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In-medium spectral change of ω mesons as a probe of QCD four-quark condensate

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Abstract

Within QCD sum rules at finite baryon density we show the crucial role of four-quark condensates, such as $\langle(\bar{q}\gamma_\mu\lambda^a q)^2\rangle_n$, for the in-medium modification of the ω meson spectral function. In particular, such a global property as the sign of the in-medium ω meson mass shift is found to be governed by a parameter which describes the strength of the density dependence of the four-quark condensate beyond mean-field approximation. To study self-consistently the broadening of the ω meson resonance we employ a hadron spectral function based on the ω meson propagator delivered by an effective chiral Lagrangian. Measurements of the ω meson spectral change in heavy-ion collisions with the HADES detector can reveal the yet unknown density dependence of the four-quark condensate.

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1. Introduction

The experiments with the detector system HADES [1] at the heavy-ion synchrotron SIS at GSI (Darmstadt) are mainly aimed at measuring in-medium modifications of the light vector mesons via the e^+e^- decay channel with high accuracy. At higher beam energies, experiments of the CERES Collaboration [2] at CERN SPS evidenced already hints to noticeable modifications of the dilepton spectrum which can be reproduced under the assumption of a strong melting

of the ρ meson in a dense, strongly interacting medium at temperature close to the chiral transition [3–5].

The great interest in studying properties of the light mesons in a hot/dense nuclear medium is caused by the expectation to find further evidences of the chiral symmetry restoration at finite temperature and baryon density. There are various theoretical indications concerning an important sensitivity of the meson spectral function to the partial restoration of the chiral symmetry in strongly interacting matter. In particular, at finite temperature the vector and axial-vector meson correlators become mixed in accordance with in-medium Weinberg sum rules [6,7]. Such a mixing causes an increasing degeneracy between vector and axial-vector

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spectral functions which would manifest themselves as a decrease of the ρ and A_1 meson mass splitting, for instance. Similarly, the degeneracy of scalar (σ channel) and pseudo-scalar (π channel) correlators found in lattice QCD [8] can lead to a considerable enhancement of the σ meson spectral function at finite temperature and density [9].

In spite of substantial efforts undertaken to understand the nature of vector mesons in a dense medium there is so far no unique and widely accepted quantitative picture of their in-medium behavior. The Brown and Rho conjecture [3] on the direct interlocking of vector meson masses and chiral quark condensate $\langle \bar{q}q \rangle_n$ supplemented by the vector manifestation of chiral symmetry in medium [10,11] predict a strong and quantitatively the same decrease of the in-medium ρ and ω meson masses.

At the same time, model calculations based on various effective Lagrangians (cf. [4]) predict rather moderate and different mass shifts for ρ and ω mesons in a dense medium. In order “to match” both sets of predictions one has to go beyond simplifications made in the above mentioned approaches: the in-medium vector meson mass shift is governed not only by $\langle \bar{q}q \rangle_n$ but also by condensates of higher order to be calculated beyond mean-field approximation. Further, the majority of effective Lagrangian models is dealing with the scattering amplitudes in free space, but effects related to the in-medium change of the QCD condensates should be included [12].

The very consistent way to incorporate in-medium QCD condensates is through QCD sum rules [13,14]. As pointed out in [15], the in-medium mass shift of the ρ and ω mesons is dominated by the dependence of the four-quark condensate on the density. In the present Letter we concentrate on the in-medium ω meson since the effect of the four-quark condensate is most pronounced for the isoscalar channel. Within the Borel QCD sum rule approach we go beyond the mean field approximation and use the linear density dependence of the four-quark condensate. We employ for the hadronic spectral function a constraint based on the general structure of the ω meson in-medium propagator with imaginary part of the self-energy delivered by an effective chiral Lagrangian. Our QCD sum rule evaluations show that the in-medium change of the four-quark condensate plays indeed a crucial role for modifications of the ω spectral function. In

particular, the sign of the ω meson mass shift is changed by variation of a parameter which describes the strength of the density dependence of the four-quark condensate. Since the difference of the vector and axial vector correlators is proportional to the four-quark condensate, the sign of the ω meson mass shift, measured via the e^+e^- channel, can serve as a tool for determining how fast the strongly interacting matter approaches the chiral symmetry restoration with increasing density.

2. QCD sum rule equations

Within QCD sum rules the in-medium ω meson is considered as a resonance in the current–current correlation function $\Pi_{\mu\nu}(q, n) = i \int d^4x e^{iqx} \langle T J_\mu(x) \times J_\nu(0) \rangle_n$, where $q_\mu = (E, \mathbf{q})$ is the ω meson four-momentum, T denotes the time ordered product of the ω meson current operators $J_\mu(x) J_\nu(0)$, and $\langle \dots \rangle_n$ stands for the expectation value in the medium. In what follows, we focus on the ground state of baryonic matter approximated by a Fermi gas with nucleon density n . In terms of quark field operators, the ω meson current is given by $J_\mu(x) = \frac{1}{2}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)$. At zero momentum, $\mathbf{q} = 0$, in the rest frame of the matter the correlator can be reduced to $\frac{1}{3}\Pi_\mu^\mu(q^2, n) = \Pi(q^2, n)$ for $q^2 < 0$. The correlator $\Pi(q^2, n)$ satisfies the twice subtracted dispersion relation, which can be written with $Q^2 \equiv -q^2 = -E^2$ as

$$\frac{\Pi(Q^2)}{Q^2} = \frac{\Pi(0)}{Q^2} - \Pi'(0) - Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}, \quad (1)$$

with $\Pi(0) = \Pi(q^2 = 0, n)$ and $\Pi'(0) = (d\Pi(q^2)/dq^2)|_{q^2=0}$ as subtraction constants and $R(s) = -\text{Im} \Pi(s, n)/(\pi s)$.

As usual in QCD sum rules (QSR) [13,14], for large values of Q^2 one can evaluate the l.h.s. of Eq. (1) by the operator product expansion (OPE) $\Pi(Q^2)/Q^2 = -c_0 \ln(Q^2) + \sum_{i=1}^\infty c_i/Q^{2i}$, where the coefficients c_i include the well-known Wilson coefficients and the expectation values of the corresponding products of quark and gluon field operators, i.e., condensates.

Performing a Borel transformation of the dispersion relation Eq. (1) with appropriate mass parameter M^2 and taking into account the OPE one gets the basic

QSR equation

$$\begin{aligned} \Pi(0) + \int_0^\infty ds R(s) e^{-s/M^2} \\ = M^2 c_0 + \sum_{i=1}^\infty \frac{c_i}{(i-1)! M^{2(i-1)}}. \end{aligned} \quad (2)$$

The general structure of the coefficients c_i up to $i = 3$ is given, for instance, in [16]. In order to calculate the density dependence of the condensates entering the coefficients $c_{0...3}$ we employ the standard linear density approximation, which is valid for not too large density. This gives for the chiral quark condensate $\langle \bar{q}q \rangle_n = \langle \bar{q}q \rangle_0 + \frac{\sigma_N}{2m_q} n$, where both the light quark masses and their condensates are taken to be the same, i.e., $m_q = m_u = m_d = 7$ MeV and $\langle \bar{q}q \rangle = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle$ with $\langle \bar{q}q \rangle_0 = -(245 \text{ MeV})^3$. The nucleon sigma term is $\sigma_N = 45$ MeV.

The gluon condensate is obtained as usual from QCD trace anomaly, $\langle \frac{\alpha_s}{\pi} G^2 \rangle_n = \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 - \frac{8}{9} M_N^0 n$, where $\alpha_s = 0.38$ is the QCD coupling constant and $M_N^0 = 770$ MeV is the nucleon mass in the chiral limit. The vacuum gluon condensate is $\langle \frac{\alpha_s}{\pi} G^2 \rangle_0 = (0.33 \text{ GeV})^4$.

The coefficient c_3 in Eq. (2) contains also the four-quark condensates

$$\langle (\bar{q}\gamma_\mu \lambda^a q)^2 \rangle_n \quad \text{and} \quad \langle (\bar{q}\gamma_\mu \gamma^5 \lambda^a q)^2 \rangle_n.$$

The standard approach to estimate their density dependence consists in the mean-field approximation. Within such an approximation the four-quark condensates are proportional to $\langle \bar{q}q \rangle_n^2$ and their density dependence is actually governed by the square of the chiral quark condensate. Keeping in mind the important role of the four-quark condensate for the in-medium modifications of the ω meson [15] we go beyond the above mean-field approximation and employ the following parameterization

$$\begin{aligned} \langle (\bar{q}\gamma_\mu \gamma^5 \lambda^a q)^2 \rangle_n \\ = -\langle (\bar{q}\gamma_\mu \lambda^a q)^2 \rangle_n \\ = \frac{16}{9} \langle \bar{q}q \rangle_0^2 \kappa_0 \left[1 + \frac{\kappa_N}{\kappa_0} \frac{\sigma_N}{m_q \langle \bar{q}q \rangle_0} n \right]. \end{aligned} \quad (3)$$

In vacuum, $n = 0$, the parameter κ_0 reflects a deviation from the vacuum saturation assumption. The case

$\kappa_0 = 1$ corresponds obviously to the exact vacuum saturation as used, for instance, in [16]. To control the deviation of the in-medium four-quark condensate from the mean-field approximation we introduce the parameter κ_N . The limit $\kappa_N = \kappa_0$ recovers the mean-field approximation, while the case $\kappa_N > \kappa_0$ ($\kappa_N < \kappa_0$) is related to a stronger (weaker) density dependence compared to the mean-field approximation. Below we vary the parameter κ_N to estimate the contribution of the four-quark condensates to the QSR with respect to the main trends of the in-medium modification of the ω meson spectral function.

Using the above condensates and usual Wilson coefficients we get the coefficients $c_{0...3}$ as

$$\begin{aligned} c_0 &= \frac{1}{8\pi^2} \left(1 + \frac{\alpha_s}{\pi} \right), \quad c_1 = -\frac{3m_q^2}{4\pi^2}, \\ c_2 &= m_q \langle \bar{q}q \rangle_0 + \frac{\sigma_N}{2} n + \frac{1}{24} \left[\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 - \frac{8}{9} M_N^0 n \right] \\ &\quad + \frac{1}{4} A_2 M_N n, \\ c_3 &= -\frac{112}{81} \pi \alpha_s \kappa_0 \langle \bar{q}q \rangle_0^2 \left[1 + \frac{\kappa_N}{\kappa_0} \frac{\sigma_N}{m_q \langle \bar{q}q \rangle_0} n \right] \\ &\quad - \frac{5}{12} A_4 M_N^3 n. \end{aligned} \quad (4)$$

The last terms in $c_{2,3}$ correspond to the derivative condensates from nonscalar operators as a consequence of the breaking of Lorentz invariance in the medium. These condensates are proportional to the moments $A_i = 2 \int_0^1 dx x^{i-1} [q_N(x, \mu^2) + \bar{q}_N(x, \mu^2)]$ of quark and antiquark distributions q_N, \bar{q}_N inside the nucleon at a scale $\mu^2 = 1$ GeV² (see for details [14]). Our choice of the moments A_2 and A_4 is 1.02 and 0.12, respectively.

To model the hadronic side of the QSR Eq. (2) we make the standard separation of the ω meson spectral density into resonance part and continuum contribution by means of the threshold parameter s_0 :

$$R(s, n) = F \frac{S(s, n)}{s} \Theta(s_0 - s) + c_0 \Theta(s - s_0), \quad (5)$$

where $S(s, n)$ stands for the resonance peak in the spectral function; the normalization F is unimportant for the following consideration. In the majority of the QCD sum rule evaluations, the zero-width approximation [14] or some schematic parameterization of S [17] are employed. In contrast to this, we use here a more

realistic ansatz for the resonance spectral density S based on the general structure of the in-medium vector meson propagator in the vicinity to the pole mass,

$$S(s, n) = -\frac{\text{Im } \Sigma(s, n)}{(s - m_\omega^2(n))^2 + (\text{Im } \Sigma(s, n))^2} \quad (6)$$

with $\text{Im } \Sigma(s, n)$ as imaginary part of the in-medium ω meson self-energy and $m_\omega(n)$ as its physical mass. In Eq. (6), the real part of the self-energy is absorbed in $m_\omega(n)$, which is determined by the QCD sum rule Eq. (2). As a result (see below), the in-medium change of the QCD condensates causes global modifications of the ω meson spectral function, in addition to the collision broadening. (An analogous approach was used in [18].)

Within the linear density approximation the ω meson self-energy is given by

$$\Sigma(E, n) = \Sigma^{\text{vac}}(E) - nT^{\omega N}(E), \quad (7)$$

where $E = \sqrt{s}$ is the ω meson energy, $\Sigma^{\text{vac}}(E) = \Sigma(E, n = 0)$ and $T^{\omega N}(E)$ is the off-shell forward ω -nucleon scattering amplitude in free space. Note, that the three-momentum of ω mesons in Eq. (7) is still zero, and nucleons are also assumed to be at rest. We take the needed $\text{Im } T^{\omega N}(E)$ from an effective Lagrangian [19], which is based on vector meson dominance and chiral SU(3) dynamics. The dominant contribution to $\text{Im } T^{\omega N}(E)$ in the region $E \lesssim 1$ GeV comes from the processes $\omega N \rightarrow \pi N$ (at $E < 0.6$ GeV) and $\omega N \rightarrow \rho N \rightarrow \pi \pi N$ (at $E > 0.6$ GeV), see [19] for details. The process $\omega N \rightarrow \pi N$ is at least partially under experimental control, while the contribution from the reaction $\omega N \rightarrow \rho N \rightarrow \pi \pi N$ appears to be sensitive to the poorly known meson–baryon form factors. However, this uncertainty does not spoil the results below. For definiteness, in Fig. 1 we plot $\text{Im } T^{\omega N}(E)$ employed in our QSR evaluations. To simplify the calculations we also take $\text{Im } \Sigma^{\text{vac}}(E) \approx \text{Im } \Sigma^{\text{vac}}(E = m_\omega^{\text{vac}}) = -m_\omega^{\text{vac}} \Gamma_\omega^{\text{vac}}$, where m_ω^{vac} and $\Gamma_\omega^{\text{vac}}$ are the vacuum ω meson mass and decay width, respectively.

Following [19] we use for the subtraction constant $\Pi(0) = 9n/(4M_N)$ which is actually the Thomson limit of the ωN scattering process, but also coincides with the Landau damping term elaborated in [20] for the hadronic spectral function entering the dispersion relation without subtractions. For details about the re-

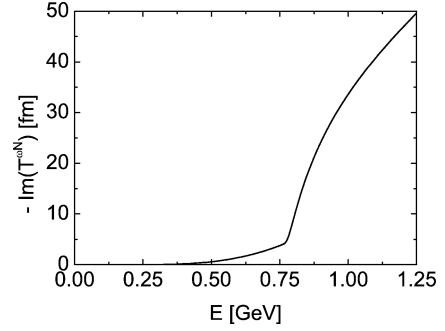


Fig. 1. Imaginary part of the off-shell ωN forward scattering amplitude.

lation of subtraction constants and the Landau damping we refer the interested reader to [21].

3. Results of QCD sum rule evaluation

The ratio of Eq. (2) and its derivative with respect to M^2 yields with Eq. (5)

$$\begin{aligned} & \frac{\int_0^{s_0} ds S(s, n) e^{-s/M^2}}{\int_0^{s_0} ds S(s, n) s^{-1} e^{-s/M^2}} \\ &= \left\{ c_0 M^2 \left[1 - (1 + s_0 M^{-2}) e^{-s_0/M^2} \right] \right. \\ & \quad \left. - c_2 M^{-2} - c_3 M^{-4} \right\} \\ & \quad \times \left\{ c_0 (1 - e^{-s_0/M^2}) + c_1 M^{-2} + c_2 M^{-4} \right. \\ & \quad \left. + \frac{1}{2} c_3 M^{-6} - \frac{9n}{4M_N} M^{-2} \right\}^{-1} \quad (8) \end{aligned}$$

with the coefficients $c_{0..3}$ from Eq. (4) and the spectral function $S(s, n)$ from Eqs. (6) and (7). One has to solve Eq. (8) to find the mass parameter $m_\omega(n, M^2, s_0)$. At a given density n , the continuum threshold s_0 is determined by requiring maximum flatness of $m_\omega(M^2)$ within the Borel window $M_{\min} \cdots M_{\max}$. The minimum Borel mass M_{\min} is obtained such that the terms of order $\mathcal{O}(M^{-6})$ on the OPE side contribute not more than 10% [17,22]. According to our experience [15], $m_\omega(M^2)$ is not very sensitive to variations of M_{\max} . We therefore fix the maximum Borel mass by $M_{\max}^2 = 2 \text{ GeV}^2$. To get the ω meson mass m_ω we average finally $m_\omega(M^2)$ within the Borel mass window.

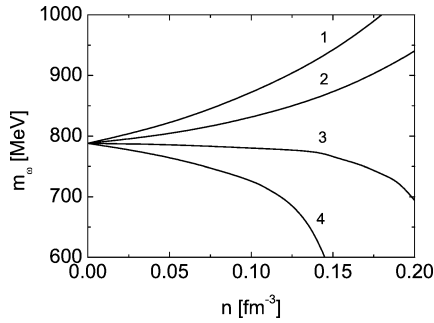


Fig. 2. Density dependence of the ω meson mass for various values of the parameter κ_N .

The value of κ_0 in Eq. (3) is related to such a choice of the chiral condensate $\langle \bar{q}q \rangle_0$ which adjusts the vacuum ω meson mass to $m_\omega(n=0) = 782$ MeV resulting in $\kappa_0 = 3$. The ratio κ_N/κ_0 in the parameterization (3) is restricted by the conditions $q_4 = \langle (\bar{q}\gamma_\mu\lambda^a q)^2 \rangle_n \rightarrow 0$ with increasing density and $q_4 \leq 0$, so that one gets $0 \leq \kappa_N \lesssim 4$ as numerical limits.

The results of our QSR evaluations for $m_\omega(n)$ for $\kappa_N = 1 \dots 4$ are exhibited in Fig. 2. The in-medium mass of the ω meson changes even qualitatively under variation of the parameter κ_N : for $\kappa_N < 2.7$, m_ω increases with density, while for $\kappa_N > 2.7$ it drops. From the above considerations one can conclude that the sign of the in-medium mass shift is directly related to the density dependence of the four-quark condensate.

In Fig. 3 we display the in-medium change of the ω meson spectral function $S(E)$, given by Eq. (6) and calculated with density dependent mass $m_\omega(n)$. The in-medium spectral change is still seen to be dominated by the density dependence of the four-quark condensate. The dependence of the peak position as a function of the density n and the four-quark parameter κ_N is the same as for m_ω . One can also observe that for the positive mass shift (say, for $\kappa_N = 2$) the width increases with density, while for a negative mass shift (say, for $\kappa_N = 3$) it appears to be approximately constant. In both cases we obtain a considerable “melting” of the in-medium ω meson: at $n = n_0$ the height of the resonance peak drops more than by a factor of 5. For $\kappa_N = 2.5$ the shift of the peak is small; also here the peak is broadened.

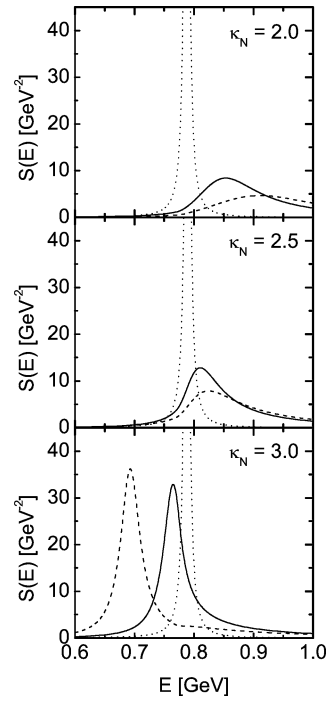


Fig. 3. ω meson spectral function for $\kappa_N = 2, 2.5, 3$. Solid curves correspond to normal nuclear density ($n = n_0 = 0.15 \text{ fm}^{-3}$), while dotted and dashed curves are for vacuum and $n = 0.2 \text{ fm}^{-3}$, respectively.

Because of the strong “melting” of the in-medium ω meson, an identification of its spectral change in matter will be an experimental challenge. Moreover, in heavy-ion collisions one can expect also an additional broadening of the signal due to the collective expansion of the matter [23]. Nevertheless, the high precision measurements planned with HADES give a chance to observe at least an “in-medium shoulder” which supplements the vacuum peak. Whether such a shoulder will occur at the right- or left-hand side of the vacuum peak is directly governed by the strength of the density dependence of the four-quark condensate.

4. Summary

In summary we have found that, within the Borel QCD sum rule approach, the in-medium spectral change of the ω meson is dominated by the density dependence of the four-quark condensate. We go

beyond the standard mean-field approximation and vary with the parameter κ_N the strength of the four-quark condensate dependence on the density. The sign of the ω meson in-medium mass shift and the resonance peak position are shown to be governed by the value of κ_N . We find a strong “melting effect” of the in-medium ω meson resonance. Only for positive mass shift we observe a considerable broadening of the in-medium ω meson spectral function.

The in-medium ω meson spectral change, in particular the sign of the mass shift, to be looked for via the e^+e^- channel with the HADES detector in heavy-ion collisions, can give an important information on the yet unknown density dependence of the four-quark condensate and consequently on the chiral symmetry restoration in a dense nuclear medium.

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