Short survey

Finite-width QCD sum rules for ρ and ω mesons

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Dedicated to the memory of O.P. Pavlenko

Abstract

We present a combined analysis of the in-medium behavior of ρ and ω mesons within the Borel QCD sum rule taking into account finite widths.
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1. Introduction

The experiments performed at the high-acceptance di-electron spectrometer HADES [1] are aimed at verifying predictions of the behavior of light vector mesons in nuclear matter. Due to the decay channel $V \to e^+e^-$ a measurement of the escaping di-electrons can reveal directly the properties of the parent mesons, $V = \rho, \omega, \ldots$, since the interaction probability of the $e^{\pm}$ with the ambient strongly interacting medium is small.

In relativistic heavy-ion collisions temperature effects play an important role, which also cause a change of the properties of vector mesons. Indeed, the experiments of the CERES collaboration at the CERN-SPS [2] can only be explained by assuming strong medium effects, in particular for the ρ meson (cf. [3, 4] and further references therein). This seems to be confirmed at higher beam energies, as delivered by the relativistic heavy-ion collider, since the ρ meson, as measured via the $\pi^+\pi^-$ decay channel, suffers some modification [5]. In contrast, in heavy-ion collisions at typical SIS18 energies, i.e., at beam energies around 1 A GeV, the in-medium behavior of vector mesons in compressed nuclear matter can be studied, where temperature effects are small and

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may be neglected. Complementary to heavy-ion collisions one can seek for in-medium modifications in reactions of hadronic projectiles [6] or real and virtual photons [7] at nuclei, as already at nuclear saturation density sizeable modifications of vector mesons are predicted.

There exists a vastly extended literature on the in-medium modification of hadrons. We mention here only the Brown–Rho scaling hypothesis, according to which a mass shift of a vector meson is directly interrelated to a change of the chiral condensate [8], and the vector manifestation [9], the effective Lagrangian approach [10], purely hadronic approaches [11, 12], and QCD sum rules [13–16]. QCD sum rules [17] follow the idea of duality (cf. [18]) by relating quantities expressed by partonic (quark and gluon) degrees of freedom with hadronic observables. We take here the attitude to assume that the partonic quantities are given and examine the QCD sum rule to elucidate the in-medium change of the ρ and ω meson on a common footing. The corresponding current operators, expressed by the interpolating quark field operators u and d have the form $J_\mu^\rho = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$ and $J_\mu^\omega = \frac{1}{2}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)$, suggesting that the same partonic quantities enter the sum rules. A common treatment of ρ and ω mesons is necessary since our schematic transport model studies [19] show that the relative shift of the peak positions and in-medium broadenings must be known to arrive at firm predictions concerning the chances to identify both mesons in experiments performed at HADES.

In a broader context such studies address the phenomenon "mass". Within the standard model, the masses of quarks and leptons are generated by spontaneous symmetry breaking (in the Higgs mode) in the electro-weak sector, while the spontaneous symmetry breaking (in the Goldstone mode) explains the features of the hadronic mass spectrum: light Goldstone bosons (π, K, η with finite masses generated by an additional explicit symmetry breaking) and heavier hadronic states emerge [20]. Within the QCD sum rule approach [17, 18] the hadron masses can be related to condensates which represent non-perturbative features of the QCD vacuum. This is highlighted by the Gell-Mann–Oakes–Renner relation, $m_\pi^2 f_\pi^2 = -2m_q \langle \bar{q}q \rangle$ (which is actually related to PCAC), and the Ioffe formula, $M_N = -8\pi^2 \langle \bar{q}q \rangle / M^2$ (cf. [21]). Both expressions, which are in leading order, suggest that hadron masses are tightly related to the chiral condensate $\langle \bar{q}q \rangle$, which in turn is a measure of the order parameter of the chiral symmetry breaking.

Since $\langle \bar{q}q \rangle$ changes with changing temperature [22] and density (cf. Fig. 1 in [23]), one can expect a simultaneous change of the hadron masses. This was the very idea of the Brown–Rho scaling hypothesis [8] which is modified according to [13]. This promoted very much the physics programme at HADES with the motivation to verify, via an observed change of the hadron masses, the chiral condensate's change.

To give a loosely spoken analogy, one is seeking for the QCD analog of the Zeeman/Stark effects: an external field (here: strong interaction mediated by surrounding hadrons) changes the excitation spectrum of an atom (here: the hadronic excitation spectrum above the QCD ground state). Very often the notion "mass shift" is used as shorthand notation for a shift of the peak in the spectral function.

Our paper addresses such mass shifts of ρ and ω mesons on the basis of the Borel QCD sum rule; it is organized as follows. In Section 2 we recapitulate the basics of the QCD sum rule approach. In Section 3 we present a combined study of the in-medium behavior
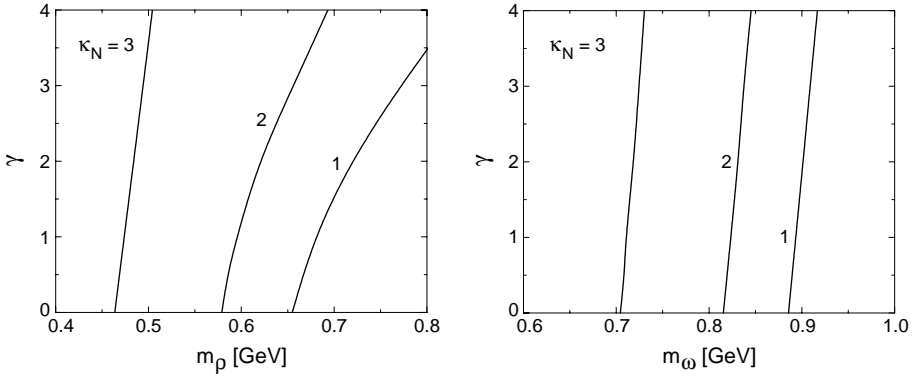
Fig. 1. Results of the QCD sum rule evaluations for ρ (left panels) and ω (right panels) at n_0. T_{VN} is discarded.

of ρ and ω mesons accounting for the effects of finite widths. The ρ–ω mass splitting is discussed in Section 4. The conclusions can be found in Section 5.

2. QCD sum rule

Within QCD sum rules (QSR) the in-medium vector mesons $V = \rho, \omega$ are considered as resonances in the current–current correlation function

$$\Pi_{\mu\nu}(q, n) = i \int \mathrm{d}^4x \, \mathrm{e}^{iq \cdot x} \langle \mathcal{T} J_\mu^V(x) J_\nu^V(0) \rangle_n, \tag{1}$$

where $q_\mu = (E, \mathbf{q})$ is the meson four momentum, $\mathcal{T}$ denotes the time ordered product of the respective meson current operators $J_\mu^V(x)$, and $\langle \cdots \rangle_n$ stands for the expectation value in medium. In what follows, we focus on the ground state of low-density baryon matter approximated by a Fermi gas with nucleon density n. We consider isospin symmetric nuclear matter, where the ρ–ω mixing effect is negligible.

The correlator (1) can be reduced to $\frac{1}{3} \Pi_\mu^\mu(q^2, n) = \Pi^{(V)}(q^2, n)$ for a vector meson at rest, $\mathbf{q} = 0$, in the rest frame of matter. In each of the vector meson channels the corresponding correlator $\Pi^{(V)}(q^2, n)$ satisfies the twice subtracted dispersion relation, which can be written with $Q^2 \equiv -q^2 = -E^2$ as

$$\frac{\Pi^{(V)}(Q^2)}{Q^2} = \frac{\Pi^{(V)}(0, n)}{Q^2} - \Pi^{(V)\prime}(0) - Q^2 \int_0^\infty \mathrm{d}s \frac{R^{(V)}(s)}{s(s + Q^2)}, \tag{2}$$

with $\Pi^{(V)}(0, n) = \Pi^{(V)}(q^2 = 0, n)$ and $\Pi^{(V)\prime}(0) = \frac{\mathrm{d}\Pi^{(V)}(q^2)}{\mathrm{d}q^2}\big|_{q^2=0}$ as subtraction constants, and $R^{(V)}(s) \equiv -\mathrm{Im}\Pi^{(V)}(s, n)/(\pi s)$.

For large values of Q^2 one can evaluate the r.h.s. of Eq. (1) by the operator product expansion (OPE) leading to

$$\frac{\Pi^{(V)}(Q^2)}{Q^2} = -c_0 \ln(Q^2) + \sum_{i=1}^\infty \frac{c_i}{Q^{2i}}, \tag{3}$$

where the coefficients c_i include the Wilson coefficients and the expectation values of the corresponding products of the quark and gluon field operators, i.e. condensates. Performing a Borel transformation of the dispersion relation (2) with appropriate parameter M^2 and taking into account the OPE (3) one gets the basic QSR equation

$$\Pi^{(V)}(0, n) + \int_0^\infty \mathrm{d}s\, R^{(V)}(s)\, \mathrm{e}^{-s/M^2} = c_0\, M^2 + \sum_{i=1}^\infty \frac{c_i}{(i-1)!\, M^{2(i-1)}}. \tag{4}$$

The advantage of the Borel transformation is (i) the exponential suppression of the high-energy part of $R^V(s)$, and (ii) the possibility to suppress higher-order contributions to the r.h.s. sum. Choosing sufficiently large values of the internal technical parameter M one can truncate the sum in a controlled way, in practice at $i = 3$.

For the subtraction constants $\Pi^{(V)}(0, n)$ in Eq. (2) we use $\Pi^{(\rho)}(0, n) = n/(4M_N)$, $\Pi^{(\omega)}(0, n) = 9n/(4M_N)$, which are actually the Thomson limit of the VN scattering processes, but also coincide with Landau damping terms elaborated in [24] for the hadronic spectral function entering the dispersion relation without subtractions. For details about the connection of subtraction constants and Landau damping term we refer the interested reader to [25].

In calculating the density dependence of the condensates entering the coefficients c_i we employ the standard linear density approximation, which is valid for not too large density values. This gives for the chiral quark condensate $\langle \overline{q}q \rangle_n = \langle \overline{q}q \rangle_0 + \frac{\sigma_N}{2m_q}n$, where we assume here isospin symmetry for the light quarks, i.e. $m_q = 5.5$ MeV and $\langle \bar{q}q \rangle_0 = -(0.24 \text{ GeV})^3$. The nucleon sigma term is $\sigma_N = 45$ MeV. The gluon condensate is obtained as usual employing the QCD trace anomaly $\langle \frac{\alpha_s}{\pi}G^2 \rangle_n = \langle \frac{\alpha_s}{\pi}G^2 \rangle_0 - \frac{8}{9}M_N^0 n$, where $\alpha_s = 0.38$ is the QCD coupling constant and $M_N^0 = 770$ MeV is the nucleon mass in the chiral limit. The vacuum gluon condensate is $\langle \frac{\alpha_s}{\pi}G^2 \rangle_0 = (0.33 \text{ GeV})^4$.

The coefficient c_3 in Eq. (4) contains also the mass dimension-6 4-quark condensates (cf. [26] for a recent calculation of corresponding matrix elements) $\langle(\bar{q}\gamma_\mu \lambda^a q)^2\rangle_n$, $\langle(\bar{u}\gamma_\mu \lambda^a u)(\bar{d}\gamma^\mu \lambda^a d)\rangle_n$, $\langle(\bar{q}\gamma_\mu \lambda^a q)(\bar{s}\gamma^\mu \lambda^a s)\rangle_n$, and $\langle(\overline{q}\gamma_\mu \gamma^5 \lambda^a q)^2\rangle_n$ which are common for ρ and ω mesons. On this level, ρ and ω mesons differ only by the condensate $\pm 2\langle(\bar{u}\gamma_\mu \gamma_5 \lambda^a u)(\bar{d}\gamma^\mu \gamma_5 \lambda^a d)\rangle_n$ (cf. [16]), causing the small ρ–ω mass splitting in vacuum [17]. Keeping in mind the important role of the 4-quark condensate [27, 28] for the in-medium modifications of the vector mesons, we employ the following parameterization:

$$\langle(\bar{q}\gamma_\mu \gamma^5 \lambda^a q)^2\rangle_n = \frac{16}{9}\langle \overline{q}q \rangle_0^2\, \hat{\kappa}_0 \left[1 + \frac{\hat{\kappa}_N}{\hat{\kappa}_0}\frac{\sigma_N}{m_q \langle \overline{q}q \rangle_0}n \right]. \tag{5}$$

The parameter $\hat{\kappa}_0$ reflects a deviation from the vacuum saturation assumption. (The case $\hat{\kappa}_0 = 1$ corresponds obviously to the exact vacuum saturation [18].) To control the deviation of the in-medium 4-quark condensate from the mean-field approximation we introduce the parameter $\hat{\kappa}_N$. An analog procedure applies for the other 4-quark condensates, each with its own $\hat{\kappa}_0$ and $\hat{\kappa}_N$, which sum up to a parameter κ_0 and a parameter κ_N. As seen in Eqs. (5) and (9) below, κ_N parameterizes the density dependence of the summed 4-quark condensates; κ_0 is adjusted to the vacuum masses. Below we vary the poorly constrained parameter κ_N to estimate the contribution of the 4-quark

condensates to the QSR with respect to the main trends of the in-medium modification of the vector meson spectral function. (Strictly speaking, κ_0 and κ_N differ for ρ and ω mesons due to contributions of the above mentioned flavor-mixing condensate; in addition, in medium a twist-4 condensate makes further ρ and ω to differ [15]. However, the differences can be estimated to be sub-dominant. Therefore, we use in the present work one parameter κ_N, keeping in mind that it may slightly differ for different light vector mesons.)

Using the above condensates and usual Wilson coefficients one gets as relevant terms for mass dimension ≤ 6 and twist ≤ 2 [27, 28]

$$c_0 = \frac{1}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right), \tag{6}$$

$$c_1 = -\frac{3m_q^2}{4\pi^2}, \tag{7}$$

$$c_2 = m_q\langle\overline{q}q\rangle_0 + \frac{\sigma_N}{2}n + \frac{1}{24}\left[\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0 - \frac{8}{9}M_N^0 n\right] + \frac{1}{4}A_2 M_N n, \tag{8}$$

$$c_3 = -\frac{112}{81}\pi\alpha_s\kappa_0\langle\overline{q}q\rangle_0^2\left[1 + \frac{\kappa_N}{\kappa_0}\frac{\sigma_N}{m_q\langle\overline{q}q\rangle_0}n\right] - \frac{5}{12}A_4 M_N^3 n$$
$$+ \frac{4\alpha_s}{81\pi f_\pi^2}\langle\overline{q}q\rangle_0^2 Q_0^2(2\pm9)\left[1 + \frac{\sigma_N}{m_q\langle\overline{q}q\rangle_0}n\right]. \tag{9}$$

The terms with $A_{2,4}$ in $c_{2,3}$ correspond to the derivative condensates from non-scalar operators as a consequence of the breaking of Lorentz invariance in the medium. These condensates are proportional to the moments of quark and anti-quark distributions inside the nucleon at scale $\mu^2 = 1$ GeV^2 (see for details [13]). Our choice of the moments A_2 and A_4 is 1.02 and 0.12, respectively. The last line in Eq. (9) stems from the flavor-mixing condensate, $\langle\overline{u}\cdots u\overline{d}\cdots d\rangle_n$, which has been evaluated with a technique similar to [17]. $Q_0 \sim \mathcal{O}(200\,\text{MeV})$ is a cut-off parameter from momentum integrals. For our purposes we can neglect terms related to Q_0.

The value of κ_0 in Eq. (9) is related to such a choice of the chiral condensate $\langle\overline{q}q\rangle_0$ to adjust the vacuum vector meson masses. In our QSR we have used $\kappa_0 = 3$, obtaining $m_{\rho,\omega}(n = 0) = 777$ MeV close to the nominal vacuum values. The ratio κ_N/κ_0 in the parameterization (5) is restricted by the condition $\langle(\overline{q}\gamma_\mu\lambda^a q)^2\rangle_n \leq 0$, so that one gets $0 \leq \kappa_N \leq 4$ as reasonable numerical limits when considering $n \leq n_0$, as dictated by our low-density approximation.

The case of finite baryon density <u>and</u> temperature has been considered in [27]. Here we focus on density effects with the reasoning that temperature effects below 100 MeV are negligible.

To model the hadronic side of the QSR (4) we make the standard separation of the vector meson spectral density $R^{(V)}$ into a resonance part and a continuum contribution by means of the threshold parameter s_V

$$R^{(V)}(s, n) = F_V\frac{S^{(V)}(s, n)}{s}\Theta(s_V - s) + c_0\,\Theta(s - s_V), \tag{10}$$

where $S^{(V)}(s, n)$ stands for the resonance peak in the spectral function; the normalization F_V is unimportant for the following consideration. In vacuum, this ansatz is justified since the time-reversed reaction, $e^+e^- \to V$, which is directly related to $R^{(V)}$, exhibits a prominent vector meson peak at low energies and a smooth continuum at higher energies. The sum rule can be then cast into the form

$$\frac{\int_0^{s_V} ds\, S^{(V)}(s, n)\, e^{-s/M^2}}{\int_0^{s_V} ds\, S^{(V)}(s, n) s^{-1} e^{-s/M^2}}$$

$$= \frac{c_0 M^2 \left[1 - \left(1 + \frac{s_V}{M^2} \right) e^{-s_V/M^2} \right] - \frac{c_2}{M^2} - \frac{c_3}{M^4}}{c_0 \left(1 - e^{-s_V/M^2} \right) + \frac{c_1}{M^2} + \frac{c_2}{M^4} + \frac{c_3}{2M^6} - \frac{\Pi^{(V)}(0,n)}{M^2}}. \tag{11}$$

Given as such, the sum rule allows one to test the consistency of a particular model for the spectral function $S^{(V)}$ as exercised, e.g., in [10]. In the zero-width approximation, $S^{(V)}(s, n) \propto \delta(s - \tilde{m}_V^2(n))$, the l.h.s. of Eq. (11) becomes simply $\tilde{m}_V^2(n)$. In this sense one could also consider the l.h.s. as averaged mass, denoted as $\bar{m}_V^2(n)$, keeping in mind that it represents a normalized moment of the hadron strength $S^{(V)}$. Then the sum rule Eq. (11) determines the parameters $\tilde{m}_V^2$ or $\bar{m}_V^2$ by the density dependence of the condensates, encoded in the coefficients $c_{1,2,3}$ from Eqs. (6)–(9), and the subtraction constant $\Pi^{(V)}(0, n)$. Without further explication of $S^{(V)}$ nothing can be deduced from Eq. (11) on in-medium mass shifts and broadening (see, however, [13] where, after determining the mass parameter $\tilde{m}_V(n)$ an estimate of the corresponding width is attempted).

3. Finite width effects

Our intention is now to evaluate the QCD sum rule Eq. (11) by taking into account the finite widths of the vector mesons. Note that already in vacuum the ρ and ω widths differ noticeably, 150.7 and 8.43 MeV, respectively. This must be reflected in the form of $S^{(V)}$.

Reference [16] made for the ρ meson the ansatz

$$S^{(V)}(s, n) = \frac{\gamma_V(n) \Gamma_V(s)}{(s - m_V^2(n))^2 + (\gamma_V(n) \Gamma_V(s))^2} \tag{12}$$

with the two parameters, $m_V(n)$ and $\gamma_V(n)$, and some parameterization of the width $\Gamma_V(s)$. Clearly, the one Eq. (11) cannot determine the two unknowns m_V and γ_V, rather only the correlation of $\gamma_V(m_V)$ is determined at a given density. In line with the above ansatz (12), one can use a more realistic form for the resonance spectral density $S^{(V)}$ based on the general structure of the in-medium vector meson propagator

$$S^{(V)}(s, n) = -\frac{\gamma_V(n)\, \text{Im}\Sigma_V(s, n)}{(s - \overset{0}{m}_V^2(n) - \text{Re}\Sigma_V(s, n))^2 + (\gamma_V(n)\text{Im}\Sigma_V(s, n))^2}, \tag{13}$$

with $\text{Re}\Sigma_V(s, n)$ and $\text{Im}\Sigma_V(s, n)$ as real and imaginary parts of the in-medium vector meson self-energy. In the spirit of Eq. (12) [16] the meson mass parameter $m_V(n)$ and the width factor $\gamma_V(n)$ become density dependent in nuclear matter to have a degree of

freedom for the "request" of the QSR. This dependence is determined by the QCD sum rule Eq. (11) and mainly governed by the QCD condensates. (An analogous approach with $\gamma_\rho = 1$ was used in [14].) The in-medium vector meson mass is determined by the pole position of the meson propagator, i.e., $m_V^2 = \overset{0}{m}\,{}_V^2(n) + \mathrm{Re}\,\Sigma_V(s = m_V^2(n), n)$.

Within the linear density approximation the vector meson self-energy is given by

$$\Sigma_V(E, n) = \Sigma_V^{\mathrm{vac}}(E) - n T_{VN}(E), \tag{14}$$

where $E = \sqrt{s}$ is the meson energy, $\Sigma_V^{\mathrm{vac}}(E) = \Sigma_V(E, n = 0)$, and $T_{VN}(E)$ is the (off-shell) forward meson–nucleon scattering amplitude in free space. The renormalized quantity $\Sigma_\rho^{\mathrm{vac}}$ is summarized in the Appendix A in [29]. For the ω meson we absorb as usual $\mathrm{Re}\,\Sigma_\omega^{\mathrm{vac}}$ in $\overset{0}{m}\,{}_\omega^2$ and put simply $\mathrm{Im}\,\Sigma_\omega^{\mathrm{vac}} = -m_\omega \Gamma_\omega \Theta(E - 3m_\pi)$ with the vacuum values of mass m_ω and width Γ_ω.

The described framework is well defined, supposed T_{VN} is reliably known.[1] Unfortunately, the determination of T_{VN} is hampered by uncertainties (cf. results in [10] and [12]). $\mathrm{Im}\,T_{VN}$ is more directly accessible, while $\mathrm{Re}\,T_{VN}$ follows by a dispersion relation with sometimes poorly known subtraction coefficients. Since our emphasis here is to include the collision broadening and other finite width effects in the spectral function, we absorb, as an intermediate step, $\mathrm{Re}\,T_{VN}$ in $\overset{0}{m}\,{}_V^2(n)$ thus neglecting a possible strong energy dependence. In such a way, the uncertainties of $\mathrm{Re}\,T_{VN}$ become milder since $m_V(n)$ is then mainly determined by the QSR. Neglecting the energy dependence of $\mathrm{Re}\,T_{VN}$ one discards a potentially rich structure of $S^{(V)}$, such as, for instance, a double-peak structure obtained in [12] for the ω meson or in [11] for the ρ meson. Afterwards the importance of $\mathrm{Re}\,T_{VN}$ is checked.

We take the needed scattering amplitude $T_{VN}(E)$ for ρ and ω mesons from results of the detailed analysis of pion- and photon–nucleon scattering data performed recently in [12] on the footing of the Bethe–Salpeter equation approach with four-point meson–baryon contact interactions and a unitary condition for the coupled channels.

To test the importance of this particular form of T_{VN} we proceed to evaluate the sum rule[2] in three steps: (i) first neglect T_{VN} at all, (ii) include only $\mathrm{Im}\,T_{VN}$, and (iii) include both $\mathrm{Im}\,T_{VN}$ and $\mathrm{Re}\,T_{VN}$ as well. The results are exhibited in Figs. 1–3. Compared

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[1] Then, $\gamma_V = 1$ and $m_V(n) = m_V(0)$, and the QSR acts merely as consistency check, as mentioned above.

[2] At a given baryon density n the continuum threshold s_V is determined by requiring maximum flatness of $m_V(n; M^2, s_V)$ as a function of M^2 within the Borel window $M_{\mathrm{min}}^2 \cdots M_{\mathrm{max}}^2$. The minimum Borel parameter M_{min}^2 is determined such that the terms of order $\mathcal{O}(M^{-6})$ on the OPE side Eq. (4) contribute not more that 10%. Selecting such sufficiently large values of M_{min}^2 suppresses higher-order contributions in the OPE Eq. (4) and justifies the truncation. Typically, $M_{\mathrm{min}}^2(10\%)$ is in the order of 0.6 GeV^2. The values for M_{max}^2 are roughly determined by the "50% rule", i.e., the continuum part of the hadronic side must not contribute more than 50% to the total hadronic side to be sufficiently sensitive to the resonance part. According to our experience [27–29], m_V is not very sensitive to variations of M_{max}^2. We can, therefore, fix the maximum Borel parameter by $M_{\mathrm{max}}^2 = 1.5(2.4)$ GeV^2 for the $\omega(\rho)$ meson, in good agreement with the "50% rule". The sensitivity of the results on these choices of the Borel window is discussed in [29]. Flat curves $m_V(n; M^2, s_V)$ within the Borel window represent a prerequisite for the stability of the sum rule analysis.
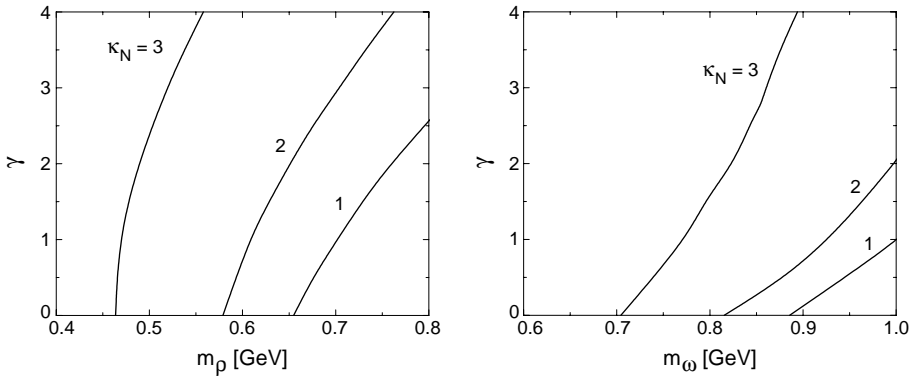
Fig. 2. As in Fig. 1 but with $\text{Im}T_{VN}$ from [12].
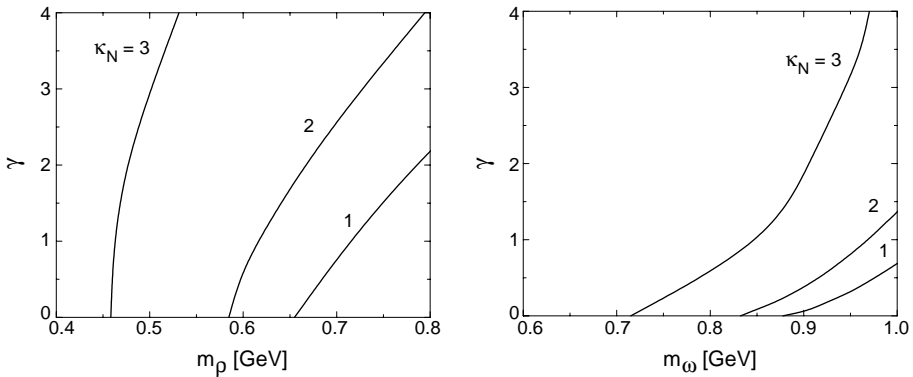
Fig. 3. As in Fig. 2 but with both $\text{Re}T_{VN}$ and $\text{Im}T_{VN}$ from [12].

to vacuum, the in-medium changes of the r.h.s. of Eq. (11) require for the ρ meson more strength at low energy which is accomplished, for the given parameterization, by a down-shift of the peak position or a larger width. With increasing γ the masses are shifted to larger values. This can be understood in the following way: a larger width gives more contribution to the integrals on the l.h.s. of Eq. (11) at lower energy; this is compensated by an up-shift of the peak position. Note that $\gamma = 0$ reproduces the zero-width approximation.

One observes in Figs. 1–3 a strong sensitivity on the density dependence of the 4-quark condensate. The sensitivity against variations of κ_N is larger than that of γ. (The displayed range of γ covers an enormous range of widths.) Remarkable is the tendency of a down-shift of the ρ mass and a related up-shift of the ω mass when restricting the width parameters γ_V to smaller values. The overall pattern seen in Figs. 1–3 is quite robust. The numerical details, however, change under variations of T_{VN}. To have a fix point let us consider $\kappa_N = 3$ (which will later turn out as relevant value). The peak position of the
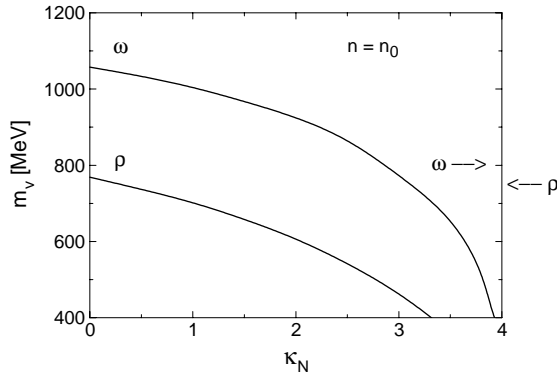
Fig. 4. The peak positions of the ω (upper curve) and ρ meson (lower curve) as a function of the parameter κ_N. Same choice of parameters as in Fig. 2 for $\gamma = 1$. The arrows depict the vacuum masses.

ρ meson is stable with respect to variations of T_{VN}. This may be attributed to the large vacuum width of ρ; changes of the amount of T_{VN} by γ cause only small changes of the ρ peak position. The ω meson, in contrast, seems to sit at some borderline: the variations of T_{VN} can cause an up-shift or a down-shift. Inclusion of $\mathrm{Im}T_{VN}$ alone results in a tiny effect, while $\mathrm{Re}T_{VN}$ pushes the peak further up. Larger values of κ_N are required to get the ω meson's peak position down-shifted.

In contrast to the universal scaling hypothesis [8] there is no unique in-medium behavior of ρ and ω mesons. Rather, the strikingly different values of the subtraction constants $\Pi_V(0, n)$ cause the different behavior of ρ and ω mesons, as stressed in [24].[3]

4. ρ–ω mass splitting

Inspecting Figs. 1–3 seems to point to a lacking predictive power of the QSR. However, if one is interested in the peak position of the spectral function (usually called "in-medium mass"), and not in the very details of the shape of $S^{(V)}$, and if one assumes that the essential features of the in-medium broadening are sufficiently accurately described by $\mathrm{Im}T_{VN}$, one has to put $\gamma_V = 1$. Then one arrives at the in-medium mass splitting of ρ and ω mesons as displayed in Fig. 4. The mass splitting is surprisingly large already at normal nuclear density n_0. The magnitude of the mass splitting can be traced back to the particular form of $\mathrm{Im}T_{VN}$ from [12]. (Using $\mathrm{Im}T_{VN}$ from [10] results in a smaller splitting.) The driving force of the ρ–ω mass splitting, however, is the difference of $\Pi^{\omega}(0, n)$ and $\Pi^{\rho}(0, n)$.

The ρ and ω mass shifts are determined by the still unconstraint parameter κ_N. Once one of the vector mesons's peak position is experimentally determined the other one is fixed by the correlation displayed in Fig. 4, up to some uncertainty caused by the actual $\mathrm{Im}T_{VN}$ or the freedom in γ_V.

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[3] The detailed analysis of further terms in the OPE side of the sum rule will be relegated to a separate study.

A presently running experiment [7] measures the reaction $\gamma + A \rightarrow X + \omega$ with identifying the ω via the 3γ Dalitz decay. First data analysis [7] seems to exclude any up-shift, and even a weak down-shift of ω strength is compatible with the data. If this result gets confirmed it would require, within the present approach, that $\kappa_N > 3$, i.e., a strong density dependence of the 4-quark condensate. In other words, this would give the first direct evidence for a change of a condensate. Even a null effect for ω mesons decaying inside the target nucleus would require a strong density dependence of the 4-quark condensate, as evidenced by Fig. 4. In fact, at n_0 and for $\kappa_N = 3$ the 4-quark condensate drops to 60% of its vacuum value.

5. Conclusion

In summary, we present here an analysis of the QCD sum rule for the in-medium behavior of ρ and ω mesons. Truncating the "QCD side" of the sum rule at mass dimension 6 and twist 2 we find a strong sensitivity of the ρ and ω peak positions on the density dependence of the 4-quark condensate. Assuming that we have at our disposal a sufficiently realistic description of the collision broadening, described in low-density approximation by the off-shell forward meson–nucleon scattering amplitude $\mathrm{Im}T_{VN}$, the in-medium mass shifts of ρ and ω mesons are related essentially to one parameter. Experiments identifying the ω decay in nuclear matter then constrain this parameter, thus fixing also the in-medium ρ mass, up to uncertainties inherent in the sum rule approach. If an up-shift of ω strength can be experimentally excluded, the present QCD sum rule analysis points to a strong reduction of the 4-quark condensate already at nuclear saturation density.

On a quantitative level there is some uncertainty caused by the actual form of $\mathrm{Im}T_{VN}$, in particular for the ρ meson which influences the ρ–ω mass splitting. Poorly known twist-4 condensates modify further this splitting and deserve additional investigations. Given the importance of the 4-quark condensate, one could be afraid of the influence of higher order condensates. Here, the hope is that an appropriate Borel window suppresses these higher orders. We also found some quantitative changes when including explicitly $\mathrm{Re}T_{VN}$ in the spectral function. But the overall pattern of the in-medium modifications of ρ and ω mesons is stable.

In contrast to ρ and ω mesons, which are insensitive against changes of the genuine chiral condensate, $\langle \bar{q}q \rangle$, the ϕ meson depends sensitively on the chiral strange condensate, $\langle \bar{s}s \rangle$, and only very weakly on the 4-quark condensate.

With respect to the future accelerator project SIS200/300 at GSI an extension of the present approach to the in-medium behavior of D mesons is challenging.

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