Identifying Elasticities in Autocorrelated Time Series Using Causal Graphs:

an Application to German Electricity Data

Silvana Tiedeman[†], Jorge Sánchez Canales[†*], Felix Schur[‡], Raffaele Sgarlato, Oliver Ruhnau[§], Lion Hirth[†], Jonas Peters[‡]

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[†]Hertie School, Centre for Sustainability, [‡] ETH Zürich, [§]Energiewirtschaftliches Institut an der Universität zu Köln

*Presenting author: j.sanchez-canales@hertie-school.org

Motivation (I)

- What is the price elasticity of electricity demand?
  - Typically thought to be inelastic or rather small
- How can we learn about the dynamics of the demand response to prices?
  - Is electricity demand price-responsive at all? Is it autocorrelated? Is there optimization across hours?
- Instrumental variables (IV) can be used to overcome endogeneity
  - Such as market equilibrium (Angrist et al., 2000)
  - Some papers have used wind generation as an instrument
    - (i.e., Arnold, 2023; Hirth et al., 2023; Fabra et al., 2021)
  - But autocorrelated instruments (i.e., wind-based time series) can introduce a new bias (Thams et al., 2022)

Motivation (II)

- We argue that Directed Acyclical Graphs (DAGs) can help us in both,
  - find valid estimators (get the right elasticity)
  - and infer dynamics
- Because we are able to derive (several) valid estimators given model assumptions, to verify these assumptions
- Caveat: It is not trivial how one expresses equilibrium in a DAG
  - (Imbens, 2020)
  - Our solution in the appendix

Our methodology

1. Propose a structural equation model (SEM)
    - Explicitly state your assumptions

$$
\begin{cases}
D_t := D_0 + \beta^P P_t + \beta^{D1} D_{t-1} + U_t^D \\
S_t := S_0 + \gamma^P P_t + \gamma^W W_t + U_t^S \\
S_t = D_t.
\end{cases}
$$

Our methodology

1. Propose a structural equation model (SEM)

$$\begin{cases} D_t := D_0 + \beta^P P_t + \beta^{D1} D_{t-1} + U_t^D \\ S_t := S_0 + \gamma^P P_t + \gamma^W W_t + U_t^S \\ S_t = D_t. \end{cases}$$

2. Solve for price

$$P_t = \frac{S_0 - D_0}{\beta^P - \gamma^P} + \frac{\gamma^W}{\beta^P - \gamma^P} W_t - \frac{\beta^{D1}}{\beta^P - \gamma^P} D_{t-1} + \frac{U_t^S - U_t^D}{\beta^P - \gamma^P}$$

Our methodology

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2. Solve for price

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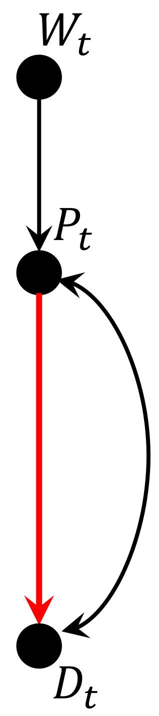
3. Create the DAG



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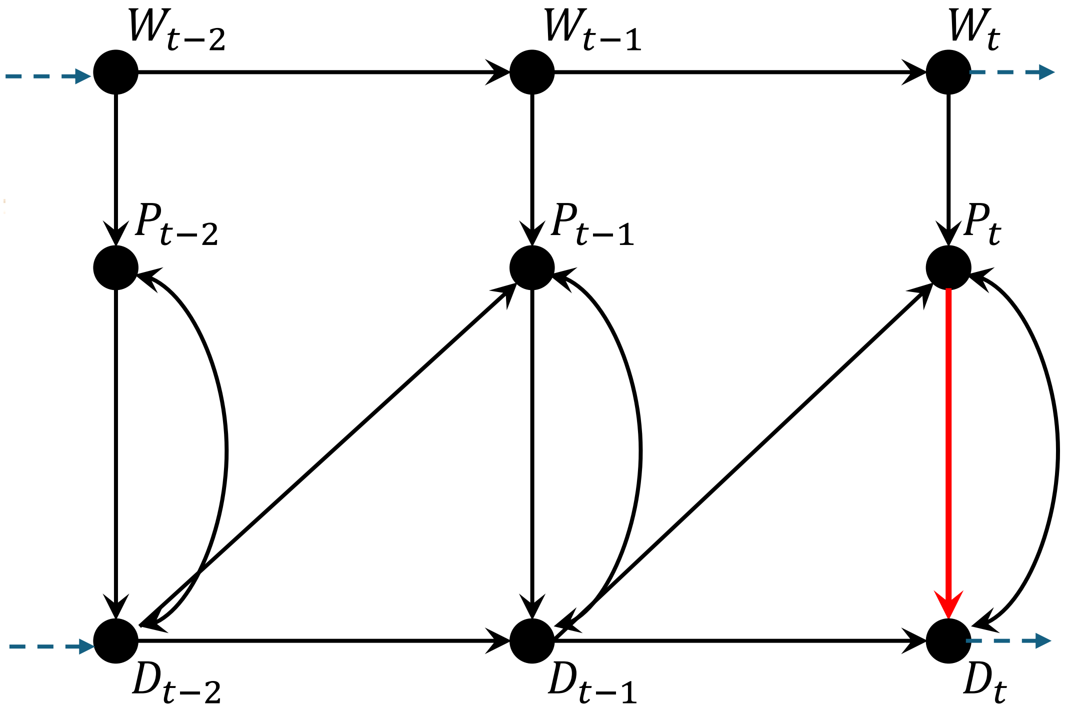
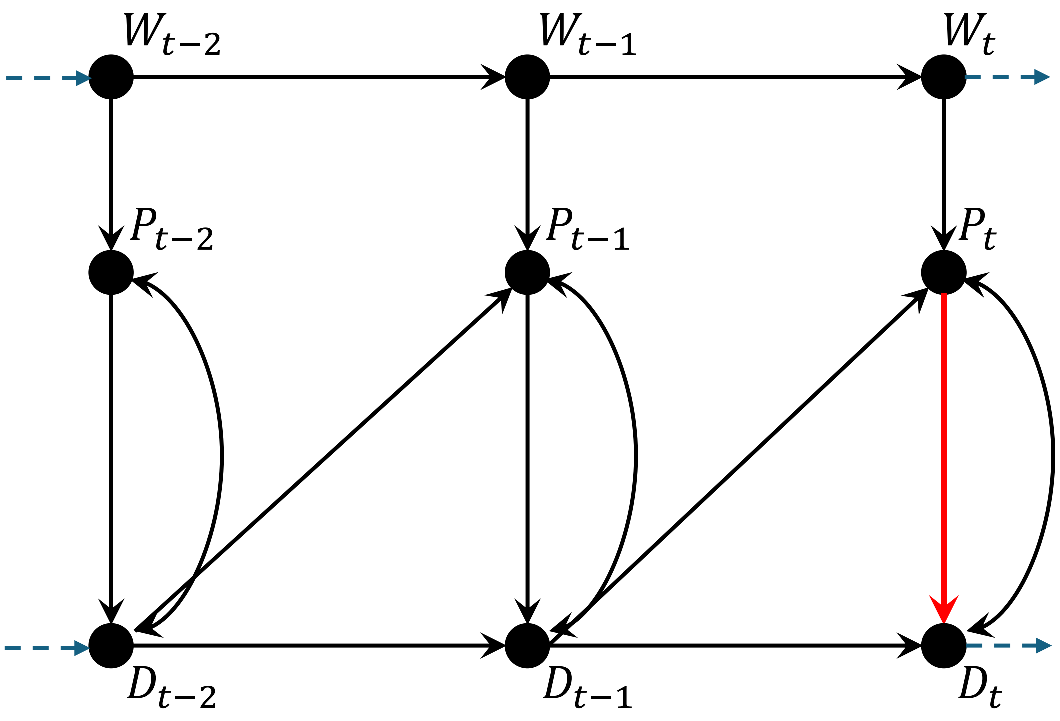


Our methodology

1. Propose SEM
2. Solve for price
3. Construct DAG
4. Derive (several) valid estimators
   - Using the CIV criteria (D-separation) (Pearl, 2009; Thams et al., 2022)
   - Valid estimators block all (information) paths between P_t and D_t except the red (our estimate)



Deriving valid estimators

- **Notation:** *CIV(instrument|estimated effect|conditioning set*)*

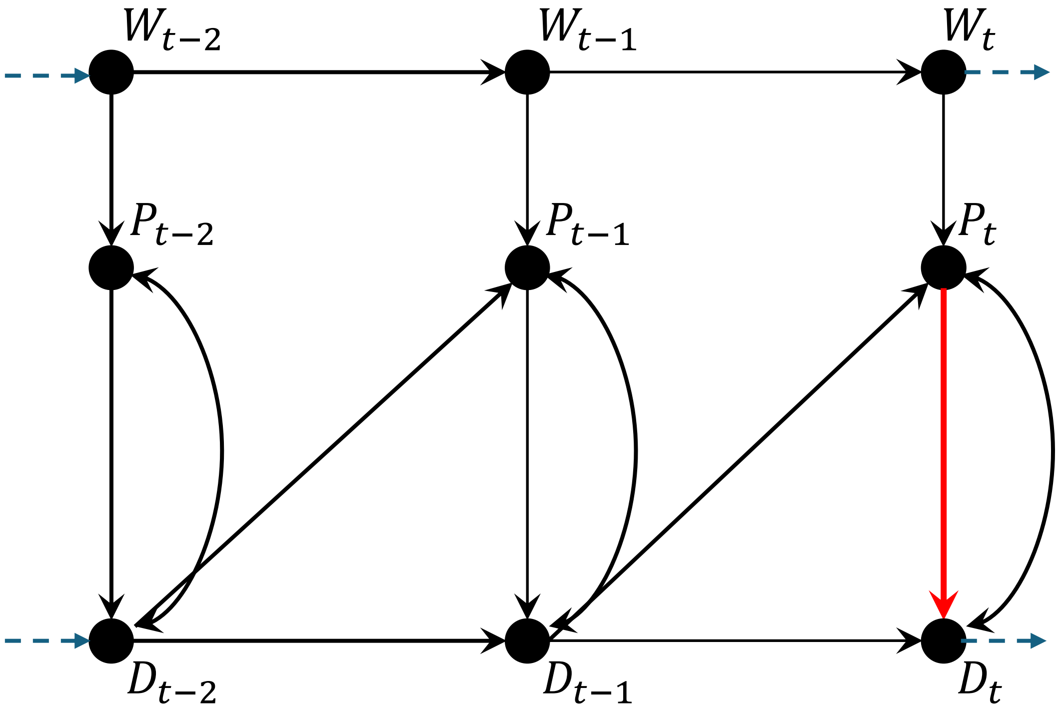
  * Typicall control variables (temperature, seasonalities, etc.) excluded

- **Criteria:** after conditioning, no open path remains between the dependent and independent variables

Deriving valid estimators

- **Notation:** *CIV(instrument|estimated effect|conditioning set)*

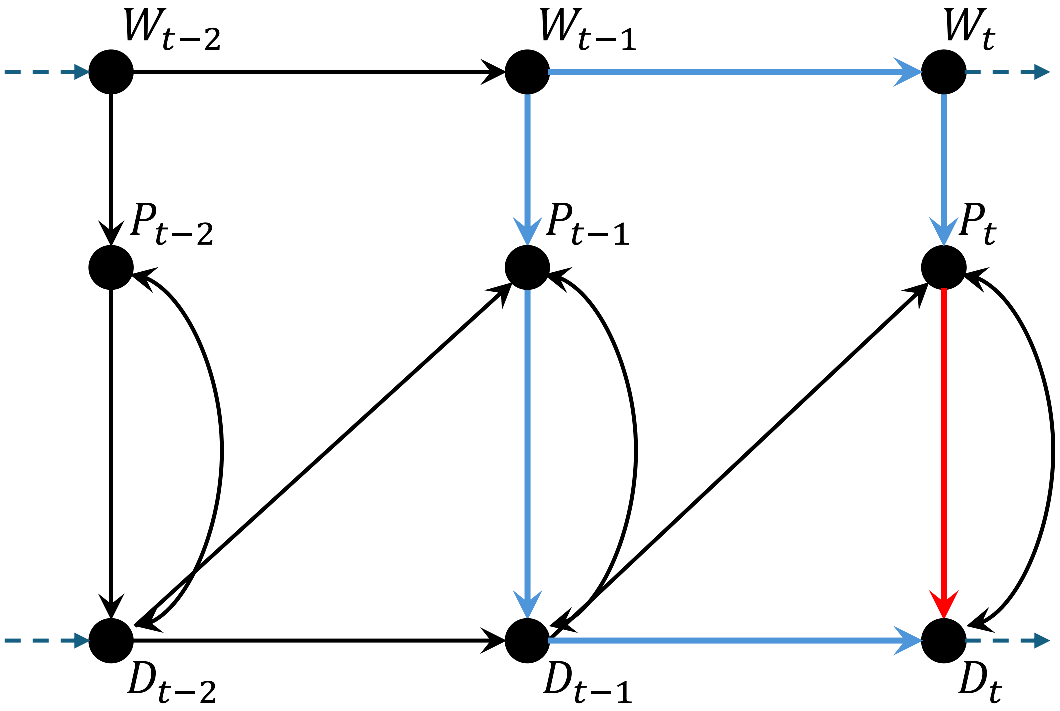
- $CIV(W_t|P_t \rightarrow D_t|\emptyset)$



Deriving valid estimators

- **Notation:** *CIV(instrument | estimated effect | conditioning set)*

- $CIV(W_t|P_t \rightarrow D_t|\emptyset)$ $\rightarrow$ invalid
  - W_{t-1} is a confounder!
  - A common cause of P_t and D_t

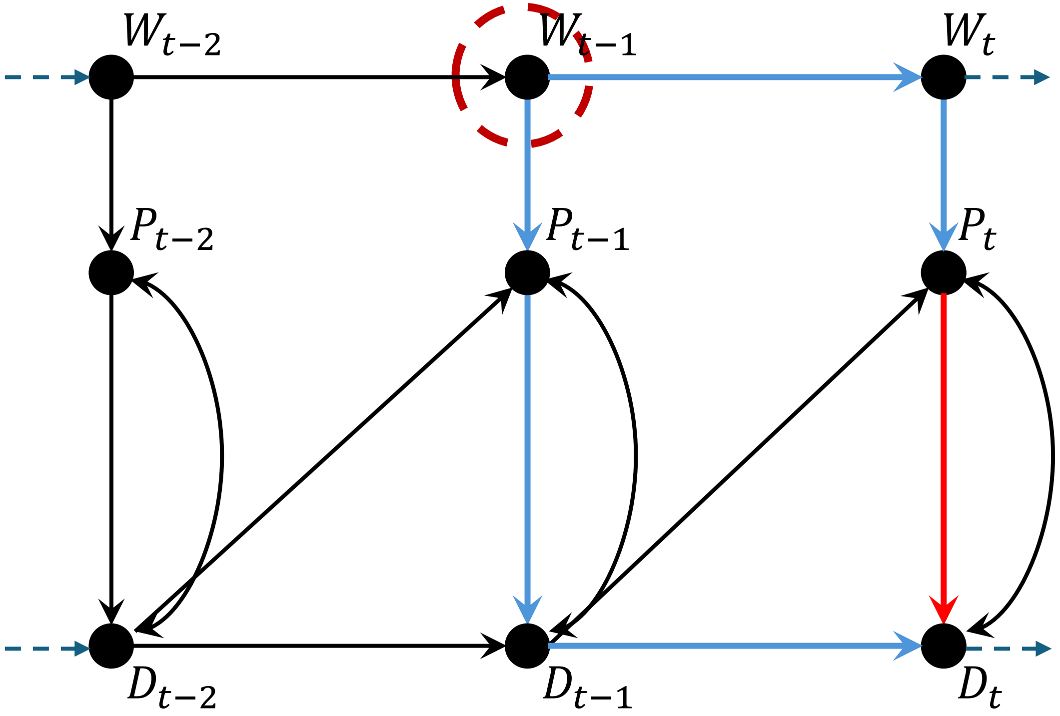


Deriving valid estimators

- **Notation:** *CIV(instrument|estimated effect|conditioning set)*

- $CIV\,(W_t|P_t \rightarrow D_t|\emptyset)$ → invalid
  - W_{t-1} is a confounder!
  - A common cause of P_t and D_t

- $CIV\,(W_t|P_t \rightarrow D_t|W_{t-1})$ → valid
  - The path through autocorrelation is now closed

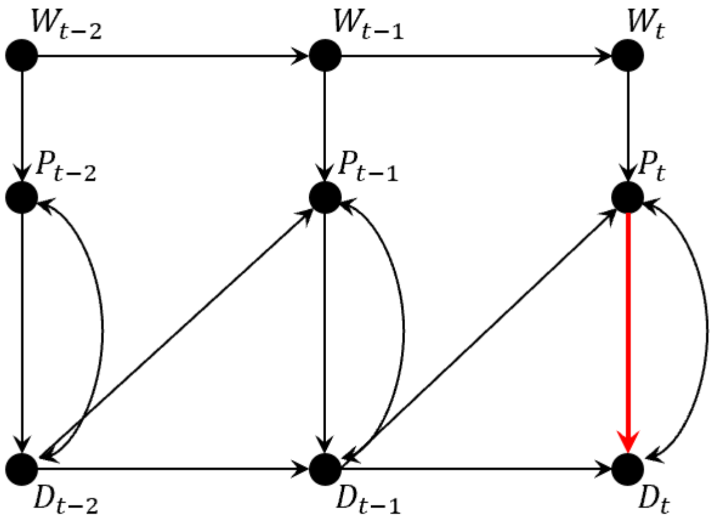


… and with valid estimators

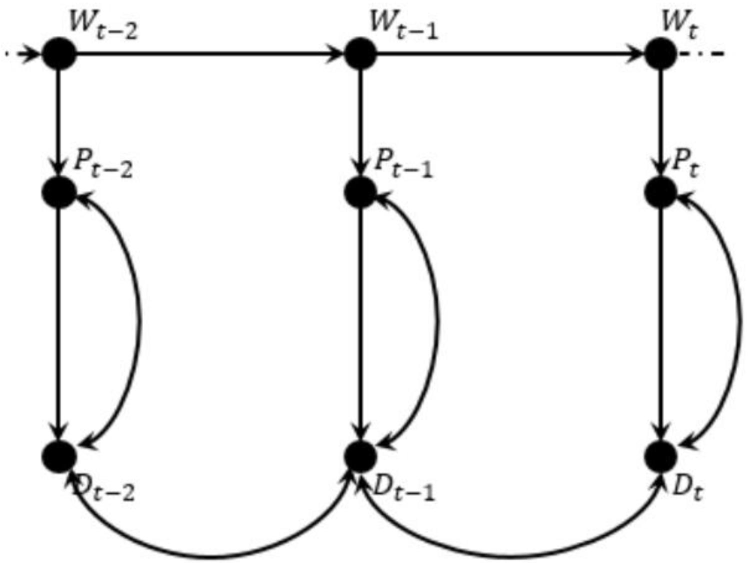
- Once you have a set of valid estimators, you can test the validity of your SEM
  - If the model is true (i.e., the data was generated following this model), then all the estimators should lead to the same estimate
    - If they lead to different estimates, we can reject the model
  - If the model is false, they could still by chance lead to the same result
    - We only “fail” to reject the model

The three models in the paper

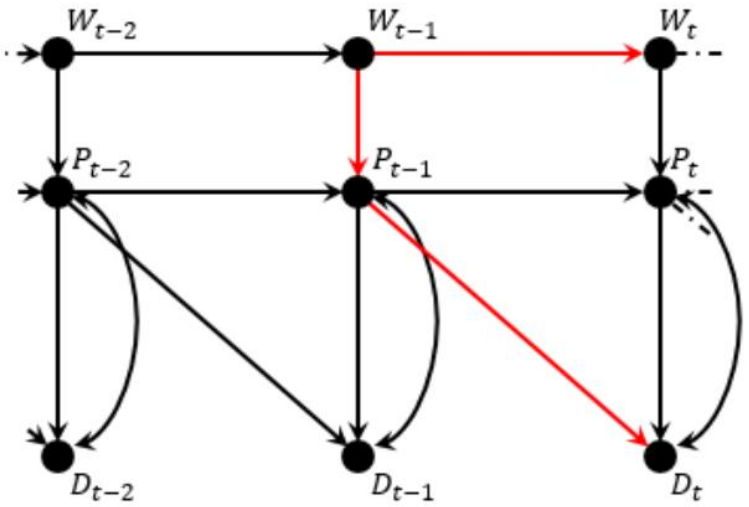
A) Structurally autocorrelated demand



B) Partially responsive demand



C) Cross-price elasticity

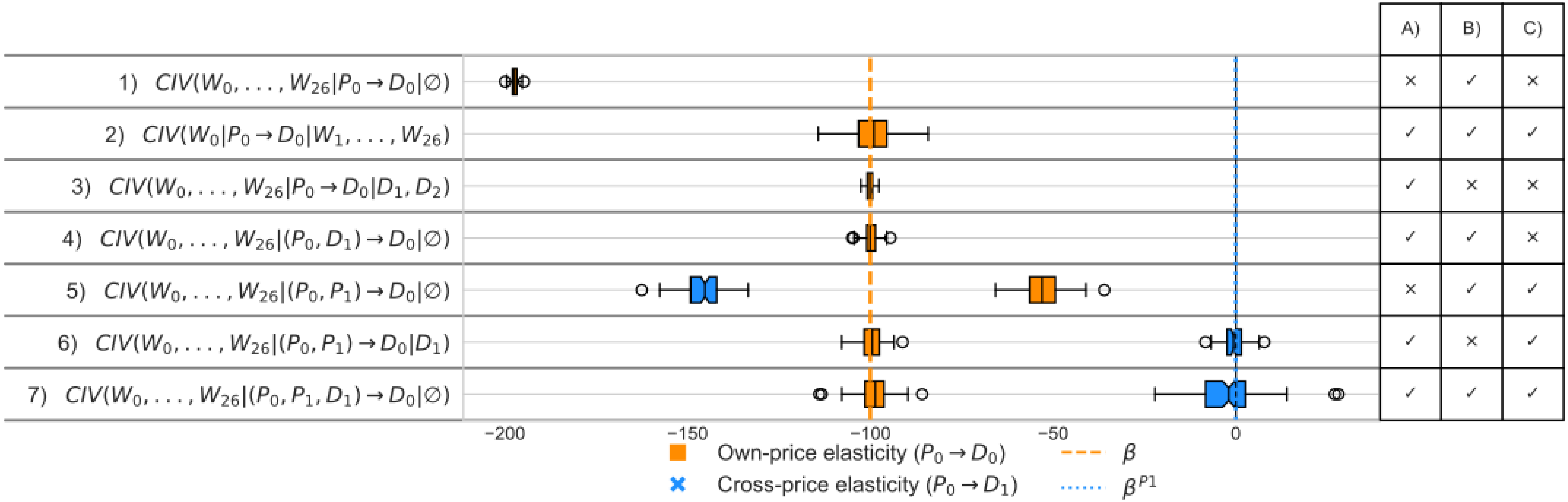


Valid estimators by model

	A)	B)	C)
1) $CIV(W_0, \dots, W_{26} \mid P_0 \rightarrow D_0 \mid \emptyset)$	×	✓	×
2) $CIV(W_0 \mid P_0 \rightarrow D_0 \mid W_1, \dots, W_{26})$	✓	✓	✓
3) $CIV(W_0, \dots, W_{26} \mid P_0 \rightarrow D_0 \mid D_1, D_2)$	✓	×	×
4) $CIV(W_0, \dots, W_{26} \mid (P_0, D_1) \rightarrow D_0 \mid \emptyset)$	✓	✓	×
5) $CIV(W_0, \dots, W_{26} \mid (P_0, P_1) \rightarrow D_0 \mid \emptyset)$	×	✓	✓
6) $CIV(W_0, \dots, W_{26} \mid (P_0, P_1) \rightarrow D_0 \mid D_1)$	✓	×	✓
7) $CIV(W_0, \dots, W_{26} \mid (P_0, P_1, D_1) \rightarrow D_0 \mid \emptyset)$	✓	✓	✓

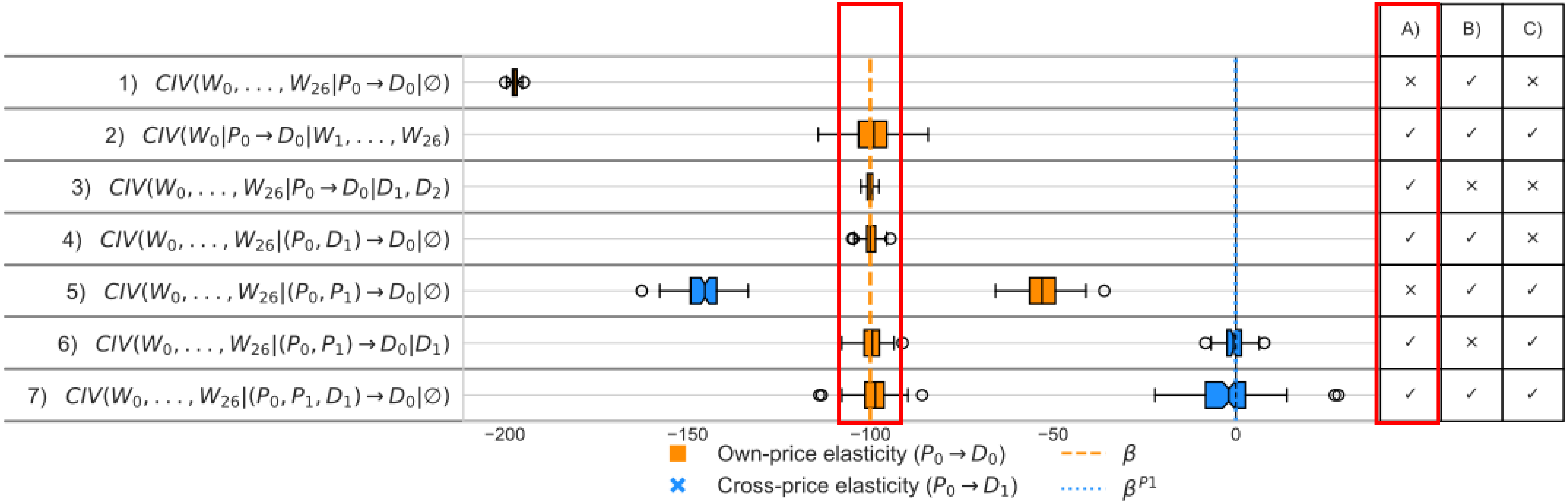
Simulations

- We verify our sets of proposed valid estimators with simulations
  - Using one SEM to generate the data



Simulations

- In this case, model A)

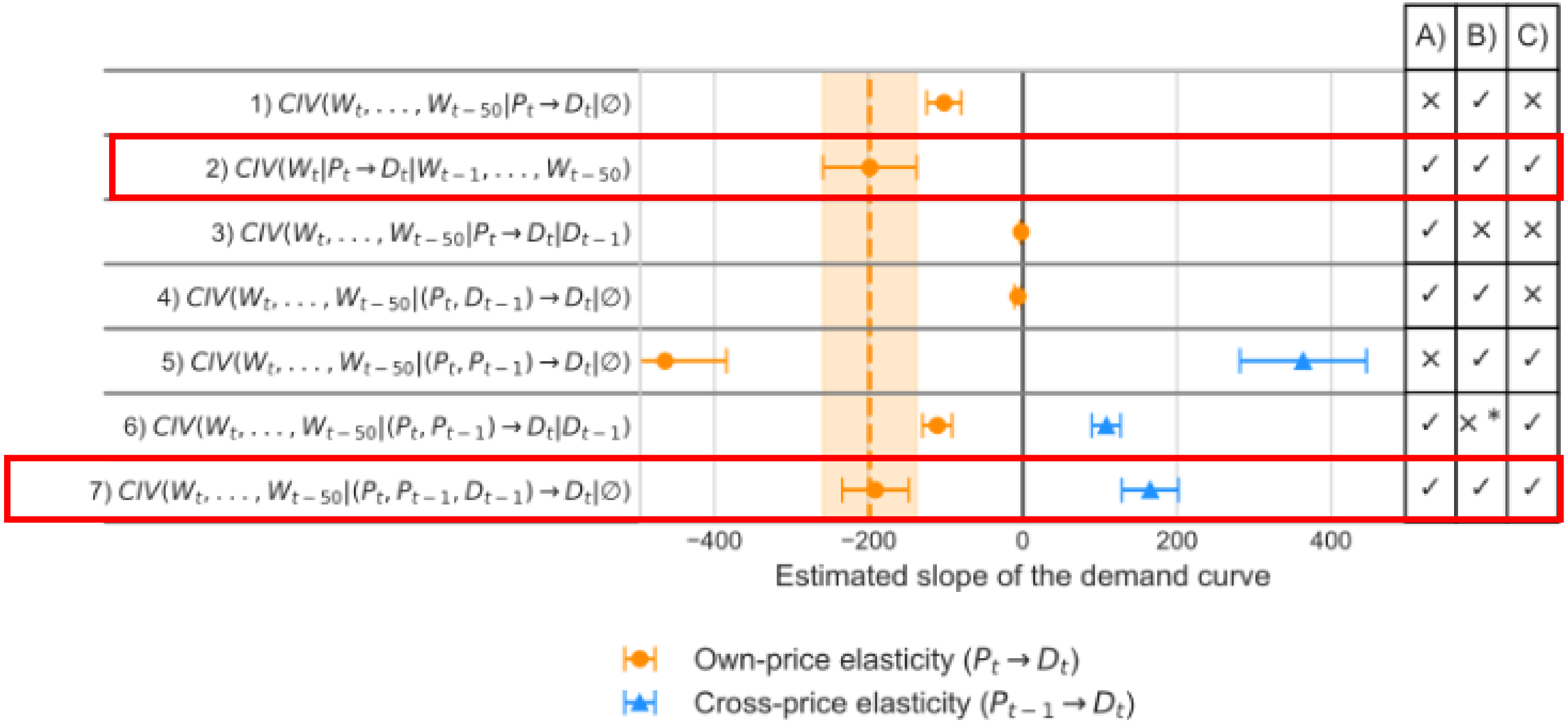


From simulation to application

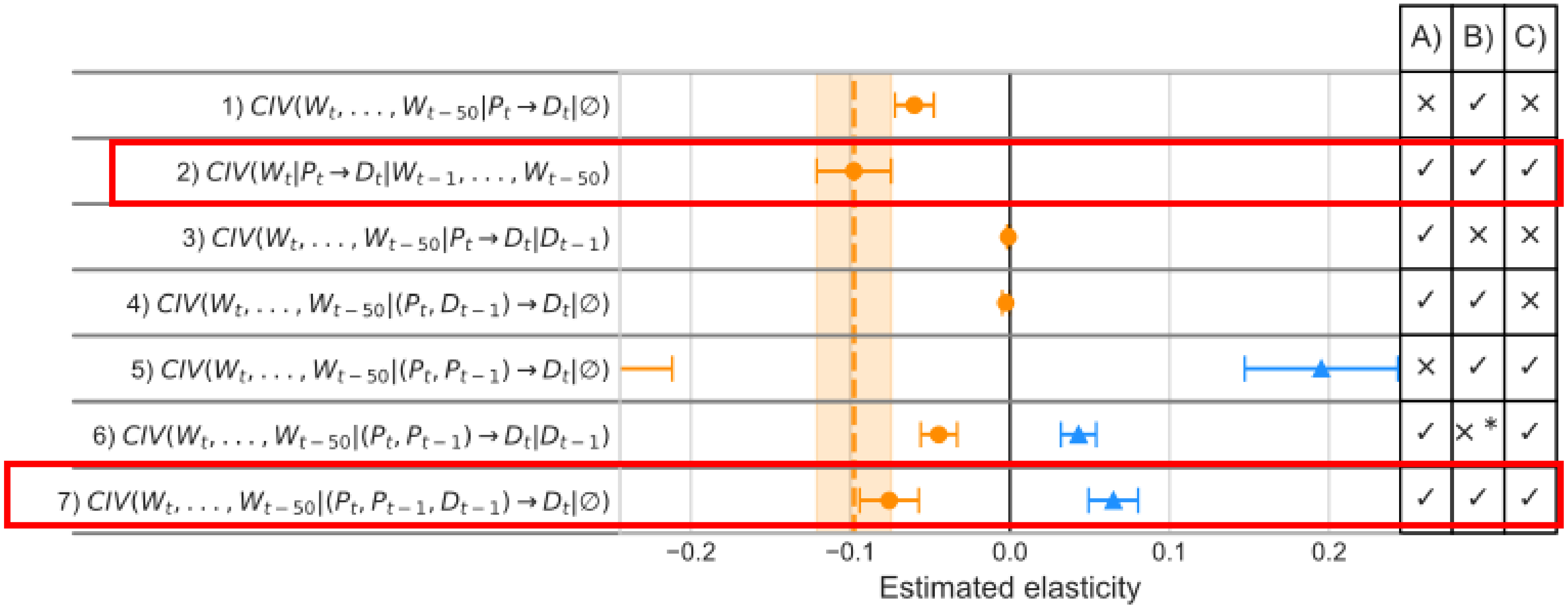
- We can then apply all our estimators to real world data (Germany 2017-2021)
  - If the pattern of estimators is inconsistent with one model, we can reject it
- We also note in all three, two estimators are consistent
  - (2) and (7)
  - Particularly (2) we believe to be unbiased in many scenarios

	A)	B)	C)
1) $CIV(W_0,...,W_{26}\|P_0 \to D_0\|\emptyset)$	×	✓	×
2) $CIV(W_0\|P_0 \to D_0\|W_1,...,W_{26})$	✓	✓	✓
3) $CIV(W_0,...,W_{26}\|P_0 \to D_0\|D_1,D_2)$	✓	×	×
4) $CIV(W_0,...,W_{26}\|(P_0,D_1) \to D_0\|\emptyset)$	✓	✓	×
5) $CIV(W_0,...,W_{26}\|(P_0,P_1) \to D_0\|\emptyset)$	×	✓	✓
6) $CIV(W_0,...,W_{26}\|(P_0,P_1) \to D_0\|D_1)$	✓	×	✓
7) $CIV(W_0,...,W_{26}\|(P_0,P_1,D_1) \to D_0\|\emptyset)$	✓	✓	✓

Application results (linear)



Application results (log-log)



Findings

- We can reject all three models for Germany
  - A regular IV (estimator 1) is biased and underestimates elasticity
- We cannot reject any model under which only estimators (2) and (7) are valid
  - This means the structure of the demand response must have all components: some elastic demand, some inelastic, with autocorrelation, and cross-price elasticities
- Since these are likely unbiased estimators, we believe that
  - The short-term own-price slope of electricity demand in Germany is -200 MWh/€ or elasticity of -0.1 (log-log)
  - Another interpretation: If there was a 1GW (unexpected) **supply shock**, up to **20% would be absorbed by demand response** in the same hour

Conclusion

- Every empirical analysis should state the assumptions about the dynamics (or structure) of the response
  - The estimators we use are not neutral, they need strong assumptions!
- The (further) formalization of these assumptions in DAG helps to
  - Defend the validity of the (IV-based) identification strategy
  - Develop a set of several valid estimators that help to verify our assumptions and thereby generate knowledge
- Regarding the price elasticity of electricity demand,
  - The price response is too complex to neglect its dynamics
  - The existing response is <u>underestimated</u> under regular IV approach
  - Interpreting the coefficients is nontrivial matter

Appendix

Funding sources

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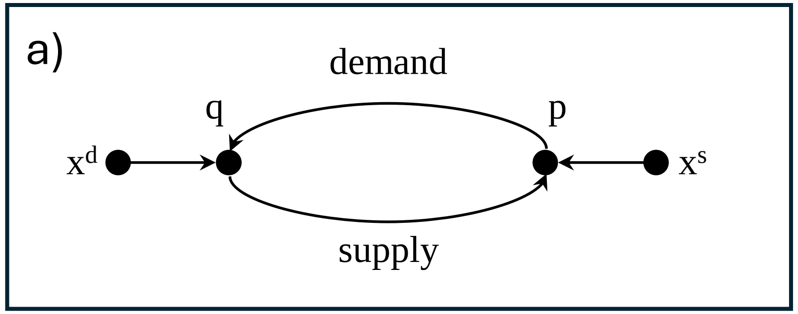
Summary of methodology

- Make assumptions about the structure of the electricity market
- Translate into structural equation model (SEM)
- Solve for the variable of interest (in this case, price)
- Express as a Directed Acyclical Graph (DAG)
- Derive valid estimators
  - (verify validity in simulations)
- Check if we reject / fail to reject SEM with real data
- Repeat!

DAGs (I)

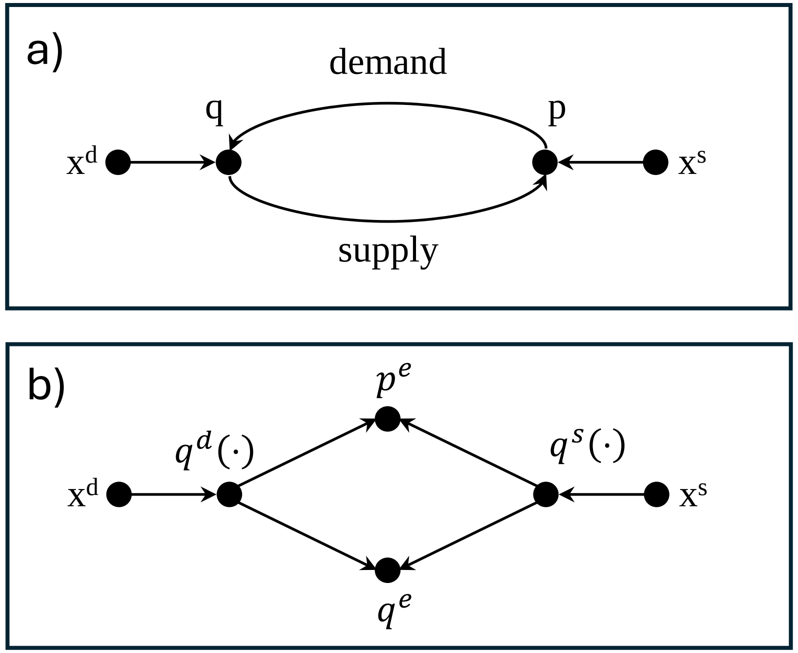
- Problem (a):
  - Arrows going from p to q and q to p do not capture the market equilibrium dynamic



DAGs (I)

- Problem (a):
  - Arrows going from p to q and q to p do not capture the market equilibrium dynamic

- But rather (b):
  - Supply and demand functions work as primitives (Imbens, 2020)



DAGs (I)

- Problem (a):
  - Arrows going from p to q and q to p do not capture the market equilibrium dynamic
- But rather (b):
  - Supply and demand functions work as primitives (Imbens 2020)
- Our proposed solution (c):
  - "Solve for" price and
  - Treat market equilibrium mechanism as an (unobserved) confounder