

***Probabilistic welfare analysis for
system adequacy – analytical and
numerical insights***

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Probabilistic welfare analysis for system adequacy – analytical and numerical insights

European Single
Market for
Electricity

requires
integration of
infrastructure

Growing share of
renewable
contribution to
generation

growing need for
infrastructure
expansion and
backup capacity

Increasing cross-
border flows

Need for
Transmission
expansion

Derive the
**value of
transmission
and
generation
capacity**
as
components
of system
adequacy

Probabilistic welfare analysis for system adequacy – analytical and numerical insights

1. Introduction

2. Methodology

- Single-node system
generation adequacy accounting for generation outages
- 2-node system
generation and transmission adequacy accounting for generation and transmission outages

3. Application

- Single-node system
- 2-node system

4. Conclusions and outlook

Probabilistic welfare analysis for system adequacy – analytical and numerical insights

- Probabilistic approach to system adequacy
 - Generation adequacy
 - Generation and transmission adequacy
- Value of transmission capacity
 - **Increasing reliability by replacing backup capacity/ pooling generation capacity**
 - More efficient use of resources/ generation potential
 - Smoothing of demand asynchronies
- Concept: Social welfare analysis
 - Perspective of a central social planner
 - As two stage decision problem
 - 1st: investment into generation (and transmission) capacity
 - 2nd: operation of existing system

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- Objective function

$$\max_{K_i, L_{i,j}, y_i(t, s(t)), f_{i,j}(t, s(t))} W = \int_0^T \sum_{s \in \mathcal{S}} \pi_{s(t)} [v \cdot (D_i(s, t) - u_i(s, t)) - c_{var,i} \cdot y_i(s, t)] dt - C_{inv}$$

- Subject to several constraints:

- $y_i(s, t) \leq \psi_i(s, t) \cdot K_i$
- $D_i(s, t) = y_i(s, t) + \sum_{j=1}^J f_{j,i}(s, t) - \sum_{j=1}^J f_{i,j}(s, t) + u_i(s, t)$
- $f_{i,j} \leq \psi_{i,j}(s, t) \cdot L_{i,j}$

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- Setting

- Generation capacity: single controllable conventional technology subject to outages
- Inelastic demand function: load duration curve approximated by linear function

- Lagrangian function

$$L(K, y, u, \lambda_E, \lambda_K) = \int_0^T \sum_{s \in S} \pi_{s(t)} [v \cdot (D - u) - c_{var} \cdot y] dt - \lambda_E \cdot [D - y - u] - \lambda_K \cdot [y - \psi K] - c_{inv} \cdot K$$

- Necessary conditions for optimal decisions

$$- \frac{\partial L}{\partial y(t, s(t))} = \pi_{s(t)} \cdot (-c_m + \lambda_E - \lambda_K) \leq 0 \quad \perp y(t, s(t)) \geq 0 \quad (1)$$

$$- \frac{\partial L}{\partial u(t, s(t))} = \pi_{s(t)} \cdot (-v + \lambda_E(t, s(t))) \leq 0 \quad \perp u(t, s(t)) \geq 0 \quad (2)$$

$$- \frac{\partial L}{\partial \lambda_E(t, s(t))} = D(t, s(t)) - y(t, s(t)) - u(t, s(t)) \stackrel{!}{=} 0 \quad (3)$$

$$- \frac{\partial L}{\partial \lambda_K(t, s(t))} = y(t, s(t)) - \psi(t, s(t))K \leq 0 \quad \perp \lambda_K \geq 0 \quad (4)$$

$$- \frac{\partial L}{\partial K} = -c_{inv} + \int_0^T \sum_{s \in S} \pi_{s(t)} \cdot \lambda_K(t, s(t)) dt \leq 0 \quad \perp K \geq 0 \quad (5)$$

- Results

- Non-binding capacity constraint: $\lambda_K = 0, \lambda_E = c_m$
 - Binding capacity constraint: $\lambda_K > 0, \lambda_E = v$
- $$\Rightarrow c_{inv} = \int_0^T \sum_{s \in S} \pi_{s(t)} \cdot \lambda_K(t, s(t)) dt$$

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- Setting

- Generation capacity: one controllable conventional technology in each country
- Assumption: $c_{m1} < c_{m2}$
- Transmission investment

- Objective function

$$W = \int_0^T \sum_{s \in S} \pi(s, t) [v \cdot (D_1(t, s) + D_2(t, s) - u_1(t, s) - u_2(t, s)) - c_{m1}y_1(t, s) - c_{m2}y_2(t, s)] dt$$

$$- c_{inv1} \cdot K_1 - c_{inv2} \cdot K_2 - c_{invl} \cdot L_{12}$$

- Necessary conditions for optimal decisions

- 6 FOCs for shadow prices
- 5 FOCs for 2nd stage decision variables
- 3 FOCs for 1st stage decision variable
- Distinguish different situations according to relevance of capacity constraints

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Different situations according to the relevance of capacity constraints

Binding transmission constraint

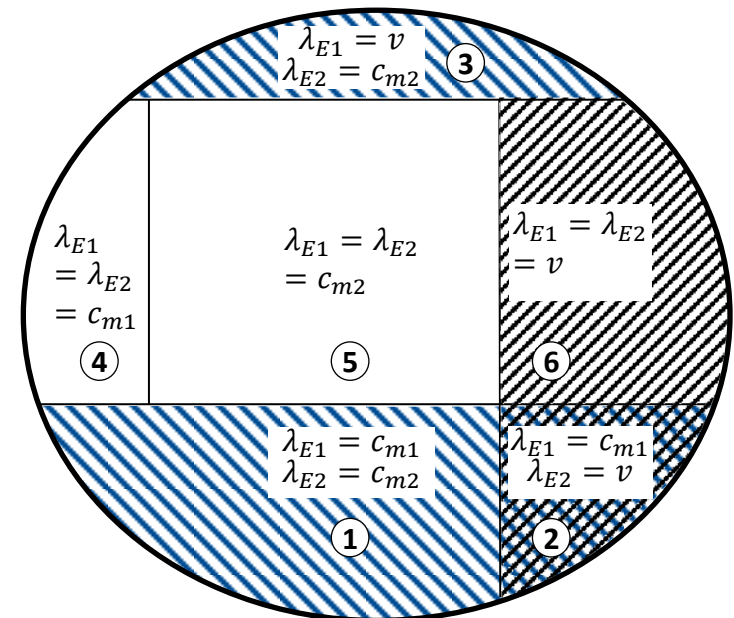
- Excess local generation capacity
 - ① $\lambda_{L1} = c_{m2} - c_{m1}$
- Binding generation capacity constraint in country 2
 - ② $\lambda_{L1} = v - c_{m1}$
- Binding generation capacity constraint in country 1
 - ③ $\lambda_{L2} = v - c_{m2}$




Excess transmission capacity ④ ⑤ ⑥

- No price differences although possibility of transmission increases overall surplus
- Zero value added by marginal transmission line
 - $\lambda_{L1} = \lambda_{L2} = 0$

→ Refinancing of investment costs

$$c_{inv,L} = \int_0^T \sum_{s \in S} \pi_{s(t)} \Psi_L(\lambda_{L1} - \lambda_{L2}) dt$$



-  Binding **overall** generation constraint
-  Binding transmission constraint
-  None of these constraints is binding

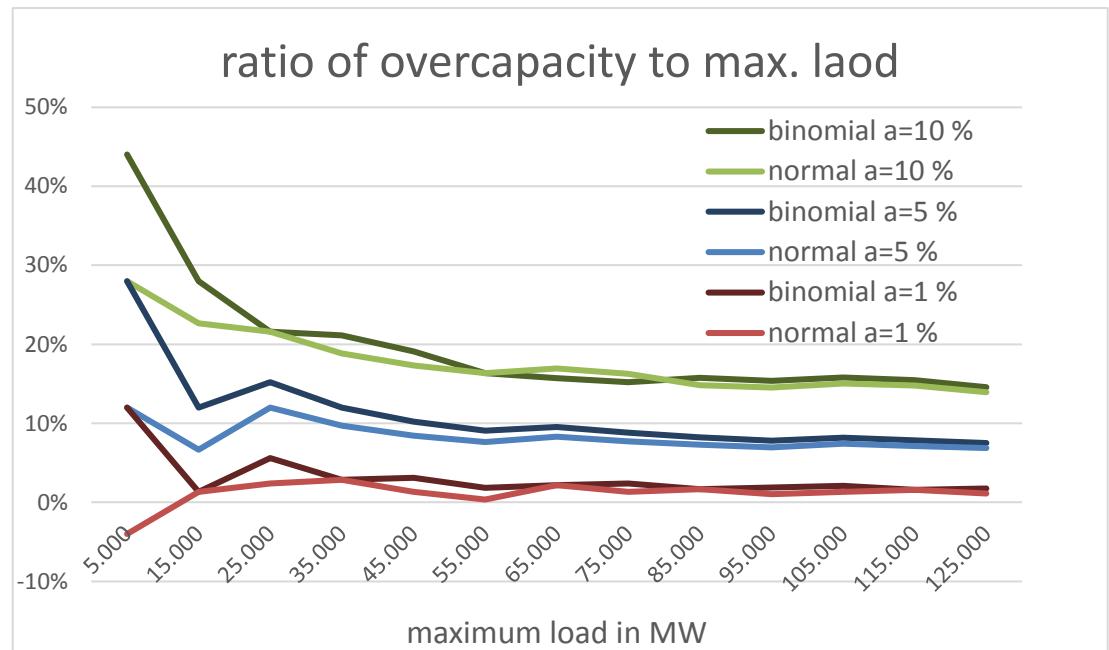
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Germany

- Simplified linearized load duration curve: max. load 86,000 MW, min. 43,000 MW
- Assumption: lumpy investment in equally sized generation units (800 MW)
- Optimal generation capacity: 117 generation units, i.e. 93,600 MW generation capacity (108 without outages)

Dependence on system size

- Comparison: binomial vs normal distribution of outages
- Small differences for large systems
- Normal distribution underestimates optimal overcapacity for small systems



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- Parameterization calibrated on Germany and the Netherlands
 - Lumpy generation (800 MW) and transmission (500 MW) investment
- Assumptions
 - Demand in both countries is perfectly correlated
 - Transmission value arising from balancing of asynchronous demand pattern is not accounted for
 - Single generation technology and unit size in both countries
 - Transmission value arising from more efficient use of generation capacity is not accounted for
- Examined cases
 - Abundant transmission capacity
 - No transmission capacity
 - Optimal transmission capacity

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Case	Region	Max. Load (MW)	n^*	Generation cap (MW)	L^*	Transmission cap (MW)	System costs (bn €)	Decision variable(s)
Copper plate	Total	95,000	128	102,400	∞	abundant*	35.362	n (# generation units)
	Total	95,000	131	104,800	0	0	35.451	
Separate systems	DE	71,250	97	77,600				n_i (# national generation units)
	NL	23,750	34	27,200				n_i
Optimal Tcap	Total	95,000	128	102,400	6	3,000	35.365	n_i , transmission capacity
	DE	71,250	95	76,000				
	NL	23,750	33	26,400				

* in copper plate case, transmission network is assumed to be existent at no costs

bold numbers highlight decision variables in the respective optimization problem

- Derive value of transmission by comparing extreme cases: abundant and no transmission capacity
- Difference in social welfare or respectively system costs btw. these (89 million €) is the max. WTP for the transmission infrastructure

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- **Conclusions**
 - Analytical approach to derive value of transmission capacity is feasible
 - Complexity already arises in simple 2-node settings
 - Analytical approach confirms expectations on synergies and allows simple quantification
 - Analytical approach may be used to get lower and/or upper bounds on benefits in more complex settings
- **Further steps**
 - Account for further differences btw. countries and corresponding value components of transmission
 - Different technologies → derive value of transmission originating from a more efficient use of resources
 - Different load patterns → derive value of transmission originating from smoothening of demand
 - Relation between components
 - Transfer to more complex setting

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Many thanks!

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