

# Flexibility in Infrastructure Markets - A Multi-Level Perspective

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# Outline

## 1 Motivation

- 1 The Role of Optimization and Data Analytics for Improving Social Welfare
- 2 Bilevel Optimization/Mathematical Programs with Equilibrium Constraints (MPECS) for Improved Infrastructure Management
- 3 Overview of Optimization Problems
- 4 Overview of Equilibrium Problems
- 5 Equilibrium Problem Example: Positional-Advantage Games in River Systems

## 2 Bilevel Optimization-Focus on Energy

- 1 Selected Examples of Bilevel Optimization in Energy
  - ★ Energy Conservation
  - ★ Wastewater-to-Energy
  - ★ Optimal Grid Tariffs
- 2 Detailed Power Generation/Transmission Infrastructure Investment Example

## 3 Summary

## References



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# Optimization and Data Analytics for Improving Social Welfare

- Use technology, management and economics to improve social welfare and achieve societal sustainability goals
- One key area to concentrate on is infrastructure, e.g., energy, transportation, water, the environment
- By improving these key areas, society as a whole, as well as individuals will see an improved quality of life
- If we consider just energy as an example (but also true of other areas of infrastructure), no "silver bullet" here, society will need a portfolio of options for
  - ▶ supply/demand (e.g., renewable but intermittent supply, demand response)
  - ▶ transmission/distribution
  - ▶ regulatory/policy incentives
 to achieve the above goals
- Will need a combination of new technologies as well as market equilibrium-based and optimization-based models to maximize benefits given limited resources and competing goals (i.e., multiobjective, user vs. system equilibria)



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# "30,000-foot" / "10,000-meter" Perspective: Modeling and Analysis of Data-Driven Systems with Autonomous Agents

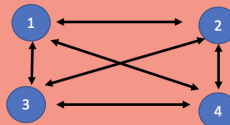
Top-level agent sets policy or incentives/coordinates activities of autonomous agents

**Optional Top-Level Agent**

Top-level agent could also be a private company with an advantageous position

Each agent solves a specific optimization problem, potentially interacting with other agents

**Autonomous Agents**



The resulting "system" behavior is not known *a priori*

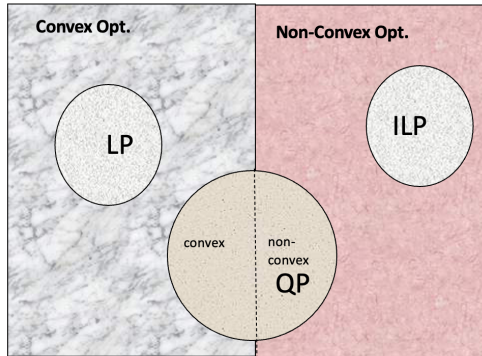
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# Overview of Optimization Problems

## The Big Picture



KEY

LP=linear program

ILP=integer linear program

QP=quadratic program

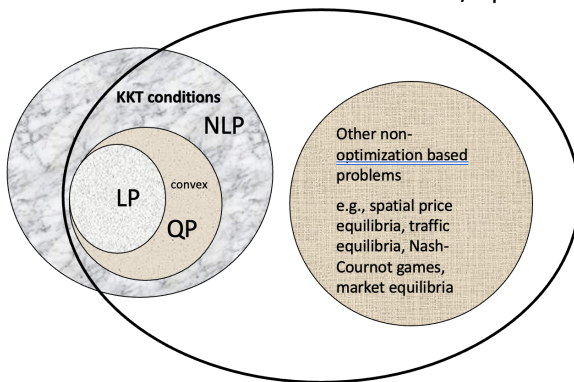
Easier  
to solve

Harder  
to solve

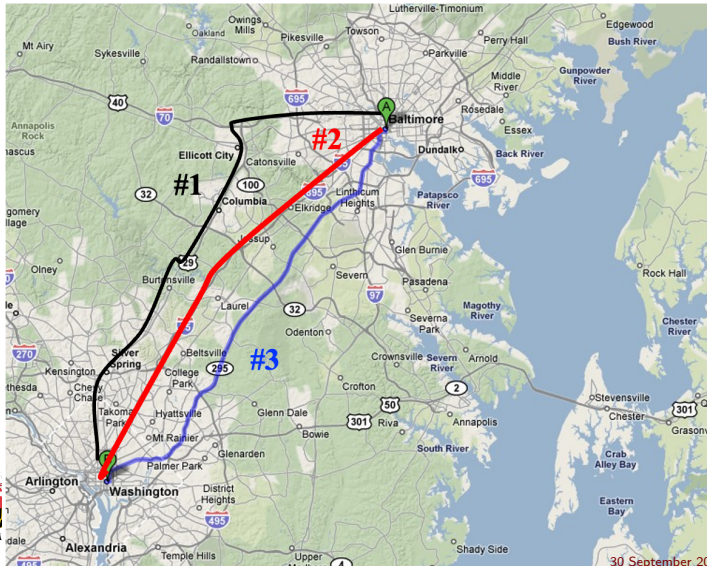
# Overview of Equilibrium Problems: Generalizing Certain Optimization and Game Theory Problems [Gabriel et al., 2013]

## The Bigger Picture

Complementarity  
Problems/Equilibrium Problems



# Wardrop Traffic Equilibrium [Gabriel & Bernstein,1997]



A->B,  
38.5 miles  
62 kilometers  
About 53  
minutes travel  
time  
Which route  
to take for  
commuting?  
**Black, red,  
blue?**

# Wardrop Traffic Equilibrium Principle

- Original formulation (from Aashtiani and Magnanti, 1981), Wardrop Equilibrium (1952)
- “At equilibrium, for each origin-destination pair the travel times on all routes serving the same OD pair, actually used are equal, and less than then travel times on all nonused routes.”
- Wardrop user equilibrium principle: users will choose the minimum cost path between each OD pair resulting in aths with positive flow all having equal costs, paths with costs higher than the minimum will have no flow

$$(C_p(h) - u_i)h_p = 0, \forall p \in P_i, i \in I$$

$$C_p(h) - u_i \geq 0, \forall p \in P_i, i \in I$$

$$\sum_{p \in P_i} h_p - D_i(u) = 0, \forall i \in I$$

$$h_p \geq 0, \forall p \in P$$

$$u_i \geq 0, \forall i \in I$$



# Bilevel Optimization Problem (or Mathematical Program with Equilibrium Constraints) [Gabriel et al., 2013]

$\min f(x,y)$   
 s.t.  $(x,y) \in \Omega$   
 $y \in S(x)$   
 where

$\Omega$  set of constraints for  $(x,y)$

$x \in R^{n_x}$  upper-level variables

$y \in R^{n_y}$  lower-level variables

$f(x,y)$  upper-level objective function

$S(x)$  solution set of lower-level problem (opt. or game)

Top level decides on the vector of decisions  $\mathbf{x}$



Bottom level decides on the vector  $\mathbf{y}$  given  $\mathbf{x}$  as fixed

Can be one more optimization problems or an equilibrium problem



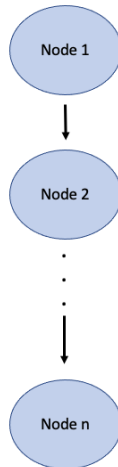
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# What is Being Modeled: Balancing Water Markets [Boyd, et al., 2022]

- Consider a river network with  $n$  users (nodes),  $I = \{1, \dots, n\}$
- Let  $D_i$  represent the nodes downstream of node  $i$ ,  $U_i$  represent the nodes upstream of node  $i$
- For all nodes in  $D_i$ , would they be willing to pay upstream nodes  $i, i-1, \dots, 1$  something to:
  - ▶ Increase volume of water?
  - ▶ Decrease volume of pollutants?
- If so, how to compute payments?

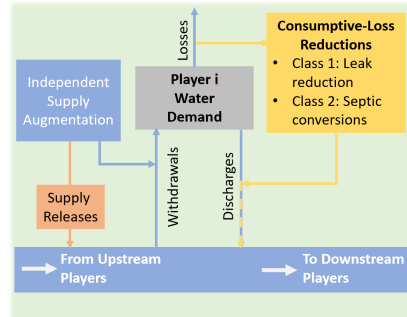


Will everyone do better with these water markets?



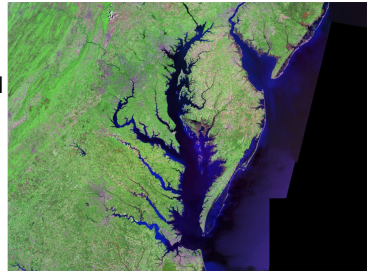
# Water Flow in a River

- Generically, each node (player) withdraws/discharges water from the river
- Each player can also add their own supply
- There is an opportunity though to be more efficient by reducing consumptive losses
- Overall, each node solves an optimization problem related to maximizing benefits less costs with possible participation in consumptive loss-reduction markets



# Nutrient Markets: Virginia Nutrient Credit Exchange Association

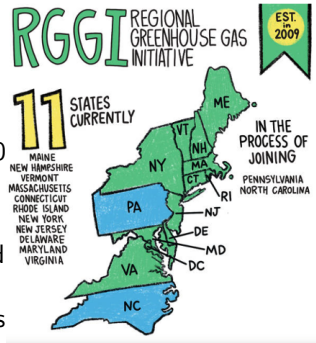
- There are other markets similar to water with success stories
- Consider the Virginia Nutrient Credit Exchange Association
  - ▶ Established in 2005 to reduce nitrogen and phosphorus discharges to the Chesapeake Bay
  - ▶ Voluntary collective of owners of 105 wastewater treatment plants (WWTPs)
  - ▶ Pollution reduction goals exceeded by over 2,000% for nitrogen and 450% for phosphorus in 2011
  - ▶ Smaller WWTPs compensated larger facilities to upgrade on their behalf



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# Carbon Allowance Markets:Regional Greenhouse Gas Initiative, <https://www.rggi.org/>

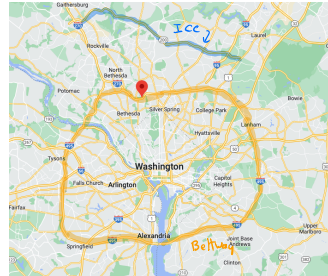
- Regional Greenhouse Gas Initiative (RGGI)
- RGGI is a cooperative, market-based effort among 11 U.S. states with a population of about 60 million to cap and reduce CO2 emissions from the power sector
- As of June 2022, RGGI states raised over \$5 billion in carbon allowance auctions for supporting communities for local energy, health and environmental goals



<https://www.nrdc.org/resources/regional-greenhouse-gas-initiative-model-nation>

# Transportation: Flow-Based Pricing

- Intercounty Connector (ICC), a way to avoid the Washington Beltway, save time
- To use the ICC, you need a transponder in your car, from which flow-based prices are charged
- The payment puts a value on free flow of travel
- There is no obligation to use the ICC, drivers can just use regular (non-tolled) roads and avoid fees



# Incentivizing Market Participation

- Clearly these infrastructure markets can work, the problem is how to incentivize everyone to participate
- Should there be some minimum participation required?
- Should there be legal mandates?
- Should participation be voluntary based on some social improvement?

# Incentivizing Water Market Participation

- What we propose for water markets is voluntary participation in the consumptive-loss reduction markets
- This means, downstream river users pay upstream ones to improve the efficiency of water lost so more water makes it downstream (water quantity) or
- A similar sort of payment but to reduce pollutants upstream (e.g., sediment deposition) for overall river benefit (e.g., better flood control)
- Each river node will be modeled as solving a particular optimization problem
- The concatenation of the resulting Karush-Kuhn-Tucker (KKT) optimality conditions plus system or market-clearing conditions gives rise to a mixed complementarity problem (MCP)
- A solution is flows and prices in the consumptive loss-reduction markets (and other items)

This problem then is the "bottom level" of the two-level perspective presented earlier





# Bilevel Optimization in Energy: Cutting Across Sustainable Energy Technologies, Markets, and Policy

**Top  
Level**

- Design decisions (e.g., what materials, size of CCS plants)
- Dominant firm generation decisions
- Government policy decisions
- Investment decisions for technologies



**Bottom  
Level**

- Operational decisions (e.g., how to operate the technologies, the CCS plants)
- Rest of the market (competitive fringe, ISO) generation and endogenous market prices
- Market responses to policy
- Market responses to investments

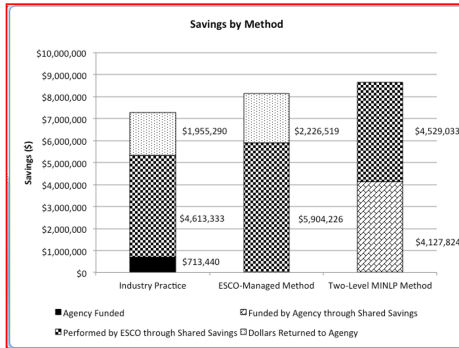
CCS=Carbon, capture, and sequestration.

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# Bilevel Optimization: Energy Conservation Example [B. R. Champion and S.A. Gabriel, 2015]



- Energy Conservation Programs
- Two-level optimization model to better manage energy conservation programs for agencies, schools
- More efficient decision-making for internal/outsourced energy project retrofits

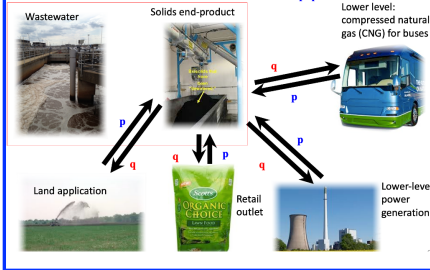
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# Bilevel Optimization: Wastewater-to-Energy Example [U-tapao et al., 2016]

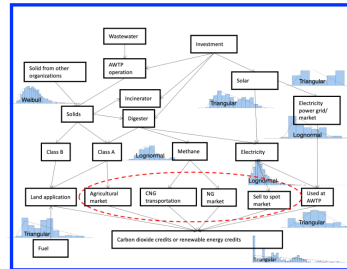
## Top and Bottom Levels

Top level: Stochastic optimization problem with recourse for wastewater treatment plant

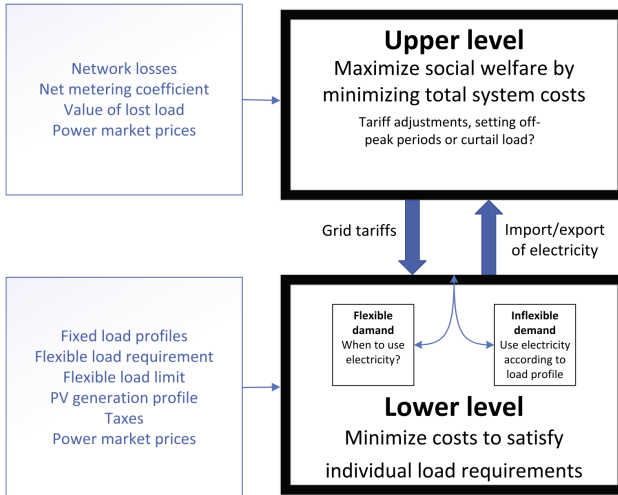
**Key:**  
 $q$ =quantities  
 $p$ =prices



## Process Diagram for Top Level Wastewater Treatment Plant



# Bilevel Optimization: Optimal Grid Tariffs [Askeland et al., 2020]



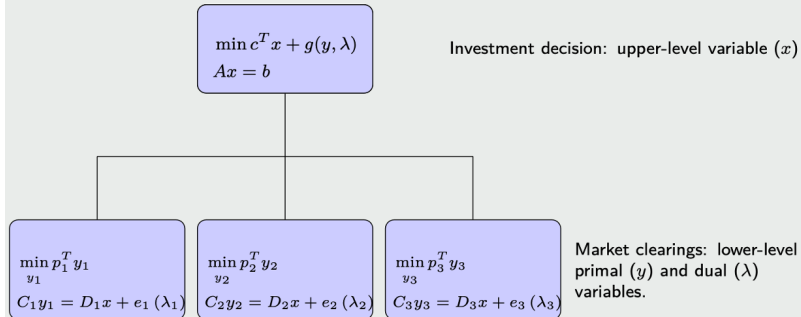
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# Overview: Power Generation/Transmission Infrastructure Investment Example [Bylling et al., 2019, 2020]

## Bilevel Investment Problem



# Mathematical Details-1

## Linear Bilevel Programming Problem

$$\min_{x,y,\lambda} c^T x + d^T y^* + k^T \lambda^* + g(y^*, \lambda^*) \quad (1a)$$

$$\text{s.t. } Ax = b \quad (1b)$$

$$x \geq 0 \quad (1c)$$

$$y^* \in \operatorname{argmin}\{p^T y \quad (1d)$$

$$\text{s.t. } Cy = Dx + e \quad (1e)$$

$$y \geq 0\} \quad (1f)$$

$$\lambda^* \in \operatorname{argmax}\{\lambda^T (Dx + e) \quad (1g)$$

$$\text{s.t. } C^T \lambda \leq p\} \quad (1h)$$

$g(y, \lambda)$  is assumed linear in  $y$ .



# Mathematical Details-2

## Linear Bilevel Programming Problem

$$\min_{x,y,\lambda} c^T x + d^T y^* + k^T \lambda^* + \lambda^{*T} M y^* \quad (1a)$$

$$\text{s.t. } Ax = b \quad (1b)$$

$$x \geq 0 \quad (1c)$$

$$y^* \in \operatorname{argmin}\{p^T y \quad (1d)$$

$$\text{s.t. } Cy = Dx + e \quad (1e)$$

$$y \geq 0\} \quad (1f)$$

$$\lambda^* \in \operatorname{argmax}\{\lambda^T (Dx + e) \quad (1g)$$

$$\text{s.t. } C^T \lambda \leq p\} \quad (1h)$$

Replace  $\lambda^T M y$  with  $\lambda^T M y$  in most cases

# Mathematical Details-3

Reformulate the problem as

$$\min_x c^T x + F(x) \quad (2a)$$

$$\text{s.t. } Ax = b \quad (2b)$$

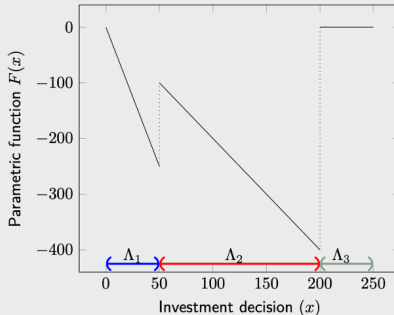
$$x \geq 0 \quad (2c)$$

with

$$F(x) = d^T y(x) + k^T \lambda(x) + \lambda(x)^T M y(x), \quad (3)$$

# Mathematical Details-4

## Investment Example



Critical regions,  $\Lambda_s$ , for each lower-level optimal basis.

## Proposition

- $F$  is a piece-wise linear function (on *critical regions*).
- $F$  is possibly discontinuous (because of the dual variables).

# Mathematical Details-5

## Data and approach

- Danish price regions DK1 and DK2 [7].
- Regions connected with 600 MW DC cable.
- Potential generation investment in DK1 and DK2.
- Full year of demand data.

## DK1 and DK2

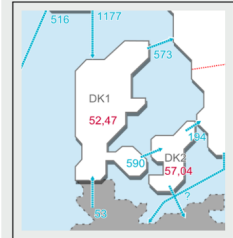
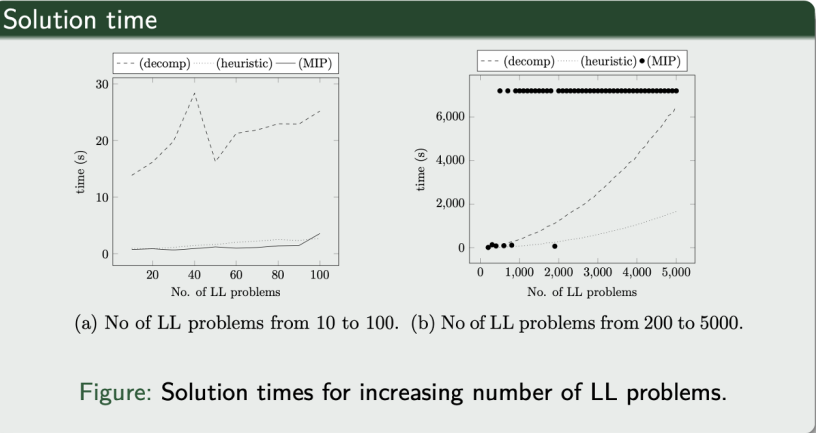


Figure: Source:  
nordpoolspot.com

# Mathematical Details-6



# Summary

- Infrastructure management can be improved through the use of analysis using bilevel optimization/MPEC models
- Users are autonomous agents with a top-level decision-maker testing out various policy regimes to seek overall best policies for social welfare
- Computational issues related to the bilevel structure– these can be overcome through the use of optimization/operations research techniques for small- or medium-scale problems
- For larger problems, there are opportunities for research to improve the related modeling and algorithmic approaches

# References

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