Evidence for In-Medium Changes of Four-Quark Condensates

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Utilizing the QCD sum rule approach to the behavior of the ω meson in nuclear matter we derive evidence for in-medium changes of particular four-quark condensates from the recent CB-TAPS experiment for the reaction \( \gamma + A \rightarrow A' + \omega (\rightarrow \pi^0 \gamma) \) with \( A = Nb \) and LH2.

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The chiral condensate \( \langle \bar{q}q \rangle \) is an order parameter for the spontaneous breaking of chiral symmetry in the theory of strong interaction (cf. e.g., Ref. [1] for introducing this topic). The role of \( \langle \bar{q}q \rangle \) is highlighted, e.g., by the Gell-Mann–Oakes–Renner relation \( m_\pi^2 f_\pi^2 \propto -\langle \bar{q}q \rangle \) (cf. Ref. [2]); the explicit chiral symmetry breaking is essential for a finite pion mass \( \langle \bar{q}q \rangle \) (Ref. [2]; the explicit chiral symmetry breaking is essential for a finite pion mass \( \langle \bar{q}q \rangle \)). There is growing evidence that the quark-gluon condensate is another order parameter [5]. The QCD trace anomaly related to scale invariance breaking gives rise to the gluon condensate. There are many other condensates characterizing the complicated structure of the QCD vacuum. In a medium, described by temperature and baryon density \( n \), these condensates change; i.e., the ground state is rearranged. Since hadrons are considered as excitations above the vacuum, a vacuum change should manifest itself as a change of the hadronic excitation spectrum. This idea triggered widespread activities to search for in-medium modifications of hadrons. Such in-medium modifications of hadronic observables are found (cf. the lists in Ref. [6,7]), and it is timely to relate them to corresponding order parameters.

We deduce here evidence for a noticeable drop of in-medium four-quark condensates in cold nuclear matter from results of the recent CB-TAPS experiment [6] for the reaction \( \gamma + A \rightarrow A' + \omega (\rightarrow \pi^0 \gamma) \). The CB-TAPS collaboration observed the occurrence of additional low-energy \( \omega \) decay strength for a Nb \((A = 93)\) target compared to a LH2 \((A = 1)\) target. The link of observables to quark and gluon condensates is established by QCD sum rules [8], which are expected to be sensitive to four-quark condensates in the vector channels [9]. Four-quark condensate combinations which contain only left-right helicity flipping terms (as the chiral condensate does) represent other order parameters of chiral symmetry.

Concentrating on the isoscalar part of the causal current-current correlator [3]

\[
\Pi_{\omega}(q, n) = \frac{i}{3} \int d^4 x e^{i q x} \langle \Omega | T j_\mu(x) j^{\mu}(0) | \Omega \rangle,
\]

here for the \( \omega \) meson with the current \( j_\mu = (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d)/2 \) and nuclear matter states \( | \Omega \rangle \) (the symbol \( T \) means time ordering, and \( u, d \) denote quark field operators), an operator product expansion and a Borel transformation (cf. Refs. [3,10] for arguments in favor of Borel sum rules) of the twice-subtracted dispersion relation result in

\[
\Pi_{\omega}(0, n) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi_{\omega}(s, n)}{s} e^{-s/M^2} = c_0 M^2 + \sum_{j=0}^\infty \frac{c_j}{(j-1)! M^{2(j-1)}},
\]

where \( \Pi_{\omega}(0, n) = 9 n / (4 M_\omega) \) with the nucleon mass \( M_N \) is a subtraction constant having the meaning of Landau damping or \( oN \) forward scattering amplitude, and the coefficients \( c_j \) contain condensates and Wilson coefficients; \( M \) is the Borel mass. The first coefficients \( c_j \) have been spelled out in many papers (cf. Ref. [11] for our notation, and Ref. [12] for an anomalous contribution) and are not reproduced here in full length. \( c_0 = (1 + \frac{Q^2}{L^2})/(8 \pi^2) \) is the perturbative term. \( c_1 \approx m_q^2 \) is exceedingly small due to the small current quark mass \( m_q \). In \( c_2 \) the gluon condensate (being less sensitive to medium effects), some moments of the parton distribution in the nucleon (combined with a density dependence), and the renormalization group invariant combination \( m_q \langle \bar{q}q \rangle \) (being numerically tiny) enter. The latter fact makes the Borel sum rule insensitive to the genuine chiral condensate, but sensitive to four-quark condensates which enter \( c_3 \), among other quantities related to expectation values of certain traceless and symmetric twist-2 and twist-4 operators. To be specific, the flavor-mixing condensates \( \frac{2}{3} \langle \bar{u} \gamma^\mu \lambda_u \bar{d} \gamma_\mu \lambda_d \rangle + \langle \bar{d} \gamma^\mu \lambda_d \bar{u} \gamma_\mu \lambda_u \rangle \) and the pure flavor four-quark condensates (for which we employ \( u - d \) isospin symmetry; \( \gamma_\mu \) and \( \lambda_u \) stand for Dirac and Gell-Mann matrices) \( \frac{2}{3} \langle \bar{q} \gamma^\mu \lambda_u \bar{q} \gamma^\nu \lambda_d \rangle + \langle \bar{q} \gamma_\mu \gamma^\nu \lambda_u \bar{q} \gamma_\nu \gamma_d \lambda_u \lambda_d \rangle \) enter \( c_3 \). (\( c_4 \) will be discussed below.) Our strategy to deal with these condensates is as follows: (i) the factorized expressions (which might fail badly [3,12]) are corrected by factors...
\( \kappa_{\Omega} \) (with \( \Omega \) being a label of the respective four-quark condensate) using \( \langle \bar{u} \gamma^\mu \lambda_{\mu} u \bar{d} \gamma^\nu \lambda_{\nu} d \rangle = - \kappa_1 \frac{2}{2 \pi} \frac{\Omega^2}{T^2} \langle \bar{q} q \rangle^2 \),
\( \langle \bar{q} \gamma^\mu \lambda_{\mu} u \bar{d} \gamma^\nu \lambda_{\nu} d \rangle = \kappa_2 \frac{4}{2 \pi} \frac{\Omega^2}{T^2} \langle \bar{q} q \rangle^2 \) (where \( Q_0 \)
is a cutoff related to the \( \rho - \omega \) mass splitting; both expressions are already beyond the ground state saturation [11]), \( \langle \bar{q} \gamma^\mu \lambda_{\mu} A q \bar{q} \gamma^\nu \lambda_{\nu} A q \rangle = - \frac{16}{9} \kappa_3 \langle \bar{q} q \rangle^2 \),
\( \langle \bar{q} \gamma^\mu \lambda_{\mu} A q \bar{q} \gamma^\nu \lambda_{\nu} A q \rangle = \frac{16}{9} \kappa_4 \langle \bar{q} q \rangle^2 \); \( \kappa_{1,2} = 0 \) and \( \kappa_{3,4} = 1 \) recover the factorized terms in the ground state saturation approximation; (ii) expand \( \kappa_{\Omega} \) in density [13], i.e., \( \kappa_{\Omega} = \kappa_{\Omega}^{(0)} + \kappa_{\Omega}^{(1)} n \), use the known sigma term \( \sigma_N \)
in \( \langle \bar{q} q \rangle = \langle \bar{q} q \rangle_0 + \xi n \) with \( \xi = \sigma_N / (2m_q) \) [3], and linearize the resulting expressions [14]; (iii) add up all contributions with their corresponding prefactors to get a common factor
\( \kappa_0 = \frac{9}{2 \pi} \frac{\Omega^2}{T^2} \langle \bar{q} q \rangle_0^2 \) and a common factor
\( \kappa_N = \kappa_0 + \frac{\langle \bar{q} q \rangle_0^2}{\langle \bar{q} q \rangle_0^2} \) for the density dependent medium contribution of the mentioned four-quark condensates [cf. the first term in Eq. (10) below]. \( \kappa_0 \) enters the vacuum sum rule and has to be adjusted properly with other quantities to get the correct vacuum \( \omega \) mass, while \( \kappa_N \) is the subject of our further consideration. Because of the mixing of density dependencies of \( \kappa_0 \) and \( \langle \bar{q} q \rangle \) even the accurate knowledge of \( \kappa_0 \) does not fix \( \kappa_N \).

No density dependence of the four-quark condensates would imply \( \kappa_N = 0 \), while strong density dependencies will result in a sizeable value of \( \kappa_N \), unless the terms contributing to \( \kappa_N \) cancel. The estimate in Ref. [11] points to small values of \( \kappa_{1,2} \) thus having essentially the density dependence of the pure flavor scalar dimension-6 condensates \( \frac{1}{8} \langle \bar{q} q \rangle_0 \lambda_{\mu} \lambda_{\nu} A q \bar{q} \gamma^\mu \lambda_{\nu} A q \rangle \) and \( \langle \bar{q} \gamma^\mu \lambda_{\mu} A q \bar{q} \gamma^\nu \lambda_{\nu} A q \rangle \) to be constrained.

Large-\( N_c \) arguments [3,15] favor \( \kappa_0 = 0 \). Previously, often the factorization \( \langle \bar{q} \cdot q \cdot q \cdot q \rangle \rightarrow \langle \bar{q} q \rangle^2 \) has been used. Here, we study explicitly, however, the role of the four-quark condensates using the square of the genuine chiral condensate only to set the scale, as outlined above. The integral in the left hand side of (2)
\[
m^2_{\omega}(n, M^2, s_\omega) = \frac{c_0 M^2 [1 - (1 + \frac{c_2}{M^2}) e^{-s_\omega/M^2}] - c_2 - \frac{c_2}{M^2}}{c_1 (1 - e^{-s_\omega/M^2}) + \frac{c_1}{M^2} + \frac{c_1}{M^4} + \frac{c_4}{6 M^6} - \frac{\Pi(0, n)}{M^2}} \]  
(4)

This sum rule is to be handled as usual (cf. Refs. [10,11]): determine the sliding Borel window by requiring that (i) the sum of the \( c_{3,4} \) terms in Eq. (2) does not contribute more than 10% to the right hand side; (ii) the continuum part defined above does not exceed 50% of the left hand side of (2) to ensure sufficient sensitivity for the resonance part; (iii) the continuum threshold is determined by the requirement of maximum flatness of \( m^2_{\omega}(n, M^2, s_\omega) \) within the Borel window; and (iv) \( m^2_{\omega}(n, M^2, s_\omega) \) follows as average with respect to \( M^2 \).

can be decomposed in a low-lying resonance part, \( f_{\omega}(s_\omega, n) e^{-s_\omega/M^2} \), and the continuum part, \( f_\omega(s_\omega, n) e^{-s_\omega/M^2} \), both depending on the continuum threshold \( s_\omega \). The quantity
\[
m^2_{\omega}(n, M^2, s_\omega) = \frac{\int_0^s ds \int_0^1 ds I_{\Pi}(s) e^{-s/M^2}}{\int_0^s ds \int_0^1 ds I_{\Pi}(s) e^{-s/M^2}} \]  
(3)

is a normalized moment with \( s \) meaning the coordinate of the center of gravity of \( \int_0^s ds \int_0^1 ds I_{\Pi}(s) e^{-s/M^2} / s \) in the interval \( s = 0 \cdots s_\omega \). Clearly, when additional strength of \( I_{\Pi} \) at lower values of \( s \) is caused by in-medium effects as observed in Ref. [6], then the center of gravity shifts to the left; i.e., \( m^2_{\omega}(n, M^2, s_\omega) \) becomes smaller. Direct use of the count rates in the middle panel of Fig. 2 in Ref. [6] as estimator of \( \int_0^s ds \int_0^1 ds I_{\Pi}(s) e^{-s/M^2} / s \) in the interval \( s = 0.41 \cdots 0.77 \) GeV\(^2\) yields \( m^2_{\omega}(N_{H_{2}}) = 0.599 \) GeV\(^2\) and \( m^2_{\omega}(N_{B}) = 0.568 \) GeV\(^2\) for \( M \sim O(1) \) GeV. Instead of testing the consistency of a particular model for \( I_{\Pi}(s, n) \) with the sum rule, we suggest here to use the experimental information on \( I_{\Pi} \) to find constraints on the QCD side of the sum rule. In fact, the \( \omega \) decay rate \( \omega \rightarrow \pi^0 \gamma \) is given by \( dR_{\omega \rightarrow \pi^0 \gamma} / dM_{\pi^0 \gamma} = (d \sigma / d \Omega)^2 \pi(3 q^2) \times (q^2 - m^2_{\omega}) \) \( \omega \rightarrow q^2 = s \) with \( d = 0.011 \). However, acceptance and efficiency corrections to the results of Ref. [6] need to be invoked and the fraction of events, where the rate \( dR_{\omega \rightarrow \pi^0 \gamma} / dM_{\pi^0 \gamma} \) is shifted to smaller values of \( M_{\pi^0 \gamma} \) (being the invariant mass of the \( \pi^0 \) and \( \gamma \) decay products of \( \omega \)) by final state interaction of the decay \( \pi^0 \) in the ambient nuclear medium [16], must be corrected for as well. We postpone such a quantitative and model dependent study for future work and consider qualitatively here the implication of the observation of Ref. [6], i.e., the occurrence of additional \( \omega \) decay strength at \( M_{\pi^0 \gamma} \ll m^2_{\omega} \) which translates into \( m_{\omega} < m^2_{\omega} \)-0.782 GeV for low-momentum \( \omega \) decaying in the Nb nucleus.

With (3) the truncated QCD sum rule (2) for the \( \omega \) meson can be arranged as [11]
\[
m^2_{\omega}(n, M^2, s_\omega) = R + \Delta n \]  
(5)

Despite the linear density expansion of the condensates entering the coefficients \( c_j \), the sum rule (4) is nonlinear in density. It is instructive to consider the linearized form. Using the notation \( s_\omega = s_{\omega}^{(0)} + s_{\omega}^{(1)} n \) and \( c_j = c_j^{(0)} + c_j^{(1)} n \) we arrive at
\[
m^2_{\omega}(n, M, s_{\omega}^{(0)}, s_{\omega}^{(1)}) = R + \Delta n \]  
with
The mass parameter \( m_{w0} \) determined by the chiral limit is required, as evidenced by Eqs. (5) and (11). Thereby, the term \( c_3^{(i)} \) provides a counterbalance to the large Landau damping [17] [first term in (8)]. (For the \( \rho \) meson the Landau damping term is 9 times smaller [17], resulting in an always negative shift parameter conforming with the dropping \( \rho \) mass scenario in Ref. [18] and in qualitative agreement with the Brown-Rho scaling [19].)

Indeed, the evaluation of the complete sum rule (4) requires for the described parameter set \( \kappa_N \geq 4 \) to have \( m_{w0}^2 (n > 0) < m_{w0}^{(0)} \); see Fig. 1. In other words, the above mentioned four-quark condensates must change drastically in the nuclear medium. With the above quoted parameters this translates into the huge amount of more than a 50% drop of the combined four-quark condensates at nuclear matter saturation density when relying on the linear density expansion up to such density. (The experiment [6] probes actually densities \(-0.6 n_0\).) Phrased differently, the density by less constrained quantities in (6)–(10) we assign \( N, R, c_0^{(0)}, c_2^{(i)}, \) and \( c_3^{(i)} \) (the term in front of \( \kappa_N \) and the remainder separately) the large uncorrelated variations of \( \pm 10\% \) and arrive at \( \Delta = (2.8 \cdots 5.3 - \kappa_N) \times (0.023 \cdots 0.035) n_0^{-1} \text{GeV}^2 \). In essence, to achieve a negative value of \( \Delta \) and thus the experimentally observed [6] dropping of \( m_w \) in medium, a sufficiently large value of \( \kappa_N \) is required, as evidenced by Eqs. (5) and (11).

\[
R = \frac{1}{N} \left[ c_0^{(0)} M^2 - \left( 1 + \frac{s_w^{(0)}}{M^2} \right) E - \frac{c_2^{(0)}}{M^2} - \frac{c_3^{(0)}}{2 M^2} \right],
\]

\[
N = c_0^{(0)} (1 - E) + \frac{c_1^{(0)}}{M^2} + \frac{c_2^{(0)}}{M^2} + \frac{c_3^{(0)}}{2 M^2} + \frac{c_4^{(0)}}{6 M^2},
\]

\[
\Delta = \frac{1}{N M^2} \left[ \frac{9 R}{4 M_N} + c_0^{(0)} E_{s_0} - c_2^{(i)} (1 + \frac{R}{M^2}) \right] - \frac{c_3^{(i)}}{M^2} \left( 1 + \frac{R}{2 M^2} \right) - \frac{c_4^{(i)}}{M^2} \left( 1 + \frac{R}{3 M^2} \right),
\]

\[
E = e^{-\omega_0^2}/M^2,
\]

which we use for illustrative purposes. The quantity \( R \) determines the vacuum properties of the \( \omega \) meson; for the sake of estimates we can put it equal to 0.002 and use \( M \sim 1 \) GeV. \( N \) contains only vacuum quantities, and \( N > 0 \) holds. Therefore, the sign of the in-medium shift of \( m_w^2 \) is determined by \( \Delta \). For its estimate we note

\[
c_2^{(i)} = \frac{1}{2} \left( 1 + \frac{1}{3} \frac{\alpha_s}{\pi} \right) \sigma_N - \frac{M_{N,0}}{27} + \left( \frac{1}{4} - \frac{5}{36} \frac{\alpha_s}{\pi} \right) A_{u+d}^2 M_N
\]

\[
- \frac{9}{16} \frac{\alpha_s}{\pi} A_A^2 M_N,
\]

\[
c_3^{(i)} = -\frac{112}{81} \frac{\alpha_s}{\pi} \sigma_N \langle \bar{q} q \rangle_0 - \frac{5}{12} \frac{\alpha_s}{144} A_{u+d}^2 M_N^3
\]

\[
+ \frac{615}{324} \frac{\alpha_s}{\pi} A_A^2 M_N^3
\]

\[
+ \frac{1}{4} M_N \left[ \frac{3}{2} K_{n,1} + \frac{3}{8} K_{n,2} + \frac{15}{16} K_{n,g} \right] - \frac{7}{144} \sigma_N M_N^2,
\]

where we include three active flavors on a 1 GeV scale; \( M_{N,0} = 0.77 \) GeV is the nucleon mass in the chiral limit. Some of the quantities in (6)–(10) are rather well-known (e.g., the twist-2 contributions), while others are individually less accurately fixed. We use for our evaluations \( \langle \bar{q} q \rangle_0 = (-0.245 \text{ GeV}^3) \), \( \sigma_N = 0.045 \text{ GeV} \), \( \alpha_s = 0.38 \), \( Q_0 = 0.15 \text{ GeV} \), \( f_\pi = 0.093 \text{ GeV} \), \( A_{u+d} = 1.02 \), \( A_A^2 = 0.83 \), \( A_{u+d}^2 = 0.12 \), \( A_A^2 = 0.04 \), \( K_{n,1} = -0.112 \text{ GeV}^2 \), \( K_{n,2} = 0.11 \text{ GeV}^2 \), \( K_{n,g} = -0.3 \text{ GeV}^2 \) [11] to get \( c_2^{(i)} \) to be calculated from data or models. The sum rule Eq. (4) is evaluated as described in the text with appropriately adjusted \( \alpha_s \). Inclusion of \( c_4^{(0)} = \hat{O}(\pm 10^{-3}) \text{ GeV}^6 \) requires a readjustment of \( \kappa_N \) in the range 1–5 to \( m_{w0}^{(0)} \). A simultaneous change of \( \kappa_N \) in the order of 20% is needed to recover the same density dependence as given by the solid curve at small values of \( n \). The effect of \( c_4^{(i)} \) term is exhibited, too \( c_4^{(i)} = \pm 10^{-5} n_0^{-1} \text{ GeV}^8 \), dashed curves; \( c_4^{(i)} = \pm 5 \times 10^{-5} n_0^{-1} \text{ GeV}^8 \), dotted curves; the upper (lower) curves are for negative (positive) signs.
dependence of the \( c_3 \) term must be stronger than the simple factorization allows.

To have some confidence in our estimate, the influence of \( c_4 \) must be evaluated. An order-of-magnitude estimate utilizing ground state saturation would yield \( c_4 = (5.48 + 0.122n_0/n_0) \times 10^{-5} \text{ GeV}^8 \) when considering only the first seven mass dimension-8 scalar condensates [20] and the two twist-2 condensates [21]. The corresponding value of \( c_4 \) is substantially smaller than the standard estimates [6] when considering only the first \( m_\omega^2 \) by \pm 10\% which should reflect the approximate character of the vacuum sum rule). In other words, as long as \( c_4^{(1)} < 1.7 \times 10^{-3}n_0^{-1} \text{ GeV}^8 \) a finite positive value of \( k_N \) is required, i.e., a noticeable density dependence of the combined four-quark condensates. Note that this statement is independent of a model for \( \text{Im}\Pi^\omega \); it is based only on the observation that \( m_\omega^{(02)} \) becomes smaller if additional strength of \( \text{Im}\Pi^\omega \) below \( m_\omega^{(02)} \) occurs in the medium, as observed in Ref. [6].

Finally, we mention that many more four-quark condensates enter other sum rules in different combinations. For instance, the investigation of the three coupled sum rule equations for the nucleon [23] points to some cancellations among the four-quark condensates when using the estimates from Ref. [24]. (The results of Ref. [24] cannot be employed directly for our \( \omega \) sum rule since the flavor-mixing four-quark condensates are delivered in color combinations suitable only for the nucleon sum rule.) Furthermore, a crucial point is that the genuine chiral condensate is not suppressed in the nucleon sum rule, while in the \( \omega \) sum rule it is.

In summary we argue that the recent CB-TAPS experiment [6] implies a noticeable drop (more than 50\%) when extrapolating to nuclear saturation density and truncating the sum rule beyond mass dimension 6) of a certain combination of four-quark condensates. Four-quark condensates are fundamental quantities, among others, characterizing the nonperturbative QCD vacuum. Specific four-quark condensates, changing under chiral transformation, represent further important order parameters for chiral symmetry restoration. Clearly, also other channels, besides the omega meson considered here, must be studied to gain more information on these particular four-quark condensates.

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