TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Traffic Sciences Chair of Traffic Flow Science

Extending Periodic Event Scheduling by Decisional Flow Transportation Networks

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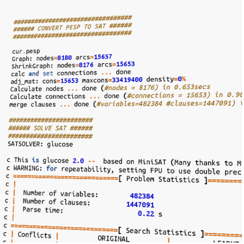
Outline

Motivation
Introduction



- timetabling of public transport railway networks (e. g., PESP) is difficult to be calculated automatically (NP-complete)

  $\rightarrow$ no existence of "default" software
- SAT solvers are universally applicable to real world problems
- best approach:

PESP instance $\rightarrow$ PESP $\leq_p$ SAT $\rightarrow$ SAT solver $\rightarrow$ SAT solution $\rightarrow$ PESP schedule

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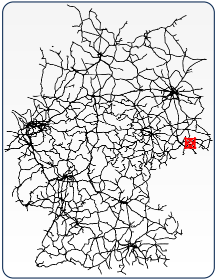
Motivation

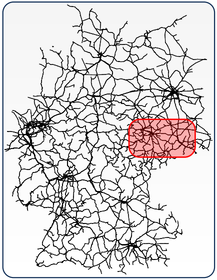
Problem Sizes with Same Computation Time

manual effort



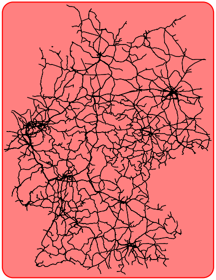
max: 20 nodes, 40 constraints

previous state-of-the-art solver



600 nodes, 3 000 constraints

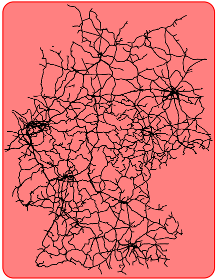
new approach (SAT)



15 000 nodes, 80 000 constraints
(up to 1 000 000 constraints)

Possible Extensions

new approach (SAT)

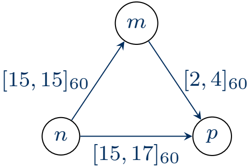


15 000 nodes,
80 000 constraints
(up to 1 000 000 constraints)

- extracting and resolving infeasible periodic event networks (local conflicts)
- flow and decision periodic event scheduling (FDPESP)
  - non-connected flow graphs (track allocation)
  - distinct chain paths (integration of routing and timetabling)
  - duplicated chain paths (periodic rail freight train paths)
- SAT-based optimization (e. g., MaxSAT, MCS)
  - minimization of weighted slacks
  - optimization in FDPESP

Periodic Event Networks

Periodic Event Scheduling Problem (PESP)



- period $T = 60$
- (periodic) events (nodes) n, m, $p \in \mathcal{V}$
  - schedule $\Pi : \mathcal{V} \to [0, T-1] \subseteq \mathbb{N}$
  - potential $\Pi(n)$, $\Pi(m)$, $\Pi(p)$
- constraints resp. activities (edges) $\mathcal{E}$
- e. g., (n, m) with $[15, 15]_{60}$
  - constraint holds iff $\Pi(m) - \Pi(n) \in [15, 15]_{60}$
  - e. g., $\Pi(n) = 2, \Pi(m) = 17$ or $\Pi(n) = 50, \Pi(m) = 5$
- tuple $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$ is called periodic event network

Periodic Event Networks
Definition PESP

- let $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$ be a periodic event network

Definition (Valid Schedule)

A schedule Π is valid for $\mathcal{N}$, iff for all $e = (n, m) \in \mathcal{E}$ with $[l, u]_T$ it holds

$$\Pi(m) - \Pi(n) \in [l, u]_T.$$

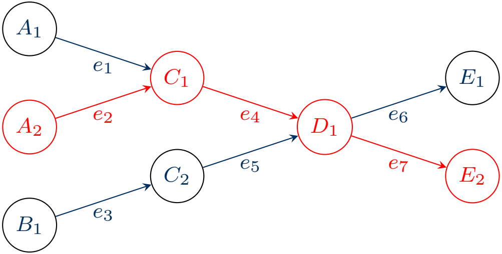
Definition (Periodic Event Scheduling Problem (PESP))

Is the decision problem whether it exists a valid schedule for $\mathcal{N}$.

- PESP is NP-complete

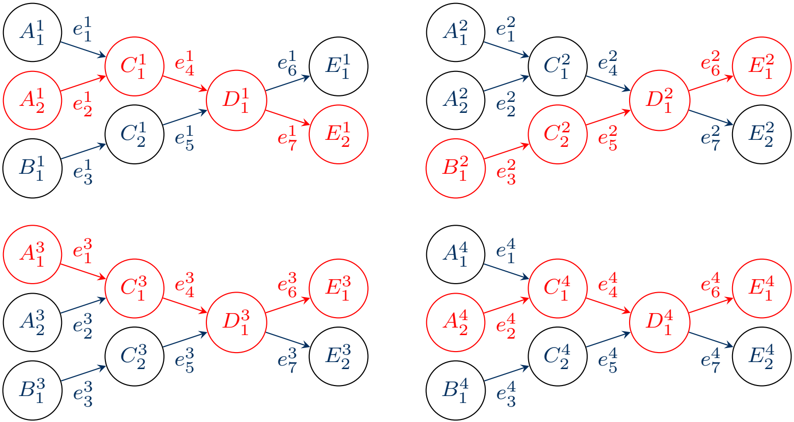
Flow Decision PESP

Flow Graphs

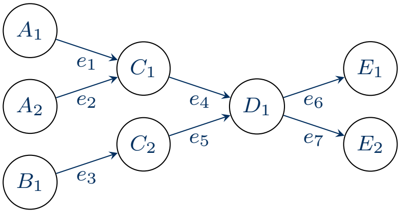


- extending PESP by further structures:
  - decision nodes and decision edges
  - transport networks (flow graphs)
- goal: path of a source to a destination node maintaining feasibility

Duplicating Flow Graphs

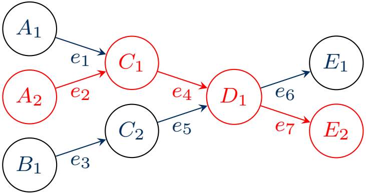


Flow Decision Periodic Event Network



- $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$ ... periodic event network
- $\mathcal{S}$ ... set of flow graphs
- $\mathcal{H}$ ... set of decision nodes ($\mathcal{H} \subseteq \mathcal{V}$)
- bijective mapping of flow edges e_i ($i \in \{1, \ldots, 7\}$) and periodic events via label function $l(e_i) = n_i$ ($n_i \in \mathcal{H}$)
- $\mathcal{D} = (\mathcal{V}, \mathcal{E}, T, \mathcal{H})$ is flow decision periodic event network for $\mathcal{S}$
- node, edge sets of $\mathcal{S}$ not necessarily disjoint (currently no application)

Flow Decision Periodic Event Network (2)



- $\mathcal{D} = (\mathcal{V}, \mathcal{E}, T, \mathcal{H})$ ... flow decision periodic event network for flow graph set $\mathcal{S}$
- $\mathcal{V}'$ ... set of periodic events ($\mathcal{V}' \subseteq \mathcal{V}$), e. g., $\mathcal{V}' = \{n_2, n_4, n_7, n_8\}$
- Π ... schedule for $\mathcal{V}'$

Definition (Valid Schedule – extended)

Π is valid for $\mathcal{D}$ under $\mathcal{S}$, iff

- exists exactly one path for all flow graphs in $\mathcal{S}$ via l^{-1} for nodes in $\mathcal{V}'$
- for all $e = (n, m) \in \mathcal{E}$: $n, m \in \mathcal{V}' \Rightarrow e$ holds under Π

Encoding to SAT

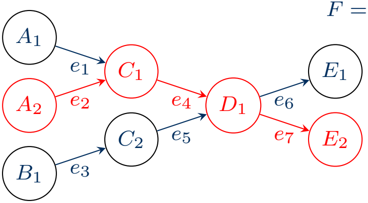
Conjunctive Normal Form – Notations

$$F = \neg p \wedge (q \vee r)$$

- literal: $L = p$ or $L = \neg p$ (variable or its negation), $p \in \mathcal{R}$
- clause c: disjunction of literals
- formula F in conjunctive normal form (CNF): conjunction of clauses
- interpretation $I : \mathcal{R} \to \{0, 1\}$
- F is satisfiable ($F^I = 1$) iff for all clauses at least one literal $L^I = 1$
- e. g., $p^I = 0$, $q^I = 1$ and $r^I = 0$ implies $F^I = 1$
- F unsatisfiable if no such I exists

Encoding to SAT

encoding flow graphs



$$
\begin{aligned}
F = {} & (q_1 \lor q_2 \lor q_3) \land (\neg q_1 \lor \neg q_2) \land (\neg q_1 \lor \neg q_3) \land (\neg q_2 \lor \neg q_3) \\
& \land (q_6 \lor q_7) \land (\neg q_6 \lor \neg q_7) \\
& \land (\neg q_6 \lor \neg q_7) \\
& \land (\neg q_1 \lor \neg q_2) \land (\neg q_4 \lor \neg q_5) \\
& \land (\neg q_1 \lor q_4) \land (\neg q_2 \lor q_4) \land (\neg q_4 \lor q_6 \lor q_7) \ldots
\end{aligned}
$$

- q_i ... propositional variable if flow edge (node) e_i is active (in path)
- exactly one source (at-least-one, at-most-one)
- exactly one destination (at-least-one, at-most-one)
- flow conservation
  - at-most-one outgoing flow edge per flow node
  - at-most-one incoming flow edge per flow node
  - at-least-one successor flow edge active if incoming active
- $q^I_2 = 1, q^I_4 = 1, q^I_7 = 1$ (remaining $= 0$) $\Rightarrow$ path (e_2, e_4, e_7)

Encoding to SAT

encoding Flow Decision PESP constraints

- $\mathcal{D} = (\mathcal{V}, \mathcal{E}, T, \mathcal{H})$ ... flow decision periodic event network for flow graph set $\mathcal{S}$
- encode constraint (edge) $e = (n_i, n_j)$ for a periodic event network with mapping $\mathrm{encodeEdge}$ (developed in 2011)
- $M = \{q_k \in \mathcal{R} \mid k \in \{i, j\} : n_k \in \mathcal{H}\}$
- encoding for constraints

$$\mathrm{encodeEdge}_M(e) = \bigvee_{q \in M} (\neg q) \vee \mathrm{encodeEdge}(e)$$

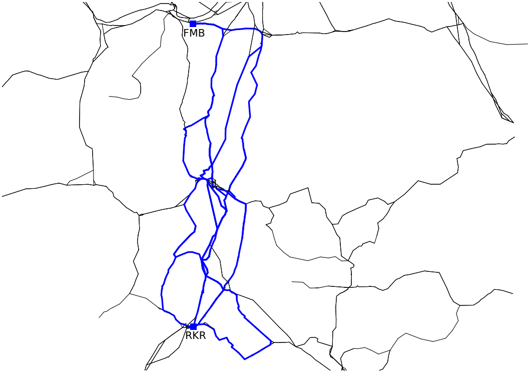
final encoding:

- encode flow graphs
- encode periodic events
- encode constraints via $\mathrm{encodeEdge}_M$

and connect all structures conjunctively

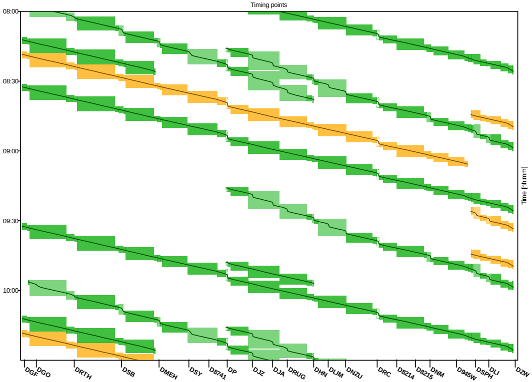
Periodic Rail Freight Planning

Problem



- input data:
  - feasible periodic event network
  - relation of the to be inserted train paths (e. g., Mainz → Karlsruhe, Germany)

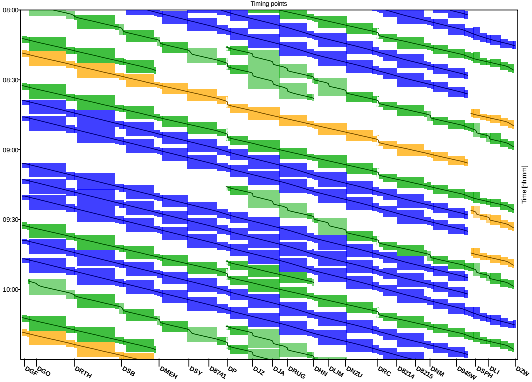
Periodic Rail Freight Planning
Problem (2)



- goal: insertion (path+timetable) of rail freight train paths based on short sub paths (e. g., Gutenfürst → Zwickau, Germany)

Periodic Rail Freight Planning

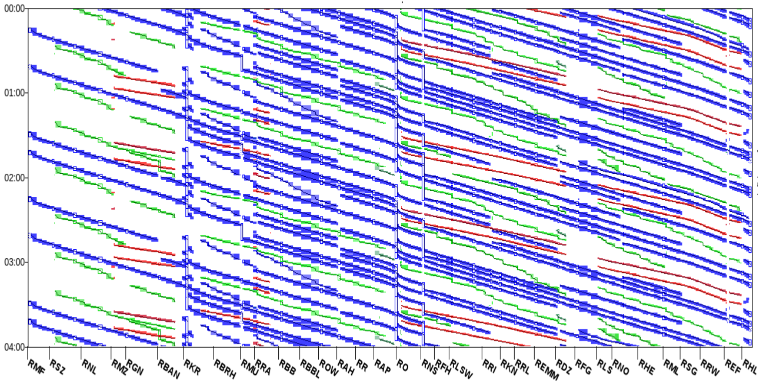
Result



- newly inserted rail freight train paths (blue) on given relation (Gutenfürst → Zwickau, Germany)

Periodic Rail Freight Planning

Result (2)



- Mannheim (Germany) $\rightarrow$ Basel (Switzerland)
- 200 flow edges per flow graph, 12 duplicated flow graphs

Conclusion

- automated timetabling of large and complex periodic event networks is possible
- reasonable extension of PESP by flow graphs:
  - track allocation
  - integration of routing and timetabling
  - periodic rail freight train paths
- outlook:
  - more efficient encoding for even better computational time
  - solver parallelization with appropriate preprocesses

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