\hat{d}}\gamma_\mu\gamma_5\hat{u}|p\rangle = \langle n|\overline{\hat{d}}\gamma_\mu\gamma_5\hat{d} - \overline{\hat{u}}\gamma_\mu\gamma_5\hat{u}|n\rangle$ [60]. Then, taking the solutions of nucleon formula for two-quark operators, Eqs. (B5) and (B6) for proton states, and Eqs. (B5) and (B6) for neutron states (see Appendix B), yields on quark level for the bare (isoduplet) nucleon

$$\langle N(k_2, \sigma_2)|\hat{A}_\mu^a|N(k_1, \sigma_1)\rangle = \overline{u}_N(k_2, \sigma_2)\gamma_\mu\gamma_5\frac{\tau^a}{2}u_N(k_1, \sigma_1). \quad (16)$$

Comparison of (16) with the result on effective hadronic level yields for the axial charge $g_A^v = 1$, in fair agreement with the value $g_A^v \simeq 0.84$ deduced from MIT Bag model evaluations and neutron β-decay experiment [61].

B. Chiral condensate in nucleon

The chiral condensate inside the nucleon is related to the pion-nucleon sigma term [62],

$$\sigma_N = \frac{m_q}{2M_N}\langle N_{\mathrm{phys}}(k, \sigma)|\overline{\hat{u}}\,\hat{u} + \overline{\hat{d}}\,\hat{d}|N_{\mathrm{phys}}(k, \sigma)\rangle, \quad (17)$$

where $2m_q = m_u + m_d$. A typical value for the pion-nucleon sigma term is $\sigma_N = 45$ MeV [63,64]. The sigma term can be decomposed, according to Eq. (2), into a valence quark contribution (bare nucleon) and a pion cloud contribution (sea quarks and gluons): $\sigma_N = \sigma_N^v + \sigma_N^\pi$. To evaluate σ_N^v we first consider the u quark chiral condensate

inside the bare proton. With Eqs. (B5) and (B6) one obtains

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\,\hat{u}\,|p(k_1, \sigma_1)\rangle = 2\bar{u}_p(k_2, \sigma_2)u_p(k_1, \sigma_1),$$
$$\langle p(k_2, \sigma_2)|\hat{\bar{d}}\,\hat{d}\,|p(k_1, \sigma_1)\rangle = 1\bar{u}_p(k_2, \sigma_2)u_p(k_1, \sigma_1). \quad (18)$$

These relations show the momentum and spin dependence of the chiral condensate inside the bare proton. Of course, for a finite-size nucleon there are additional momentum dependences for which is accounted for by nucleon form-factors. An application of the nucleon formula to the bare neutron reveals the isospin symmetry relations

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\,\hat{u}\,|n(k_1, \sigma_1)\rangle = \langle p(k_2, \sigma_2)|\hat{\bar{d}}\,\hat{d}\,|p(k_1, \sigma_1)\rangle,$$
$$\langle n(k_2, \sigma_2)|\hat{\bar{d}}\,\hat{d}\,|n(k_1, \sigma_1)\rangle = \langle p(k_2, \sigma_2)|\hat{\bar{u}}\,\hat{u}\,|p(k_1, \sigma_1)\rangle. \quad (19)$$

Accordingly, it is only necessary to compare the findings for the proton with results reported in the literature. For the special case $k_1 = k_2$ and $\sigma_1 = \sigma_2$ Eq. (18) simplifies to

$$\frac{1}{2M_N}\langle p(k, \sigma)|\hat{\bar{u}}\,\hat{u}\,|p(k, \sigma)\rangle = 2\ (2.1),$$
$$\frac{1}{2M_N}\langle p(k, \sigma)|\hat{\bar{d}}\,\hat{d}\,|p(k, \sigma)\rangle = 1\ (1.4). \quad (20)$$

The parenthesized values are the findings of Ref. [65] for the valence quark contribution which well agree with our results. From (20) and isospin symmetry relations (19) one may now deduce the valence quark contribution to the nucleon sigma term within the algebraic approach,

$$\sigma_N^v = \frac{m_q}{2M_N}\langle N(k, \sigma)|\hat{\bar{u}}\,\hat{u} + \hat{\bar{d}}\,\hat{d}\,|N(k, \sigma)\rangle = 3m_q. \quad (21)$$

We compare this result with Ref. [66], where the valence quark contribution to the sigma term has been estimated to be $\sigma_N^v = \sigma_N/(1 + G_S f_\sigma^2)$. By using the given values $G_S = 7.91$ GeV^{-2} and $f_\sigma = 0.393$ GeV one obtains $\sigma_N^v = 20$ MeV. Accordingly, our result (21) is, for $m_q \simeq 7$ MeV, in good numerical agreement with [66].

Finally, by assuming that the contribution of the pion cloud for the physical proton is the same for the chiral u and d quark condensates one can get rid of the term σ_N^π by subtracting the chiral d quark from the chiral u quark condensate. That means the following approximation should be valid

$$\langle p_{\text{phys}}(k, \sigma)|\hat{\bar{u}}\,\hat{u} - \hat{\bar{d}}\,\hat{d}\,|p_{\text{phys}}(k, \sigma)\rangle$$
$$\simeq \langle p(k, \sigma)|\hat{\bar{u}}\,\hat{u} - \hat{\bar{d}}\,\hat{d}\,|p(k, \sigma)\rangle = 2M_N, \quad (22)$$

where we have used (18). Indeed, the result (22) is in fair agreement with $\langle p_{\text{phys}}(k, \sigma)|\hat{\bar{u}}\,\hat{u} - \hat{\bar{d}}\,\hat{d}\,|p_{\text{phys}}(k, \sigma)\rangle = 2M_N(M_\Xi - M_\Sigma)/m_s = 1.3$ GeV obtained in [65].

C. Four-quark condensates

Four-quark condensates seem to be quite important in predicting the properties of light vector mesons within the QCD sum rule method [67]. This is related to the fact that in leading-order the chiral condensate is numerically suppressed since it appears in a renormalization invariant contribution $m_q\langle\bar{q}\,\hat{q}\rangle$. Therefore, the gluon condensate and four-quark condensates become numerically more important. However, the numerical values of four-quark condensates are poorly known, and up to now it remains a challenge to estimate their magnitude in a more reliable way. Accordingly, the evaluation of four-quark condensates inside the nucleon is an important issue. Such quantities have been evaluated in [64] within the groundstate saturation approximation, noting the importance of four-quark condensates also for properties of the nucleon within the QCD sum rule approach. An attempt to go beyond the groundstate saturation approximation has been presented in [30], where, by using the Nambu-Jona-Lasinio model and including pions and σ mesons, correction terms have been obtained. Further evaluations of four-quark condensates beyond the groundstate saturation approximation have been performed in [8] by using a perturbative chiral quark model for describing the nucleons. Later, the results of [8] have been used for evaluating nucleon parameter at finite density within QCD sum rules [68]. In [9,52] lattice evaluations for scalar and traceless four-quark operators with nonvanishing twist have been reported. In view of these very few results obtained so far further insight into such condensates is desirable.

Before considering this important issue we notice a general decomposition of four-quark condensates. Let $\hat{A}$ and $\hat{B}$ two arbitrary two-quark operators. Then the nucleon expectation value of $\hat{O} = \hat{A}\hat{B}$ can be decomposed as [69,70]

$$\langle N_{\text{phys}}|\hat{A}\,\hat{B}\,|N_{\text{phys}}\rangle = \langle\hat{A}\rangle_0\langle N_{\text{phys}}|\hat{B}|N_{\text{phys}}\rangle$$
$$+ \langle N_{\text{phys}}|\hat{A}|N_{\text{phys}}\rangle\langle\hat{B}\rangle_0$$
$$+ \langle N_{\text{phys}}|\hat{A}\,\hat{B}\,|N_{\text{phys}}\rangle^{\text{C}}, \quad (23)$$

where the first two terms refer to the so called factorization approximation, while the last term is a correction term to the factorization approximation and describes the scattering of a nucleon with $\hat{B}$ into a nucleon and $\hat{A}$, i.e. it is a sum over all connected scattering Feynman diagrams $N_{\text{phys}} + \hat{B} \rightarrow N_{\text{phys}} + \hat{A}$. The decomposition (23) is matched with the decompositions (24), (25), and (45) given below, as it is seen in [70] where we consider an explicit example for the vector channel. The first two terms in (23) scale with N_c^2 (N_c denotes the number of colors, for the moment being taken as a free parameter of QCD), while the correction term scales with N_c [71,72]. Dividing both sides of (23) by N_c^2 one recognizes that the last term has to be considered as a correction term of the order $1/N_c$ [71,72]. That means that a factorization of four-quark operators in a cold medium is consistent with the large-N_c limit [71,72], a statement which is also valid in vacuum [73].

1. Flavor-unmixed four-quark condensates

We start our investigation with the flavor-unmixed four-quark condensates and consider the general expression of two different kinds of flavor-unmixed condensates inside the nucleon, namely, condensates without and with Gell-

Mann matrices λ^a ($a = 1, \ldots, 8$ are the Gell-Mann indices, which should not be confused with the color indices (in roman style) a, b, c, (later also i..., n) of quark fields) [30,64,74]

$$\langle N_{\text{phys}}|\hat{\bar{q}}\Gamma_1\hat{q}\,\hat{\bar{q}}\,\Gamma_2\hat{q}|N_{\text{phys}}\rangle = \frac{1}{8}[\text{Tr}(\Gamma_1)\text{Tr}(\Gamma_2) - \frac{1}{3}\text{Tr}(\Gamma_1\Gamma_2)]\langle\hat{\bar{q}}\,\hat{q}\rangle_0\langle N_{\text{phys}}|\hat{\bar{q}}\,\hat{q}|N_{\text{phys}}\rangle + \frac{1}{16}\Big[\text{Tr}(\Gamma_1)\text{Tr}(\gamma^\mu\Gamma_2)$$

$$+ \text{Tr}(\Gamma_2)\text{Tr}(\gamma^\mu\Gamma_1) - \frac{1}{3}\text{Tr}(\Gamma_1\gamma^\mu\Gamma_2) - \frac{1}{3}\text{Tr}(\Gamma_2\gamma^\mu\Gamma_1)\Big]\langle\hat{\bar{q}}\,\hat{q}\rangle_0\langle N_{\text{phys}}|\hat{\bar{q}}\gamma_\mu\hat{q}|N_{\text{phys}}\rangle$$

$$+ \langle N_{\text{phys}}|\hat{\bar{q}}\Gamma_1\hat{q}\,\hat{\bar{q}}\,\Gamma_2\hat{q}|N_{\text{phys}}\rangle^{\text{C}}, \tag{24}$$

$$\langle N_{\text{phys}}|\hat{\bar{q}}\Gamma_1\lambda^a\hat{q}\,\hat{\bar{q}}\,\Gamma_2\lambda^a\hat{q}|N_{\text{phys}}\rangle = -\frac{2}{9}\text{Tr}(\Gamma_1\Gamma_2)\langle\hat{\bar{q}}\,\hat{q}\rangle_0\langle N_{\text{phys}}|\hat{\bar{q}}\,\hat{q}|N_{\text{phys}}\rangle - \frac{1}{9}[\text{Tr}(\Gamma_1\gamma^\mu\Gamma_2) + \text{Tr}(\Gamma_2\gamma^\mu\Gamma_1)]\langle\hat{\bar{q}}\,\hat{q}\rangle_0$$

$$\times \langle N_{\text{phys}}|\hat{\bar{q}}\gamma_\mu\hat{q}|N_{\text{phys}}\rangle + \langle N_{\text{phys}}|\hat{\bar{q}}\Gamma_1\lambda^a\hat{q}\,\hat{\bar{q}}\,\Gamma_2\lambda^a\hat{q}|N_{\text{phys}}\rangle^{\text{C}}, \tag{25}$$

where $\hat{\bar{q}}\ldots\hat{q}$ is either $\hat{\bar{u}}\ldots\hat{u}$ or $\hat{\bar{d}}\ldots\hat{d}$ (the dots stand for Γ or $\Gamma\lambda^a$). For the chiral condensate we take $\langle\hat{\bar{q}}\,\hat{q}\rangle_0 = -(0.250\ \text{GeV})^3$. The decompositions of Eqs. (24) and (25) are related to (23) by means of a Fierz rearrangement; an explicit example for the vector channel is given in [70]. The last term on the right side of Eqs. (24) and (25) is a correction term [30] to the groundstate saturation approximation [64], describing the scattering process $N_{\text{phys}} + \hat{\bar{q}}\ldots\hat{q} \rightarrow N_{\text{phys}} + \hat{\bar{q}}\ldots\hat{q}$. To get an idea about the magnitude of these correction terms we consider two typical examples. The factorization approximation (25) (i.e. without the correction term) yields for the scalar channel $\langle N_{\text{phys}}|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\hat{q}|N_{\text{phys}}\rangle = -0.173\ \text{GeV}^4$. In [30] the correction term to this groundstate saturation approximation has been found to be $\langle N_{\text{phys}}|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\hat{q}|N_{\text{phys}}\rangle^{\text{C}} = 0.011\ \text{GeV}^4$. As another example we consider the vector channel with Gell-Mann matrices. The factorization approximation (25) (i.e. without the correction term) yields $\langle N_{\text{phys}}|\hat{\bar{q}}\gamma_\mu\lambda^a\hat{q}\,\hat{\bar{q}}\,\gamma^\mu\lambda^a\hat{q}|N_{\text{phys}}\rangle = 0.335\ \text{GeV}^4$, while the correction term in [30] is $\langle N_{\text{phys}}|\hat{\bar{q}}\gamma_\mu\lambda^a\times \hat{q}\,\hat{\bar{q}}\,\gamma^\mu\lambda^a\hat{q}|N_{\text{phys}}\rangle^{\text{C}} = -0.139\ \text{GeV}^4$. Accordingly, the correction to the groundstate saturation approximation in the scalar channel turns out to be less than 10%, while in the vector channel with Gell-Mann matrices it is about 30%. As we will see, from (24) and (25) the valence quark contribution can be extracted in a unique way.

Now we evaluate the valence quark contribution of four-quark condensates, and start to consider the u quark inside the bare proton. Application of the nucleon formula (8) with the composite proton field operator (12) yields

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}|p(k_1, \sigma_1)\rangle$$

$$= \bar{u}_p^{\beta_2}(k_2, \sigma_2)(\gamma_0)_{\beta_2\alpha_2}(\gamma_0)_{\alpha_1\beta_1}u_p^{\beta_1}(k_1, \sigma_1)$$

$$\times (\Gamma_1)^{\alpha\beta}(\Gamma_2)^{\gamma\delta}\int d^3\mathbf{r}_1 e^{i\mathbf{k}_1\mathbf{r}_1}\int d^3\mathbf{r}_2 e^{-i\mathbf{k}_2\mathbf{r}_2}$$

$$\times \langle 0|[\hat{\psi}_p^{\alpha_2}(\mathbf{r}_2, 0), [\hat{\bar{u}}_\alpha^i\hat{u}_\beta^i\hat{\bar{u}}_\gamma^j\hat{u}_\delta^j, \hat{\bar{\psi}}_p^{\alpha_1}(\mathbf{r}_1, 0)]_-]_+|0\rangle. \tag{26}$$

By inserting the expression given in Eq. (B9) in the

Appendix B into (26) one obtains for the bare proton

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}|p(k_1, \sigma_1)\rangle$$

$$= \frac{1}{6}\langle\hat{\bar{u}}\,\hat{u}\rangle_0(3\text{Tr}(\Gamma_1)\bar{u}_p(k_2, \sigma_2)\Gamma_2 u_p(k_1, \sigma_1)$$

$$+ 3\text{Tr}(\Gamma_2)\bar{u}_p(k_2, \sigma_2)\Gamma_1 u_p(k_1, \sigma_1)$$

$$- \bar{u}_p(k_2, \sigma_2)\Gamma_1\Gamma_2 u_p(k_1, \sigma_1)$$

$$- \bar{u}_p(k_2, \sigma_2)\Gamma_2\Gamma_1 u_p(k_1, \sigma_1)). \tag{27}$$

In a similar way one obtains for the four-quark condensates involving Gell-Mann matrices

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\,\Gamma_2\lambda^a\hat{u}|p(k_1, \sigma_1)\rangle$$

$$= -\frac{8}{9}\langle\hat{\bar{u}}\,\hat{u}\rangle_0(\bar{u}_p(k_2, \sigma_2)\Gamma_1\Gamma_2 u_p(k_1, \sigma_1)$$

$$+ \bar{u}_p(k_2, \sigma_2)\Gamma_2\Gamma_1 u_p(k_1, \sigma_1)). \tag{28}$$

For the d flavor we have

$$\langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\hat{d}\,\hat{\bar{d}}\,\Gamma_2\hat{d}|p(k_1, \sigma_1)\rangle$$

$$= \frac{1}{2}\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}|p(k_1, \sigma_1)\rangle, \tag{29}$$

$$\langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\lambda^a\hat{d}\,\hat{\bar{d}}\,\Gamma_2\lambda^a\hat{d}|p(k_1, \sigma_1)\rangle$$

$$= \frac{1}{2}\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\,\Gamma_2\lambda^a\hat{u}|p(k_1, \sigma_1)\rangle. \tag{30}$$

An analog evaluation of these four-quark operators inside the bare neutron gives

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}|n(k_1, \sigma_1)\rangle$$

$$= \langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\hat{d}\,\hat{\bar{d}}\,\Gamma_2\hat{d}|p(k_1, \sigma_1)\rangle, \tag{31}$$

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\,\Gamma_2\lambda^a\hat{u}|n(k_1, \sigma_1)\rangle$$

$$= \langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\lambda^a\hat{d}\,\hat{\bar{d}}\,\Gamma_2\lambda^a\hat{d}|p(k_1, \sigma_1)\rangle, \tag{32}$$

which, like in the case of chiral condensate, reflect the isospin symmetry. By interchanging $\hat{u} \leftrightarrow \hat{d}$ on both sides in Eq. (31) and (32) one also gets the d flavor four-quark condensates inside neutron. The results (27)–(30) for the four-quark condensates inside the bare nucleon distinguish between proton and neutron, and they also contain the dependence of these condensates on the momentum of the (pointlike) nucleon. Of course, as in the case of two-quark matrix elements, for a finite-size nucleon there are additional momentum dependences implemented in formfactors.

We compare now these findings of Eqs. (27), (28), (28), and (30)–(32) with Ref. [64], i.e. we set $k_1 = k_2$ and $\sigma_1 = \sigma_2$, and average over proton and neutron to get the nucleon condensates, $2\langle N|\hat{O}|N\rangle = \langle p|\hat{O}|p\rangle + \langle n|\hat{O}|n\rangle$. First we consider the case of scalar four-quark condensate, i.e. $\Gamma_1 = \Gamma_2 = \mathbb{1}$. Then, for the bare nucleon one obtains from Eqs. (27), (29), and (31)

$$\langle N(k, \sigma)|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\hat{q}\,|N(k, \sigma)\rangle = \frac{11}{2}\langle \hat{\bar{q}}\,\hat{q}\rangle_0 M_N, \qquad (33)$$

while, according to the first term in Eq. (24) (the second term vanishes in this special case), for the physical nucleon the result

$$\langle N_{\mathrm{phys}}(k, \sigma)|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\hat{q}\,|N_{\mathrm{phys}}(k, \sigma)\rangle = \frac{11}{6}\langle \hat{\bar{q}}\,\hat{q}\rangle_0 M_N \frac{\sigma_N}{m_q}$$

$$= \frac{11}{6}\langle \hat{\bar{q}}\,\hat{q}\rangle_0 M_N \left(\frac{\sigma_N^v}{m_q} + \frac{\sigma_N^\pi}{m_q}\right) \qquad (34)$$

has been obtained in [64]. For the last line of Eq. (34) we have used the decomposition $\sigma_N = \sigma_N^v + \sigma_N^\pi$, which has already been considered in section IV B. Comparing both results, Eqs. (33) and (34), one recognizes, by means of relation $\sigma_N^v/m_q = 3$ [cf. [66] and Eq. (21)], that the result (33) is nothing else but just the valence quark contribution of the scalar four-quark condensate inside the nucleon; it is in agreement with the separated valence quark term of Eq. (34).

Because of its importance and its instructive property we will also consider the case $\Gamma_1 = \mathbb{1}$, $\Gamma_2 = \gamma_\rho$. From Eqs. (27), (29), and (31) we find

$$\langle N(k, \sigma)|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\gamma_\rho\hat{q}|N(k, \sigma)\rangle$$
$$= \frac{5}{4}\langle \hat{\bar{q}}\,\hat{q}\rangle_0 \bar{u}_N(k, \sigma)\gamma_\rho u_N(k, \sigma), \qquad (35)$$

which is the contribution of the bare nucleon, i.e. the valence quark contribution. To compare it with the corresponding result of Ref. [64] we first deduce from Eq. (24) that

$$\langle N_{\mathrm{phys}}(k, \sigma)|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\gamma_\rho\hat{q}|N_{\mathrm{phys}}(k, \sigma)\rangle$$
$$= \frac{5}{6}\langle \hat{\bar{q}}\,\hat{q}\rangle_0\langle N_{\mathrm{phys}}(k, \sigma)|\hat{\bar{q}}\gamma_\rho\hat{q}|N_{\mathrm{phys}}(k, \sigma)\rangle. \qquad (36)$$

From (36) we have to extract the valence quark contribution by virtue of Eq. (2) (with $Z_N \simeq 1$)

$$\langle N_{\mathrm{phys}}(k, \sigma)|\hat{\bar{q}}\gamma_\rho\hat{q}|N_{\mathrm{phys}}(k, \sigma)\rangle$$
$$= \langle N(k, \sigma)|\hat{\bar{q}}\gamma_\rho\hat{q}|N(k, \sigma)\rangle + \sum_n \phi_n^2\langle Nn\pi|\hat{\bar{q}}\gamma_\rho\hat{q}|Nn\pi\rangle. \qquad (37)$$

The first term on the right side, which is in fact an average over proton and neutron, is the valence quark term we are interested in, while the second term is the pion cloud contribution. With isospin invariance one obtains

$$\langle N(k, \sigma)|\hat{\bar{q}}\gamma_\rho\hat{q}|N(k, \sigma)\rangle$$
$$= \frac{1}{2}(\langle p(k, \sigma)|\hat{\bar{u}}\gamma_\rho\hat{u}|p(k, \sigma)\rangle + \langle p(k, \sigma)|\hat{\bar{d}}\gamma_\rho\hat{d}|p(k, \sigma)\rangle)$$
$$= \frac{1}{2}(2\bar{u}_p(k, \sigma)\gamma_\rho u_p(k, \sigma) + 1\bar{u}_p(k, \sigma)\gamma_\rho u_p(k, \sigma))$$
$$= \frac{3}{2}\bar{u}_N(k, \sigma)\gamma_\rho u_N(k, \sigma), \qquad (38)$$

where for the last term we have set $u_p = u_N$ because of $M_p = M_N$. By inserting (37) into (36) and using (38) we obtain

$$\langle N_{\mathrm{phys}}(k, \sigma)|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\gamma_\rho\hat{q}|N_{\mathrm{phys}}(k, \sigma)\rangle$$
$$= \frac{5}{4}\langle \hat{\bar{q}}\,\hat{q}\rangle_0 \bar{u}_N(k, \sigma)\gamma_\rho u_N(k, \sigma)$$
$$+ \frac{5}{6}\langle \hat{\bar{q}}\,\hat{q}\rangle_0 \sum_n \phi_n^2\langle Nn\pi|\hat{\bar{q}}\gamma_\rho\hat{q}|Nn\pi\rangle. \qquad (39)$$

The first term on the right side of Eq. (39) agrees with our result (35), while the second term on the right side of Eq. (39) is a factorization approximation of the expression $\sum_n\phi_n^2\langle Nn\pi|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\gamma_\rho\hat{q}|Nn\pi\rangle$. From that it becomes obvious that our result (35) is nothing else but just the valence quark contribution of (39).

Other combinations of Clifford matrices, like $\Gamma_1 = \gamma_5$ and $\Gamma_2 = \gamma_\rho$, with or without Gell-Mann matrices, can be evaluated and compared in the same way [75]. This means that within the algebraic approach (8) for evaluating bare nucleon matrix elements we find an agreement for all of the flavor-unmixed four-quark condensates if one takes from the corresponding results of Ref. [64] the valence quark contribution, for instance by means of the decomposition $\sigma_N = \sigma_N^v + \sigma_N^\pi$. Therefore, one actually may consider our evaluation as a re-evaluation of the valence quark contribution of the four-quark condensates of Ref. [64] and a confirmation of their results within an independent microscopic approach (quark-diquark picture of the nucleon). But we have to be aware that such an agreement between our algebraic approach and the factorization approximation applies only for the valence quark contribution of nucleon matrix elements. Especially, such an agreement is not expected when taking into account the pion cloud contributions according to Eq. (2).

Having found the remarkable agreement with valence quark contribution of the factorization approximation it becomes interesting to compare our results also with other evaluations in the literature. However, it turns out that a comparison with the recent lattice data of Ref. [9] is quite involved since in [9] the condensates have been evaluated at a renormalization scale of about $\mu_{\text{lattice}}^2 \simeq 5 \text{ GeV}^2$, which is considerably higher than our renormalization point of about $\mu^2 \simeq 1 \text{ GeV}^2$ (our renormalization point is hidden in the chiral condensate, i.e. $\langle \hat{\bar{q}} \hat{q} \rangle_0 (\mu^2)$). To scale the lattice data from 5 GeV^2 down to the hadronic scale of 1 GeV^2 one needs to know the matrix of anomalous dimension for all of the four-quark operators which accounts for the effect of operator mixing among the four-quark condensates. This operator mixing may change considerably the numerical values and even the signs of the four-quark condensates given in [9]. Another method which seems also capable to compare our results with lattice data would be to scale our renormalization point μ^2 up to the lattice scale μ_{lattice}^2. Such a procedure requires, however, the knowledge of renormalization scale dependence of the chiral condensate. Altogether, performing these procedures is out of the scope of the present paper and we therefore abandon a comparison of our results with the ones given in Ref. [9].

In view of the mentioned points and especially in view of an acceptable lucidity of our paper, we restrict ourselves to a comparison with the recently obtained four-quark condensates of Ref. [8]. Because of the specific notation for the four-quark condensates choosen in Ref. [8] we list our results for the valence quark contribution of scalar four-quark condensates for all channels in the same way as in Ref. [8]. Our results for the valence quark contribution can be obtained from Eqs. (27)–(32) by averaging over proton and neutron ($k_1 = k_2$, $\sigma_1 = \sigma_2$):

$$\left\langle N \left| \frac{2}{3} \hat{\bar{q}} \hat{q} \, \hat{\bar{q}} \hat{q} - \frac{1}{2} \hat{\bar{q}} \lambda^a \hat{q} \, \hat{\bar{q}} \lambda^a \hat{q} \right| N \right\rangle$$
$$= -0.0733 \text{ GeV}^4 (-0.117 \text{ GeV}^4), \qquad (40)$$

$$\left\langle N \left| \frac{2}{3} \hat{\bar{q}} \gamma_5 \hat{q} \, \hat{\bar{q}} \gamma_5 \hat{q} - \frac{1}{2} \hat{\bar{q}} \gamma_5 \lambda^a \hat{q} \, \hat{\bar{q}} \gamma_5 \lambda^a \hat{q} \right| N \right\rangle$$
$$= -0.0147 \text{ GeV}^4 (-0.0567 \text{ GeV}^4), \qquad (41)$$

$$\left\langle N \left| \frac{2}{3} \hat{\bar{q}} \gamma_\mu \hat{q} \, \hat{\bar{q}} \gamma^\mu \hat{q} - \frac{1}{2} \hat{\bar{q}} \gamma_\mu \lambda^a \hat{q} \, \hat{\bar{q}} \gamma^\mu \lambda^a \hat{q} \right| N \right\rangle$$
$$= -0.0586 \text{ GeV}^4 (-0.0582 \text{ GeV}^4), \qquad (42)$$

$$\left\langle N \left| \frac{2}{3} \hat{\bar{q}} \gamma_\mu \gamma_5 \hat{q} \, \hat{\bar{q}} \gamma^\mu \gamma_5 \hat{q} - \frac{1}{2} \hat{\bar{q}} \gamma_\mu \gamma_5 \lambda^a \hat{q} \, \hat{\bar{q}} \gamma^\mu \gamma_5 \lambda^a \hat{q} \right| N \right\rangle$$
$$= +0.0586 \text{ GeV}^4 (+0.0567 \text{ GeV}^4), \qquad (43)$$

$$\left\langle N \left| \frac{2}{3} \hat{\bar{q}} \sigma_{\mu\nu} \hat{q} \, \hat{\bar{q}} \sigma^{\mu\nu} \hat{q} - \frac{1}{2} \hat{\bar{q}} \sigma_{\mu\nu} \lambda^a \hat{q} \, \hat{\bar{q}} \sigma^{\mu\nu} \lambda^a \hat{q} \right| N \right\rangle$$
$$= -0.176 \text{ GeV}^4 (-0.356 \text{ GeV}^4). \qquad (44)$$

The values parenthesized are the results for these condensates as given in Ref. [8], but for the physical nucleon, i.e. for a nucleon which contains the valence quark, sea quark and gluon contributions [76]. Since we have compared our evaluated valence quark contribution with the total contribution for the physical nucleon of Ref. [8] it becomes obvious that one may actually not expect a perfect numerical agreement. The more interesting fact is to notice that the valence quark contribution for the vector and axial vector channel, (42) and (43), respectively, agrees very well with the total contribution for the physical nucleon. For the scalar, axial scalar and tensor channel the signs for the valence quark contribution and total contribution of Ref. [8] agree, while the numerical values differ. This illustrates that the sea quark and gluon contributions are expected to give noticeable contributions.

2. Flavor-mixed four-quark condensates

For the flavor-mixed four-quark condensates the general decomposition reads [64]

$$\langle N_{\text{phys}} | \hat{\bar{u}} \Gamma_1 \hat{u} \, \hat{\bar{d}} \Gamma_2 \hat{d} | N_{\text{phys}} \rangle = \frac{1}{8} \langle \hat{\bar{q}} \hat{q} \rangle_0 \langle N_{\text{phys}} | \hat{\bar{q}} \hat{q} | N_{\text{phys}} \rangle \text{Tr}(\Gamma_1) \text{Tr}(\Gamma_2) + \langle \hat{\bar{q}} \hat{q} \rangle_0 \langle N_{\text{phys}} | \hat{\bar{q}} \gamma_\mu \hat{q} | N_{\text{phys}} \rangle [\text{Tr}(\Gamma_1) \text{Tr}(\gamma^\mu \Gamma_2)$$
$$+ \text{Tr}(\Gamma_2) \text{Tr}(\gamma^\mu \Gamma_1)] + \langle N_{\text{phys}} | \hat{\bar{u}} \Gamma_1 \hat{u} \, \hat{\bar{d}} \Gamma_2 \hat{d} | N_{\text{phys}} \rangle^{\text{C}}, \qquad (45)$$

where the isospin symmetry relations (19) have been used. The last term in (45) is a correction term to the factorization approximation, describing the scattering process $N_{\text{phys}} + \bar{d} \Gamma_2 \hat{d} \rightarrow N_{\text{phys}} + \hat{\bar{u}} \Gamma_1 \hat{u}$. The decomposition (45) is, like (24) and (25), matched with Eq. (23) by means of a Fierz transformation. The flavor-mixed condensates with Gell-Mann matrices vanish in the factorization approximation, $\langle N_{\text{phys}} | \hat{\bar{u}} \Gamma_1 \lambda^a \hat{u} \, \bar{d} \Gamma_2 \lambda^a \hat{d} | N_{\text{phys}} \rangle = 0$ [64] (in [77] we have found small corrections to the factorization approximation of flavor-mixed condensates for the vector and axial vector channel with Gell-Mann matrices).

Now we consider the valence quark contribution of the flavor-mixed four-quark condensates, i.e. the contribution of the bare nucleon. Application of our nucleon formula (8) with (12) yields for the bare proton

$$\langle p(k_2, \sigma_2) | \hat{\bar{u}} \Gamma_1 \hat{u} \, \hat{\bar{d}} \Gamma_2 \hat{d} | p(k_1, \sigma_1) \rangle$$
$$= \frac{1}{4} (2 \langle \hat{\bar{u}} \hat{u} \rangle_0 \text{Tr}(\Gamma_1) \bar{u}_p(k_2, \sigma_2) \Gamma_2 u_p(k_1, \sigma_1)$$
$$+ 1 \langle \hat{\bar{d}} \hat{d} \rangle_0 \text{Tr}(\Gamma_2) \bar{u}_p(k_2, \sigma_2) \Gamma_1 u_p(k_1, \sigma_1)), \qquad (46)$$

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1 \lambda^a \hat{u} \hat{\bar{d}} \Gamma_2 \lambda^a \hat{d}|p(k_1, \sigma_1)\rangle = 0, \qquad (47)$$

while for the bare neutron we find

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1 \hat{u} \hat{\bar{d}} \Gamma_2 \hat{d}|n(k_1, \sigma_1)\rangle$$

$$= \frac{1}{4}(1\langle \hat{\bar{d}} \hat{d}\rangle_0 \text{Tr}(\Gamma_1) \bar{u}_n(k_2, \sigma_2) \Gamma_2 u_n(k_1, \sigma_1)$$

$$+ 2\langle \hat{\bar{u}} \hat{u}\rangle_0 \text{Tr}(\Gamma_2) \bar{u}_n(k_2, \sigma_2) \Gamma_1 u_n(k_1, \sigma_1)), \qquad (48)$$

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1 \lambda^a \hat{u} \hat{\bar{d}} \Gamma_2 \lambda^a \hat{d}|n(k_1, \sigma_1)\rangle = 0. \qquad (49)$$

The Eqs. (46) and (48) are generalized expressions of the factorization approximation given in [64] because of the dependence on nucleon momentum and the distinction between proton and neutron. The results of Eqs. (47) and (49) are in agreement with the factorization approximation [64], but, as mentioned above, beyond the factorization approximation these condensates are nonvanishing [77].

To compare the obtained results of Eqs. (46) and (48) with the factorization approximation, i.e. neglecting the correction term in (45), we consider the special case $k_1 = k_2$, $\sigma_1 = \sigma_2$, and $\Gamma_1 = \Gamma_2 = \mathbb{1}$. For the bare nucleon one obtains by averaging over the bare proton and bare neutron,

$$\langle N(k, \sigma)|\hat{\bar{u}} \hat{u} \hat{\bar{d}} \hat{d}|N(k, \sigma)\rangle = 6\langle \hat{\bar{q}} \hat{q}\rangle_0 M_N. \qquad (50)$$

This result has to be compared with the corresponding finding of [64] which according to Eq. (45) reads

$$\langle N_{\text{phys}}(k, \sigma)|\hat{\bar{u}} \hat{u} \hat{\bar{d}} \hat{d}|N_{\text{phys}}(k, \sigma)\rangle = 2\langle \hat{\bar{q}} \hat{q}\rangle_0 M_N \frac{\sigma_N}{m_q}$$

$$= 2\langle \hat{\bar{q}} \hat{q}\rangle_0 M_N \left(\frac{\sigma_N^v}{m_q} + \frac{\sigma_N^\pi}{m_q}\right), \qquad (51)$$

where in the last expression we have used the decomposition $\sigma_N = \sigma_N^v + \sigma_N^\pi$. By means of the relation $\sigma_N^v/m_q = 3$ [cf. [66] and Eq. (21)] the result (50) is in agreement with the separated valence quark contribution of (51). As in the cases considered in the previous Sec. IV C 1 such an agreement with Ref. [64] can be achieved for all the other combinations of Γ_1 and Γ_2 of the Clifford algebra.

As in the case of flavor-unmixed four-quark operators we compare our findings for the valence quark contribution of flavor-mixed condensates, (46) and (48), with the total result for the physical nucleon of Ref. [8] to examine the magnitude and sign of our results. According to Eqs. (46)–(49) only the valence quark contribution of the scalar channel does not vanish. Its numerical magnitude

$$\left\langle N \left| \frac{2}{3}\hat{\bar{u}} \hat{u} \hat{\bar{d}} \hat{d} - \frac{1}{2}\hat{\bar{u}}\lambda^a \hat{u} \hat{\bar{d}} \lambda^a \hat{d} \right| N \right\rangle$$

$$= -0.0586 \text{ GeV}^4(-0.094 \text{ GeV}^4), \qquad (52)$$

turns out to be comparable with the evaluation of [8] for the total contribution of the physical nucleon given in the parentheses in (52). The numerical difference in magnitude is caused by sea quark and gluon contributions.

To summarize this section, we have evaluated the valence quark contribution to flavor-unmixed and flavor-mixed four-quark condensates for the u and d flavor inside proton and neutron. The results for the flavor-unmixed operators are given by the Eqs. (27)–(32), and the results for the flavor-mixed operators are given by the Eqs. (46)–(49). We have seen that our findings for the four-quark condensates within the algebraic approach, which is by far a different method than the used one of Ref. [64], are in agreement with the large-N_c limit [71,72] and with the results of Ref. [64] when taking from there the valence quark contribution only. It seems admissible, especially in view of the agreement with valence quark contribution of factorization, that the nucleon formula yields reliable results for the valence quark contribution of four-quark condensates inside the nucleon.

V. SIX-QUARK CONDENSATES

Six-quark condensates become important mainly for two reasons. First, in the operator product expansion (OPE) of current correlators one usually takes into account all terms up to the order of the four-quark condensates and neglects the contributions of higher order. Such an approximation may work or may not work, depending on the specific physical system under consideration. Accordingly, one has to be aware about the contribution of the next order to decide how good such an approximation is. This is also necessary for the more involved case of finite density, where a Gibbs average over all hadronic states of the correlator under consideration has to be taken. Indeed, a very recent estimate of such higher contributions beyond the four-quark condensate for the nucleon correlator in matter underlines also the importance of an estimate for the six-quark condensates inside the nucleon [78]. And second, it is well known that instantons give rise to corrections to the Wilson coefficients of six-quark condensates [79,80]. These corrections cause a substantial enhancement of the vacuum contribution of six-quark condensates in the OPE of current correlators. To investigate such current correlators at finite density implies the knowledge of the nucleon matrix elements of six-quark condensates. Here, after getting confidence on our proposed approach in the previous sections, we will use the nucleon formula to evaluate the valence quark contribution of six-quark condensates inside the nucleon.

Within our algebraic approach by using the nucleon formula (8) with (12) we obtain for the u flavor inside

the bare proton

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}\,\hat{\bar{u}}\,\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle = \bar{u}_p^{\beta_2}(k_2, \sigma_2)(\gamma_0)_{\beta_2\alpha_2}(\gamma_0)_{\alpha_1\beta_1}u_p^{\beta_1}(k_1, \sigma_1)(\Gamma_1)^{\alpha\beta}(\Gamma_2)^{\gamma\delta}(\Gamma_3)^{\epsilon\zeta}$$

$$\times \int d^3\mathbf{r}_1 e^{i\mathbf{k}_1\mathbf{r}_1} \int d^3\mathbf{r}_2 e^{-i\mathbf{k}_2\mathbf{r}_2}\langle 0|[\hat{\psi}_p^{\alpha_2}(\mathbf{r}_2, 0), [\hat{\bar{u}}_\alpha^i\hat{u}_\beta^i\hat{\bar{u}}_\gamma^k\hat{u}_\delta^k\hat{\bar{u}}_\epsilon^m\hat{u}_\zeta^m, \hat{\bar{\psi}}_p^{\alpha_1}(\mathbf{r}_1, 0)]_-]_+|0\rangle. \quad (53)$$

The commutator-anticommutator is given in Eq. (B11) in Appendix B for a more general case. According to this result the six-quark condensate inside the bare proton is reduced to a four-quark condensate in vacuum. We note one of these four-quark condensates in vacuum saturation approximation [1]

$$\langle\hat{u}_\beta^j\hat{\bar{u}}_\gamma^k\hat{u}_\delta^l\hat{\bar{u}}_\epsilon^m\rangle_0 = \frac{1}{(12)^2}\langle\hat{\bar{u}}\,\hat{u}\rangle_0^2(\delta_{\beta\gamma}\delta_{\delta\epsilon}\delta^{jk}\delta^{lm} - \delta_{\beta\epsilon}\delta_{\gamma\delta}\delta^{jm}\delta^{kl}). \quad (54)$$

When evaluating all of the four-quark condensates of Eq. (B11) in the same way one obtains, by using the normalization (13),

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}\,\hat{\bar{u}}\,\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle = \frac{2}{(12)^2}\langle\hat{\bar{u}}\,\hat{u}\rangle_0^2[\bar{u}_p(k_2, \sigma_2)(\Gamma_1\Gamma_2\Gamma_3 + \Gamma_1\Gamma_3\Gamma_2 + \Gamma_2\Gamma_1\Gamma_3)u_p(k_1, \sigma_1)$$

$$+ \bar{u}_p(k_2, \sigma_2)(\Gamma_2\Gamma_3\Gamma_1 + \Gamma_3\Gamma_1\Gamma_2 + \Gamma_3\Gamma_2\Gamma_1)u_p(k_1, \sigma_1)$$

$$- 3\bar{u}_p(k_2, \sigma_2)\Gamma_1\Gamma_3 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_2) - 3\bar{u}_p(k_2, \sigma_2)\Gamma_2\Gamma_3 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_1)$$

$$- 3\bar{u}_p(k_2, \sigma_2)\Gamma_1\Gamma_2 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_3) - 3\bar{u}_p(k_2, \sigma_2)\Gamma_3\Gamma_2 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_1)$$

$$- 3\bar{u}_p(k_2, \sigma_2)(\Gamma_2\Gamma_1)u_p(k_1, \sigma_1)\text{Tr}(\Gamma_3) - 3\bar{u}_p(k_2, \sigma_2)(\Gamma_3\Gamma_1)u_p(k_1, \sigma_1)\text{Tr}(\Gamma_2)$$

$$+ 9\bar{u}_p(k_2, \sigma_2)\Gamma_3 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_1)\text{Tr}(\Gamma_2) + 9\bar{u}_p(k_2, \sigma_2)\Gamma_2 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_1)\text{Tr}(\Gamma_3)$$

$$+ 9\bar{u}_p(k_2, \sigma_2)\Gamma_1 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_2)\text{Tr}(\Gamma_3) - 3\bar{u}_p(k_2, \sigma_2)\Gamma_3 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_1\Gamma_2)$$

$$- 3\bar{u}_p(k_2, \sigma_2)\Gamma_2 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_1\Gamma_3) - 3\bar{u}_p(k_2, \sigma_2)\Gamma_1 u_p(k_1, \sigma_1)\text{Tr}(\Gamma_2\Gamma_3)]. \quad (55)$$

For the d flavor six-quark condensate we get

$$\langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\hat{d}\,\hat{\bar{d}}\,\Gamma_2\hat{d}\,\hat{\bar{d}}\,\Gamma_3\hat{d}|p(k_1, \sigma_1)\rangle$$

$$= \frac{1}{2}\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}\,\hat{\bar{u}}\,\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle. \quad (56)$$

These findings are, to the best of our knowledge, the first attempts to estimate the size of six-quark condensates inside a (bare) nucleon. An analog evaluation for the bare neutron reveals the isospin symmetry relations

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}\,\hat{\bar{u}}\,\Gamma_3\hat{u}|n(k_1, \sigma_1)\rangle$$

$$= \langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\hat{d}\,\hat{\bar{d}}\,\Gamma_2\hat{d}\,\hat{\bar{d}}\,\Gamma_3\hat{d}|p(k_1, \sigma_1)\rangle, \quad (57)$$

$$\langle n(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\hat{d}\,\hat{\bar{d}}\,\Gamma_2\hat{d}\,\hat{\bar{d}}\,\Gamma_3\hat{d}|n(k_1, \sigma_1)\rangle$$

$$= \langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\hat{u}\,\hat{\bar{u}}\,\Gamma_2\hat{u}\,\hat{\bar{u}}\,\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle. \quad (58)$$

Finally, by averaging over the proton matrix elements, Eqs. (55) and (56), and the neutron matrix elements, Eqs. (57) and (58), one gets the six-quark condensates inside a bare nucleon. For instance, the six-quark condensate for the scalar channel is found to be

$$\langle N(k, \sigma)|\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\hat{q}\,\hat{\bar{q}}\,\hat{q}|N(k, \sigma)\rangle = \frac{55}{8}\langle\hat{\bar{q}}\,\hat{q}\rangle_0^2 M_N. \quad (59)$$

Further, we present results for six-quark condensates which contain Gell-Mann matrices. Note that only nucleon matrix elements of colorless operators do not vanish, i.e. only two Gell-Mann matrices may occur. With the general result (B11) in Appendix B and normalization (13) we obtain

$$\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_2\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle = \frac{2}{(12)^2}\langle\hat{\bar{u}}\hat{u}\rangle_0^2\Bigg[\frac{16}{3}\bar{u}_p(k_2, \sigma_2)(\Gamma_1\Gamma_2\Gamma_3 + \Gamma_1\Gamma_3\Gamma_2 + \Gamma_2\Gamma_1\Gamma_3)u_p(k_1, \sigma_1)$$

$$+ \frac{16}{3}\bar{u}_p(k_2, \sigma_2)(\Gamma_2\Gamma_3\Gamma_1 + \Gamma_3\Gamma_1\Gamma_2 + \Gamma_3\Gamma_2\Gamma_1)u_p(k_1, \sigma_1)$$

$$- 16\bar{u}_p(k_2, \sigma_2)\Gamma_2\Gamma_1\bar{u}_p(k_1, \sigma_1)\text{Tr}(\Gamma_3) - 16\bar{u}_p(k_2, \sigma_2)\Gamma_1\Gamma_2\bar{u}_p(k_1, \sigma_1)\text{Tr}(\Gamma_3)$$

$$- 16\bar{u}_p(k_2, \sigma_2)\Gamma_3\bar{u}_p(k_1, \sigma_1)\text{Tr}(\Gamma_1\Gamma_2)\Bigg]. \tag{60}$$

For the d flavor we get

$$\langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\lambda^a\hat{d}\,\hat{\bar{d}}\Gamma_2\lambda^a\hat{d}\,\hat{\bar{d}}\Gamma_3\hat{d}|p(k_1, \sigma_1)\rangle$$

$$= \frac{1}{2}\langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_2\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle. \tag{61}$$

Finally, evaluating these operators inside the bare neutron we obtain the isospin symmetry relations

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_2\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_3\hat{u}|n(k_1, \sigma_1)\rangle$$

$$= \langle p(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\lambda^a\hat{d}\,\hat{\bar{d}}\Gamma_2\lambda^a\hat{d}\,\hat{\bar{d}}\Gamma_3\hat{d}|p(k_1, \sigma_1)\rangle, \tag{62}$$

$$\langle n(k_2, \sigma_2)|\hat{\bar{d}}\Gamma_1\lambda^a\hat{d}\,\hat{\bar{d}}\Gamma_2\lambda^a\hat{d}\,\hat{\bar{d}}\Gamma_3\hat{d}|n(k_1, \sigma_1)\rangle$$

$$= \langle p(k_2, \sigma_2)|\hat{\bar{u}}\Gamma_1\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_2\lambda^a\hat{u}\,\hat{\bar{u}}\Gamma_3\hat{u}|p(k_1, \sigma_1)\rangle. \tag{63}$$

As before, by averaging over the proton matrix elements, Eqs. (60) and (61), and the neutron matrix elements, Eqs. (62) and (63), one obtains the six-quark condensates containing Gell-Mann matrices inside a bare nucleon. The presented findings for six-quark condensates inside the bare nucleon provide basic results for further investigations beyond the order of four-quark condensates within the QCD sum rule approach for the nucleon in matter (for the nucleon sum rule in vacuum see [54,55], and for the nucleon sum rule in matter see [63,64,81]). Investigations aiming at predictions beyond the order of four-quark condensates, however, imply in addition to the evaluation of the six-quark condensates also the knowledge of the Wilson coefficients for all of these six-quark condensates. So far, these coefficients in the OPE for the nucleon correlator have been determined up to the order of the four-quark condensates [78].

VI. SUMMARY

An algebraic approach for evaluating bare nucleon matrix elements has been presented. The supposed nucleon formula (8) relates bare nucleon matrix elements to vacuum matrix elements and, therefore, new parameters for evaluating them are not needed. A feature of the algebraic method is that the valence quark contribution of two-quark, four-quark and six-quark condensates can be evaluated on the same footing. One aim of the present paper is to demonstrate how the nucleon formula works and to test it in several cases. In doing so, the nucleon has been considered as a composite pointlike object, described by a va-

lence quark and a valence diquark approximated by two classical Dirac spinors. Neither sea quarks nor gluons, or in a hadronic language no meson cloud, have been taken into account here. Accordingly, the results presented have to be considered as pure valence quark contribution to the matrix elements under consideration.

A consideration of the electromagnetic current (15) and the axial vector current (16) for the bare nucleon reveals the expected current structure for a pointlike object. We have evaluated then the valence quark contribution of the chiral condensate (21), finding the relation $\sigma_N^v = 3m_q$ which turns out to be in numerical agreement with results obtained in an earlier work [66].

Furthermore, the valence quark contribution of four-quark condensates has been investigated. Our results are given in Eqs. (27)–(32) for flavor-unmixed operators, and in Eqs. (46)–(49) for flavor-mixed operators. In the special case $k_1 = k_2, \sigma_1 = \sigma_2$ we find an interesting agreement with the groundstate saturation approximation explored in Ref. [64] if one separates the valence quark contribution of four-quark condensates from that results. In this respect our approach yields an independent re-evaluation and confirmation of the results of Ref. [64], because both methods are different from the conceptional point of view, which is already interesting in itself. Even more, our algebraic approach presented recovers the dependence of condensates on momentum for a pointlike nucleon and distinguishes between proton and neutron matrix elements. In this respect it goes beyond the common factorization approximation. In this context it is worth to underline that the agreement between our algebraic approach and the ground-state saturation approximation has been found for the bare nucleon, and not for the physical nucleon. In Eqs. (40)–(44) and (52) we have compared our results with values of four-quark condensates inside the physical nucleons recently obtained within a chiral quark model [8].

As a further application of nucleon formula we have presented results for six-quark condensates inside the bare nucleon, given in Eqs. (55)–(58) and Eqs. (60)–(63), respectively. These values obtained are, to the best of our knowledge, the first evaluation of six-quark condensates inside (bare) nucleons. Finally, in Eqs. (21), (33), and (59) we have given more explicit examples for the scalar channel of two-quark, four-quark and six-quark operators, respectively, inside the bare nucleon, showing up an interesting alternative change in the algebraic sign from

two-quark to four-quark and from four-quark to six-quark condensates.

A remark should also be in order about nucleon matrix elements of gluonic operators. Evaluating such operators within the algebraic approach presented requires, in general, the implementation of gluonic degrees of freedom into the composite nucleon field operator (12). Such an implementation might be provided by the quark-gluon interpolating nucleon field discussed in another context in [82].

The algebraic approach can be extended into several di rections. First, the description of the proton core with the field operator (12), and the corresponding one for the bare neutron, can be improved, e.g. by implementing an effective potential between the valence quarks. And second, the pion cloud of nucleon, accounting for virtual sea quarks and gluons inside the physical nucleon, can be implemented within the Tamm-Dancoff method. To get an algebraic approach also for such a case one has to combine the nucleon formula (8) with the soft pion theorem (A8). This implies the evaluation of the coefficients ϕ_n in (1) within the renormalizable pion-nucleon interaction Hamiltonian, which is therefore a topic of further investigations. Accordingly, for the time being the application of nucleon formula (8) in combination with the field operator (12) has to be considered as a first step in determining more accurately nucleon matrix elements of quark operators. In summary, we arrive at the conclusion that a reliable evaluation of quark operators inside nucleons can be based on a purely algebraic approach. This triggers the hope that predictions of in-medium properties of hadrons become more precise in future.

ACKNOWLEDGMENTS

APPENDIX A: SOFT PION THEOREM

In this Appendix we recall a soft pion theorem relevant for our purposes to show the similarity in derivation and final form of it with the nucleon formula (8). Let us

consider the general pion matrix element of an operator $\hat{\mathcal{O}}(x)$ which in general may depend on space and time [14]

$$
\langle\pi^b(p_2)|\hat{\mathcal{O}}(x)|\pi^a(p_1)\rangle = iZ_\varphi^{-1/2}\int d^4x_1 e^{-ip_1x_1}(\Box_{x_1} + m_\pi^2)
$$
$$
\times \langle\pi^b(p_2)|\mathrm{T}_W(\hat{\mathcal{O}}(x)\hat{\varphi}^a(x_1))|0\rangle,
$$
(A1)

where the LSZ reduction formalism has been applied on pion state $|\pi^a(p_1)\rangle$. Here, the letters $a, b = 1, 2, 3$ denote isospin indices. The normalization of pion state is $\langle\pi^b(p_2)|\pi^a(p_1)\rangle = 2E_{p_1}(2\pi)^3\delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2)\delta^{ab}$, where $E_{p_1} = \sqrt{\mathbf{p}_1^2 + m_\pi^2}$. The wave function renormalization constant is $0 \leq Z_\varphi^{-1/2} \leq 1$. The normalization of nonperturbative QCD vacuum is $\langle0|0\rangle = 1$. Here, $x_1 = (\mathbf{r}_1, t_1)$ is the space-time four-vector, and T_W denotes the Wick time ordering. The states $|\pi^a(p_i)\rangle$ are, of course, on-shell states, i.e. solutions of the Klein-Gordon equation for noninteracting pions, while the field operator $\hat{\varphi}^a$, in general, is the interacting field, i.e. it is off-shell. The four momenta in (A1) are on-shell, i.e. $p_1^2 = p_2^2 = m_\pi^2$. The soft pion theorem is valid for a noninteracting pion field (i.e. $Z_\varphi^{-1/2} = 1$), wich is a solution of the Klein-Gordon equation $(\Box_x + m_\pi^2)\hat{\varphi}^a(x)$. In order to be complete in the representation we will also specify the noninteracting pion field operator

$$
\hat{\varphi}^a(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}(\hat{a}^a(p)e^{-ipx} + \hat{b}^{a\dagger}(p)e^{ipx}), \quad (A2)
$$

where the creation and annihilation operators obey the following commutator relations

$$
[\hat{a}^a(p_1), \hat{a}^{b\dagger}(p_2)]_- = [\hat{b}^a(p_1), \hat{b}^{b\dagger}(p_2)]_-
$$
$$
= 2E_{p_1}(2\pi)^3\delta^{(3)}(\mathbf{p}_1 - \mathbf{p}_2)\delta^{ab}. \quad (A3)
$$

Accordingly, $|\pi^a(p)\rangle = \hat{a}^{a\dagger}(p)|0\rangle$. From (A2) and (A3) we deduce the equal-time commutator for the noninteracting pion fields,

$$
[\hat{\varphi}^a(\mathbf{r}_1, t), \partial_0\hat{\varphi}^b(\mathbf{r}_2, t)]_- = i\delta^{(3)}(\mathbf{r}_1 - \mathbf{r}_2)\delta^{ab}. \quad (A4)
$$

In addition, the soft pion theorem is only valid in case of vanishing four-momentum $p_1^\mu \to 0$ which implies $m_\pi = 0$. Then we get

$$
\lim_{p_1\to0}\langle\pi^b(p_2)|\hat{\mathcal{O}}(x)|\pi^a(p_1)\rangle
$$
$$
= i\int d^4x_1\Box_{x_1}\langle\pi^b(p_2)|\mathrm{T}_W(\hat{\mathcal{O}}(x)\hat{\varphi}^a(x_1))|0\rangle
$$
$$
= -i\langle\pi^b(p_2)|\left[\hat{\mathcal{O}}(x), \frac{\partial}{\partial t_1}\hat{\varphi}^a(x_1)\right]_-|0\rangle\delta(t - t_1). \quad (A5)
$$

In the last line we have used the equation of motion for the massless pion field, i.e. $\Box_{x_1}\hat{\varphi}^a(x_1) = 0$. Now, the PCAC hypothesis, which asserts a relation between the axial current and the pion field ($f_\pi \simeq 92.4$ MeV is the pion decay constant),

$$\hat{A}_\mu^a(x) = -f_\pi \partial_\mu \hat{\varphi}^a(x), \tag{A6}$$

is inserted into Eq. (A5) (by means of the field equation for the noninteracting pion it becomes obvious from (A6) that in the limit of vanishing pion mass the PCAC goes over to a conserved axial vector current (see also [58])). The axial vector current $\hat{A}_\mu^a(x)$ obeys the well known current algebra commutation relations [12] which directly leads to the soft pion theorem relevant for our purposes [14]

$$\lim_{p_1 \to 0}\langle \pi^b(p_2)|\hat{\mathcal{O}}(x)|\pi^a(p_1)\rangle = \frac{i}{f_\pi}\int d^4x_1 \langle \pi^b(p_2)|[\hat{\mathcal{O}}(x), \hat{A}_0^a(x_1)]_-|0\rangle\delta(t-t_1). \tag{A7}$$

One may apply the same steps as before on the other pion state as well, ending up with the soft pion theorem

$$\lim_{p_2 \to 0}\lim_{p_1 \to 0}\langle \pi^b(p_2)|\hat{\mathcal{O}}(x)|\pi^a(p_1)\rangle$$

$$= \frac{1}{f_\pi^2}\int d^4x_1 \int d^4x_2 \delta(t-t_1)\delta(t-t_2)$$

$$\times \langle 0|[\hat{A}_0^b(x_2), [\hat{\mathcal{O}}(x), \hat{A}_0^a(x_1)]_-]_-|0\rangle$$

$$= \frac{1}{f_\pi^2}\langle 0|[\hat{Q}_A^b, [\hat{\mathcal{O}}(x), \hat{Q}_A^a]_-]_-|0\rangle. \tag{A8}$$

where $\hat{Q}_A^a$ is the (time independent) axial charge, $\hat{Q}_A^a = \int d^3\mathbf{r}\hat{A}_0^a(\mathbf{r}, t)$. It seems expedient to emphasize that the QCD quark degrees of freedom were not necessary for deriving the soft pion theorem. If one expresses the axial vector current by quark fields, $\hat{A}_\mu^a(x) = \overline{\hat{\Psi}}(\tau^a/2)\gamma_\mu\gamma_5\hat{\Psi}$

with $\hat{\Psi} = (\hat{u}\,\hat{d})^{\mathrm{T}}$, and the pion fields as well by means of their interpolating fields (i.e. composite quark fields which have the quantum numbers of pions), then the relation (A6), the current algebra and, therefore, the soft pion theorem (A8) can also be established within QCD degrees of freedom. This theorem can then also be used for evaluating pion matrix elements of quark operators (see, for instance, [21,22]). Summarizing, the soft pion theorem is valid for a noninteracting pion field with vanishing pion four-momentum. In many applications such a restriction is not problematic since the pion mass is small compared to a typical hadronic scale of about 1 GeV.

APPENDIX B: EVALUATING EQ. (53)

To evaluate Eq. (53) we start with the case of two-quark field operators. The nucleon formula (8) with (12) yields

$$\langle p(k_2, \sigma_2)|\overline{\hat{u}}_\alpha^{\mathrm{i}}\hat{u}_\beta^{\mathrm{i}}|p(k_1, \sigma_1)\rangle = \overline{u}_p^{\beta_2}(k_2, \sigma_2)(\gamma_0)_{\beta_2\alpha_2}(\gamma_0)_{\alpha_1\beta_1}u_p^{\beta_1}(k_1, \sigma_1)\int d^3\mathbf{r}_1 \int d^3\mathbf{r}_2 e^{i\mathbf{k}_1\mathbf{r}_1}e^{-i\mathbf{k}_2\mathbf{r}_2}$$

$$\times \langle 0|[\hat{\psi}_p^{\alpha_2}(\mathbf{r}_2, 0), [\overline{\hat{u}}_\alpha^{\mathrm{i}}\hat{u}_\beta^{\mathrm{i}}, \overline{\hat{\psi}}_p^{\alpha_1}(\mathbf{r}_1, 0)]_-]_+|0\rangle. \tag{B1}$$

Inserting the proton field operator (12) and using $[\hat{A}\hat{B}, \hat{C}]_- = \hat{A}[\hat{B}, \hat{C}]_+ - [\hat{A}, \hat{C}]_+\hat{B}$ for the commutator we obtain

$$[\overline{\hat{u}}_\alpha^{\mathrm{i}}\hat{u}_\beta^{\mathrm{i}}, \overline{\hat{\psi}}_p^{\alpha_1}(\mathbf{r}_1, 0)]_- = A_p^* \epsilon^{abc}(\overline{u}^{a\mathrm{T}}(\mathbf{r}_1)C\gamma_5\overline{d}^b(\mathbf{r}_1))$$

$$\times \delta^{\mathrm{ic}}(\gamma_0)_{\beta\alpha_1}\delta^{(3)}(\mathbf{r}_1)\overline{\hat{u}}_\alpha^{\mathrm{i}}. \tag{B2}$$

In the same way we get

$$[\hat{\psi}_p^{\alpha_2}(\mathbf{r}_2, 0), [\overline{\hat{u}}_\alpha^{\mathrm{i}}\hat{u}_\beta^{\mathrm{i}}, \overline{\hat{\psi}}_p^{\alpha_1}(\mathbf{r}_1, 0)]_-]_+$$

$$= |A_p|^2 \epsilon^{abc}\epsilon^{a'b'c'}(\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha\alpha_2}\delta^{cc'}\delta^{(3)}(\mathbf{r}_1)\delta^{(3)}(\mathbf{r}_2)$$

$$\times (u^{a'\mathrm{T}}(\mathbf{r}_2)C\gamma_5 d^{b'}(\mathbf{r}_2))(\overline{u}^{a\mathrm{T}}(\mathbf{r}_1)C\gamma_5\overline{d}^b(\mathbf{r}_1)). \tag{B3}$$

Integrating over both delta-functions and then using the normalization (13) we obtain

$$\int d^3\mathbf{r}_1 e^{i\mathbf{k}_1\mathbf{r}_1}\int d^3\mathbf{r}_2 e^{-i\mathbf{k}_2\mathbf{r}_2}$$

$$\times \langle 0|[\hat{\psi}_p^{\alpha_2}(\mathbf{r}_2, 0), [\overline{\hat{u}}_\alpha^{\mathrm{i}}\hat{u}_\beta^{\mathrm{i}}, \overline{\hat{\psi}}_p^{\alpha_1}(\mathbf{r}_1, 0)]_-]_+|0\rangle$$

$$= 2(\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha\alpha_2}, \tag{B4}$$

and with (B1)

$$\langle p(k_2, \sigma_2)|\overline{\hat{u}}_\alpha^{\mathrm{i}}\hat{u}_\beta^{\mathrm{i}}|p(k_1, \sigma_1)\rangle = 2\overline{u}_p^\alpha(k_2, \sigma_2)u_p^\beta(k_1, \sigma_1). \tag{B5}$$

We note an analog relation for the d quark

$$\langle p(k_2, \sigma_2)|\overline{\hat{d}}_\alpha^{\mathrm{i}}\hat{d}_\beta^{\mathrm{i}}|p(k_1, \sigma_1)\rangle = 1\overline{u}_p^\alpha(k_2, \sigma_2)u_p^\beta(k_1, \sigma_1), \tag{B6}$$

while for the neutron we have

$$\langle n(k_2, \sigma_2)|\hat{\bar{u}}^i_\alpha \hat{u}^i_\beta|n(k_1, \sigma_1)\rangle = 1\bar{u}^\alpha_n(k_2, \sigma_2)u^\beta_n(k_1, \sigma_1),$$
(B7)

$$\langle n(k_2, \sigma_2)|\hat{\bar{d}}^i_\alpha \hat{d}^i_\beta|n(k_1, \sigma_1)\rangle = 2\bar{u}^\alpha_n(k_2, \sigma_2)u^\beta_n(k_1, \sigma_1).$$
(B8)

Using relations like $[\hat{A}, \hat{B}\,\hat{C}]_+ = [\hat{A}, \hat{B}]_+ \hat{C} - \hat{B}[\hat{A}, \hat{C}]_-$ analog equations for the four-quark condensates can be obtained. Two illustrative examples are given for the flavor-unmixed four-quark condensate for the proton

$$\int d^3\mathbf{r}_1 e^{i\mathbf{k}_1\mathbf{r}_1} \int d^3\mathbf{r}_2 e^{-i\mathbf{k}_2\mathbf{r}_2}$$

$$\times \langle 0|[\hat{\psi}^{\alpha_2}_p(\mathbf{r}_2, 0), [\hat{\bar{u}}^i_\alpha \hat{u}^i_\beta \hat{\bar{u}}^j_\gamma \hat{u}^j_\delta, \hat{\bar{\psi}}^{\alpha_1}_p(\mathbf{r}_1, 0)]_-]_+|0\rangle$$

$$= \frac{1}{6}\langle\bar{u}\,u\rangle_0(3(\gamma_0)_{\delta\alpha_1}(\gamma_0)_{\alpha_2\gamma}\delta_{\alpha\beta} - (\gamma_0)_{\delta\alpha_1}(\gamma_0)_{\alpha_2\alpha}\delta_{\beta\gamma}$$

$$+ 3(\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha_2\alpha}\delta_{\gamma\delta} - (\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha_2\gamma}\delta_{\alpha\delta}),$$ (B9)

where the normalization (13) and $\langle\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta\rangle_0 = \frac{1}{12}\delta^{ij}\delta_{\alpha\beta}\langle\bar{u}\,u\rangle_0$ [1] has been used. For four-quark condensates with Gell-Mann matrices involved we find

$$\int d^3\mathbf{r}_1 e^{i\mathbf{k}_1\mathbf{r}_1} \int d^3\mathbf{r}_2 e^{-i\mathbf{k}_2\mathbf{r}_2}\langle 0|[\hat{\psi}^{\alpha_2}_p(\mathbf{r}_2, 0), [\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta \hat{\bar{u}}^k_\gamma \hat{u}^l_\delta, \hat{\bar{\psi}}^{\alpha_1}_p(\mathbf{r}_1, 0)]_-]_+|0\rangle(\lambda^a)^{ij}(\lambda^a)^{kl}$$

$$= |A_p|^2 \epsilon^{abc}\epsilon^{a'b'c'}(u^{a'T}C\gamma_5 d^{b'})(\bar{u}^{aT}C\gamma_5\bar{d}^b)\left(2\delta^{il}\delta^{kj} - \frac{2}{3}\delta^{ij}\delta^{kl}\right)((\gamma_0)_{\delta\alpha_1}(\gamma_0)_{\alpha_2\gamma}\delta_{\alpha\beta}\delta^{cl}\delta^{c'k}\delta^{ij}$$

$$- (\gamma_0)_{\delta\alpha_1}(\gamma_0)_{\alpha_2\alpha}\delta_{\beta\gamma}\delta^{cl}\delta^{ic'}\delta^{jk} + (\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha_2\alpha}\delta_{\gamma\delta}\delta^{jc}\delta^{ic'}\delta^{kl} - (\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha_2\gamma}\delta_{\alpha\delta}\delta^{jc}\delta^{c'k}\delta^{il})$$

$$= -\frac{8}{9}[(\gamma_0)_{\delta\alpha_1}(\gamma_0)_{\alpha_2\alpha}\delta_{\beta\gamma} + (\gamma_0)_{\beta\alpha_1}(\gamma_0)_{\alpha_2\gamma}\delta_{\alpha\delta}]\langle\bar{u}\,u\rangle_0.$$ (B10)

By using the same technique the general result for the six-quark condensates is obtained as

$$\int d^3\mathbf{r}_1 e^{i\mathbf{k}_1\mathbf{r}_1} \int d^3\mathbf{r}_2 e^{-i\mathbf{k}_2\mathbf{r}_2}\langle 0|[\hat{\psi}^{\alpha_2}_p(\mathbf{r}_2, 0), [\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta \hat{\bar{u}}^k_\gamma \hat{u}^l_\delta \hat{\bar{u}}^m_\epsilon \hat{u}^n_\zeta, \hat{\bar{\psi}}^{\alpha_1}_p(\mathbf{r}_1, 0)]_-]_+|0\rangle$$

$$= |A_p|^2\epsilon^{abc}\epsilon^{a'b'c'}(u^{a'T}C\gamma_5 d^{b'})(\bar{u}^{aT}C\gamma_5\bar{d}^b)(\langle\hat{u}^j_\beta \hat{\bar{u}}^k_\gamma \hat{u}^l_\delta \hat{\bar{u}}^m_\epsilon\rangle_0(\gamma_0)_{\alpha_1\zeta}(\gamma_0)_{\alpha_2\alpha}\delta^{cn}\delta^{c'i} + \langle\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta \hat{u}^l_\delta \hat{\bar{u}}^m_\epsilon\rangle_0(\gamma_0)_{\alpha_1\zeta}(\gamma_0)_{\alpha_2\gamma}\delta^{cn}\delta^{c'k}$$

$$+ \langle\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta \hat{\bar{u}}^k_\gamma \hat{u}^l_\delta\rangle_0(\gamma_0)_{\alpha_1\zeta}(\gamma_0)_{\alpha_2\epsilon}\delta^{cn}\delta^{c'm} + \langle\hat{u}^j_\beta \hat{\bar{u}}^k_\gamma \hat{\bar{u}}^m_\epsilon \hat{u}^n_\zeta\rangle_0(\gamma_0)_{\alpha_1\delta}(\gamma_0)_{\alpha_2\alpha}\delta^{cl}\delta^{c'i} + \langle\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta \hat{\bar{u}}^m_\epsilon \hat{u}^n_\zeta\rangle_0(\gamma_0)_{\alpha_1\delta}(\gamma_0)_{\alpha_2\gamma}\delta^{cl}\delta^{c'k}$$

$$- \langle\hat{\bar{u}}^i_\alpha \hat{u}^j_\beta \hat{\bar{u}}^k_\gamma \hat{u}^n_\zeta\rangle_0(\gamma_0)_{\alpha_1\delta}(\gamma_0)_{\alpha_2\epsilon}\delta^{cl}\delta^{c'm} + \langle\hat{\bar{u}}^k_\gamma \hat{u}^l_\delta \hat{\bar{u}}^m_\epsilon \hat{u}^n_\zeta\rangle_0(\gamma_0)_{\alpha_1\beta}(\gamma_0)_{\alpha_2\alpha}\delta^{cj}\delta^{c'i} - \langle\hat{\bar{u}}^i_\alpha \hat{u}^l_\delta \hat{\bar{u}}^m_\epsilon \hat{u}^n_\zeta\rangle_0(\gamma_0)_{\alpha_1\beta}(\gamma_0)_{\alpha_2\gamma}\delta^{cj}\delta^{c'k}$$

$$- \langle\hat{\bar{u}}^i_\alpha \hat{\bar{u}}^k_\gamma \hat{u}^l_\delta \hat{u}^n_\zeta\rangle_0(\gamma_0)_{\alpha_1\beta}(\gamma_0)_{\alpha_2\epsilon}\delta^{cj}\delta^{c'm}).$$ (B11)

Note that for applying the normalizations (13) and (14) one needs a term $\delta^{cc'}$, which naturally arises when evaluating a specific matrix element under consideration.

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[57] When coupling two quarks to a diquark one obtains $\{3\}_c \otimes \{3\}_c = \{6\}_c \oplus \{\bar{3}\}_c$, i.e. a totally symmetric color sextet $\{6\}_c$ and a totally antisymmetric color antitriplet $\{\bar{3}\}_c$. To get a color singlet nucleon state only the antisymmetric color states $\{\bar{3}\}_c$, which are also color attractive, cf. [83], can contribute. Therefore we have $u^{aT}C\gamma_5 d^b = -u^{bT}C\gamma_5 d^a$. Then, together with the relation $\bar{u}^{aT}C\gamma_5 \bar{d}^b = (u^{aT}C\gamma_5 d^b)^*$ one may easily show that the term (13) is a positive real number. In the same way one proves that (14) is also a positive real number. Note that A_p and B_p have (mass dimension)$^{-3}$ and are adjusted such that the normalizations (13) and (14) are obtained.

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[61] From neutron β decay experiment, the total axial charge is $g_A = 1.255$. A numerical value for the valence quark contribution of the axial charge can be obtained by means of the decomposition $g_A = g_A^v + g_A^\pi$ and the results of the MIT Bag model (for massless pions), $g_A^\pi = 0.5 g_A^v$ [14], which yields $g_A^v \simeq 0.84$.

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[69] Taking into account the first two terms in (23) refers to the so called factorization approximation. These terms, however, vanish when $\hat{A}$ and $\hat{B}$ are nonscalar operators. On the other side, the decomposition (23) is, by virtue of a Fierz transformation of $\hat{A}\hat{B}$, not unique (see also endnote [70]). Then, in particular, the factorization approximation does not vanish, even in the case when $\hat{A}$ and $\hat{B}$ are nonscalar operators, cf. Eqs. (24) and (25).

[70] As an explicit example of footnote [69] we consider the vector channel, $\hat{\mathcal{O}}_V = \hat{\bar{q}}^i\gamma_\mu\hat{q}^i\hat{\bar{q}}^j\gamma^\mu\hat{q}^j$. Because of $\langle 0|\hat{\bar{q}}^i\gamma_\mu\hat{q}^i|0\rangle = 0$ the factorization term vanishes according to Eq. (23), i.e. only the correction term remains: (1) $\langle N_{\text{phys}}|\hat{\mathcal{O}}_V|N_{\text{phys}}\rangle = 0 +$ correction (a). On the other side, a Fierz rearrangement of $\hat{\mathcal{O}}_V$ and afterwards the use of (23) leads to (2) $\langle N_{\text{phys}}|\hat{\mathcal{O}}_V|N_{\text{phys}}\rangle = -2\langle 0|\hat{\bar{q}}^i\hat{q}^i|0\rangle \times \langle N_{\text{phys}}|\hat{\bar{q}}^j\hat{q}^j|N_{\text{phys}}\rangle +$ correction (b), i.e. the factorization term is not vanishing anymore. The seeming difference concerning the factorization term between the expression (1) and (2) is absorbed into the correction terms (a) and

(b) which differ from each other. Furthermore, while the expression (1) is in line with Eq. (23), the expression (2) just coincides with Eq. (24). This concrete example elucidates the fact, that (23) on one side and (24) and (25) on the other side are matched with each other. Note that $\langle 0|\hat{\bar{q}}^i\hat{q}^j|0\rangle = 1/3\delta^{ij}\langle 0|\hat{\bar{q}}\,\hat{q}\,|0\rangle$, and $\langle N_{\text{phys}}|\hat{\bar{q}}^i\hat{q}^j|N_{\text{phys}}\rangle = 1/3\delta^{ij}\langle N_{\text{phys}}|\hat{\bar{q}}\,\hat{q}\,|N_{\text{phys}}\rangle$.

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[75] For combinations containing $\sigma_{\mu\nu}$ we remind that $\bar{u}_N(k,\sigma)\sigma_{\mu\nu}u_N(k,\sigma) = 0$. This can be proven, for instance, by inserting the completeness relation $2M_N\mathbb{1} = \sum_{\sigma=1}^2 u_N(k,\sigma)\bar{u}_N(k,\sigma) - v_N(k,\sigma)\bar{v}_N(k,\sigma)$.

[76] We compare the sign and magnitude of our evaluated valence quark contribution with the total contribution for the physical nucleon, because in Ref. [8] the valence quark contribution vanishes for all nonscalar four-quark condensates. This vanishing is neither in contradiction with our findings nor with the valence quark contribution extracted from the factorization approximation in Ref. [64] (see also [69,70]).

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