Parametrized post-post-Newtonian analytical solution for light propagation

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An analytical solution for light propagation in the post-post-Newtonian approximation is given for the Schwarzschild metric in harmonic gauge augmented by PPN and post-linear parameters β, γ and ϵ. The solutions of both Cauchy and boundary problem are given. The Cauchy problem is posed using the initial position of the photon $\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$ and its propagation direction $\boldsymbol{\sigma}$ at minus infinity: $\boldsymbol{\sigma} = \frac{1}{c} \lim\limits_{t\to-\infty} \dot{\boldsymbol{x}}(t)$. An analytical expression for the total light deflection is given. The solutions for $t-t_0$ and $\boldsymbol{\sigma}$ are given in terms of boundary conditions $\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$ and $\boldsymbol{x} = \boldsymbol{x}(t)$.

Contents

I. INTRODUCTION

The goal of this note is to derive a rigorous analytical solution for light propagation in the gravitational field of one spherically symmetric body in the framework of parametrized post-Newtonian (PPN) formalism extended by a non-linear parameter for the component of order c^{-4} in g_{ij}. Our interest to the analytical post-post-Newtonian solution for light propagation is caused by the inability of the post-Newtonian solution to predict the light deflection with an accuracy of 1 μas for solar system objects observed close to Jupiter and Saturn [1]. The maximal errors of the standard post-Newtonian solution obtained by a comparison of that solution with numerical integrations of the geodetic equations for light propagation may in some cases attain 16 μas (the details of these deviations will be discussed in a subsequent report). Because of this drawback of the post-Newtonian solution the current implementation [1, 2] of the Gaia baseline model GREM [3] uses a kind of time-consuming numerical inversion procedure to compute the light deflection effect for solar system bodies if the accuracy below ~ 20 μas is requested. Although it does not seem to be a problem for the practical use of the model, a fully analytical solution for light propagation is certainly desirable.

The geodetic equation for the light ray in Schwarzschild metric can in principle be integrated exactly [4]. However, such an analytical solution is given in terms of elliptic integrals and is not very suitable for massive calculations. Besides that, only the trajectory of the photon is readily available from the literature, but not the position and velocity of a photon as functions of time. Fortunately, in many cases of interest approximate solutions are sufficient. The standard way to solve the geodetic equation is the well-known post-Newtonian approximation scheme. Normally, in current practical applications of light propagation in the relativistic models the first post-Newtonian solution is used. Post-post-Newtonian effects have been also sometimes considered [5, 6], but in a way which cannot be called self-consistent since no rigorous solution in the post-post-Newtonian approximation has been used. Such a rigorous post-post-Newtonian analytical solution for light propagation in the Schwarzschild metric of general relativity has been derived by Brumberg [7, 8]. Although the author has considered the solution in a class of gauges introducing gauge parameters, this parametrization does not cover alternative theories of gravity and therefore, a post-post-Newtonian solution for light propagation within the PPN formalism and its extension to the second post-Newtonian approximation is not known. However, it is clearly advantageous to have such a solution within the PPN formalism. The goal of this note is to repeat the post-post-Newtonian solution of Brumberg [7] and to extend it for the boundary problem.

II. DIFFERENTIAL EQUATIONS OF LIGHT PROPAGATIONS AND THEIR INTEGRAL

The purpose of this Section is to derive the differential equations of light propagations with PPN and post-linear parameters. We also find an integral of these equations from the condition that the solution should represent an isotropic geodetic.

A. Metric tensor

It is well known that in harmonic gauge

$$\frac{\partial\left(\sqrt{-g}\,g^{\alpha\beta}\right)}{\partial x^\beta} = 0 \tag{1}$$

the components of the covariant metric tensor of the Schwarzschild solution are given by

$$\begin{aligned} g_{00} &= -\frac{1-a}{1+a}, \\ g_{0i} &= 0, \\ g_{ij} &= (1+a)^2\;\delta_{ij}\;+\;\frac{a^2}{x^2}\,\frac{1+a}{1-a}\,x^i\,x^j \end{aligned} \tag{2}$$

with

$$a = \frac{m}{x}, \tag{3}$$

where $m = \frac{GM}{c^2}$ is the Schwarzschild radius of a body with mass M, G is the Newtonian gravitational constant, c is the light velocity, and $x = |\boldsymbol{x}| = \sqrt{\delta_{ij}\,x^i\,x^j} = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$ is the Euclidean absolute value of vector $\boldsymbol{x}$. Expanding this metric in powers of c^{-1}, retaining only the terms relevant for the post-post-Newtonian solution for the light propagation, and introducing the PPN parameters β and γ [9] and the post-linear parameter ϵ one gets

$$\begin{aligned} g_{00} &= -1 + 2\,a - 2\,\beta\,a^2 + \mathcal{O}(c^{-6}), \\ g_{0i} &= 0, \\ g_{ij} &= \delta_{ij} + 2\,\gamma\,a\,\delta_{ij} + \epsilon\,\left(\delta_{ij} + \frac{x^i\,x^j}{x^2}\right)\,a^2 + \mathcal{O}(c^{-6}). \end{aligned} \tag{4}$$

In general relativity one has $\beta = \gamma = \epsilon = 1$. Parameter ϵ should be considered as a formal way to trace, in the following calculations, the terms coming from the terms c^{-4} in g_{ij}. No physical meaning of ϵ is claimed in this report. However, this parameter is equivalent to parameter Λ of Richter & Matzner [10, 11, 12] and parameter ϵ of Epstein & Shapiro [13].

The corresponding contravariant components of metric tensor can be deduced from (4) and are given by

$$\begin{aligned} g^{00} &= -1 - 2\,a + 2\,(\beta - 2)\,a^2 + \mathcal{O}(c^{-6}), \\ g^{0i} &= 0, \\ g^{ij} &= \delta_{ij} - 2\,\gamma\,a\,\delta_{ij} + \left((4\gamma^2 - \epsilon)\,\delta_{ij} - \epsilon\,\frac{x^i\,x^j}{x^2}\right)\,a^2 + \mathcal{O}(c^{-6}). \end{aligned} \tag{5}$$

The determinant of metric tensor reads

$$g = -1 - 2\,(3\,\gamma - 1)\,a - 2\,(\beta + 2\,\epsilon + 6\,\gamma\,(\gamma - 1))\,a^2 + \mathcal{O}(c^{-6}), \tag{6}$$

$$\sqrt{-g} = 1 + (3\,\gamma - 1)\,a + (2\,\beta + 4\,\epsilon - 1 + 3\,\gamma\,(\gamma - 2))\,a^2 + \mathcal{O}(c^{-6}). \tag{7}$$

Metric (4) is obviously harmonic for $\gamma=\beta=\epsilon=1$ since the harmonic conditions (1) take the form

$$\left(\sqrt{-g}\,g^{0\alpha}\right)_{,\,\alpha} = 0,$$
$$\left(\sqrt{-g}\,g^{i\alpha}\right)_{,\,\alpha} = (1-\gamma)\,\frac{a\,x^i}{x^2} + ((1+\gamma)^2 - 2\beta - 2\epsilon)\,\frac{a^2\,x^i}{x^2} + \mathcal{O}(c^{-6}). \tag{8}$$

B. Christoffel symbols

The Christoffel symbols of second kind, defined as

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}\,g^{\mu\nu}\left(\frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} + \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right), \tag{9}$$

can be derived from metric (4)–(5):

$$\Gamma^0{}_{00} = 0, \tag{10}$$
$$\Gamma^0{}_{0i} = \frac{a\,x^i}{x^2} + (1-\beta)\,\frac{2\,a^2\,x^i}{x^2} + \mathcal{O}(c^{-6}), \tag{11}$$
$$\Gamma^0{}_{ik} = 0, \tag{12}$$
$$\Gamma^i{}_{00} = \frac{a\,x^i}{x^2} - (\beta+\gamma)\,\frac{2\,a^2\,x^i}{x^2} + \mathcal{O}(c^{-6}), \tag{13}$$
$$\Gamma^i{}_{0k} = 0, \tag{14}$$
$$\begin{aligned}\Gamma^i{}_{kl} = {} & \gamma\,\left(x^i\,\delta_{kl} - x^k\,\delta_{il} - x^l\,\delta_{ik}\right)\,\frac{a}{x^2} \\ & + \left(2\,(\epsilon-\gamma^2)\,x^i\,\delta_{kl} - (\epsilon - 2\,\gamma^2)\,\left(x^k\,\delta_{il} + x^l\,\delta_{ik}\right) - 2\,\epsilon\,\frac{x^i\,x^k\,x^l}{x^2}\right)\,\frac{a^2}{x^2} + \mathcal{O}(c^{-6}).\end{aligned} \tag{15}$$

C. Isotropic condition for the null geodetic

From now on x^{α} denote the coordinates of a photon, x^i denote the spatial coordinates of the photon, and $x=|\boldsymbol{x}|$ is the distance of the photon from the gravitating body that is situated at the origin of the used reference system. The conditions that a photon follows an isotropic geodetic can be formulated as an equation for the four components of the coordinate velocity of that photon $\dot{x}^{\alpha}$:

$$g_{\alpha\beta}\,\frac{d\,x^{\alpha}}{d\,\lambda}\,\frac{d\,x^{\beta}}{d\,\lambda} = 0, \tag{16}$$

λ being the canonical parameter, or

$$g_{00} + \frac{2}{c}\,g_{0i}\,\dot{x}^i + \frac{1}{c^2}\,g_{ij}\,\dot{x}^i\,\dot{x}^j = 0\,, \tag{17}$$

where a dot denotes the derivative with respect to coordinate time t. Eq. (17) is a first integral of motion for the differential equation for light propagation and must be valid for any point of an isotropic geodetic. Substituting the ansatz $\dot{\boldsymbol{x}} = c\, s\, \boldsymbol{\mu}$, where $\boldsymbol{\mu}$ is a unit coordinate direction of light propagation ($\boldsymbol{\mu} \cdot \boldsymbol{\mu} = 1$) and $s = |\dot{\boldsymbol{x}}|/c$, into Eq. (17) one gets for metric (4)

$$s = 1 - (1+\gamma)\, a + \frac{1}{2}\left(-1 + 2\,\beta - \epsilon + \gamma\,(2+3\gamma) - \epsilon\left(\frac{\boldsymbol{\mu}\cdot\boldsymbol{x}}{x}\right)^2\right) a^2 + \mathcal{O}(c^{-6})\,. \qquad (18)$$

This formula allows one to compute the absolute value of coordinate velocity of light in the chosen reference system if the position of the photon x^i and the coordinate direction of its propagation μ^i are given.

D. Equations of light propagation

The geodetic equation is given by

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}\,\frac{dx^\alpha}{d\lambda}\,\frac{dx^\beta}{d\lambda} = 0\,. \qquad (19)$$

Using the relation of the affine parameter λ and the coordinate time $t = x^0/c$

$$\frac{d^2 t}{d\lambda^2} + \frac{1}{c}\,\Gamma^0_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\,\frac{dx^\beta}{d\lambda} = 0 \qquad (20)$$

one can reparametrize the geodetic equation by coordinate time t,

$$\ddot{x}^i = -c^2\,\Gamma^i{}_{00} - 2\,c\,\Gamma^i{}_{0j}\,\dot{x}^j - \Gamma^i_{jk}\,\dot{x}^j\,\dot{x}^k + \dot{x}^i\left(c\,\Gamma^0{}_{00} + 2\,\Gamma^0{}_{0j}\,\dot{x}^j + \frac{1}{c}\,\Gamma^0_{jk}\,\dot{x}^j\,\dot{x}^k\right)\,. \qquad (21)$$

Inserting the Christoffel symbols (10)–(15), one gets the following equations of light propagation in post-post-Newtonian approximation,

$$\begin{aligned}\ddot{\boldsymbol{x}} = &-\left(c^2 + \gamma\,\dot{\boldsymbol{x}}\cdot\dot{\boldsymbol{x}}\right)\,\frac{a\,\boldsymbol{x}}{x^2} + 2\,(1+\gamma)\,\frac{a\,\dot{\boldsymbol{x}}\,(\dot{\boldsymbol{x}}\cdot\boldsymbol{x})}{x^2}\\ &+2\,\left((\beta+\gamma)\,c^2 + (\gamma^2-\epsilon)\,(\dot{\boldsymbol{x}}\cdot\dot{\boldsymbol{x}})\right)\,\frac{a^2\,\boldsymbol{x}}{x^2} + 2\,\epsilon\,\frac{a^2\,\boldsymbol{x}\,(\dot{\boldsymbol{x}}\cdot\boldsymbol{x})^2}{x^4}\\ &+2\,(2(1-\beta)+\epsilon-2\,\gamma^2)\,\frac{a^2\,\dot{\boldsymbol{x}}\,(\dot{\boldsymbol{x}}\cdot\boldsymbol{x})}{x^2} + \mathcal{O}(c^{-4})\,.\end{aligned} \qquad (22)$$

Here, for estimating the analytical order of smallness of the terms we take into account that $|\dot{\boldsymbol{x}}| = \mathcal{O}(c)$. Using (18) and $\dot{\boldsymbol{x}}\cdot\dot{\boldsymbol{x}} = c^2\,s^2$ one can simplify (22) to get

$$\begin{aligned}\ddot{\boldsymbol{x}} = &-(1+\gamma)\,c^2\,\frac{a\,\boldsymbol{x}}{x^2} + 2\,(1+\gamma)\,\frac{a\,\dot{\boldsymbol{x}}\,(\dot{\boldsymbol{x}}\cdot\boldsymbol{x})}{x^2}\\ &+2\,c^2\,\left(\beta-\epsilon+2\,\gamma\,(1+\gamma)\right)\,\frac{a^2\,\boldsymbol{x}}{x^2} + 2\,\epsilon\,\frac{a^2\,\boldsymbol{x}\,(\dot{\boldsymbol{x}}\cdot\boldsymbol{x})^2}{x^4}\\ &+2\,(2(1-\beta)+\epsilon-2\,\gamma^2)\,\frac{a^2\,\dot{\boldsymbol{x}}\,(\dot{\boldsymbol{x}}\cdot\boldsymbol{x})}{x^2} + \mathcal{O}(c^{-4})\,.\end{aligned} \qquad (23)$$

E. Equations of light propagation with additional empirical parameter α

For our purposes it is advantageous to have one more additional parameter that can be used to trace terms in the folowing calculations which come from the post-post-Newtonian terms in the equations of motion of a photon. We denote this parameter by α and introduce it in the above equation simply as a factor for all the post-post-Newtonian terms:

$$\begin{aligned}
\ddot{\boldsymbol{x}} = & -(1+\gamma)\, c^2\, \frac{a\, \boldsymbol{x}}{x^2} + 2\,(1+\gamma)\, \frac{a\, \dot{\boldsymbol{x}}\, (\dot{\boldsymbol{x}} \cdot \boldsymbol{x})}{x^2} \\
& + 2\, c^2\, \alpha\, \left(\beta - \epsilon + 2\, \gamma\, (1+\gamma)\right)\, \frac{a^2\, \boldsymbol{x}}{x^2} + 2\, \alpha\, \epsilon\, \frac{a^2\, \boldsymbol{x}\, (\dot{\boldsymbol{x}} \cdot \boldsymbol{x})^2}{x^4} \\
& + 2\, \alpha\, (2(1-\beta) + \epsilon - 2\, \gamma^2)\, \frac{a^2\, \dot{\boldsymbol{x}}\, (\dot{\boldsymbol{x}} \cdot \boldsymbol{x})}{x^2} + \mathcal{O}(c^{-4})\,. \qquad (24)
\end{aligned}$$

Setting $\alpha = 0$ in the solution of Eq. (24) one can get formally a second-order solution for the post-Newtonian equations of light propagation.

III. INITIAL VALUE PROBLEM

Let us now solve analytically an initial value problem for the derived equations.

A. Analytical post-post-Newtonian solution

For two initial conditions

$$\begin{aligned}
\boldsymbol{x}_0 &= \boldsymbol{x}(t_0)\,, \\
\boldsymbol{\sigma} &= \lim_{t \to -\infty} \frac{\dot{\boldsymbol{x}}(t)}{c}\,, \qquad (25)
\end{aligned}$$

using the same approach as was used by Brumberg [7], one gets a formal solution $\boldsymbol{x}(t)$ and $\dot{\boldsymbol{x}}(t)$ of Eq. (24):

$$\frac{1}{c}\, \dot{\boldsymbol{x}}_N = \boldsymbol{\sigma}, \qquad (26)$$

$$\boldsymbol{x}_N = \boldsymbol{x}_0 + c\,(t-t_0)\, \boldsymbol{\sigma}, \qquad (27)$$

$$\frac{1}{c}\, \dot{\boldsymbol{x}}_{\rm pN} = \boldsymbol{\sigma} + m\, \boldsymbol{A}_1(\boldsymbol{x}_N), \qquad (28)$$

$$\boldsymbol{x}_{\rm pN} = \boldsymbol{x}_N + m\, \left(\boldsymbol{B}_1(\boldsymbol{x}_N) - \boldsymbol{B}_1(\boldsymbol{x}_0)\right), \qquad (29)$$

$$\frac{1}{c}\, \dot{\boldsymbol{x}}_{\rm ppN} = \boldsymbol{\sigma} + m\, \boldsymbol{A}_1(\boldsymbol{x}_{\rm pN}) + m^2\, \boldsymbol{A}_2(\boldsymbol{x}_N), \qquad (30)$$

$$\boldsymbol{x}_{\rm ppN} = \boldsymbol{x}_N + m\, \left(\boldsymbol{B}_1(\boldsymbol{x}_{\rm pN}) - \boldsymbol{B}_1(\boldsymbol{x}_0)\right) + m^2\, \left(\boldsymbol{B}_2(\boldsymbol{x}_N) - \boldsymbol{B}_2(\boldsymbol{x}_0)\right), \qquad (31)$$

with

$$\boldsymbol{A}_1(\boldsymbol{x}) = -(1+\gamma)\, \left(\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x(x - \boldsymbol{\sigma} \cdot \boldsymbol{x})} + \frac{\boldsymbol{\sigma}}{x}\right), \qquad (32)$$

$$\boldsymbol{B}_1(\boldsymbol{x}) = -(1+\gamma)\, \left(\frac{\boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}{x - \boldsymbol{\sigma} \cdot \boldsymbol{x}} + \boldsymbol{\sigma}\, \log\left(x + \boldsymbol{\sigma} \cdot \boldsymbol{x}\right)\right), \qquad (33)$$

$$
\begin{aligned}
\boldsymbol{A}_2(\boldsymbol{x}) = & -\frac{1}{2}\,\alpha\,\epsilon\,\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x^4}\,\boldsymbol{x} + 2\,(1+\gamma)^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x^2\;(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})} + (1+\gamma)^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x\,(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})^2} \\
& -(1+\gamma)^2\,\frac{\boldsymbol{\sigma}}{x\;(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})} + \left(2(1-\alpha+\gamma)\,(1+\gamma) + \alpha\,\beta - \frac{1}{2}\,\alpha\,\epsilon\right)\,\frac{\boldsymbol{\sigma}}{x^2} \\
& -\frac{1}{4}\,\left(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma) - 4\,\alpha\,\beta + 3\,\alpha\,\epsilon\right)\,(\boldsymbol{\sigma}\cdot\boldsymbol{x})\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x^2\,|\boldsymbol{\sigma}\times\boldsymbol{x}|^2} \\
& -\frac{1}{4}\,\left(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma) - 4\,\alpha\,\beta + 3\,\alpha\,\epsilon\right)\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{|\boldsymbol{\sigma}\times\boldsymbol{x}|^3}\;\times \\
& \times\left(\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|} + \frac{\pi}{2}\right), \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{B}_2(\boldsymbol{x}) = & -(1+\gamma)^2\,\frac{\boldsymbol{\sigma}}{x-\boldsymbol{\sigma}\cdot\boldsymbol{x}} + (1+\gamma)^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})^2} + \frac{1}{4}\,\alpha\,\epsilon\,\frac{\boldsymbol{x}}{x^2} \\
& -\frac{1}{4}\,\alpha\,\left(8\,(1+\gamma) - 4\,\beta + 3\,\epsilon\right)\,\frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|}\,\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|} \\
& -\frac{1}{4}\,\left(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma) - 4\,\alpha\,\beta + 3\,\alpha\,\epsilon\right)\,(\boldsymbol{\sigma}\cdot\boldsymbol{x})\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{|\boldsymbol{\sigma}\times\boldsymbol{x}|^3}\;\times \\
& \times\left(\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|} + \frac{\pi}{2}\right), \qquad (35)
\end{aligned}
$$

or, alternatively, for $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$

$$
\boldsymbol{B}_1(\boldsymbol{x}) = -(1+\gamma)\,\left(\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x-\boldsymbol{\sigma}\cdot\boldsymbol{x}} - \boldsymbol{\sigma}\,\log\,(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})\right), \qquad (36)
$$

$$
\begin{aligned}
\boldsymbol{B}_2(\boldsymbol{x}) = & +(1+\gamma)^2\,\frac{\boldsymbol{\sigma}}{x-\boldsymbol{\sigma}\cdot\boldsymbol{x}} + (1+\gamma)^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})^2} + \frac{1}{4}\,\alpha\,\epsilon\,\frac{\boldsymbol{x}}{x^2} \\
& -\frac{1}{4}\,\alpha\,\left(8\,(1+\gamma) - 4\,\beta + 3\,\epsilon\right)\,\frac{\boldsymbol{\sigma}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|}\,\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|} \\
& -\frac{1}{4}\,\left(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma) - 4\,\alpha\,\beta + 3\,\alpha\,\epsilon\right)\,(\boldsymbol{\sigma}\cdot\boldsymbol{x})\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{|\boldsymbol{\sigma}\times\boldsymbol{x}|^3}\;\times \\
& \times\left(\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|} + \frac{\pi}{2}\right). \qquad (37)
\end{aligned}
$$

With these definitions the solution of (24) reads

$$
\begin{aligned}
\boldsymbol{x} &= \boldsymbol{x}_{\rm ppN} + \mathcal{O}(c^{-6}), \\
\frac{1}{c}\dot{\boldsymbol{x}} &= \frac{1}{c}\dot{\boldsymbol{x}}_{\rm ppN} + \mathcal{O}(c^{-6}).
\end{aligned} \qquad (38)
$$

It is easy to check that the solution for coordinate velocity of light $\dot{\boldsymbol{x}}_{\rm ppN}$ satisfies the integral (18). In order to demonstrate this fact, it is important to understand that position $\boldsymbol{x}$ in (18) lies on the trajectory of the photon and must be therefore considered as $\boldsymbol{x}_{\rm pN}$ in the post-Newtonian terms and as $\boldsymbol{x}_{\rm N}$ in the post-post-Newtonian terms of (30).

B. Total light deflection

In order to derive the total light deflection, we have to consider the limits of coordinate light velocity $\dot{\boldsymbol{x}}$ for $t \to \pm\infty$:

$$\lim_{t\to-\infty} \frac{1}{c}\,\dot{\boldsymbol{x}}(t) = \boldsymbol{\sigma}, \tag{39}$$

$$\begin{aligned}
\lim_{t\to+\infty} \frac{1}{c}\,\dot{\boldsymbol{x}}(t) \equiv \boldsymbol{\nu} \\
&= \boldsymbol{\sigma} - 2\,(1+\gamma)\,m\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}_0\times\boldsymbol{\sigma})}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|^2} - 2\,(1+\gamma)^2\,m^2\,\frac{\boldsymbol{\sigma}}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|^2} \\
&\quad -\frac{1}{4}\,\pi\,\left(8(1+\gamma-\alpha\,\gamma)(1+\gamma) - 4\alpha\,\beta + 3\alpha\,\epsilon\right)\,m^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}_0\times\boldsymbol{\sigma})}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|^3} \\
&\quad +2\,(1+\gamma)^2\,m^2\,\left(x_0 + \boldsymbol{\sigma}\cdot\boldsymbol{x}_0\right)\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}_0\times\boldsymbol{\sigma})}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|^4} + \mathcal{O}(c^{-6}).
\end{aligned} \tag{40}$$

Accordingly, the total light deflection reads

$$\begin{aligned}
|\boldsymbol{\sigma}\times\boldsymbol{\nu}| &= 2\,(1+\gamma)\,m\,\frac{1}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|} - 2\,(1+\gamma)^2\,m^2\,(x_0 + \boldsymbol{\sigma}\cdot\boldsymbol{x}_0)\,\frac{1}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|^3} \\
&\quad +\frac{1}{4}\,\left(8(1+\gamma-\alpha\,\gamma)(1+\gamma) - 4\,\alpha\,\beta + 3\alpha\,\epsilon\right)\,\pi\,m^2\,\frac{1}{|\boldsymbol{x}_0\times\boldsymbol{\sigma}|^2} + \mathcal{O}(c^{-6})\,.
\end{aligned} \tag{41}$$

Eq. (41) defines the sine of the angle of total light deflection in post-post-Newtonian approximation. The first term in (41) is the post-Newtonian expression of total light deflection $\sim 4m/d$, where $d = |\boldsymbol{x}_0\times\boldsymbol{\sigma}|$ being the impact parameter for the light trajectory. The other two terms are the post-post-Newtonian corrections. Note that although the total light deflection $|\boldsymbol{\sigma}\times\boldsymbol{\nu}|$ is a coordinate-independent quantity, $\boldsymbol{x}_0$ and, therefore, the impact parameter $d = |\boldsymbol{x}_0\times\boldsymbol{\sigma}|$ are coordinate-dependent. In order to compare Eq. (41) with results expressed through the coordinate-independent impact parameter d' one has to find a transformation between that impact parameter and d. The coordinate-independent impact parameter is defined as a limit

$$d' = \lim_{t\to-\infty} |\boldsymbol{x}(t)\times\boldsymbol{\sigma}| = \lim_{t\to+\infty} |\boldsymbol{x}(t)\times\boldsymbol{\nu}|\,. \tag{42}$$

Substituting the post-Newtonian coordinates (29) of the photon $\boldsymbol{x}(t)$ into this definition one gets

$$d' = d + (1+\gamma)\,m\,\frac{x_0 + \boldsymbol{\sigma}\cdot\boldsymbol{x}_0}{d} + \mathcal{O}(c^{-4})\,. \tag{43}$$

It is now clear that the second term in the right-hand side of (41) just "corrects" the main post-Newtonian term converting it to $2(1+\gamma)m/d'$. Using d' one can write (41) as

$$|\boldsymbol{\sigma}\times\boldsymbol{\nu}| = 2\,(1+\gamma)\,\frac{m}{d'} + \frac{1}{4}\,\left(8(1+\gamma-\alpha\,\gamma)(1+\gamma) - 4\,\alpha\,\beta + 3\alpha\,\epsilon\right)\,\pi\,\frac{m^2}{d'^2} + \mathcal{O}(c^{-6})\,. \tag{44}$$

This result with $\alpha = 1$ coincides with Eq. (4) of Epstein & Shapiro [13] and also agrees with the results of Richter & Matzner [10], Cowling [14] and Brumberg [7] in the corresponding limits.

IV. BOUNDARY PROBLEM

For practical modeling of observations of solar system objects it is not sufficient to consider the initial value problem for light propagation. Two-point boundary value problem is of interest here. Let us consider that the light ray must propagate between two points being emitted at a position $\boldsymbol{x}_0$ at time moment t_0 and received at position $\boldsymbol{x}$ at a time moment t

$$
\begin{aligned}
\boldsymbol{x}_0 &= \boldsymbol{x}(t_0)\,, \\
\boldsymbol{x} &= \boldsymbol{x}(t)\,.
\end{aligned}
\tag{45}
$$

Initial time moment t_0 can be considered here to be known (although for any stationary metric like that considered here this moment plays no role), but the final moment t is unknown. We also denote

$$
\boldsymbol{R} = \boldsymbol{x} - \boldsymbol{x}_0\,, \tag{46}
$$

$$
\boldsymbol{k} = \frac{\boldsymbol{R}}{R}\,, \tag{47}
$$

where $R = |\boldsymbol{R}|$ is the absolute value. In the following, the solution of geodetic equation (24) will be expressed as a function of the boundary values $\boldsymbol{x}_0$ and $\boldsymbol{x}$.

A. Formal expressions

An iterative solution of (26)–(31) for the propagation time $\tau = t - t_0$ and unit direction $\boldsymbol{\sigma}$ reads as follows:

$$
\begin{aligned}
c\,\tau = R - m\,\boldsymbol{k}\cdot[\boldsymbol{B}_1(\boldsymbol{x}) - \boldsymbol{B}_1(\boldsymbol{x}_0)] - m^2\,\boldsymbol{k}\cdot[\boldsymbol{B}_2(\boldsymbol{x}) - \boldsymbol{B}_2(\boldsymbol{x}_0)] \\
+ \frac{m^2}{2\,R}\,\left|\boldsymbol{k}\times(\boldsymbol{B}_1(\boldsymbol{x}) - \boldsymbol{B}_1(\boldsymbol{x}_0))\right|^2 + \mathcal{O}(c^{-6}),
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
\boldsymbol{\sigma} = \boldsymbol{k} &+ m\,\frac{1}{R}\,(\boldsymbol{k}\times[\boldsymbol{k}\times(\boldsymbol{B}_1(\boldsymbol{x}) - \boldsymbol{B}_1(\boldsymbol{x}_0))]) \\
&+ m^2\,\frac{1}{R}\,(\boldsymbol{k}\times[\boldsymbol{k}\times(\boldsymbol{B}_2(\boldsymbol{x}) - \boldsymbol{B}_2(\boldsymbol{x}_0))]) \\
&+ m^2\,\frac{1}{R^2}\,(\boldsymbol{B}_1(\boldsymbol{x}) - \boldsymbol{B}_1(\boldsymbol{x}_0))\times[\boldsymbol{k}\times(\boldsymbol{B}_1(\boldsymbol{x}) - \boldsymbol{B}_1(\boldsymbol{x}_0))] \\
&- \frac{3}{2}\,m^2\,\frac{1}{R^2}\,\boldsymbol{k}\,\left|\boldsymbol{k}\times(\boldsymbol{B}_1(\boldsymbol{x}) - \boldsymbol{B}_1(\boldsymbol{x}_0))\right|^2 + \mathcal{O}(c^{-6}).
\end{aligned}
\tag{49}
$$

These expressions are still implicit since for the post-post-Newtonian accuracy one should use the post-Newtonian relation between $\boldsymbol{\sigma}$ and $\boldsymbol{k}$ in $\boldsymbol{B}_1$ appearing in the post-Newtonian terms. That relation can be again obtained from (49) by neglecting all terms of order $\mathcal{O}(m^2)$. On the contrary, in the terms in (48) and (49) of the order of $\mathcal{O}(m^2)$ one can use the Newtonian relation $\boldsymbol{\sigma} = \boldsymbol{k}$.

B. Propagation time

Substituting (33) and (35) into (48) one can derive an explicit formula for the time of light propagation:

$$
\begin{aligned}
c\,\tau = R &+ (1+\gamma)\,m\,\log\frac{x+x_0+R}{x+x_0-R} \\
&+\frac{1}{8}\,\alpha\,\epsilon\,\frac{m^2}{R}\left(\frac{x_0^2-x^2-R^2}{x^2}+\frac{x^2-x_0^2-R^2}{x_0^2}\right) \\
&+\frac{1}{4}\,\alpha\,\left(8(1+\gamma)-4\beta+3\epsilon\right)\,m^2\,\frac{R}{|\boldsymbol{x}\times\boldsymbol{x}_0|}\,\arctan\frac{x^2-x_0^2+R^2}{2|\boldsymbol{x}\times\boldsymbol{x}_0|} \\
&-\frac{1}{4}\,\alpha\,\left(8(1+\gamma)-4\beta+3\epsilon\right)\,m^2\,\frac{R}{|\boldsymbol{x}\times\boldsymbol{x}_0|}\,\arctan\frac{x^2-x_0^2-R^2}{2|\boldsymbol{x}\times\boldsymbol{x}_0|} \\
&+\frac{1}{2}\,(1+\gamma)^2\,m^2\,\frac{R}{|\boldsymbol{x}\times\boldsymbol{x}_0|^2}\,(x-x_0-R)\,(x-x_0+R)+\mathcal{O}(c^{-6})\,.
\end{aligned}
\tag{50}
$$

This formula allows one to compute time of light propagation τ for given boundary conditions $\boldsymbol{x}_0$ and $\boldsymbol{x}$.

C. Transformation of $\boldsymbol{k}$ to $\boldsymbol{\sigma}$

In the same way substituting (33) and (35) into (49) one gets:

$$
\begin{aligned}
\boldsymbol{\sigma} = \boldsymbol{k} &+ (1+\gamma)\,m\,\frac{x-x_0+R}{|\boldsymbol{x}\times\boldsymbol{x}_0|^2}\,\boldsymbol{k}\times(\boldsymbol{x}_0\times\boldsymbol{x}) \\
&-\frac{(1+\gamma)^2}{2}\,m^2\,\frac{(x-x_0+R)^2}{|\boldsymbol{x}\times\boldsymbol{x}_0|^2}\,\boldsymbol{k} \\
&+m^2\,\boldsymbol{k}\times(\boldsymbol{x}_0\times\boldsymbol{x})\,\Bigg[-\frac{1}{4}\,\alpha\,\epsilon\,\frac{1}{R^2}\left(\frac{1}{x^2}-\frac{1}{x_0^2}\right) \\
&+\frac{1}{8}\,\left(8(1+\gamma-\alpha\,\gamma)(1+\gamma)-4\,\alpha\,\beta+3\alpha\,\epsilon\right)\frac{1}{|\boldsymbol{x}\times\boldsymbol{x}_0|^3} \\
&\qquad\times\Bigg(\pi\,R^2+\left(x^2-x_0^2+R^2\right)\,\arctan\frac{x^2-x_0^2+R^2}{2\,|\boldsymbol{x}\times\boldsymbol{x}_0|} \\
&\qquad\qquad-\left(x^2-x_0^2-R^2\right)\,\arctan\frac{x^2-x_0^2-R^2}{2\,|\boldsymbol{x}\times\boldsymbol{x}_0|}\Bigg) \\
&+\frac{1}{2}\,(1+\gamma)^2\,\frac{1}{|\boldsymbol{x}\times\boldsymbol{x}_0|^4}\,(x+x_0)\,(x-x_0-R)\,(x-x_0+R)^2\Bigg]+\mathcal{O}(c^{-6})\,.
\end{aligned}
\tag{51}
$$

This formula allows one to compute $\boldsymbol{\sigma}$ for given boundary conditions $\boldsymbol{x}_0$ and $\boldsymbol{x}$.

D. Transformation of $\boldsymbol{\sigma}$ to $\boldsymbol{n}$

Considering the relativistic model of positional observations (e.g., Klioner [3]) it is clear that a unit direction of light propagation at the point of reception $\boldsymbol{n}$ plays here an important role. This vector is defined as

$$\boldsymbol{n} = \frac{\dot{\boldsymbol{x}}(t)}{|\dot{\boldsymbol{x}}(t)|}\,. \tag{52}$$

Expanding Eq. (30) up to order $\mathcal{O}(c^{-6})$ leads in post-post-Newtonian approximation

$$\boldsymbol{n} = \boldsymbol{\sigma} + m\,\boldsymbol{C}_1(\boldsymbol{x}_{\rm pN}) + m^2\,\boldsymbol{C}_2(\boldsymbol{x}_{\rm N}) + \mathcal{O}(c^{-6}) \tag{53}$$

with

$$\begin{aligned}
\boldsymbol{C}_1(\boldsymbol{x}) &= \boldsymbol{A}_1(\boldsymbol{x}) \;-\; \boldsymbol{\sigma}\,(\boldsymbol{\sigma}\cdot\boldsymbol{A}_1(\boldsymbol{x})) \;=\; -(1+\gamma)\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x\,(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})}\,,\\
\boldsymbol{C}_2(\boldsymbol{x}) &= \boldsymbol{A}_2(\boldsymbol{x}) \;-\; \boldsymbol{A}_1(\boldsymbol{x})\;(\boldsymbol{\sigma}\cdot\boldsymbol{A}_1(\boldsymbol{x})) \;-\; \frac{1}{2}\boldsymbol{\sigma}\;(\boldsymbol{A}_1(\boldsymbol{x})\cdot\boldsymbol{A}_1(\boldsymbol{x})) \;-\; \boldsymbol{\sigma}\;(\boldsymbol{\sigma}\cdot\boldsymbol{A}_2(\boldsymbol{x}))\\
&\quad +\frac{3}{2}\boldsymbol{\sigma}\;(\boldsymbol{\sigma}\cdot\boldsymbol{A}_1(\boldsymbol{x}))^2\\
&= -\frac{1}{2}\,\alpha\,\epsilon\,\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x^4}\,\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma}) +\;(1+\gamma)^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x^2\,(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})}\\
&\quad +(1+\gamma)^2\,\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x\,(x-\boldsymbol{\sigma}\cdot\boldsymbol{x})^2} \;-\; \frac{1}{2}\,(1+\gamma)^2\,\frac{\boldsymbol{\sigma}}{x^2}\,\frac{x+\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x-\boldsymbol{\sigma}\cdot\boldsymbol{x}}\\
&\quad -\frac{1}{4}\;(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma)-4\,\alpha\,\beta+3\,\alpha\,\epsilon)\;(\boldsymbol{\sigma}\cdot\boldsymbol{x})\;\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{x^2\,|\boldsymbol{\sigma}\times\boldsymbol{x}|^2}\\
&\quad -\frac{1}{4}\;(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma)-4\,\alpha\,\beta+3\,\alpha\,\epsilon)\;\frac{\boldsymbol{\sigma}\times(\boldsymbol{x}\times\boldsymbol{\sigma})}{|\boldsymbol{\sigma}\times\boldsymbol{x}|^3}\;\left(\arctan\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{|\boldsymbol{\sigma}\times\boldsymbol{x}|}+\frac{\pi}{2}\right).
\end{aligned} \tag{54}$$

Noting that $\boldsymbol{x}_{\rm pN} = \boldsymbol{x} + \mathcal{O}(c^{-4})$, $\boldsymbol{x} = \boldsymbol{x}_0 + R\,\boldsymbol{k}$ and using Eq. (51) for $\boldsymbol{\sigma}$ in $\boldsymbol{C}_1(\boldsymbol{x}_{\rm pN})$ we get

$$\begin{aligned}
\boldsymbol{n} &= \boldsymbol{\sigma} \;-\; (1+\gamma)\,m\,\boldsymbol{k}\times(\boldsymbol{x}_0\times\boldsymbol{x})\frac{R}{|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^2}\;\left(1\;+\;\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{x}\right)\\
&\quad +\frac{1}{4}\,(1+\gamma)^2\,m^2\,\frac{\boldsymbol{k}}{|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^2}\,\frac{R}{x}\,\left(1\;+\;\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{x}\right)\,(3x-x_0-R)\,(x-x_0+R)\\
&\quad +m^2\,\boldsymbol{k}\times(\boldsymbol{x}_0\times\boldsymbol{x})\,\Bigg[\,(1+\gamma)^2\,\frac{R}{|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^2}\,\left(1+\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{x}\right)\,\left(\frac{R\;(R^2-(x-x_0)^2)}{2\,|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^2}+\frac{1}{x}\right)\\
&\quad -\frac{1}{2}\,\alpha\,\epsilon\,\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{R\,x^4}-\frac{1}{4}\;(8\,(1+\gamma-\alpha\,\gamma)(1+\gamma)\;-4\,\alpha\,\beta\;+3\,\alpha\,\epsilon)\,\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{x^2}\,\frac{R}{|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^2}\\
&\quad -\frac{1}{4}\;(8(1+\gamma-\alpha\,\gamma)(1+\gamma)\;-4\,\alpha\,\beta\;+3\,\alpha\,\epsilon)\;\frac{R^2}{|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^3}\;\left(\arctan\frac{x^2-x_0^2+R^2}{2\,|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|}\;+\;\frac{\pi}{2}\right)\,\Bigg]\\
&\quad +\mathcal{O}(c^{-6})\,.
\end{aligned} \tag{55}$$

This expression allows one to compute the difference between the vectors $\boldsymbol{n}$ and $\boldsymbol{\sigma}$, starting from the boundary conditions $\boldsymbol{x}_0$ and $\boldsymbol{x}$.

E. Transformation of $\boldsymbol{k}$ to $\boldsymbol{n}$

Finally, a direct relation between vectors $\boldsymbol{k}$ and $\boldsymbol{n}$ should be derived. To this end, we directly combine Eqs. (51) and (55) to get

$$\begin{aligned}
\boldsymbol{n} = \boldsymbol{k} &- (1+\gamma)\, m\, \frac{\boldsymbol{k}\times(\boldsymbol{x}_0\times\boldsymbol{x})}{x\,(x\,x_0+\boldsymbol{x}\cdot\boldsymbol{x}_0)}\,(1+F) \\
&-\frac{1}{8}\,(1+\gamma)^2\,\frac{m^2}{x^2}\,\boldsymbol{k}\,\frac{\left((x-x_0)^2-R^2\right)^2}{|\boldsymbol{x}\times\boldsymbol{x}_0|^2} \\
&+m^2\,\boldsymbol{k}\times(\boldsymbol{x}_0\times\boldsymbol{x})\left[\frac{1}{2}\,(1+\gamma)^2\,\frac{R^2-(x-x_0)^2}{x^2\,|\boldsymbol{x}\times\boldsymbol{x}_0|^2}\right. \\
&+\frac{1}{4}\,\alpha\,\epsilon\,\frac{1}{R}\left(\frac{1}{R\,x_0^2}-\frac{1}{R\,x^2}-2\,\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{x^4}\right) \\
&-\frac{1}{4}\,\left(8(1+\gamma-\alpha\gamma)(1+\gamma)-4\alpha\beta+3\,\alpha\,\epsilon\right)\,R\,\frac{\boldsymbol{k}\cdot\boldsymbol{x}}{x^2\,|\,\boldsymbol{x}\times\boldsymbol{x}_0\,|^2} \\
&+\frac{1}{8}\,\left(8(1+\gamma-\alpha\,\gamma)(1+\gamma)-4\,\alpha\,\beta+3\alpha\,\epsilon\right)\,\frac{x^2-x_0^2-R^2}{|\boldsymbol{x}\times\boldsymbol{x}_0|^3} \\
&\qquad\left.\times\left(\arctan\frac{x^2-x_0^2+R^2}{2\,|\boldsymbol{x}\times\boldsymbol{x}_0|}-\arctan\frac{x^2-x_0^2-R^2}{2\,|\boldsymbol{x}\times\boldsymbol{x}_0|}\right)\right]+\mathcal{O}\left(c^{-6}\right)
\end{aligned}\tag{56}$$

with

$$F=-(1+\gamma)\,m\,\frac{x+x_0}{x\,x_0+\boldsymbol{x}\cdot\boldsymbol{x}_0}\,. \tag{57}$$

This formula allows one directly to compute the unit coordinate direction of light propagation $\boldsymbol{n}$ at the point of reception starting from the positions of the source $\boldsymbol{x}_0$ and observer $\boldsymbol{x}$. The reason for writing the post-post-Newtonian term proportional to F together with the main post-Newtonian term is that it is the only post-post-Newtonian term which cannot be estimated as const $\cdot\, m^2$. This will be discussed in a subsequent report.

V. CONCLUSION

The analytical post-post-Newtonian solution for light propagation derived in this note will be used in a subsequent report to obtain an analytical formula for light deflection for solar system bodies. Not all post-post-Newtonian terms in the above formulas should be used to obtain the goal accuracy of 1 μas. Analytical estimations of individual terms can be used to find those of them which are numerically relevant at the level of 1 μas. Detailed estimations and comparison to numerical solutions of the boundary problem will be given elsewhere.

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