On the Systemic Nature of Weather Risk

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Agricultural Ins. Systems

Country	Ins. coverage	Premium subsidies	Catastrophe aid	Partici-pation	Reinsu-rance
Germany	hail, suppl. ins.	none	only for uninsure-able risks	approx.35% hail <1% MPCI	pri. ins.
France	multiple peril crop ins.	60%	government aid for natu-ral disasters (drought, earthquake, flooding)	20%	pri. ins.
Greece	comprehensive ins.	50%	n.a.	n.a.	n.a.
Italy	hail, frost, drought	60% for hail 80% for MPCI	only for uninsure-able risks	n.a.	pri. ins.
Luxem-bourg	comprehensive ins.	up to 50%	n.a.	10%	n.a.
Austria	comprehensive ins.	50% for hail- and frost ins.	only for uninsure-able risks	78% hail 56% MPCI	priv. ins. exclusively
Spain	comprehensive ins.	55%	only for extreme disasters	approx. 42%	pri.and pub. ins.
Canada	multiple peril crop ins.	50%	for extreme and uninsurable disasters	50%	pri. and pub. ins.
USA	multiple peril crop ins.	$\sim$60%	only for unin-surable disasters	80%	pri. and pub. ins.

Table 1: Agricultural Insurances Systems

Pearson Correlation Coefficients vs. Distance: normal yield years

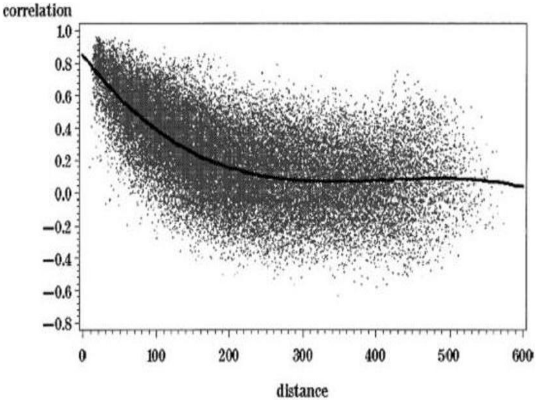


Figure 1: Goodwin, B.K.(2001)

Pearson Correlation Coefficients vs. Distance: extreme yield years

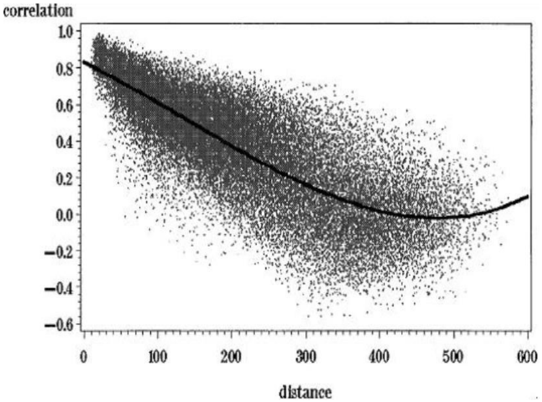


Figure 2: Goodwin, B.K.(2001)

Objectives & Research Questions

- Quantification of the dependence structure of weather events at different locations
- Does the dependence of weather events fade out with increasing distance?
- Is spatial diversification of systemic weather risk possible?
- How to measure systemic weather risk correctly?

Outline

1. Motivation ✓
2. Copula: Modeling and Estimation
3. Application
4. Conclusion

Target: Buffer Fund

$$BF = Var_{\alpha}(NTL), \quad NTL = \sum_{i=1}^{n} w_i \cdot (L_i - \Pi_i),$$

$$L_i = f(I_i, K_i) \cdot V, \qquad \Pi_i = E(L_i),$$

- BF – buffer fund,
- NTL – net total loss,
- L – loss,
- Π – fair premium,
- w – weight,
- I – weather index,
- K – trigger level,
- V – tick size,
- α – confidence level,
- i – region.

Copulae

- A copula maps a n-dimensional unit hypercube into the unit interval:

$$C(u) = C(u_1, \dots, u_n), C : [0,1]^n \rightarrow [0,1]. \tag{1}$$

- A copula can be understood as a multivariate distribution function with all marginals being uniformly distributed

$$C(u_1, \dots, u_n) = P(U_1 \leq u_1, \dots, U_n \leq u_n). \tag{2}$$

- Sklar's Theorem: If F is a multivariate distribution function with marginals $F_1, \dots, F_n$ then there exists a (unique) copula C such that

$$F(x_1, \dots, x_n) = C(F(x_1), \dots, F(x_n)). \tag{3}$$

Copula Classes

1. independence
2. perfect dependence
3. implicit copulae
   $\rightarrow$ e.g. Gaussion, Student
4. explicit copulae
   $\rightarrow$ e.g. Gumbel, Clayton, Frank

Simplest Copulae

1. independence

$$\Pi\left(u_1, u_2\right) = u_1 \cdot u_2 \tag{4}$$

2. perfect dependence

$$\begin{aligned} M\left(u_1, u_2\right) &= \min\left(u_1, u - 2\right) \\ W\left(u_1, u_2\right) &= \max\left(u_1 + u_2 - 1, 0\right) \end{aligned} \tag{5}$$

3. implicit copulas
e.g. Gaussian, Student

Archimedean Copulae

4. explicit copulas

Multivariate Archimedean Copula $C : [0,1]^d \rightarrow [0,1]$ defined as

$$C(u_1, \ldots, u_d) = \phi\{\phi^{-1}(u_1) + \cdots + \phi^{-1}(u_d)\} \tag{6}$$

where $\phi(0) = 1$, $\phi(\text{inf}) = 0$ and ϕ^{-1} its pseudo-inverse.

Example:

$\phi_{Clayton}(u, \theta) = (\theta u + 1)^{-\frac{1}{\theta}}$, where $\theta \in [-1, \infty) \setminus \{0\}$

$\phi_{Gumbel}(u, \theta) = \exp\left(-u^{\frac{1}{\theta}}\right)$, where $1 \leq \theta < \infty$

Disadvantages: too restrictive, single parameter, exchangeable

Copulae

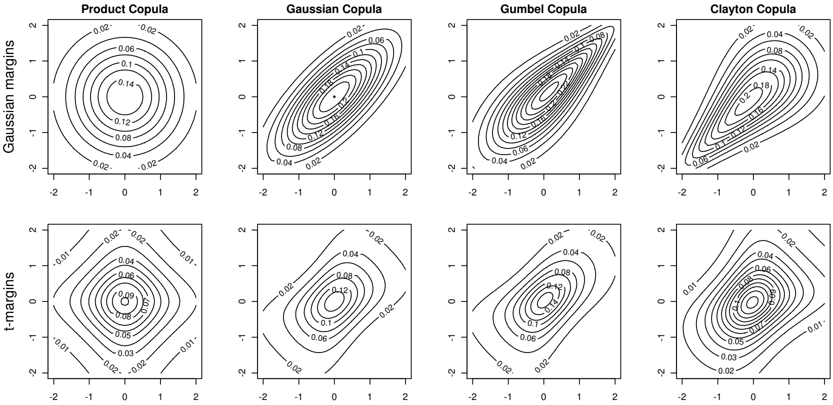


Figure 3: Different meta distributions

Estimation of Copulae I (parametric)

a) exact maximum likelihood

$$\widehat{\Lambda} = (\widehat{\theta}, \widehat{\alpha}_1, \dots, \widehat{\alpha}_n) = \arg\max_{\Lambda} \{l(\theta)\}$$
$$= \sum_{t=1}^{T} \ln \left[c\{F_1(x_{1t};\alpha_1), \dots, F_n(x_{nt};\alpha_n)\}\right] + \sum_{t=1}^{T}\sum_{j=1}^{n} \ln \{F_j(x_{jt};\alpha_j)\}. \quad (7)$$

b) IFM

- estimate parameters for marginals $F_j(x_{jt}, \alpha_j)$
- estimate θ with ML conditional on $\widehat{\alpha}_j$

Estimation of Copulae II (semiparametric)

Knowledge about Margins

$$\widehat{F}_k(x) = \frac{1}{1+n}\sum_{i=1}^{n} \mathrm{I}\{X_{ik} \leq x\}, \tag{8}$$

$$\widetilde{F}_k(x) = \frac{1}{1+n}\sum_{i=1}^{n} K\left(\frac{x - X_{ik}}{h}\right), \tag{9}$$

with $K(x) = \int_{-\infty}^{x} \kappa(t)\,dt$ and $\kappa : \Re \rightarrow \Re, \int \kappa = 1, h > 0$
$F_k(x, \widehat{\alpha})$ - parametric distribution $F_k(x)$ - known distribution

$$\breve{F}_k \in \{\widehat{F}_k(x), \widetilde{F}_k(x), F_k(x, \widehat{\alpha}), F_k(x)\}$$

Estimation of Copulae III (nonparametric)

$$\widehat{C}(u_1,\ldots,u_d) = \frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{d} \mathrm{I}\{\check{F}(X_{ik}) \leq u_k\}, \quad (10)$$

$$\widetilde{C}(u_1,\ldots,u_d) = \frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{d} K_k\{\frac{u_k - \check{F}_k(X_{ik})}{h_k}\}. \quad (11)$$

If $d = 2$,one uses other local linear Kernel to avoid bias, see Chen and Huang, 2007.
$C(u_1,\ldots,u_d;\widehat{\theta})$ - parametric copula

Estimation of Copulae IV (nonparametric)

if $\check{F}_k(x) = F_k(x, \alpha)$

$$
\begin{aligned}
l(\theta, \alpha_1 \dots \alpha_d) &= \sum_{i=1}^{n} \log c\{F_1(X_{i1}\alpha_1), \dots, F_d(X_{id}; \alpha_d); \theta\} \\
&+ \sum_{i=1}^{n} \sum_{k=1}^{d} \log f_k(X_{ik}; \alpha_k) \qquad (12) \\
\widehat{\theta}, \widehat{\alpha}_1, \cdots, \widehat{\alpha}_d &= \arg \max_{\theta, \alpha_1, \dots, \alpha_d} l(\theta, \alpha_1, \dots, \alpha_d)
\end{aligned}
$$

if $\check{F}_k \in \{\widehat{F}_k(x), \widetilde{F}_k(x), F_k(x, \widehat{\alpha})\}$)

$$
\begin{aligned}
l(\theta) &= \sum_{i=1}^{n} \log c\{\check{F}_1(X_{i1}), \dots, \check{F}_d(X_{id}); \theta\} \qquad (13) \\
\widehat{\theta} &= \arg \max_{\theta} l(\theta)
\end{aligned}
$$

Copula: Goodness-of-Fit Tests

Hypothesis

$$H_0 : C_\theta \in \mathcal{C}_0; \theta \in \Theta \;\; vs \;\; H_1 : C_\theta \notin \mathcal{C}_0; \theta \in \Theta, \tag{14}$$

Cramér von Mises

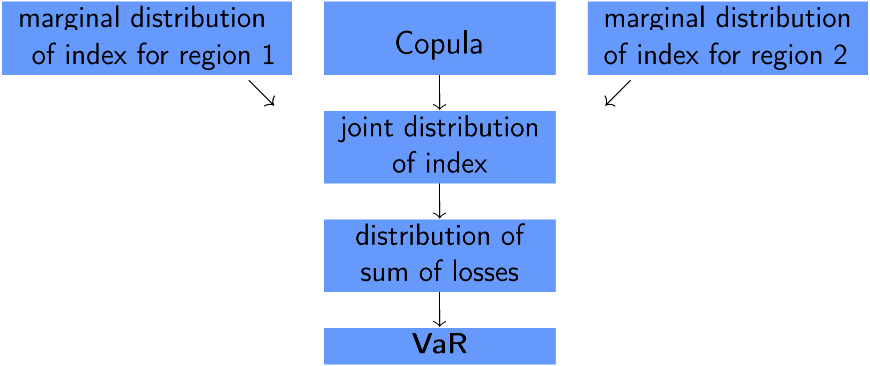
$$S = n \int\limits_{[0,1]^d} \{\widehat{C}(u_1, \ldots, u_d) - C(u_1, \ldots, u_d; \widehat{\theta})\}^2 d\widehat{C}(u - 1, \ldots, u_d) \tag{15}$$

Kolmogorov-Smirnov

$$T = \sqrt{n} \sup_{u_1, \ldots, u_d \in [0,1]} |\widehat{C}(u_1, \ldots, u_d) - C(u_1, \ldots, u_d; \widehat{\theta})| \tag{16}$$

in practice p-values are calculated using the bootstrap methods described in Genest and Remillard (2008)

Simulation with Copulas



Application

Figure 4: Location of Weather Stations

Scen.	Description
S1	**Brandenburg** (4 contracts, 4 single stations)
S2	**Germany** (4 contracts, 4 single stations, underlined)
S3	**Germany** (4 contracts, average of station (colored))

Table 2: Definition of Scenarios

Weather Indices

- Cumulative Rainfall Index (CRI)
- Potential Flood Index (PFI)
- Growing Degree Days (GDD)
- Two alternative strike levels: 50% and 15% quantiles of the index distribution

Cumulative Rainfall Index (CRI)

$$CRI_t = \sum_{j=\tau_B}^{\tau_E} P_{j,t}, \tag{17}$$

where τ_B is the first of April and τ_E is June 30

- Loss function for drought risk

$$L_t = \max(0, K_{CRI} - CRI_t) \cdot V, \tag{18}$$

where
$P_{j,t}$ is the daily precipitation at day j in year t, and
τ_B and τ_E are the beginning and the end of the vegetation period, respectively.

Potential Flood Indicator (PFI)

$$PFI_t = \max_{\tau \in \{1,\dots,365-2+1\}+1(t-1)\cdot 365} \left(\sum_{j=\tau}^{s+\tau-1} P_j \right), \tag{19}$$

with $s = 5$;

- Loss function for the risk of excessive rainfall

$$L_t = \max\left(0, PFI_t - K_{PFI}\right) \cdot V, \tag{20}$$

where *PFI* is the rainfallsum of the wettest s- day - period within a year;

Growing Degree Days (GDD)

$$GDD_t = \sum_{j=\tau_{B,t}}^{\tau_{E,t}} \max\left(0, T_j - \widehat{T}\right), \tag{21}$$

where $\tau_{B,t}$ is the first of March, $\tau_{E,t}$ is October 31, L_{GDD} is $5^\circ C$;

- Loss function for the risk of insufficient temperature

$$L_t = \max(0, K_{GDD} - GDD_t) \cdot V, \tag{22}$$

where $\widehat{T}$ is the triggering temperature.

Marginal Distributions of Index[1]

Contract	Scenario	A	B	C	D
	S1	Lognormal	Gamma	Lognormal	Weibull
CRI	S2	Lognormal	**Beta**	Gamma	Gamma
	S3	Normal	**Beta**	Logistic	Gamma
	S1	Lognormal	Lognormal	Lognormal	Lognormal
PFI	S2	Lognormal	Lognormal	Lognormal	Gamma
	S3	Lognormall	Lognormal	**Beta**	**Beta**
	S1	Weibull	**Beta**	Weibull	Weibull
GDD	S2	Weibull	Gamma	Weibull	Gamma
	S3	Weibull	Weibull	Weibull	Weibull

Table 3: Trading Area

[1] According to KS test, Chi-square test Anderson-Darling test

Estimation Results for Parametric Copulas I

Scenario	Copula	θ	t-value	BIC	Test statistics	P-value	Rank
S1	Gumbel	1.56	11.36	-63.99	0.09	0.02	**2**
	Clayton	1.95	7.00	-98.35	0.15	0.01	**3**
	Frank	**6.69**	**8.49**	**-105.53**	**0.06**	**0.27**	**1**
S2	**Gumbel**	**1.18**	**20.97**	**-9.25**	**0.05**	**0.39**	**3**
	Clayton	**0.65**	**3.90**	**-21.01**	**0.03**	**0.68**	**1**
	Frank	**2.38**	**3.63**	**-16.93**	**0.04**	**0.62**	**2**
S3	**Frank**	**1.30**	**16.54**	**-23.74**	**0.04**	**0.60**	**2**
	Frank	**1.10**	**5.28**	**-42.39**	**0.04**	**0.56**	**3**
	Frank	**3.52**	**4.80**	**-37.00**	**0.05**	**0.62**	**1**

Table 4: Cumulative Rainfall Index (CRI)

Estimation Results for Parametric Copulas II

Scenario	Copula	θ	t-value	BIC	Test statistics	P-value	Rank
	Gumbel	**1.24**	**19.68**	**-21.01**	**0.07**	**0.12**	**3**
S1	**Clayton**	**0.85**	**4.72**	**-32.73**	**0.07**	**0.12**	**2**
	Frank	**3.32**	**5.19**	**-35.33**	**0.05**	**0.57**	**1**
	Gumbel	1.01	32.79	3.29	0,05	NA	**1**
S2	Clayton	0.10	0.98	3.05	0,22	NA	**2**
	Frank	0.29	0.73	3.02	0,62	NA	**3**
	Gumbel	**1.14**	**23.90**	**-12.79**	**0.07**	**0.11**	**1**
S2	Clayton	0.35	2.28	-3.85	0.12	0.004	**3**
	Frank	1.71	2.89	-12.12	0.09	0.03	**2**

Table 5: Potential Flood Indicator (PFI)

Estimation Results for Parametric Copulas III

Scenario	Copula	θ	t-value	BIC	Test statistics	P-value	Rank
	Gumbel	**7.19**	**5.01**	**-377.49**	**0.04**	**0.33**	**1**
S1	Clayton	10.55	6.53	-323.37	0.09	0.07	**2**
	Frank	29.97	44.02	-353.13	0,12	NA	**3**
	Gumbel	1.94	4.73	-107.29	0.10	0.01	**2**
S2	Clayton	2.45	5.76	-125.41	0.26	0.001	**3**
	Frank	**11.34**	**12.18**	**-175.77**	**0.06**	**0.38**	**1**
	Gumbel	2.09	3.34	-110.63	0.09	0.03	**2**
S3	Clayton	3.25	5.95	-154.41	0.18	0.01	**3**
	Frank	**12.51**	**10.34**	**-196.11**	**0.05**	**0.43**	**1**

Table 6: Growing Degree Days (GDD)

Expected Payoff for Different Indices and Regions I

Contract	Scenario	Trading Area A	B	C	D
CRI	S1	15.52	12.81	13.62	17.62
	S2	13.62	14.69	21.11	19.16
	S3	13.47	11.78	16.11	13.66
PFI	S1	7.91	9.18	8.72	11.59
	S2	8.72	6.44	5.22	8.60
	S3	7.15	4.31	5.89	4.71
GDD	S1	73.89	59.81	78.89	72.76
	S2	78.89	70.20	61.78	73.46
	S3	65.08	70.75	73.44	72.06

Table 7: Indemnity Trigger = 50% Quantile

Expected Payoff for Different Indices and Regions II

Contract	Scenario	Trading Area A	B	C	D
CRI	S1	1.54	0.77	2.42	2.39
	S2	2.42	2.45	3.35	1.19
	S3	1.79	0.82	3.53	1.55
PFI	S1	3.04	2.15	2.84	3.69
	S2	2.84	0.90	1.08	1.47
	S3	1.22	0.95	0.72	0.92
GDD	S1	8.06	4.11	3.24	4.56
	S2	3.24	3.52	7.59	14.68
	S3	4.40	2.46	11.21	13.17

Table 8: Indemnity Trigger = 15% Quantile

Buffer Load and Diversification Effect I

Scenario	Method	Trading Area A	A+B	A+B+C	A+B+C+D	Effect of Diversification
S1	**Lin. Corr.**	**56.01**	**53.01**	**55.14**	**60.63**	**88.87**
	Gumbel	56.01	52.14	51.86	56.83	83.54
	Clayton	56.01	59.20	61.06	68.65	98.58
	Frank	**56.01**	**51.67**	**50.15**	**52.94**	**76.34**
S2	**Lin. Corr**	**64.79**	**65.83**	**60.84**	**54.46**	**69.85**
	Gumbel	64.79	60.11	54.44	49.83	65.01
	Clayton	**64.79**	**69.88**	**69.44**	**69.43**	**88.31**
	Frank	64.79	61.97	54.48	48.59	61.97
S3	**Lin. Corr.**	**69.66**	**53.56**	**60.25**	**55.90**	**79.37**
	Gumbel	69.66	48.03	52.48	44.51	64.00
	Clayton	69.66	58.52	70.49	65.88	93.41
	Frank	**69.66**	**50.14**	**53.56**	**46.93**	**66.69**

Table 9: Contract based on CRI, Trigger = 50% Quantile

Buffer Load and Diversification Effect II

Scenario	Method	Trading Area A	A+B	A+B+C	A+B+C+D	Effect of Diversification
S1	**Lin. Corr.**	**57.32**	**39.50**	**38.26**	**40.98**	**73.62**
	Gumbel	57.32	48.79	47.30	49.55	85.15
	Clayton	57.32	39.08	32.69	31.26	53.41
	Frank	**57.32**	**39.91**	**36.39**	**36.36**	**62.82**
S2	**Lin. Corr.**	**50.30**	**37.84**	**26.16**	**21.42**	**52.25**
	Gumbel	**54.30**	**32.54**	**23.79**	**19.19**	**45.56**
	Clayton	54.30	32.88	23.10	18.87	44.39
	Frank	54.30	32.28	23.14	19.42	46.52
S3	**Lin. Corr.**	**35.33**	**24.40**	**20.11**	**18.05**	**71.67**
	Gumbel	**35.33**	**25.20**	**22.66**	**21.15**	**83.54**
	Clayton	35.33	23.53	20.22	18.48	72.67
	Frank	35.33	22.14	17.71	15.22	59.14

Table 10: Contract based on PFI, Trigger = 50% Quantile

Buffer Load and Diversification Effect III

Scenario	Method	Trading Area A	A+B	A+B+C	A+B+C+D	Effect of Diversification
S1	**Lin. Corr.**	**500.87**	**352.71**	**402.48**	**425.30**	**99.12**
	Gumbel	500.87	359.38	407.28	422.00	96.21
	Clayton	500.87	351.14	400.14	423.17	99.58
	Frank	500.87	277.03	312.93	328.25	90.14
S2	**Lin. Corr.**	**500.37**	**375.26**	**380.90**	**351.43**	**95.05**
	Gumbel	503.37	348.60	348.09	318.28	86.17
	Clayton	503.37	367.09	367.19	349.90	97.85
	Frank	503.37	339.84	327.92	293.35	79.67
S3	**Lin. Corr.**	**446.08**	**42.63**	**451.33**	**447.77**	**96.25**
	Gumbel	446.08	430.90	422.04	411.71	87.23
	Clayton	446.08	458.71	469.42	470.76	98.94
	Frank	446.08	410.04	395.84	379.40	80.11

Table 11: Contract based on GDD, Trigger = 50% Quantile

Conclusions I

- Weather risk in Germany has a systemic component on a state level as well as on a national level
- The possibility of regional diversification depends on the type of weather index (*temperature* < *drought* < *flooding*)
- Weather risks should be globally diversified of transferred to the capital market (e.g. *weather bonds*)

Conclusions II

- Linear correlation may under- or overestimate systemic weather risks
- Copulas allow a flexible modeling of the dependence structure of joint weather risks
- But: problem of misspecification

References

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  Journal of Risk and Insurance 70: 111-124, 2003
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  Problems with Market Insurance in Agriculture
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Buffer Load I

Scenario	Method	Trading Area A	A+B	A+B+C	A+B+C+D	Effect of Diversification
	Lin. Corr.	**31.66**	**27.80**	**27.97**	**31.12**	**81.45**
S1	Gumbel	31.66	26.34	24.96	27.94	73.31
	Clayton	31.66	32.96	32.86	37.89	97.45
	Frank	**31.66**	**26.18**	**22.83**	**23.67**	**60.91**
	Lin. Corr.	**33.24**	**30.68**	**27.97**	**24.52**	**81.45**
S2	Gumbel	33.24	26.70	24.61	21.82	51.26
	Clayton	**33.24**	**34.65**	**34.37**	**35.57**	**80.10**
	Frank	33.23	27.61	23.12	21.44	48.57
	Lin. Corr.	**44.16**	**27.22**	**33.42**	**29.93**	**69.72**
S3	Gumbel	44.16	23.28	28.88	24.02	56.64
	Clayton	44.16	31.64	41.97	39.34	91.14
	Frank	**44.16**	**25.46**	**29.37**	**24.32**	**56.59**

Table 12: Contract based on CRI, Trigger = 15% Quantile

Buffer Load II

Scenario	Method	Trading Area A	A+B	A+B+C	A+B+C+D	Effect of Diversification
	Lin. Corr.	**39.58**	**26.39**	**24.96**	**27.08**	**68.25**
S1	Gumbel	39.58	33.04	30.88	32.63	79.06
	Clayton	39.58	26.05	21.04	19.40	46.27
	Frank	**39.58**	**26.80**	**21.91**	**22.43**	**51.68**
	Lin. Corr.	**37.91**	**24.87**	**17.34**	**13.83**	**48.59**
S2	**Gumbel**	**37.91**	**22.56**	**15.53**	**12.41**	**42.47**
	Clayton	37.91	21.93	15.20	12.45	41.66
	Frank	37.91	22.57	15.43	12.14	41.91
	Lin. Corr.	**25.22**	**15.76**	**11.70**	**9.70**	**61.94**
S3	**Gumbel**	**25.22**	**15.43**	**11.79**	**9.98**	**62.74**
	Clayton	25.22	15.10	10.23	7.75	46.54
	Frank	25.22	14.68	10.16	7.95	49.82

Table 13: Contract based on PFI, Trigger = 15% Quantile

Buffer Load III

Scenario	Method	Trading Area A	A+B	A+B+C	A+B+C+D	Effect of Diversification
S1	**Lin. Corr.**	**346.2**	**220.3**	**264.3**	**283.5**	**98.9**
	Gumbel	**346.2**	**223.9**	**266.9**	**278.1**	**95.2**
	Clayton	346.2	221.2	265.4	285.6	99.4
	Frank	346.2	174.1	202.6	213.2	86.0
S2	**Lin. Corr.**	**358.4**	**261.6**	**261.8**	**233.5**	**92.9**
	Gumbel	358.4	239.4	235.1	207.2	80.8
	Clayton	358.4	250.4	255.5	229.9	96.8
	Frank	**358.4**	**220.9**	**208.6**	**175.1**	**70.8**
S3	**Lin. Corr**	**317.3**	**314.9**	**315.6**	**310.0**	**94.9**
	Gumbel	317.3	297.0	282.5	270.7	81.8
	Clayton	317.3	330.5	333.5	334.6	98.5
	Frank	**317.3**	**276.5**	**256.8**	**243.0**	**72.9**

Table 14: Contract based on GDD, Trigger = 15% Quantile

Buffer Load IV

Contract based on	Trading Area	S1 LC[2]	S1 Copula[3]	S2 LC	S2 Copula	S3 LC	S3 Copula
	A	31.66		33.34		44.16	
CRI	A+B+C+D	31.12	23.67	24.52	35.57	29.93	24.32
	Change (%)	−1.7	−25.2	−26.5	**6.7**	−32.2	−44.9
	A	39.58		37.91		25.22	
PFI	A+B+C+D	27.08	21.43	13.83	12.14	9.70	9.98
	Change (%)	−31.6	−45.9	−63.5	−68.0	−61.5	−60.4
	A	346.20		358.43		317.26	
GDD	A+B+C+D	283.54	278.11	233.5	175.12	309.9	243.03
	Change (%)	−18.1	−19.7	−34.9	−51.1	−2.3	−23.4

Table 15: Trigger = 15% Quantile

[2] Linear Correlation

[3] best ranked copula

Measurement of Dependency

- **Fundamental task of economists**
  - Portfolio selection (CAMP)
  - Insurance (VaR; hedging effectiveness)
- **Measurement**
  - Multivariate distribution function
  - Linear correlation coefficient (Pearson)
  - Rank correlation coefficient (Spearman, Kendall)
  - Copulas

Pitfalls of Linear Correlation

- Measures only linear dependency
- Invariant only under linear transformations
- Perfect dependence does not always imply a linear correlation of 1; Zero correlation does not necessarily imply stochastic independence
- Marginal distributions and linear correlation of two random variables do not determine their joint distribution
- The interval $[-1, 1]$ is not attainable for the linear correlation coefficient for arbitrary distributions

Buffer Load

Contract based on	Trading Area	S1 LC[4]	S1 Copula[5]	S2 LC	S2 Copula	S3 LC	S3 Copula
CRI	A	56.01		64.79		69.66	
	A+B+C+D	60.63	52.94	54.46	69.43	55.90	46.93
	Change (%)	8.2	−5.5	−15.9	7.2	−19.8	−32.6
PFI	A	57.32		54.30		35.33	
	A+B+C+D	40.98	36.36	21.42	19.42	18.05	21.15
	Change (%)	−28.5	−36.3	−60.6	−64.2	−48.9	−40.1
GDD	A	500.87		503.37		446.08	
	A+B+C+D	425.3	422.0	351.4	293.4	447.8	379.4
	Change (%)	−15.1	−15.7	−30.2	−41.7	0.4	−14.9

Table 16: Trigger = 50% Quantile

[4] Linear Correlation
[5] best ranked copula

Diversification Effect

Contract based on	Trading Area	**S1**		**S2**		**S3**	
		LC[6]	**Copula**[7]	**LC**	**Copula**	**LC**	**Copula**
	50% quantile	88.87	76.34	69.85	88.31	79.37	66.69
CRI	15% quantile	81.45	60.91	56.07	80.10	69.72	56.59
	Change (%)	−8.3	−20.2	−19.7	−9.3	−12.2	−15.1
	50% quantile	73.62	62.82	52.25	46.25	71.67	83.54
CRI	15% quantile	68.25	51.68	48.59	41.91	61.94	62.74
	Change (%)	−7.3	−17.7	−7.0	−9.9	−13.6	−24.9
	50% quantile	99.21	96.78	95.05	79.67	96.25	80.11
CRI	15% quantile	98.90	95.19	92.91	70.79	94.87	72.94
	Change (%)	−0.3	−1.6	−2.3	−11.1	−1.4	−9.0

[6] Linear Correlation
[7] best ranked copula