ρ-*ω* splitting and mixing in nuclear matter

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We investigate the splitting and mixing of ρ and ω mesons in nuclear matter. The calculations were performed on the basis of QCD sum rules and include all operators up to mass dimension-6 twist-4 and up to first order in the coupling constants. Special attention is devoted to the impact of the scalar four-quark condensates on both effects. In nuclear matter the Landau damping governs the ρ-ω mass splitting while the scalar four-quark condensates govern the strength of individual mass shifts. A strong in-medium mass splitting causes the disappearance of the ρ-ω mixing.

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I. INTRODUCTION

The investigation of in-medium modifications of hadrons is currently a topic of wide interest. This is because the issue is related to chiral symmetry restoration as well as to a change of vacuum properties, and the phenomenon "mass of particles." Among the promising candidates for a search for changed hadron properties in an ambient strongly interacting medium are vector mesons. Due to their decay mode $V \to \gamma^* \to e^+e^-$ and the negligible interaction of the escaping e^+e^- one can expect to probe directly the parent vector meson V. Indeed, strong evidence for changes of the ρ meson are found in relativistic heavy-ion collisions, where a meson-rich hot medium is transiently created (cf. [1]). As the vector meson properties are coupled to various condensates [2–4], which change as a function of both the baryon density and the temperature, complementary investigations of their behavior via the e^+e^- decay channel in compressed nuclear matter is also required. Experimentally this will be done in a systematic way with the detector system HADES [5]. The situation is quite challenging since various predictions differ in details.

In the invariant mass region up to 1 GeV there are various sources of e^+e^- [6,7]: Dalitz decays of many hadrons, bremsstrahlung, and the direct decays $V \to e^+e^-$ mentioned above. One important channel for di-electron production is the reaction $\pi\pi \to \gamma^\star \to e^+e^-$. This channel has been evaluated with increasing sophistication over the last years (cf. [1]). The corresponding di-electron production rate $R = dN^{ee}/d^4x$ in a medium characterized by the baryon density n and temperature T is given by [8,9]

$$\frac{dR}{dM_{\rm ee}}(M_{ee},n,T) = \frac{\sigma(M_{ee}^2,n)}{(2\pi)^4} M_{ee}^4\, T\, K_1\left(\frac{M_{ee}}{T}\right)\left(1 - \frac{4\, m_\pi^2}{M_{ee}^2}\right). \tag{1}$$

Here K_1 is a modified Bessel function, M_{ee} stands for the invariant mass of the di-electron pair, and σ is the total cross section of the process $\pi^+\pi^- \to \gamma^* \to e^+e^-$,

$$\sigma(q^2,n) = \frac{4}{3}\pi\frac{\alpha_{\rm em}^2}{q^2}\sqrt{1 - \frac{4\, m_\pi^2}{q^2}}|F_\pi(q^2,n)|^2, \tag{2}$$

where $F_\pi(q^2)$ is the pion form factor and $q^2 = M_{ee}^2$ is the momentum squared of the decaying virtual photon γ^*.

The ρ-ω mixing in vacuum was discovered as a particular form of the pion form factor $|F_\pi(q^2)|$ a few decades ago [10]. Since that time much work has been done aiming at a theoretical understanding of this mixing effect (for a review see [11]). Despite that there is still some debate concerning details of the ρ-ω mixing in vacuum [12–14] the experimental pion form factor in vacuum can well be reproduced by means of several theoretical approaches. However, up to now the mixing effect has not been studied systematically in the medium. One may argue that the ρ-ω mixing is a tiny effect in evaluating the di-electron emission rate of warm nuclear matter. Moreover, inspired by [15], it seems to be a generic effect of its own interest, which should be analyzed in a dense medium. This is one issue of the present paper. In addition, we are going to investigate the ρ-ω mass splitting and mixing effect simultaneously on the same footing, i.e., we use the same parameter set and the same approach for evaluating both effects.

References [15–17] showed that ρ and ω mesons experience, within the QCD sum rule approach, quite a different in-medium behavior. Even in zero-width approximation a large ρ-ω mass splitting was found when neglecting terms in the operator product expansion (OPE) which differ for ρ and ω mesons. The individual mass shifts depend on the yet poorly known density dependence of the four-quark condensate, assuming the same effective four-quark condensate for both the ρ and the ω mesons. A further goal of the present paper is to include all terms in the OPE up to mass dimension-6 and twist-4 (up to order α_s) and to study the importance of condensates which make ρ and ω differ. We extend our previous studies [16,17] to consider here the yet unexplored effect of the density dependence of the four-quark condensate on the ρ-ω mixing in medium.

Our paper is organized as follows. In Sec. II we determine the mass shifts of ρ and ω mesons. We spell out the basic steps of QCD sum rules in low-density approximation and list all terms of the operator product expansion (OPE) up to mass dimension-6 and twist-4. We then present a numerical evaluation of the QCD sum rules and show that terms in the OPE which make ρ and ω differ are small at nuclear matter saturation density. The ρ-ω mass splitting is found to be determined by different Landau damping terms, while the individual mass shifts are governed by the density dependence of the four-quark condensate. The knowledge of the in-medium ρ,ω mass parameters is a prerequisite of a consistent treatment of the ρ-ω mixing studied in Sec. III. We define the phenomenology of the mixing and explain how this effect is related to observables. Afterwards we specify the QCD sum rule for the ρ-ω mixing and present details of the evaluation. The summary can be found in Sec. IV.

II. ρ-ω MASS SPLITTING

The masses of ρ and ω mesons differ in vacuum by a small amount, $\Delta m = m_\omega - m_\rho = 11$ MeV. It was one success of the QCD sum rule method to explain this mass splitting in vacuum by differences in the OPE of ρ and ω current-current correlators [3]. Indeed, up to mass dimension-6 there is only one operator in vacuum, the so-called flavor mixing scalar operator, which differs in sign in the OPE of ρ and ω correlators. In the following we will investigate the behavior of this splitting at finite density, where nonscalar condensates also play a role.

A. QCD sum rule

Within QCD sum rules the in-medium vector mesons $V=\rho,\omega$ are considered as resonances in the current-current correlation function

$$\Pi^{(V)}_{\mu\nu}(q,n) = i\int d^4x\, e^{iqx}\,\langle\Omega|\, T\, J^V_\mu(x) J^V_\nu(0)|\Omega\rangle, \tag{3}$$

where $q_\mu=(q_0,\boldsymbol{q})$ is the meson four momentum, T denotes the time ordered product of the meson current operators $J^V_\mu(x)$, and $|\Omega\rangle$ stands for a state of the nuclear medium. In the following, we focus on the ground state of baryonic matter approximated by a Fermi gas with nucleon density n (in Ref. [16] it was shown that temperature effects for $T \leqslant 100$ MeV are of subleading order and may be neglected for our purposes). We first study isospin symmetric nuclear matter and extend later on our approach to asymmetric nuclear matter. In terms of quark field operators, the vector meson currents are given by

$$J^V_\mu = \tfrac{1}{2}(\bar{u}\gamma_\mu u \mp \bar{d}\gamma_\mu d), \tag{4}$$

where the upper sign stands for the ρ meson while the lower sign stands for the ω meson. We will keep this notation throughout the paper. Note that the interpolating currents (4) are based on the same field operators u,d. Therefore, evaluating the right-hand side of Eq. (3) will deliver the same condensates, however a few of them with different signs. To highlight this point we spell out all terms arising from Eqs. (3) and (4) in the following.

We consider the nucleon and vector meson at rest, i.e., $q_\mu=(q_0,\boldsymbol{q}=0)$ and $k_\mu=(M_N,\boldsymbol{k}=0)$, which implies the vector meson to be off shell while the nucleon is on shell. Then the correlator (3) can be reduced to $\frac{1}{3}\Pi^\mu_\mu(q^2,n) = \Sigma_{V=\rho,\omega}\Pi^{(V)}(q^2,n)$. In each of the vector meson channels the correlator $\Pi^{(V)}(q^2,n)$ satisfies the twice subtracted dispersion relation, which can be written with $Q^2\equiv -q^2 = -E^2$ as

$$\frac{\Pi^{(V)}(Q^2,n)}{Q^2} = \frac{\Pi^{(V)}(0,n)}{Q^2} - \Pi^{(V)\prime}(0) + Q^2\frac{1}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}\,\Pi^{(V)}(s,n)}{s^2(s+Q^2)} \tag{5}$$

with $\Pi^{(V)}(0,n)=\Pi^{(V)}(q^2=0,n)$ and $\Pi^{(V)\prime}(0)=[d\Pi^{(V)}(q^2,n)/dq^2]|_{q^2=0}$ as subtraction constants. We use $\Pi^{(\rho)}(0,n)=n/(4M_N)$ and $\Pi^{(\omega)}(0,n)=9\,n/(4M_N)$ [15,18], respectively, which are the Thomson limit of the VN scattering process and correspond to Landau damping terms [19].

As usual in QCD sum rules [3,4], for large values of Q^2 one can evaluate the nonlocal operator of Eq. (3) by OPE. We truncate the OPE beyond mass dimension-6 and twist-4 and include all terms up to the first order in α_s in the SU(2) flavor sector:

$$\Pi^{(V)}(Q^2,n) = \Pi^{(V)}_{\mathrm{scalar}} + \Pi^{(V)}_{d=4,\tau=2} + \Pi^{(V)}_{d=6,\tau=2} + \Pi^{(V)}_{d=6,\tau=4} + \cdots, \tag{6}$$

$$\Pi^{(V)}_{\mathrm{scalar}} = -\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}C_F\frac{3}{4}\right) Q^2 \ln\left(\frac{Q^2}{\mu^2}\right) - \frac{3}{8\pi^2}(m_u^2+m_d^2) \tag{7}$$

$$+\frac{1}{2}\left(1+\frac{\alpha_s}{\pi}C_F\frac{1}{4}\right)\frac{1}{Q^2}\langle\Omega|\left(m_u\bar{u}u + m_d\bar{d}d\right)|\Omega\rangle + \frac{1}{24}\frac{1}{Q^2}\langle\Omega|\frac{\alpha_s}{\pi}\mathrm{G}^2|\Omega\rangle \tag{8}$$

$$-\frac{1}{2}\pi\alpha_s\frac{1}{Q^4}\langle\Omega|(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{u}\gamma^\mu\gamma_5\lambda^a u + \bar{d}\gamma_\mu\gamma_5\lambda^a d\bar{d}\gamma^\mu\gamma_5\lambda^a d)|\Omega\rangle \tag{9}$$

$$\pm\pi\alpha_s\frac{1}{Q^4}\langle\Omega|(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{d}\gamma^\mu\gamma_5\lambda^a d)|\Omega\rangle \tag{10}$$

$$-\frac{1}{9}\pi\alpha_s\frac{1}{Q^4}\langle\Omega|(\bar{u}\gamma_\mu\lambda^a u\bar{u}\gamma^\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d\bar{d}\gamma^\mu\lambda^a d)|\Omega\rangle \tag{11}$$

$$-\frac{2}{9}\pi\alpha_s\frac{1}{Q^4}\langle\Omega|(\bar{u}\gamma_\mu\lambda^a u\bar{d}\gamma^\mu\lambda^a d)|\Omega\rangle, \tag{12}$$

$$+g_s\frac{1}{12}\frac{1}{Q^6}(m_u^2\langle\Omega|m_u\bar{u}\sigma_{\mu\nu}\,G^{\mu\nu}u|\Omega\rangle + m_d^2\,\langle\Omega|m_d\bar{d}\sigma_{\mu\nu}\,G^{\mu\nu}d|\Omega\rangle), \tag{13}$$

$$\Pi^{(V)}_{d=4,\tau=2} = \frac{1}{2}\frac{\alpha_s}{\pi} n_f \frac{1}{Q^4} q^\mu q^\nu \langle \Omega | \hat{S}\hat{T}(G_\mu^{\ \alpha} G_{\alpha\nu}) | \Omega \rangle \tag{14}$$

$$- \left(\frac{2}{3} - \frac{\alpha_s}{\pi} C_F \frac{5}{18} \right) i \frac{1}{Q^4} q^\mu q^\nu \times \langle \Omega | \hat{S}\hat{T}(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d) | \Omega \rangle, \tag{15}$$

$$\Pi^{(V)}_{d=6,\tau=2} = -\frac{41}{27}\frac{\alpha_s}{\pi} n_f \frac{1}{Q^8} q^\mu q^\nu q^\lambda q^\sigma \langle \Omega | \hat{S}\hat{T}(G_\mu^{\ \rho} D_\nu D_\lambda G_{\rho\sigma}) | \Omega \rangle \tag{16}$$

$$+ \left(\frac{8}{3} + \frac{\alpha_s}{\pi} C_F \frac{67}{30} \right) i \frac{1}{Q^8} q^\mu q^\nu q^\lambda q^\sigma \times \langle \Omega | \hat{S}\hat{T}(\bar{u}\gamma_\mu D_\nu D_\lambda D_\sigma u + \bar{d}\gamma_\mu D_\nu D_\lambda D_\sigma d) | \Omega \rangle, \tag{17}$$

$$\Pi^{(V)}_{d=6,\tau=4} = \pm \frac{1}{3}\frac{1}{Q^6} q^\mu q^\nu \langle \Omega | g_s^2 \hat{S}\hat{T}(\bar{u}\gamma_\mu \gamma_5 \lambda^a u \bar{d}\gamma_\nu \gamma_5 \lambda^a d) | \Omega \rangle \tag{18}$$

$$- \frac{1}{6}\frac{1}{Q^6} q^\mu q^\nu \langle \Omega | g_s^2 \hat{S}\hat{T}(\bar{u}\gamma_\mu \gamma_5 \lambda^a u \bar{u}\gamma_\nu \gamma_5 \lambda^a u + \bar{d}\gamma_\mu \gamma_5 \lambda^a d \bar{d}\gamma_\nu \gamma_5 \lambda^a d) | \Omega \rangle \tag{19}$$

$$- \frac{1}{24}\frac{1}{Q^6} q^\mu q^\nu \langle \Omega | g_s^2 \hat{S}\hat{T}(\bar{u}\gamma_\mu \lambda^a u (\bar{u}\gamma_\nu \lambda^a u + \bar{d}\gamma_\nu \lambda^a d)) | \Omega \rangle \tag{20}$$

$$- \frac{1}{24}\frac{1}{Q^6} q^\mu q^\nu \langle \Omega | g_s^2 \hat{S}\hat{T}(\bar{d}\gamma_\mu \lambda^a d (\bar{u}\gamma_\nu \lambda^a u + \bar{d}\gamma_\nu \lambda^a d)) | \Omega \rangle \tag{21}$$

$$- \frac{5}{12}\frac{1}{Q^6} q^\mu q^\nu \langle \Omega | i g_s \hat{S}\hat{T}(\bar{u}[D_\mu, \tilde{G}_{\nu\alpha}]_+ \gamma^\alpha \gamma_5 u + \bar{d}[D_\mu, \tilde{G}_{\nu\alpha}]_+ \gamma^\alpha \gamma_5 d) | \Omega \rangle \tag{22}$$

$$- \frac{7}{3}\frac{1}{Q^6} q^\mu q^\nu \langle \Omega | \hat{S}\hat{T}(m_u \bar{u} D_\mu D_\nu u + m_d \bar{d} D_\mu D_\nu d) \times | \Omega \rangle, \tag{23}$$

where $n_f=3$ is the number of active flavors at a scale of 1 GeV, and $C_F=(n_c^2-1)/(2n_c)=4/3$ with $n_c=3$ as number of colors; $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]_-$. The strong couplings are related by $\alpha_s=g_s^2/(4\pi)$.

The SU(3) color matrices are normalized as $\mathrm{Tr}(\lambda^a\lambda^b)=2\delta^{ab}$, the covariant derivative is defined as $D_\mu=\partial_\mu+igA_\mu^a\lambda^a/2$ and $G^2=G^a_{\mu\nu}G^{a\ \mu\nu}$ where $G^a_{\mu\nu}$ is the gluon field strength tensor ($G^{\mu\nu}=G^{a\ \mu\nu}\lambda^a/2$). The dual gluon field strength tensor is defined by $\tilde{G}_{\mu\nu}=\epsilon_{\mu\nu\rho\sigma}G^{a\ \rho\sigma}\lambda^a/2$.

The OPE for scalar operators up to mass dimension-6 can be found in [3]. For the twist-2 condensates we have included all singlet operators with even parity up to order α_s (nucleon matrix elements of operators with odd parity vanish). Their Wilson coefficients can be deduced from [20].[1]

The Wilson coefficients of the twist-4 operators in lines (18)–(22) are given in [8], and for the twist-4 operator in line (23) it can be deduced from [21], where it has been found that the term (23) has some relevance for twist-4 effects of nucleon structure functions. The Wilson coefficient of an additional dimension-6 twist-4 operator, $\hat{S}\hat{T}\bar{q}[D_\mu, G_{\nu\alpha}]_-\gamma^\alpha q$ (for an estimate of this condensate see [22]), vanishes [21,23].

We emphasize that the only difference between ρ and ω mesons in the truncated OPE consists in the terms in lines (10) and (18). As mentioned above the term in line (10) is responsible for the ρ-ω mass splitting in vacuum; the term in line (18) vanishes in vacuum. It is now our goal to analyze the in-medium difference of ρ and ω mesons stemming from the OPE side. Most terms in lines (7)–(23) may be evaluated using standard techniques [24]. However, what remains to be considered are the flavor-mixing condensates in the lines (10), (12), and (18), the mixed quark-gluon condensate in line (13), the pure gluonic condensates in the lines (14) and (16) and the twist-4 condensate in line (23). The QCD corrections to order α_s of the twist-2 condensates in lines (15) and (17) have not been taken into account in previous analyses.

The chiral condensate and scalar gluon condensate have been discussed in some detail in [25]. Details for the scalar four-quark condensates in lines (9)–(12) are given in Appendices A and B, where also further notations are explained. The twist-2 quark condensates [lines (15) and (17)] and the gluonic twist-2 condensates [lines (14) and (16)] are explicitly given in Appendix C. The twist-4 condensates [lines (18)–(23)] are listed in Appendix D.

Performing a Borel transformation [3] of the dispersion relation (5) with appropriate mass parameter M^2 and taking into account the OPE (6) one gets the QCD sum rule

$$\Pi^{(V)}(0,n) - \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,\Pi^{(V)}(s,n)}{s} e^{-s/M^2} = c_0\, M^2 + \sum_{i=1}^\infty \frac{c_i}{(i-1)!\,M^{2(i-1)}}. \tag{24}$$

For nuclear matter we utilize the one-particle dilute gas approximation in order to evaluate all relevant condensates in the nuclear medium i.e.,

$$\langle \Omega | \hat{\mathcal{O}} | \Omega \rangle = \langle \hat{\mathcal{O}} \rangle_0 + \frac{n}{2M_N} \langle N(\boldsymbol{k}) | \hat{\mathcal{O}} | N(\boldsymbol{k}) \rangle, \tag{25}$$

where the nucleon states are normalized by $\langle N(\boldsymbol{k}) | N(\boldsymbol{k}') \rangle = (2\pi)^3 2\, E_k \delta(\boldsymbol{k}-\boldsymbol{k}')$ with $E_k=\sqrt{\boldsymbol{k}^2+M_N^2}$. The scalar

[1]Despite the fact that twist-2 non-singlet operators occur in the OPE for electromagnetic currents [20], they are absent in the OPE of Eq. (3). Twist-2 non-singlet condensates contribute, however, to ρ-ω mixing.

dimension-4 and dimension-5 condensates are given by [26]

$$m_u \langle\Omega|\overline{u}u|\Omega\rangle = m_u\langle\overline{u}u\rangle_0 + \tfrac{1}{2}\sigma_N^u\, n, \tag{26}$$

$$\langle\Omega|\frac{\alpha_s}{\pi}G^2|\Omega\rangle = \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0 - \tfrac{8}{9}M_N^0\, n, \tag{27}$$

$$\langle\Omega|g_s\overline{u}\sigma_{\mu\nu}G^{\mu\nu}u|\Omega\rangle = \lambda^2\langle\overline{u}u\rangle_0 + \tfrac{1}{2}\lambda^2\frac{\sigma_N^u}{m_u}n, \tag{28}$$

where we have introduced the sigma term $\sigma_N^u = m_u\langle N(k)|\overline{u}u|N(k)\rangle/M_N$ and $\lambda^2 \simeq 1\ \mathrm{GeV}^2$. The chiral d quark condensate follows by replacing the u quark by a d quark. The nucleon sigma term [26] is $2\sigma_N = \sigma_N^u + \sigma_N^d$.

Inserting the explicit expressions for all condensates (cf. Appendixes A–D) one gets for the coefficients $c_{1,2,3}$ in Eq. (24)

$$c_0 = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}C_F\frac{3}{4}\right), \tag{29}$$

$$c_1 = -\frac{3}{8\pi^2}(m_u^2 + m_d^2), \tag{30}$$

$$\begin{aligned} c_2 = {}& \frac{1}{2}\left(1 + \frac{\alpha_s}{\pi}C_F\frac{1}{4}\right)(m_u\langle\overline{u}u\rangle_0 + m_d\langle\overline{d}d\rangle_0 + \sigma_N\, n) \\ & + \frac{1}{24}\left[\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0 - \frac{8}{9}M_N^0\, n\right] + \left(\frac{1}{4}\right. \\ & \left. - \frac{5}{48}\frac{\alpha_s}{\pi}C_F\right)A_2^{(u+d)}\, M_N\, n - \frac{3}{16}n_f\frac{\alpha_s}{\pi}A_2^G\, M_N\, n, \end{aligned} \tag{31}$$

$$\begin{aligned} c_3 = {}& -\frac{112}{81}\pi\, \alpha_s\, \kappa_0\, \langle\overline{q}q\rangle_0^2\left[1 + \frac{\kappa_N}{\kappa_0}\frac{\sigma_N}{m_q\langle\overline{q}q\rangle_0}n\right] \\ & + \frac{(8\pm36)}{81}\frac{Q_0^2}{f_\pi^2}\frac{\alpha_s}{\pi}\langle\overline{q}q\rangle_0^2\left[1 + \frac{\sigma_N}{m_q}\frac{1}{\langle\overline{q}q\rangle_0}n\right] \\ & - \left(\frac{5}{12} + \frac{\alpha_s}{\pi}C_F\frac{67}{192}\right)A_4^{(u+d)}M_N^3 n + \frac{205}{864}\frac{\alpha_s}{\pi}n_f A_4^G M_N^3 n \\ & + \frac{1}{4}M_N^3 n\left(\frac{3}{8}K_u^2 + \frac{3}{2}K_u^1 - (1\pm1)K_{ud}^1 + \frac{15}{16}K_u^g\right) \\ & - \frac{7}{144}\sigma_N M_N^2 n, \end{aligned} \tag{32}$$

where $A_n^{(u+d)} = A_n^u + A_n^d$ with $n=2,4$, $2\langle\overline{q}q\rangle_0 = \langle\overline{u}u\rangle_0 + \langle\overline{d}d\rangle_0$, and $2m_q = m_u + m_d$.

B. Evaluation

We define a ratio of weighted moments

$$m_V^2(n,M^2,s_V) \equiv \frac{\displaystyle\int_0^{s_V} ds\ \mathrm{Im}\,\Pi^{(V)}(s,n)e^{-s/M^2}}{\displaystyle\int_0^{s_V} ds\ \ \mathrm{Im}\,\Pi^{(V)}(s,n)s^{-1}\, e^{-s/M^2}} \tag{33}$$

for which the desired sum rule follows by taking the ratio of Eq. (24) to its derivative with respect to $1/M^2$ as

$$m_V^2(n,M^2,s_V) = \frac{c_0\, M^2\left[1 - \left(1 + \dfrac{s_V}{M^2}\right)e^{-s_V/M^2}\right] - \dfrac{c_2}{M^2} - \dfrac{c_3}{M^4}}{c_0\,(1 - e^{-s_V/M^2}) + \dfrac{c_1}{M^2} + \dfrac{c_2}{M^4} + \dfrac{c_3}{2M^6} - \dfrac{\Pi^{(V)}(0,n)}{M^2}}, \tag{34}$$

where we have identified the highlying (continuum) contributions as $-\mathrm{Im}\,\Pi^{(V)}(s \geqslant s_V,n)/s = \pi c_0$ (s_V is the continuum threshold). The meaning of the parameter m_V^2 as normalized first moment of the spectral function $\mathrm{Im}\,\Pi^{(V)}$ becomes immediately clear in zero-width approximation, $-\mathrm{Im}\,\Pi^{(V)}(s \leqslant s_V,n) = \pi F_V\delta(s - m_V^{*2})$, from where $m_V = m_V^*$ follows. Equations (33) and (34) are the corresponding generalizations for the case of finite width, in the spirit of a resonance + continuum ansatz. The mass equation (34) is commonly used for describing m_V^2 in vacuum [3,27–31], at finite temperature [8] and at finite density [4,15,16] and will be subject of our further considerations.

The sum rule is reliable only in a Borel window $M_{\mathrm{min}}^2 \leqslant M^2 \leqslant M_{\mathrm{max}}^2$. If M^2 is too small the expansion (24) breaks down. On the other side, if M^2 is too large the contribution of perturbative QCD terms completely dominate the sum rule. We adopt the following rules for determining the Borel window [24,32–35]: The minimum Borel mass, M_{min}^2, is determined such that the terms of order $O(1/M^6)$ on the OPE side contribute not more than 10%. The maximum Borel mass, M_{max}^2, is evaluated within zero-width approximation by requiring that the continuum part is not larger than the contribution of the resonance part, i.e.,

$$\frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)M_{\mathrm{max}}^2\, e^{-s_V/M_{\mathrm{max}}^2} \leqslant \frac{F_V}{m_V^2}e^{-m_V^2/M_{\mathrm{max}}^2}. \tag{35}$$

The parameter F_V can be evaluated by means of the QCD sum rule (24). The obtained results for vacuum, $F_\rho = 0.0110\ \mathrm{GeV}^4$, $F_\omega = 0.0117\ \mathrm{GeV}^4$, are in good agreement with the relations $F_\rho = m_\rho^4/g_{\rho\gamma}^2 = 0.0130\ \mathrm{GeV}^4$ and $F_\omega = 9\ m_\omega^4/g_{\omega\gamma}^2 = 0.0138\ \mathrm{GeV}^4$, respectively, which follow from the Vector Meson Dominance (VMD) [3,11,32].

The threshold s_V is determined by maximum flatness of $m_V(n,M^2,s_V)$ as a function of M^2. These requirements give a coupled system of equations for the five unknowns M_{min}^2, M_{max}^2, F_V, s_V, m_V. The final parameters $\overline{F}_V$ and $\overline{m}_V$ are averaged to get Borel mass independent quantities. For any parameter P this average is defined by

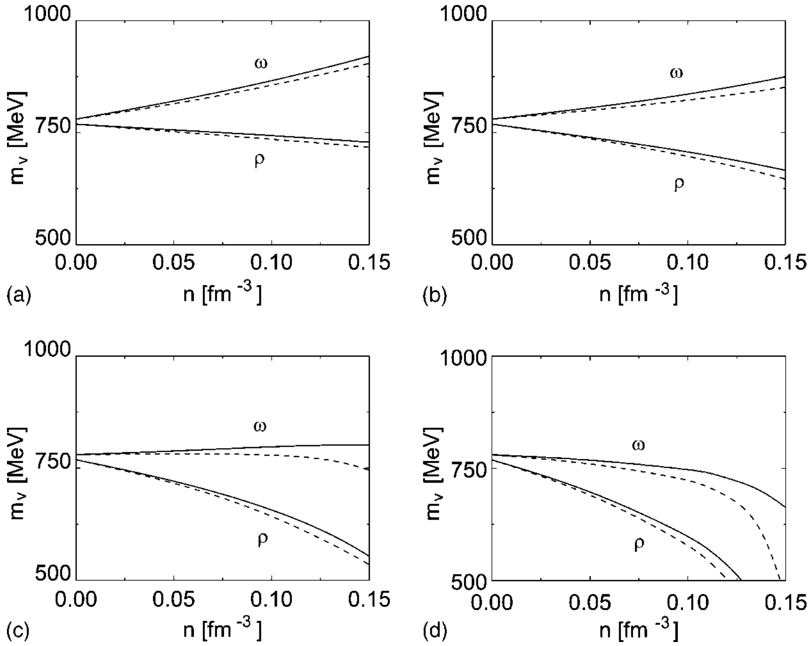

FIG. 1. Mass parameter m_V of ω meson (upper curves) and ρ meson (lower curves) as a function of the density for various values of the parameter κ_N ((a) $\kappa_N=1$, (b) $\kappa_N=2$, (c) $\kappa_N=3$, (d) $\kappa_N=4$). The solid curves are for the full set of terms in Eqs. (29)–(32), while for the dotted curves the twist-4 condensates are discarded, i.e., $K^{1,2,g}_{u,d,ud}=0$.

$$\overline{P} = \frac{1}{M^2_{\max} - M^2_{\min}} \int_{M^2_{\min}}^{M^2_{\max}} P(M^2) dM^2. \tag{36}$$

In the following we will skip the average sign.

C. Results

For considering the mass parameter splitting effect there is no need to distinguish between isospin symmetric and isospin asymmetric nuclear matter since all operators in the OPE Eq. (6) are isospin symmetric operators. Accordingly, in the following we study isospin symmetric nuclear matter.

Twist-4 condensates have been estimated in [36] where data of lepton-nucleon forward scattering amplitude has been used to fix the parameters K_u^1, K_u^2, K_u^g and K_{ud}^1 in Eq. (32). The corresponding system of equations is under-determined and therefore various sets for these parameters can be obtained. We have investigated all six sets from [36] for these parameters and find only very small changes of the results. In Fig. 1 we show the results obtained with the parameter set given in Appendix D. Since in [16,17,28,37] a strong effect of the density dependence of the four-quark condensate was found we show here results for various possibilities, parametrized by κ_N introduced in Appendix A, Eq. (A5). The mass parameter of the ρ meson decreases with increasing density for all κ_N, while the ω meson mass parameter decreases only for sufficiently large κ_N. Other QCD sum rule analyses [4,34,16] have obtained also a decreasing ρ mass parameter. An increase of the ω meson mass parameter has been found in [15,16], where the correct Landau damping term was implemented.

The flavor mixing scalar operators (i.e., $M^{ud}_{A,V}$, see Appendix B), while responsible for the mass splitting in vacuum, play only a minor rule in matter. That means, discarding the terms $\sim Q_0^2$ in Eq. (32) yields curves which are nearly identical to those represented in Fig. 1. The poorly known scalar four-quark condensate governs the strength of individual mass shifts, while the strong mass splitting in matter originates mainly from the Landau damping terms $\Pi^{(V)}(0,n)$, which differ by a factor 9 for ρ and ω [15]. The outcome of our study is that terms in the OPE, which cause a difference of ρ and ω mesons, are small in matter since the mass splitting is mainly determined by the Landau damping terms.

III. ρ-ω MIXING

First, we briefly describe the mixing scenario considered in the following. We follow the arguments given in [11]. The mixing can be accomplished by

$$\begin{pmatrix} \rho \\ \omega \end{pmatrix} = \begin{pmatrix} 1 & -\epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \rho_I \\ \omega_I \end{pmatrix}, \tag{37}$$

where the subscript I denotes isospin-pure states, and ϵ is the mixing parameter. The mixing formula (37) is quite general. Extending the mixed propagator approach described in [11] to the case of finite density one can obtain the following relation between the complex mixing parameter ϵ and the nondiagonal self-energy $\delta_{\rho\,\omega}(q^2,n)$ via (cf. [11] for vacuum, cf. [15] for matter)

$$\epsilon(n) = \frac{\delta_{\rho\,\omega}(q^2,n)}{m^2_\omega(n) - m^2_\rho(n) + i\ \mathrm{Im}\ \Sigma_\omega(q^2,n) - i\ \mathrm{Im}\ \Sigma_\rho(q^2,n)}. \tag{38}$$

The nondiagonal self-energy $\delta_{\rho\,\omega}(q^2,n)$, and therefore also the mixing parameter ϵ, is directly related to the pion form factor, given by

$$F_\pi(q^2,n) = 1 - \frac{q^2}{g_{\rho\gamma}}\frac{g_{\rho\pi\pi}}{q^2 - m_\rho^2(n) - i\ \mathrm{Im}\Sigma_\rho(q^2,n)} - \frac{q^2}{g_{\omega\gamma}}\frac{1}{q^2 - m_\omega^2(n) - i\ \mathrm{Im}\Sigma_\omega(q^2,n)} \times \delta_{\rho\ \omega}(q^2,n)\frac{g_{\rho\pi\pi}}{q^2 - m_\rho^2 - i\ \mathrm{Im}\Sigma_\rho(q^2,n)}. \quad (39)$$

Since the main contribution of the second line stems from the region $q^2 \sim m_\rho^2, m_\omega^2$ one usually approximates the nondiagonal self-energy $\delta_{\rho\omega}(q^2,n)$ in the pion form factor by its on-shell value at $q^2 = \overline{m}^2 = 0.5(m_\rho^2 + m_\omega^2)$.

The nondiagonal self-energy consists of an electromagnetic part and a hadronic part, $\delta_{\rho\ \omega}(\overline{m}^2,n) = \delta_{\rho\ \omega}^{\mathrm{EM}}(\overline{m}^2,n) + \delta_{\rho\ \omega}^{\mathrm{H}}(\overline{m}^2,n)$. Both contributions can consistently be isolated in theoretical as well as experimental analyses. The electromagnetic part comes from the process $\rho \to \gamma^* \to \omega$ and can be evaluated analytically [39]. In the following we are going to investigate the density dependence of $\delta_{\rho\ \omega}(\overline{m}^2,n)$.

A. QCD sum rule

The basic object for the ρ-ω mixing in matter is the mixed correlator

$$\Pi_{\mu\nu}^{\rho\ \omega}(q,n) = i\int d^4x\ e^{iqx}\langle T\ J_\mu^\rho(x)J_\nu^\omega(0)\rangle_n, \quad (40)$$

with the isotriplet and isosinglet currents from Eq. (4). It is straightforward to recognize that

$$\Pi^{\rho\ \omega}(q,n) \equiv \tfrac{1}{3}g^{\mu\nu}\ \Pi_{\mu\nu}^{\rho\ \omega}(q,n) = \Pi^{\mathrm{u}}(q,n) - \Pi^{\mathrm{d}}(q,n) \quad (41)$$

with

$$\Pi^{\mathrm{q}}(q,n) = \frac{i}{12}\int d^4x\ e^{iqx}\langle T\overline{q}(x)\gamma_\mu\ q(x)\overline{q}(0)\gamma^\mu\ q(0)\rangle_n. \quad (42)$$

This scalar function satisfies the twice subtracted dispersion relation The subtraction constant

$$\frac{\Pi^{\rho\omega}(q,n)}{Q^2} = \frac{\Pi^{\rho\omega}(0,n)}{Q^2} - \Pi^{\rho\omega\prime}(0,n) + Q^2\frac{1}{\pi}\int_0^\infty ds\frac{\mathrm{Im}\,\Pi^{\rho\omega}(s,n)}{s^2(s+Q^2)}. \quad (43)$$

The subtraction constant $\Pi^{\rho\ \omega}(0,n)$ vanishes in vacuum [11] as well as in case of symmetric nuclear matter [15]. For asymmetric nuclear matter, $\Pi^{\rho\ \omega}(0,n) = -3\ \alpha_{np}\ n/(4M_N)$ with $\alpha_{np} = (n_n - n_p)/n$ [15], where n_n and n_p are the neutron and proton densities, respectively, and $n = n_n + n_p$.

The other subtraction constant does not contribute to the sum rule after a Borel transformation. It is convenient [38] to subtract the pure electromagnetic contribution $\rho \to \gamma^* \to \omega$ from hadronic and OPE sides of the dispersion relation (43). In doing so we arrive at a new function, denoted by $\widetilde{\Pi}^{\rho\ \omega}$, which satisfies the same dispersion relation (43).

For large values of Q^2 one evaluates the left-hand side of Eq. (43) by the OPE. Due to large cancellations of the pure QCD terms according to Eq. (41) one has now to include also the electromagnetic contributions to the OPE in contrast to Eq. (6), where the electromagnetic terms are neglegible compared to the QCD terms. Accordingly, up to mass dimension-6 twist-4, and up to first order in α_s and α_{em} the OPE is given by (for vacuum cf. [12,38], for matter cf. [15])

$$\widetilde{\Pi}^{\rho\ \omega}(Q^2) = \widetilde{\Pi}_{\mathrm{scalar}}^{\rho\ \omega} + \widetilde{\Pi}_{d=4,\tau=2}^{\rho\ \omega} + \widetilde{\Pi}_{d=6,\tau=2}^{\rho\ \omega} + \widetilde{\Pi}_{d=6,\tau=4}^{\rho\ \omega} + \cdots, \quad (44)$$

$$\widetilde{\Pi}_{\mathrm{scalar}}^{\rho\ \omega} = -\frac{1}{64\pi^3}Q^2\ \alpha_{\mathrm{em}}\ \ln\left(\frac{Q^2}{\mu^2}\right) - \frac{3}{8\pi^2}(m_u^2 - m_d^2) \quad (45)$$

$$+\frac{1}{2}\left(1 + \frac{\alpha_s}{\pi}C_F\frac{1}{4}\right)\frac{1}{Q^2}\langle\Omega|(m_u\overline{u}u - m_d\overline{d}d)|\Omega\rangle \quad (46)$$

$$+\frac{1}{72}\frac{\alpha_{\mathrm{em}}}{\pi}\frac{1}{Q^2}\langle\Omega|(4\ m_u\overline{u}u - m_d\overline{d}d)|\Omega\rangle \quad (47)$$

$$-\frac{1}{2}\pi\alpha_s\frac{1}{Q^4}(\langle\Omega|\overline{u}\gamma_\mu\gamma_5\lambda^a u\overline{u}\gamma^\mu\gamma_5\lambda^a u|\Omega\rangle - \langle\Omega|\overline{d}\gamma_\mu\gamma_5\lambda^a d\overline{d}\gamma^\mu\gamma_5\lambda^a d|\Omega\rangle) \quad (48)$$

$$-\frac{1}{9}\pi\alpha_s\frac{1}{Q^4}(\langle\Omega|\overline{u}\gamma_\mu\lambda^a u\overline{u}\gamma^\mu\lambda^a u|\Omega\rangle - \langle\Omega|\overline{d}\gamma_\mu\lambda^a d\overline{d}\gamma^\mu\lambda^a d|\Omega\rangle) \quad (49)$$

$$-\frac{2}{9}\pi\alpha_{\mathrm{em}}\frac{1}{Q^4}(4\langle\Omega|\overline{u}\gamma_\mu\gamma_5 u\overline{u}\gamma^\mu\gamma_5 u|\Omega\rangle - \langle\Omega|\overline{d}\gamma_\mu\gamma_5 d\overline{d}\gamma^\mu\gamma_5 d|\Omega\rangle) \quad (50)$$

$$-\frac{4}{81}\pi\alpha_{\mathrm{em}}\frac{1}{Q^4}(4\langle\Omega|\overline{u}\gamma_\mu u\overline{u}\gamma^\mu u|\Omega\rangle - \langle\Omega|\overline{d}\gamma_\mu d\overline{d}\gamma^\mu d|\Omega\rangle) \quad (51)$$

$$+g_s\frac{1}{12}\frac{1}{Q^6}(m_u^2\langle\Omega|m_u\overline{u}\sigma_{\mu\nu}G^{\mu\nu}u|\Omega\rangle - m_d^2\ \langle\Omega|m_d\overline{d}\sigma_{\mu\nu}G^{\mu\nu}d|\Omega\rangle) \quad (52)$$

$$+e\frac{1}{12}\frac{1}{Q^6}\left(\frac{2}{3}m_u^2\langle\Omega|m_u\overline{u}\sigma_{\mu\nu}F^{\mu\nu}u|\Omega\rangle + \frac{1}{3}m_d^2\langle\Omega|m_d\overline{d}\sigma_{\mu\nu}F^{\mu\nu}d|\Omega\rangle\right), \quad (53)$$

$$\widetilde{\Pi}^{\rho\ \omega}_{d=4,\tau=2} = -\left(\frac{2}{3} - \frac{\alpha_s}{\pi} C_F \frac{5}{18}\right)\frac{i}{Q^4} q^\mu q^\nu \langle\Omega|\hat{S}\hat{T}(\bar{u}\gamma_\mu D_\nu u - \bar{d}\gamma_\mu D_\nu d)|\Omega\rangle \tag{54}$$

$$+ \frac{\alpha_{\rm em}}{\pi}\frac{5}{162}\frac{i}{Q^4} q^\mu q^\nu \langle\Omega|\hat{S}\hat{T}(4\bar{u}\gamma_\mu D_\nu u - \bar{d}\gamma_\mu D_\nu d)|\Omega\rangle, \tag{55}$$

$$\widetilde{\Pi}^{\rho\ \omega}_{d=6,\tau=2} = \left(\frac{8}{3} + \frac{\alpha_s}{\pi} C_F \frac{67}{30}\right)\frac{i}{Q^8} q^\mu q^\nu q^\lambda q^\sigma \langle\Omega|\hat{S}\hat{T}(\bar{u}\gamma_\mu D_\nu D_\lambda D_\sigma u - \bar{d}\gamma_\mu D_\nu D_\lambda D_\sigma d)|\Omega\rangle \tag{56}$$

$$+ \frac{\alpha_{\rm em}}{\pi}\frac{67}{270}\frac{i}{Q^8} q^\mu q^\nu q^\lambda q^\sigma \langle\Omega|\hat{S}\hat{T}(4\bar{u}\gamma_\mu D_\nu D_\lambda D_\sigma u - \bar{d}\gamma_\mu D_\nu D_\lambda D_\sigma d)|\Omega\rangle, \tag{57}$$

$$\widetilde{\Pi}^{\rho\ \omega}_{d=6,\tau=4} = -\frac{1}{24}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|g_s^2 \hat{S}\hat{T}(\bar{u}\gamma_\mu \lambda^a u \bar{u}\gamma_\nu \lambda^a u - \bar{d}\gamma_\mu \lambda^a d \bar{d}\gamma_\nu \lambda^a d)|\Omega\rangle \tag{58}$$

$$-\frac{1}{6}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|g_s^2 \hat{S}\hat{T}(\bar{u}\gamma_\mu\gamma_5 \lambda^a u \bar{u}\gamma_\nu\gamma_5 \lambda^a u - \bar{d}\gamma_\mu\gamma_5 \lambda^a d \bar{d}\gamma_\nu\gamma_5 \lambda^a d)|\Omega\rangle \tag{59}$$

$$-\frac{5}{12}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|i g_s \hat{S}\hat{T}(\bar{u}[D_\mu,\widetilde{G}_{\nu\alpha}]_+ \gamma^\alpha\gamma_5 u - \bar{d}[D_\mu,\widetilde{G}_{\nu\alpha}]_+ \gamma^\alpha\gamma_5 d)|\Omega\rangle \tag{60}$$

$$-\frac{7}{3}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|\hat{S}\hat{T}(m_u \bar{u} D_\mu D_\nu u - m_d \bar{d} D_\mu D_\nu d) \times|\Omega\rangle \tag{61}$$

$$-\frac{1}{54}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|g_e^2 \hat{S}\hat{T}(4\bar{u}\gamma_\mu u \bar{u}\gamma_\nu u - \bar{d}\gamma_\mu d \bar{d}\gamma_\nu d)|\Omega\rangle \tag{62}$$

$$-\frac{2}{27}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|g_e^2 \hat{S}\hat{T}(4\bar{u}\gamma_\mu\gamma_5 u \bar{u}\gamma_\nu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d \bar{d}\gamma_\nu\gamma_5 d)|\Omega\rangle \tag{63}$$

$$-\frac{5}{12}\frac{1}{Q^6} q^\mu q^\nu \langle\Omega|i\ e\hat{S}\hat{T}\left(\frac{2}{3}\bar{u}[D^{\rm em}_\mu,\widetilde{F}_{\nu\alpha}]_+ \gamma^\alpha\gamma_5 u + \frac{1}{3}\bar{d}[D^{\rm em}_\mu,\widetilde{F}_{\nu\alpha}]_+ \gamma^\alpha\gamma_5 d\right)|\Omega\rangle. \tag{64}$$

$\alpha_{\rm em}=e^2/(4\pi)$ is the electromagnetic fine structure constant, $F_{\nu\alpha}$ stands for the electromagnetic field strength tensor, and the dual electromagnetic field strength tensor is defined by $\widetilde{F}_{\mu\nu}=\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. The covariant derivative of QED is defined as $D^{\rm em}_\mu=\partial_\mu+ieA_\mu$. The QED contributions may be deduced from the QCD terms by the replacements $\lambda^a/2\to 1$ (which implies $C_F\to 1$) and $g_s\to e_q$ (e_q is the electric charge of quark q), respectively. Not all of the condensates given above have been taken into account in previous evaluations: the terms in lines (47), (52), and (53), the QCD corrections in lines (46), (54), and (56), the QED corrections given in lines (55) and (57), and all twist-4 contributions in lines (58)–(64) have not been considered yet.

The isospin breaking of the scalar u and d quark condensates is usually parametrized by

$$\gamma + 1 = \frac{\langle 0|\bar{d}d|0\rangle}{\langle 0|\bar{u}u|0\rangle} \simeq \frac{\langle N|\bar{d}d|N\rangle}{\langle N|\bar{u}u|N\rangle} \simeq \frac{\langle\Omega|\bar{d}d|\Omega\rangle}{\langle\Omega|\bar{u}u|\Omega\rangle}, \tag{65}$$

where we have generalized the corresponding relation for vacuum [15,12,38] to the case of nuclear matter.

The four-quark condensates are given in Appendix A. The twist-2 quark condensates are listed in Appendix C and the corresponding parameters can be found in Appendix E. The twist-4 condensates, listed here for the sake of completeness, are neglected in our analysis since they are strongly suppressed in the chosen Borel window.

Performing a Borel transformation of Eq. (43) leads to

$$\Pi^{\rho\ \omega}(0,n) - \frac{1}{\pi}\int_0^\infty ds \frac{{\rm Im}\widetilde{\Pi}^{\rho\omega}(s,n)}{s} e^{-s/M^2} = d_0\ M^2 + \sum_{i=1}^{\infty} \frac{d_i}{(i-1)\,!\,M^{2(i-1)}}. \tag{66}$$

The coefficients $d_{1,2,3}$ in linear density approximation and neglecting all twist-4 condensates are given by

$$d_0 = \frac{1}{64\pi^3}\alpha_{\rm em}, \tag{67}$$

$$d_1 = -\frac{3}{8\pi^2}(m_u^2 - m_d^2), \tag{68}$$

$$d_2 = \left[\frac{1}{2}\left(1 + \frac{\alpha_s}{\pi}C_F\frac{1}{4}\right)(m_u - m_d) + \frac{1}{72}\frac{\alpha_{\rm em}}{\pi}(4m_u - m_d)\right] \times\left(\langle\bar{q}q\rangle_0 + \frac{\sigma_N}{2m_q}n\right) - \frac{\alpha_{\rm em}}{\pi}\frac{5}{288}M_N\ n\ (A_2^{u,p} + A_2^{d,p}) + d_2^{\rm AS}, \tag{69}$$

$$d_3 = \frac{14}{81}\pi\kappa_0(8\gamma\alpha_s - \alpha_{\rm em})\langle\bar{q}q\rangle_0^2\left(1 + \frac{\kappa_N}{\kappa_0}n\frac{\sigma_N}{m_q\langle\bar{q}q\rangle_0}\right) - \frac{\alpha_{\rm em}}{\pi}\frac{67}{1152}M_N^3\ n(A_4^{u,p} + A_4^{d,p}) + d_3^{\rm AS}, \tag{70}$$

where further terms proportional to $m_q\ \gamma, \gamma^2$ and $\gamma\ \alpha_{\rm em}$ have been neglected. The terms $d_l^{\rm AS}(l=2,3)$ are proportional to α_{np} and account for isospin asymmetric matter. Their impact on mixing will be considered separately in Sec. III C 3.

Finally, we specify the hadronic side of the QCD sum rule (66) (cf. [3,12,13] for vacuum, [15] for matter), where the ϕ meson has been implemented in accordance with [13]

$$-\frac{1}{\pi}\frac{\mathrm{Im}\tilde{\Pi}^{\rho\ \omega}(s,n)}{s} = \frac{1}{4}[f_\rho\ \delta(s-m_\rho^2) - f_\omega\ \delta(s-m_\omega^2) + f_\phi\ \delta(s-m_\phi^2)] + \frac{1}{4}[f_{\rho'}\ \delta(s-m_{\rho'}^2) - f_{\omega'}\ \delta(s-m_{\omega'}^2)] + \frac{\alpha_{\mathrm{em}}}{64\pi^3}\Theta(s-s_V). \tag{71}$$

The necessity for including the higher resonances ρ' and ω' is discussed below.

We mention that the term f_ϕ is allowed since the ϕ meson is not a pure $\overline{s}s$ state but mixed with the ω meson. Even more, it has been found in [13] that the ϕ meson gives a significant contribution in vacuum due to large cancellations between f_ρ and f_ω. Accordingly, we drop the assumption of ideal mixing and take into account such a term.[2]

The five parameters $f_\rho, f_{\rho'}, f_\omega, f_{\omega'}$ and f_ϕ have to be evaluated self-consistently within the QCD sum rule approach. What we still need is a connection between these new parameters and the parameter $\delta_{\rho\omega}(\overline{m}^2)$ which enters physical observables like the pion form factor in Eq. (39). Such a relationship can be obtained by means of VMD [11]

$$J_\mu^\rho(x) = \frac{m_\rho^2}{g_{\rho\gamma}}\varphi_\mu^\rho(x), \quad J_\mu^\omega(x) = 3\frac{m_\omega^2}{g_{\omega\gamma}}\varphi_\mu^\omega(x), \tag{72}$$

where $\varphi_\mu^V(x)$ is the field operator of the respective vector meson $V=\rho,\omega$. If one inserts these relations into the correlator (40) one gets an expression which relates $\Pi_{\mu\nu}^{\rho\omega}$ with the mixed propagator (keeping in mind the zero-width approximation at all stages). Another expression for $\Pi_{\mu\nu}^{\rho\omega}$ can be obtained by inserting Eq. (71) into the dispersion relation (43). Equating both expressions leads to the searched relation

$$\delta_{\rho\omega}^{\mathrm{H}}(\overline{m}^2) = -(f_\rho + f_\omega)\frac{1}{24}g_{\rho\gamma}\, g_{\omega\gamma}\frac{\Delta m^2}{\overline{m}^2}, \tag{73}$$

which is valid to order $O(\Delta m^4/\overline{m}^4)$ being a fair approximation even when taking into account the strong mass splitting between ρ and ω mesons. We mention that for evaluating the momentum dependence of $\delta_{\rho\omega}^{\mathrm{H}}(q^2)$ the applicability of VMD has been debated in [13] due to the impact of the ϕ meson. On the other side, the reliablility of VMD for a momentum dependence of $\delta_{\rho\omega}^{\mathrm{H}}(q^2)$ has been confirmed in [14], where the finite width of vector mesons is taken into account. Anyhow, in zero-width approximation VMD, which leads to Eq. (73), is applicable as long as one restricts oneself to the on-shell value of this quantity, i.e., to $\delta_{\rho\omega}^{\mathrm{H}}(\overline{m}^2)$.

[2] In finite width QCD sum rule, which is necessary when considering the momentum dependence of mixing $\delta_{\rho\ \omega}(q^2)$, the ϕ contribution is negligible [14]. We mention also that there is no need to use unphysical values for $m_{\rho'}$ and $m_{\omega'}$, as been pointed out in [13].

B. Evaluation

Since the most relevant parameter $\delta_{\rho\omega}^{\mathrm{H}}$ enters the approach via the combination $\zeta \sim f_\rho + f_\omega$ it is convenient to rewrite the sum rule (66) as

$$\frac{1}{4}\zeta\frac{\overline{m}^2}{M^2}\left(\frac{\overline{m}^2}{M^2}-\beta\right)e^{-\overline{m}^2/M^2} + \frac{1}{4}\zeta'\frac{\overline{m}'^2}{M^2}\left(\frac{\overline{m}'^2}{M^2}-\beta'\right)e^{-\overline{m}'^2/M^2} + \frac{1}{4}\frac{1}{M^2}f_\phi\, e^{-m_\phi^2/M^2} + \frac{\Pi^{\rho\omega}(0,n)}{M^2} + \frac{\alpha_{\mathrm{em}}}{64\ \pi^3}e^{-s_V/M^2} = d_0 + \sum_{i=1}^{\infty}\frac{d_i}{(i-1)!\, M^{2i}}, \tag{74}$$

where we have introduced [13]

$$\zeta = \frac{\Delta m^2}{\overline{m}^4}\left(\frac{f_\rho + f_\omega}{2}\right), \quad \zeta' = \frac{\Delta m'^2}{\overline{m}'^4}\left(\frac{f_{\rho'} + f_{\omega'}}{2}\right),$$

$$\beta = 2\frac{f_\omega - f_\rho}{f_\rho + f_\omega}\frac{\overline{m}^2}{\Delta m^2}, \quad \beta' = 2\frac{f_{\omega'} - f_{\rho'}}{f_{\rho'} + f_{\omega'}}\frac{\overline{m}'^2}{\Delta m'^2} \tag{75}$$

with $2\overline{m}^2 = (m_\rho^2 + m_\omega^2)$, $2\overline{m}'^2 = (m_{\rho'}^2 + m_{\omega'}^2)$, $\Delta m^2 = m_\omega^2 - m_\rho^2$ and $\Delta m'^2 = m_{\omega'}^2 - m_{\rho'}^2$, respectively. We stress that Eq. (74) is valid to order $\mathcal{O}(\Delta m^4/M^4)$. Despite the observed strong mass splitting found in the previous section, Eq. (74) is a good approximation: The terms of order $\mathcal{O}(\Delta m^4/M^4)$ would give less than 10% correction to terms of order $\mathcal{O}(\Delta m^2/M^2)$, even at such a small Borel mass like $M \approx 1$ GeV.

The residues in the hadronic model (71) can be expressed by the new variables (75) to give

$$f_\rho = \left(\frac{\overline{m}^2}{\Delta m^2} - \frac{\beta}{2}\right)\zeta\overline{m}^2, \quad f_{\rho'} = \left(\frac{\overline{m}'^2}{\Delta m'^2} - \frac{\beta'}{2}\right)\zeta' m'^2, \tag{76}$$

$$f_\omega = \left(\frac{\overline{m}^2}{\Delta m^2} + \frac{\beta}{2}\right)\zeta\overline{m}^2, \quad f_{\omega'} = \left(\frac{\overline{m}'^2}{\Delta m'^2} + \frac{\beta'}{2}\right)\zeta'\overline{m}'^2. \tag{77}$$

Finally we give an expression for the mixing parameter ϵ in zero-width approximation (i.e., Im $\Sigma_{\rho,\omega}=0$) which can be deduced from (73) and (38),

$$\epsilon = -\frac{\overline{m}^2}{\Delta m^2}\frac{g_{\rho\gamma}\, g_{\omega\gamma}}{12}\zeta. \tag{78}$$

We need five equations for the five unknowns $\zeta, \zeta', \beta, \beta', f_\phi$. One could perform a Taylor expansion of Eq. (66) ending up with an equation system for these five parameters. This is the frame work of Finite Energy Sum Rules (FESR). Instead, here we use a combined FESR and Borel analysis, following the approach described in [12,13] which we extend to finite density. Accordingly, the first equation comes from a local duality relation [12] which results into

$$4\ \Pi^{\rho\omega}(0,n) - \beta\zeta\overline{m}^2 - \beta'\zeta'\overline{m}'^2 = 4\ d_0\ s_V + 4\ d_1 - f_\phi \tag{79}$$

and agrees with the first equation of the FESR approach [12,13]. This equation makes clear why the higher reso-

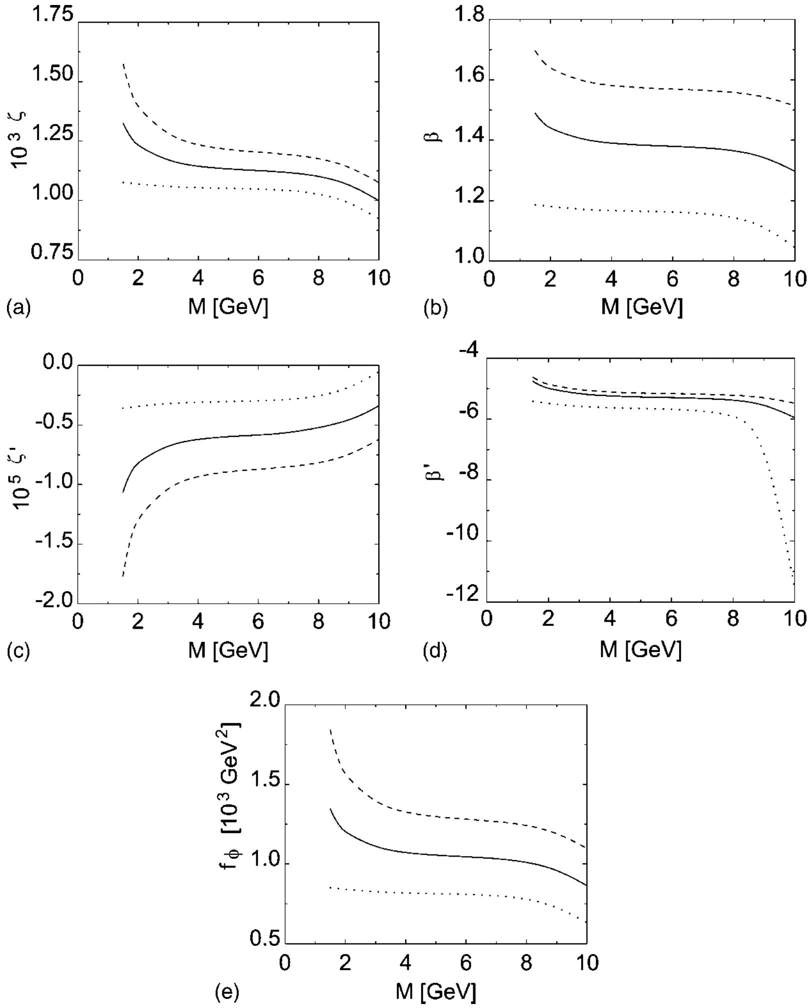

FIG. 2. Parameters ζ (a), β (b), ζ' (c), β' (d) and f_ϕ (e) as a function of the Borel mass. Dotted curves are for vacuum, solid curves stand for $n=n_0$ and dashed curves depict $n=2\ n_0$. All curves are evaluated for $\kappa_N=\kappa_0=3$ (here the mass shift of vector mesons has not been taken into account).

nances ρ' and ω' have to be taken into account: Without these higher resonances one would get either $\beta\approx 0$ or $\zeta\approx 0$, which would be in contradiction with experimental findings. The second equation is just the sum rule (66). Two equations are obtained by the first and second derivatives with respect to $1/M^2$ of Eq. (66), cf. [12] (due to the high Borel mass and the small contribution of the threshold term the second derivative sum rule is applicable, in contrast to the mass splitting, investigated in the previous section, where a second derivative sum rule becomes unstable [34]).

For evaluating f_ϕ we still need a fifth equation. In [13] a third derivative of sum rule has been used which could cause instabilities due to the truncation of OPE [34,35]. To avoid such instabilities the individual contributions of ρ' and ω' have been approximated by an effective strength $f_{\rho'\omega'}$ at the averaged mass of $m_{\rho'\ \omega'}$ in [14]. However, at finite density β is density dependent. Therefore, in order to improve this approximation we apply the second FESR for the parameter β, i.e.,

$$(1+\beta)\zeta\ \overline{m}^4+(1+\beta')\zeta'\overline{m}'^4-f_\phi\ m_\phi^2=-2\ d_0\ s_V^2+4\ d_2. \tag{80}$$

The resulting system of equations has to be solved self-consistently giving the five unknowns as function of the Borel mass, $\zeta(M^2),\zeta'(M^2),\beta(M^2),\beta'(M^2)$ and $f_\phi(M^2)$.

In Fig. 2 we have plotted these parameters as a function of the Borel mass for different densities. Like in the Borel analysis for the ρ-ω mass splitting we have to find an appropriate Borel window $M^2_{\rm min},M^2_{\rm max}$. To determine the minimal Borel window one could again use the 10% rule getting $M_{\rm min}\approx 1$ GeV, while in [12] a 25% rule has been used getting $M_{\rm min}\approx 1.3$ GeV. But it turns out that in such a region around $M_{\rm min}$ the sum rule is unstable for a wide range of parameters [12,13]. Nevertheless, the curves in Fig. 2 evidence that a stable region for all five unknowns exists in the interval $4\leqslant M\leqslant 8$ GeV. This observation confirms a corresponding stability investigation in [13]. Therefore, in line with [13], we will use a static Borel window $M_{\rm min}=4\leqslant M\leqslant M_{\rm max}=8$ GeV over which we have to average [using Eq. (36)] to get Borel mass independent quantities. The result found in [12–14] that the threshold parameter s_0 in vacuum turns out to play a subdominant rule is also valid in case of finite density. Accordingly, we may use a fixed value, $s_V=2.0$ GeV, for all densities.

C. Results

1. Vacuum

First of all, let us briefly discuss the pion form factor in vacuum, given by (39) with $n=0$. Our sum rule analysis for

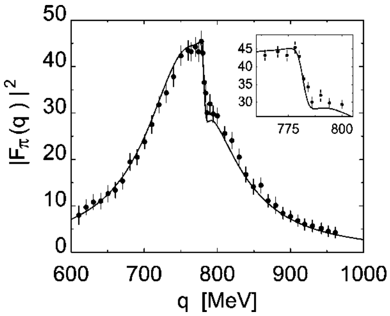

FIG. 3. Comparision of the form factor evaluated within the QCD sum rule method (solid curve) and the results of the CMD-2 experiment (symbols) [41].

vacuum results in $\zeta = 1.055 \times 10^{-3}$, in good agreement with [12,15]. Using (78) gives for the mixing parameter $\epsilon = -0.21$. The hadronic contribution of the nondiagonal on-shell self-energy which enters the pion form factor, is given, via Eq. (73), by $\delta^{\mathrm{H}}_{\rho\omega}(\overline{m}^2) = -\overline{m}^2 g_{\rho\gamma}\, g_{\omega\gamma}\, \zeta/12 = -4289\ \mathrm{MeV}^2$ which amounts, by taking into account the electromagnetic nondiagonal on-shell self-energy $\delta^{\mathrm{EM}}_{\rho\ \omega}(\overline{m}^2) = 610\ \mathrm{MeV}^2$ [39,12], in total to $\delta_{\rho\ \omega}(\overline{m}^2) = -3679\ \mathrm{MeV}^2$, in fair agreement with experiment [40]. Using this value we get the pion form factor in vacuum as shown in Fig. 3. It reproduces very well recently obtained experimental data [41]. In [13] it was argued that the sum rules might not give a good agreement with data when taking the parameter set of [12]. Our analysis shows, however, that the sum rules are in agreement with experimental data when using appropriate parameters.

2. Isospin symmetric nuclear matter

After reproducing the pion form factor in vacuum we now turn to the density dependence of the mixing effect. Due to the small effect of mixing compared to splitting and the large impact of the four-quark condensate and Landau damping terms on mass parameter splitting, it becomes obvious that mixing does not strongly influence the mass parameter splitting effect. But on the other side, the mass parameter splitting effect could strongly influence the mixing effect. To study the effect of the ρ-ω mass parameter splitting on the ρ-ω mixing we have to implement in the five equations for the five unknowns $\zeta, \beta, \zeta', \beta', f_\phi$ the density dependent mass parameters, i.e., $m_\rho(n)$, $m_\omega(n)$ and $m_\phi(n)$, respectively. For $m_\rho(n)$, $m_\omega(n)$ we use the values obtained in the previous section, while for the density dependence of the ϕ meson we will take the relation $m_\phi(n) = (1 - \alpha\, n/n_0) m_\phi(0)$ with $\alpha = 0.03$, which turns out to be almost independent of κ_N [16]. The results for the five parameters are shown in Fig. 4.

From the density behavior of the parameter ζ [see Fig. 4(a)] one might conclude that the mixing effect remains in matter. But this is actually not the case. In view of Eq. (78)

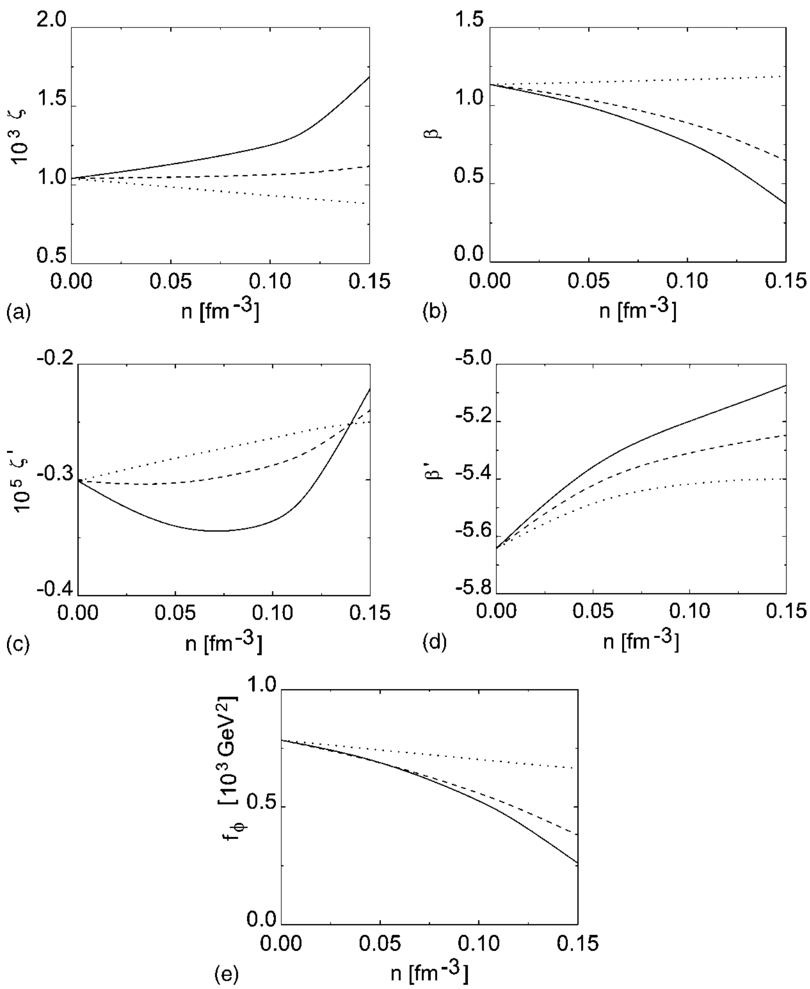

FIG. 4. Parameters ζ (a), β (b), ζ' (c), β' (d) and f_ϕ (e) at finite density. Dotted curves are for $\kappa_N = 2$, dashed curves are for $\kappa_N = 3$ and solid curves are for $\kappa_N = 4$. The density dependence of the mass parameters of ρ, ω and ϕ mesons (without the twist-4 condensates) has been taken into account consistently.

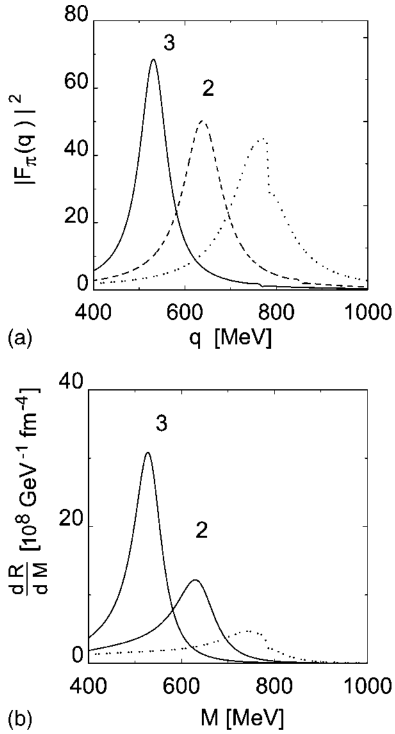

FIG. 5. (a) Pion form factor at saturation density $n = n_0$. Mass shifts of vector mesons (without twist-4 condensates) are taken into account. The dotted curve is for vacuum, while the solid curves are for saturation density $n = n_0$. The labels 2,3 denote $\kappa_N = 2, 3$, respectively. (b) Di-electron production rate from pion-pion annihilation at finite density and $T = 100$ MeV. The dotted curve is for a hot pion gas and baryonic vacuum $n = 0$, while the solid curves are for saturation density $n = n_0$.

we recognize that the mixing angle ϵ is strongly suppressed by the factor $1/\Delta m^2$. Additionally, the mass shift of the ρ meson modifies significantly the pion form factor. Using Eqs. (39) and (1) for the pion form factor and di-electron production rate, respectively, we get the results shown in Fig. 5.

These figures show that the mixing effect in the pion form factor as well as in the di-electron production rate is washed out due to the mass shifts of the vector mesons. But one has to keep in mind that global changes of vector mesons in matter like mass shift and width broadening turn out to be correlated in nuclear matter [32,33,37]. Taking into account such broadening effects needs further investigations.

We also show results without the mass shifts of ρ, ω and ϕ mesons. The corresponding density dependence of the five parameters $\zeta, \beta, \zeta', \beta', f_\phi$ is shown in Fig. 6. One observes noticeable changes for the parameters. The dashed curve (i.e., $\kappa_N = \kappa_0$) in Fig. 6(a) recovers the density-independence of ζ for isospin symmetric nuclear matter as anticipated in [15]. Otherwise, depending on the parameter κ_N which governs the density dependence of the four-quark condensate, ζ may slightly increase (large κ_N) or decrease (smaller κ_N) with increasing density. The resulting pion form factor and the di-electron production rate are plotted in Fig. 7. At finite density one obtains a very small modification of the form factor compared to the vacuum, while the modification of the rate is nearly invisible.

In Figs. 5 and 7 the meson peaks are assumed to be distributed with a schematic width $\mathrm{Im}\,\Sigma_\rho(E) = -[g^2_{\rho\pi\pi}/(48\pi)E](E^2 - 4m_\pi^2)^{3/2}\,\Theta(E - 2m_\pi)$ and $\mathrm{Im}\Sigma_\omega(E) = -m_\omega \Gamma_\omega\,\Theta(E - 3m_\pi)$, respectively. In Fig. 5 the density dependence of $m_{\rho,\omega}$ is taken into account, while in Fig. 7 no shifts of $m_{\rho,\omega}$ are assumed. Obviously, the di-electron rates shown in Figs. 5 and 7 differ significantly.

There is the possibility, advocated in [42], that in-medium the original (vacuum) ρ peak is not shifted, but additional strengths develop below the ρ peak. A similar possibility has been reported in [7] for the ω meson. In such cases the ρ-ω mixing remains, similar to Fig. 7, but the weighted ρ strength is shifted down, as required by the sum rule considered in Sec. II. A proper handling of this situation deserves further investigations with explicit knowledge of the ρ and ω in medium spectral functions. Experimentally, precision measurements with HADES [5] can deliver information on the in-medium behavior of the ρ-ω mixing.

3. Isospin asymmetric nuclear matter

So far we have considered isospin symmetric nuclear matter. While it is not necessary to study isospin asymmetric matter for the mass splitting effect, finite values of α_{np} have some relevance for the mixing effect [15]. Therefore, in this subsection we concentrate on isospin asymmetric nuclear matter. The needed proton and neutron condensates are given in the Appendix E. Accordingly, the coefficients d_i^{AS} in lines (69) and (70) contain the following terms proportional to α_{np}:

$$
\begin{aligned}
d_2^{\mathrm{AS}} = &-\frac{1}{2}\left(1 + \frac{\alpha_s}{\pi} C_F \frac{1}{4}\right)\frac{n}{2M_N} m_q\ \alpha_{np}\langle p|\overline{u}u - \overline{d}d|p\rangle \\
&- \frac{1}{72}\frac{\alpha_{\mathrm{em}}}{\pi}\frac{1}{M_N}\frac{n}{4}\alpha_{np} 5\ m_q\langle p|\overline{u}u - \overline{d}d|p\rangle \\
&- \left(\frac{1}{4} - \frac{5}{48}\frac{\alpha_s}{\pi}C_F\right)(A_2^{u,p} - A_2^{d,p})\ M_N\ \alpha_{np}\ n \\
&+ \frac{\alpha_{\mathrm{em}}}{\pi}\frac{25}{864} M_N\ n\ \alpha_{np}(A_2^{u,p} - A_2^{d,p}),
\end{aligned}
\tag{81}
$$

$$
\begin{aligned}
d_3^{\mathrm{AS}} = &\frac{56}{81}\pi\ \alpha_s\ \alpha_{np}\ n\frac{1}{M_N}\langle\overline{q}q\rangle_0\langle p|\overline{u}u - \overline{d}d|p\rangle \\
&+ \frac{7}{81}\pi\ \alpha_{np}\ n\frac{1}{M_N}\kappa_N\ \langle\overline{q}q\rangle_0\langle p|\overline{u}u - \overline{d}d|p\rangle \\
&+ \frac{\alpha_{\mathrm{em}}}{\pi}\frac{335}{3456}M_N^3\ n\ \alpha_{np}\ (A_4^{u,p} - A_4^{d,p}),
\end{aligned}
\tag{82}
$$

where terms of order $\mathcal{O}((m_d - m_u)\alpha_{np})$, $\mathcal{O}(\gamma\alpha_{\mathrm{em}})$ and $\mathcal{O}(\gamma\alpha_{np})$ are neglected.

The dependence of the parameter ζ, Eq. (75), which governs the mixing effect of the pion form factor (39) via (73), on the asymmetry parameter α_{np} is seen in Fig. 8(a). For

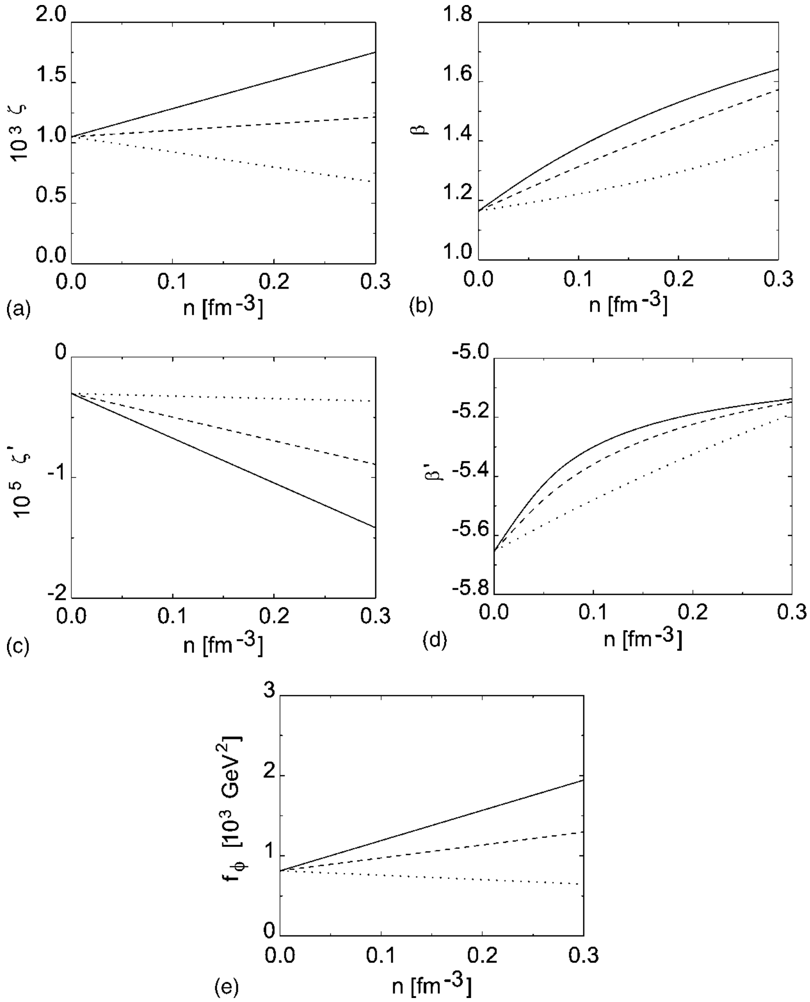

FIG. 6. Parameter ζ (a), β (b), ζ' (c), β' (d) and f_ϕ (e) at finite density. The density dependence of the mass parameters has not been taken into account. Same notation as in Fig. 4.

strong asymmetry one obtains a remarkable increase of ζ, roughly linear with α_{np}. We note that the dashed curve of Fig. 8(a) is in good agreement with [15] where an asymmetry dependence $\zeta=\zeta^{(0)}+\zeta^{(1)}\ \alpha_{np}\ n/(0.2\ n_0)$ with $\zeta^{(0)}=1.1\times 10^{-3}$ and $\zeta^{(1)}=1.5\times 10^{-3}$ has been reported, while our findings correspond to $\zeta^{(0)}=1.05\times 10^{-3}$ and $\zeta^{(1)}=1.9\times 10^{-3}$.

Altogether, without accounting for the mass shifts, an amplification of the mixing effect in the pion form factor is obtained [see the dashed curve in Fig. 8(b)]. In contrast, when accounting for the individual mass parameter shifts the mixing effect is washed out [see the solid curve of Fig. 8(b)].

Finally, it is expedient to summarize the differences between the analysis presented here and Ref. [15], which are, so far, the only investigations where the QCD sum rule approach has been applied to the mixing effect at finite density. Besides the usage of a complete OPE up to mass-dimension-6 twist-2 for the mixing effect and a self-consistent Borel analysis for all unknowns at finite density in our work, the improvements are the following: First, we have implemented the individual mass parameter shifts of the vector mesons in a consistent way and have studied their impact on the mixing effect. A second difference consists in taking into account the ϕ meson on the hadronic side, which is necessary due to large cancellations between ρ and ω meson contributions (this has been pointed out for vacuum in [13]). Third, we have investigated the relevance of ζ by considering the influence of the mixing on pion form factor and di-electron production rate.

IV. SUMMARY

In summary, we have investigated the mass parameter splitting and the mixing of ρ and ω mesons in nuclear matter within the QCD sum rule approach, starting from a complete OPE of the current-current correlator up to mass dimension-6 twist-4 and up to the first order in the coupling constant. Special attention is devoted to the impact of the poorly known scalar four-quark condensates. We have found a strong ρ-ω mass parameter splitting. The scalar flavor mixing condensate has been evaluated at finite density using quite general assumptions. It turns out that this condensate, while responsible for the ρ-ω mass parameter splitting in vacuum, plays a subdominant role in matter. Instead, the individual mass parameter splitting of ρ and ω mesons is mainly governed by the Landau damping terms. The scalar four-quark condensates have a strong impact on the individual strengths of the mass parameter shifts, while the amount of the splitting is fairly insensitive to these condensates.

We emphasize that the mass parameters are weighted moments of the spectral functions. A mass parameter shift in medium does not necessarily mean a simple shift of the peak

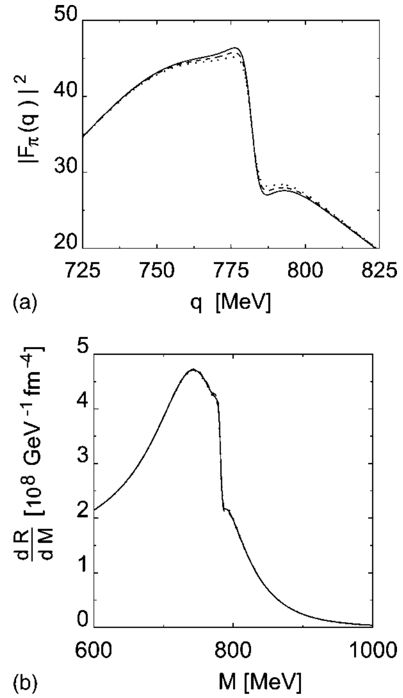

FIG. 7. (a) Form factor at finite density. Dotted line denotes vacuum, dashed line represents $n=n_0$ and solid line means $n=2n_0$. (b) Di-lepton production rate for pion-pion annihilation at finite density for $T=100$ MeV. The plotted curves are for $\kappa_N=3$. No mass shifts

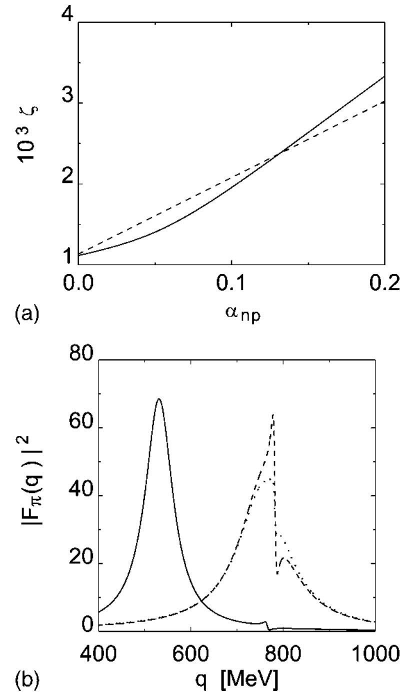

FIG. 8. (a) Parameter ζ as a function of α_{np} at saturation density n_0 (solid line: individual mass shifts of vector mesons have been taken into account; dashed line: without mass shifts of vector mesons). (b) Pion form factor for $\kappa_N=3$ and $\alpha_{np}=0.2$ (dotted line: vacuum; solid line: $n=n_0$ with mass shifts of vector mesons; dashed line: $n=n_0$ without mass shifts of vector mesons).

position of a spectral function, rather additional strength may occur at lower or higher energies causing a shift of the weighted moment. The presently employed form of the QCD sum rule approach is not sensitive to such details. Only a detailed modeling of the hadronic in-medium spectral function with parametric dependencies allows for more concise statements [46].

Another physical effect investigated concerns the ρ-ω mixing at finite density and the impact of the ρ-ω mass parameter splitting. Starting with the vacuum we find excellent agreement with experimental data recently obtained. In medium, the nondiagonal self-energy $\delta_{\rho\omega}(\overline{m},n)$, which drives the mixing effect, is only weakly amplified in isospin symmetric nuclear matter. The mixing parameter ζ, however, is remarkably enlarged for strongly isospin asymmetric nuclear matter, such as in uranium nuclei with $\alpha_{np}=0.2$. Therefore, not taking into account the individual mass shifts of the ρ and ω meson would indeed result in an in-medium amplification of the mixing effect. In contrast, if one takes into account the strong mass parameter splitting of ρ and ω mesons as a pronounced splitting of the corresponding peaks then the mixing effect in the pion form factor as well as in the di-electron production rate disappears in medium, both for isospin symmetric and isospin asymmetric nuclear matter. Upcoming measurements at HADES can deliver valuable information on these issues.

ACKNOWLEDGMENTS

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APPENDIX A: SCALAR FLAVOR-UNMIXING FOUR-QUARK CONDENSATES

In lines (9) and (11) one recognizes two different types of scalar flavor-unmixing four-quark condensates $(q=u,d)$

$$M_A^{qq}=\langle\Omega|\overline{q}\gamma_\mu\gamma_5\lambda^a q\overline{q}\gamma^\mu\gamma_5\lambda^a q|\Omega\rangle \tag{A1}$$

and

$$M_V^{qq}=\langle\Omega|\overline{q}\gamma_\mu\lambda^a q\overline{q}\gamma^\mu\lambda^a q|\Omega\rangle. \tag{A2}$$

Previous studies employed a factorization for the scalar flavor-unmixing four-quark condensates [26]. We go beyond such approximation, $M_A^{qq}=16/9\,\kappa\langle\Omega|\overline{q}q|\Omega\rangle^2$, pointing out that κ is uncertain and might even have a density dependence. In the spirit of the linear density approximation (25), a Taylor expansion results in

$$M_A^{qq}=\frac{16}{9}\langle\overline{q}q\rangle_0^2\ \kappa_0^{(1)}\left[1+\frac{\kappa_N^{(1)}}{\kappa_0^{(1)}}\frac{\sigma_N\,n}{m_q\,\langle\overline{q}q\rangle_0}\right]. \tag{A3}$$

The first term, i.e., $\frac{16}{9}\langle\overline{q}q\rangle_0^2\ \kappa_0^{(1)}$, is merely an expression for $\langle\overline{q}\gamma_\mu\gamma_5\lambda^a q\overline{q}\gamma^\mu\gamma_5\lambda^a q\rangle_0$. The second term, proportional to $\kappa_N^{(1)}$, parametrizes the poorly known four-quark condensate in the nucleon $\langle N(\boldsymbol{k})|\overline{u}\gamma_\mu\gamma_5\lambda^a u\overline{u}\gamma^\mu\gamma_5\lambda^a u|N(\boldsymbol{k})\rangle$. Similarly, for the other four-quark condensate we obtain

$$M_V^{qq}=-\frac{16}{9}\langle\overline{q}q\rangle_0^2\ \kappa_0^{(2)}\left[1+\frac{\kappa_N^{(2)}}{\kappa_0^{(2)}}\frac{\sigma_N\,n}{m_q\,\langle\overline{q}q\rangle_0}\right]. \tag{A4}$$

Accumulating all flavor-unmixing four-quark condensates, with the right weight given from the OPE, one obtains in linear density approximation finally

$$-\frac{1}{2}M_A^{uu}-\frac{1}{2}M_A^{dd}-\frac{1}{9}M_V^{uu}-\frac{1}{9}M_V^{dd} = -\frac{7}{18}\frac{16}{9}\langle\bar{q}q\rangle_0^2\ \kappa_0\left[1+\frac{\kappa_N}{\kappa_0}\frac{\sigma_N\ n}{m_q\ \langle\bar{q}q\rangle_0}\right]. \tag{A5}$$

Note that $\kappa_N=\kappa_0$ conforms to the large-N_c limit [43]. Since we are interested in medium effects, we adjust the value of κ_0 to the vacuum masses, yielding $\kappa_0=3$ both for ρ and ω, and study the impact of the unknown parameter κ_N. As stressed in [16,37], only a comparison with experimental data can pin down κ_N.

For treating the ρ-ω mixing we also need

$$N_A^{qq}=\langle\Omega|\bar{q}\gamma_\mu\gamma_5 q\bar{q}\gamma^\mu\gamma_5 q|\Omega\rangle \tag{A6}$$

and

$$N_V^{qq}=\langle\Omega|\bar{q}\gamma_\mu q\bar{q}\gamma^\mu q|\Omega\rangle. \tag{A7}$$

With the same steps as above we arrive at

$$-4N_A^{uu}+N_A^{dd}-\frac{8}{9}N_V^{uu}+\frac{2}{9}N_V^{dd} = -\frac{7}{9}\langle\bar{q}q\rangle_0^2\ \kappa_0\left[1+\frac{\kappa_N}{\kappa_0}\frac{\sigma_N\ n}{m_q\ \langle\bar{q}q\rangle_0}\right]. \tag{A8}$$

APPENDIX B: SCALAR FLAVOR-MIXING FOUR-QUARK CONDENSATES

Now we estimate the two scalar flavor-mixing condensates in lines (10) and (12) at finite density. (For the vacuum such an estimate is given in [29].) Let us first consider the condensate in line (10). To evaluate such a condensate we insert a complete set of QCD eigenstates after a Fierz transformation

$$\begin{aligned}M_A^{ud}&=\langle\Omega|\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{d}\gamma^\mu\gamma_5\lambda^a d|\Omega\rangle\\ &=\sum_n\langle\Omega|\bar{u}_i^\alpha d_l^\delta|n\rangle\langle n|\bar{d}_k^\gamma u_j^\beta|\Omega\rangle\ (\lambda^a)_{ij}\ (\lambda^a)_{kl}\Big(\delta^{\alpha\delta}\delta^{\gamma\beta}\\ &\quad+\tfrac{1}{2}(\gamma_\mu)^{\alpha\delta}(\gamma^\mu)^{\gamma\beta}+\tfrac{1}{2}(\gamma_\mu\gamma_5)^{\alpha\delta}(\gamma^\mu\gamma_5)^{\gamma\beta}\\ &\quad-(\gamma_5)^{\alpha\delta}(\gamma_5)^{\gamma\beta}\Big),\end{aligned} \tag{B1}$$

and approximate the sum by

$$\sum_n|n\rangle\langle n|\approx|\Omega\rangle\langle\Omega|+|\Omega^*\rangle\langle\Omega^*| +\sum_{b=1}^{3}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}|\Omega\ \pi^b(p)\rangle\langle\pi^b(p)\Omega|, \tag{B2}$$

where $|\Omega\rangle$ is the ground state of matter, $|\Omega^*\rangle$ denotes low-lying excitations (e.g., particle-hole excitations), and $|\Omega\ \pi^b\rangle$ means ground state plus pion with isospin index b (other states with mesons heavier than pions are suppressed by their larger masses). The matrix elements $\langle\Omega|\bar{u}_i^\alpha d_l^\delta|\Omega\rangle$ and $\langle\Omega|\bar{u}_i^\alpha d_l^\delta|\Omega^*\rangle$ vanish due to quark flavor conservation yielding

$$\begin{aligned}M_A^{ud}&=\sum_{b=1}^{3}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\langle\Omega|\bar{u}_i^\alpha d_l^\delta|\Omega\pi^b(p)\rangle\langle\Omega\pi^b(p)|\bar{d}_k^\gamma u_j^\beta|\Omega\rangle\\ &\quad\times(\lambda^a)_{ij}\ (\lambda^a)_{kl}(\delta^{\alpha\delta}\delta^{\gamma\beta}+\tfrac{1}{2}(\gamma_\mu)^{\alpha\delta}(\gamma^\mu)^{\gamma\beta}\\ &\quad+\tfrac{1}{2}(\gamma_\mu\gamma_5)^{\alpha\delta}(\gamma^\mu\gamma_5)^{\gamma\beta}-(\gamma_5)^{\alpha\delta}(\gamma_5)^{\gamma\beta}).\end{aligned} \tag{B3}$$

The soft pion theorem [8,44] allows one to calculate the needed terms in linear density approximation as

$$\langle\Omega|\bar{u}_i^\alpha d_l^\delta|\Omega\ \pi^1(0)\rangle=\frac{i}{f_\pi}\frac{1}{12}\delta_{il}(\gamma_5)^{\alpha\delta}\langle\Omega|\bar{q}q|\Omega\rangle, \tag{B4}$$

$$\langle\Omega|\bar{u}_i^\alpha d_l^\delta|\Omega\ \pi^2(0)\rangle=-\frac{1}{f_\pi}\frac{1}{12}\delta_{il}(\gamma_5)^{\alpha\delta}\langle\Omega|\bar{q}q|\Omega\rangle \tag{B5}$$

and $\langle\Omega|\bar{u}_i^\alpha d_l^\delta|\Omega\ \pi^3(0)\rangle=0$. Inserting these matrix elements into (B3) results in

$$M_A^{ud}=\frac{4}{9\pi^2}\frac{1}{f_\pi^2}\langle\Omega|\bar{q}q|\Omega\rangle^2\ Q_0^2 \tag{B6}$$

with the cutoff Q_0 coming from the momentum integral. With Eq. (25) and $\langle N(\boldsymbol{k})|\bar{q}q|N(\boldsymbol{k})\rangle=M_N\ \sigma_N/m_q$ one gets the final result

$$M_A^{ud}=\frac{4}{9\pi^2}\frac{Q_0^2}{f_\pi^2}\langle\bar{q}q\rangle_0^2\left[1+\frac{\sigma_N\ n}{m_q\ \langle\bar{q}q\rangle_0}\right]. \tag{B7}$$

Using the same technique for the matrix element in line (12) one finds

$$M_V^{ud}=\langle\Omega|\bar{u}\gamma_\mu\lambda^a u\bar{d}\gamma^\mu\lambda^a d|\Omega\rangle=-M_A^{ud}. \tag{B8}$$

Q_0 is adjusted to the vacuum ρ-ω mass splitting. Using $Q_0=150$ MeV we get the right experimental vacuum masses, i.e., $m_\rho(0)=771$ MeV and $m_\omega(0)=782$ MeV [45] for our chosen parameters, $\langle\bar{q}q\rangle_0=(-0.245\text{ GeV})^3$, $\langle\alpha_s/\pi\ G^2\rangle_0=(0.33\text{ GeV})^4$, $\alpha_s=0.38$, $m_u=4$ MeV, $m_d=7$ MeV, $M_N^0=770$ MeV, $\sigma_N=45$ MeV. Furthermore, we take the known vacuum values of M_N, f_π and m_π.

APPENDIX C: TWIST-2 CONDENSATES

The quark twist-2 condensates appear in lines (15) and (17), respectively, while the gluonic twist-2 condensates appear in lines (14) and (16), respectively. The operator $\hat{S}\hat{T}$ creates a symmetric and traceless expression with respect to the Lorentz indices, i.e., for spin-2 $\hat{S}\hat{T}(O_{\alpha\beta})=(1/2!)(\mathcal{O}_{\alpha\beta}+\mathcal{O}_{\beta\alpha})-1/4g_{\alpha\beta}\,\mathcal{O}_\gamma^\gamma$ and analogously for spin-4 condensates. These condensates vanish in vacuum and therefore, according to the low-density approximation (25), we need only the nucleon matrix elements which can generally be written as [26]

$$\langle N(\boldsymbol{k})|\hat{S}\hat{T}\bar{q}\gamma_\mu D_\nu q|N(\boldsymbol{k})\rangle = -iS_{\mu\nu}\, A_2^q(\mu^2), \tag{C1}$$

$$\langle N(\boldsymbol{k})|\hat{S}\hat{T}G_\mu{}^\alpha G_{\alpha\nu}|N(\boldsymbol{k})\rangle = S_{\mu\nu}\, A_2^G(\mu^2) \tag{C2}$$

for spin-2 operators and

$$\langle N(\boldsymbol{k})|\hat{S}\hat{T}\bar{q}\gamma_\mu D_\nu D_\lambda D_\sigma q|N(\boldsymbol{k})\rangle = iS_{\mu\nu\lambda\sigma}\, A_4^q(\mu^2), \tag{C3}$$

$$\langle N(\boldsymbol{k})|\hat{S}\hat{T}G_\mu{}^\rho D_\nu D_\lambda G_{\rho\sigma}|N(\boldsymbol{k})\rangle = -S_{\mu\nu\lambda\sigma}\, A_4^G(\mu^2) \tag{C4}$$

for spin-4 operators, respectively. The Lorentz structures are defined as

$$S_{\mu\nu} = k_\mu k_\nu - \tfrac{1}{4}k^2\, g_{\mu\nu}, \tag{C5}$$

$$S_{\mu\nu\lambda\sigma} = \Bigg[k_\mu k_\nu k_\lambda k_\sigma + \frac{k^4}{48}(g_{\mu\nu}g_{\lambda\sigma} + g_{\mu\lambda}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\lambda}) - \frac{k^2}{8}(k_\mu k_\nu g_{\lambda\sigma} + k_\mu k_\lambda g_{\nu\sigma} + k_\mu k_\sigma g_{\lambda\nu} + k_\nu k_\lambda g_{\mu\sigma} + k_\nu k_\sigma g_{\mu\lambda} + k_\lambda k_\sigma g_{\mu\nu}) \Bigg]. \tag{C6}$$

The reduced matrix elements of quark twist-2 condensates are defined as

$$A_i^q(\mu^2) = 2\int_0^1 dx\, x^{i-1}[q_N(x,\mu^2) + (-1)^i\, \bar{q}_N(x,\mu^2)],$$

where $q_N(x,\mu^2)$ and $\bar{q}_N(x,\mu^2)$ are the quark and antiquark distribution function inside the nucleon. We take $A_2^{(u+d)} \times(1\ \mathrm{GeV}^2)=1.02$ and $A_4^{(u+d)}(1\ \mathrm{GeV}^2)=0.12$ [4], respectively. The reduced matrix elements of gluon twist-2 condensates are defined by $A_i^G(\mu^2)=2\int_0^1 dx\, x^{i-1}G_N(x,\mu^2)$, with $G_N(x,\mu^2)$ as gluon distribution function inside the nucleon at the scale μ^2. We use $A_2^G(1\ \mathrm{GeV}^2)=0.83$ and $A_4^G(1\ \mathrm{GeV}^2)=0.04$ [24], respectively.

APPENDIX D: TWIST-4 CONDENSATES

Twist-4 condensates appear in the lines (18), (19), (21), and (22). All twist-4 operators vanish in vacuum and therefore, according to the low-density approximation (25), one needs only the nucleon matrix elements. The nucleon matrix elements of symmetric and traceless twist-4 operators can be decomposed as [36]

$$\langle N(\boldsymbol{k})|ig_s\hat{S}\hat{T}(\bar{u}[D_\mu,\tilde{G}_{\nu\alpha}]_+\gamma^\alpha\gamma_5 u)|N(\boldsymbol{k})\rangle = \tfrac{1}{2}\, S_{\mu\nu}(K_u^g + K_d^g), \tag{D1}$$

$$\langle N(\boldsymbol{k})|g_s^2\hat{S}\hat{T}(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{u}\gamma_\nu\gamma_5\lambda^a u)|N(\boldsymbol{k})\rangle = 2\, S_{\mu\nu}(K_u^1 + K_d^1 - K_{ud}^1), \tag{D2}$$

$$\langle N(\boldsymbol{k})|g_s^2\hat{S}\hat{T}(\bar{u}\gamma_\mu\gamma_5\lambda^a u\bar{d}\gamma_\nu\gamma_5\lambda^a d)|N(\boldsymbol{k})\rangle = 2\, S_{\mu\nu}(K_{ud}^1), \tag{D3}$$

$$\langle N(\boldsymbol{k})|g_s^2\hat{S}\hat{T}(\bar{u}\gamma_\mu\lambda^a u(\bar{u}\gamma_\nu\lambda^a u + \bar{d}\gamma_\nu\lambda^a d))|N(\boldsymbol{k})\rangle = 2\, S_{\mu\nu}\,(K_u^2 + K_d^2) \tag{D4}$$

with $S_{\mu\nu}$ defined in Eq. (C5). The other twist-4 condensates, where u and d are interchanged, are equal to the given ones due to the assumed flavor symmetry. As pointed out in [36] the coefficients $K_{u,d,ud}^{1,2}$ are related to the nucleon forward scattering amplitude of the electromagnetic current. We take the following parameter set: $K_u^1=-0.112\ \mathrm{GeV}^2$, $K_u^2 =0.110\ \mathrm{GeV}^2$, $K_u^g=-0.300\ \mathrm{GeV}^2$, $K_{ud}^1=-0.084\ \mathrm{GeV}^2$ as default. For the d quark we use $K_d^{1,2,g}=\beta K_u^{1,2,g}$ with $\beta=0.476$ from [36].

We remark that the parameters $K_{u,d,ud}^{1,2,g}$ should be taken at a hadronic scale of $\mu=1$ GeV. Unfortunately, twist-4 condensates are poorly known and even available only at a scale of $\mu=2.25$ GeV. To evolve these parameters down to $\mu =1$ GeV would require the knowledge of anomalous dimensions which are not available. Here we use the above condensates, expressed by K_u^1, K_u^2, K_u^g and K_{ud}^1, to demonstrate that accounting for these condensates has little influence on the mass splitting and individual mass shifts of ρ and ω mesons.

For the twist-4 operator in line (23) we use the estimate [21]:

$$\langle N(\boldsymbol{k})|m_q\bar{q}D_\mu D_\nu q|N(\boldsymbol{k})\rangle \simeq -P_\mu^q P_\nu^q \langle N(\boldsymbol{k})|m_q\bar{q}q|N(\boldsymbol{k})\rangle, \tag{D5}$$

where P_μ^q is the average momentum carried by the quark q inside the nucleon. Taking $P_\mu^q \sim k_\mu/6$ [21] (k_μ is the momentum of nucleon) and making the operator symmetric and traceless we get

$$\langle N(\boldsymbol{k})|\hat{S}\hat{T}m_q\bar{q}D_\mu D_\nu q|N(\boldsymbol{k})\rangle \simeq -S_{\mu\nu}\tfrac{1}{36}m_q\, \langle N(\boldsymbol{k})|\bar{q}q|N(\boldsymbol{k})\rangle. \tag{D6}$$

APPENDIX E: PARAMETERS FOR ρ-ω MIXING

For the ρ-ω mixing we have to distinguish between proton and neutron matrix elements. In particular we need the following twist-2 and twist-4 condensates:

$$\langle p(\boldsymbol{k})|\hat{S}\hat{T}\bar{q}\gamma_\mu D_\nu q|p(\boldsymbol{k})\rangle = -iS_{\mu\nu}A_2^{q,p}, \tag{E1}$$

$$\langle p(\boldsymbol{k})|\hat{S}\hat{T}\bar{u}\gamma_\mu D_\nu D_\lambda D_\sigma u|p(\boldsymbol{k})\rangle = iS_{\mu\nu\lambda\sigma}A_4^{q,p} \tag{E2}$$

with the proton state $|p(\boldsymbol{k})\rangle$ and analog expressions for the neutron. The reduced matrix elements $A_i^{q,p}(\mu^2)$ are defined as $A_i^{q,p}(\mu^2)=2\int_0^1 dx\, x^{i-1}[q_p(x,\mu^2)+(-1)^i\, \overline{q_p}(x,\mu^2)]$, where $q_p(x,\mu^2)$ and $\overline{q_p}(x,\mu^2)$ are the quark and antiquark distribution functions inside the proton. We use the following parameters: $A_2^{u,p}(1\ \mathrm{GeV}^2)=0.67$, $A_2^{d,p}(1\ \mathrm{GeV}^2)=0.35$, $A_4^{u,p}(1\ \mathrm{GeV}^2)=0.091$, $A_4^{d,p}(1\ \mathrm{GeV}^2)=0.029$; $A_2^{u,n}=A_2^{d,p}$ and $A_2^{d,n}=A_2^{u,p}$, which follow from $u_n(x,\mu^2)=d_p(x,\mu^2)$ and $u_p(x,\mu^2)=d_n(x,\mu^2)$, respectively [48].

Another needed matrix element is [47]

$$\langle p|\overline{u}u-\overline{d}d|p\rangle = 2\ M_N \frac{m_\Xi - m_\Sigma}{m_s} = 1.3\ \mathrm{GeV}. \tag{E3}$$

The isospin symmetry breaking parameter for the quark condensate is $\gamma=-0.008$ (cf. [12]), and the mass parameters of higher resonances are $m_{\rho'}=1.465$ GeV and $m_{\omega'}=1.649$ GeV [45], respectively. For the coupling constants we take the values $g_{\rho\pi\pi}=6.0$, $g_{\rho\gamma}=5.2$ and $g_{\omega\gamma}=3g_{\rho\gamma}$ [11].

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