Farmer's Income Risk and Risk Management by Cross-hedging: A Note

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Abstract: The purpose of this study is to provide theoretical insights into the optimal hedging strategies in farmers contracts usage. We study the hedging decisions of a risk-averse farmer. The farmer faces multiple sources of price uncertainty. Cross-hedging is plausible in that one of these two commodities has a futures market. We show that the farmer's optimal futures market position is a full-hedge, an over-hedge, or an under-hedge, depending on whether the two random prices are strongly positively correlated, uncorrelated, or negatively correlated, respectively.

Keywords: agricultural price risk, cross-hedging, correlation

JEL classification: Q12, Q14

1. Introduction

Developing countries vary widely in resources, climate, population, and in their political, cultural, and religious traditions. Some have achieved striking economic growth over the last several generations (Korea, Taiwan, and Brazil), while others have lingered in economic stagnation and poverty (most of Sub-Saharan Africa). The gap between different groups of developing countries is widening as the newly industrialized countries pull away from the truly poor of the world. The world recession and the subsequent debt crisis and weakness in oil and other commodity prices, raised barriers to the progress of all but the most resilient developing countries.

Despite these difficulties, developing countries have made great strides in expanding agricultural production and consumption over the last several decades. Not all have been successful, however: famine has struck repeatedly in several sub-Saharan African countries, and hundreds fo millions of people throughout the third world remain undernourished. Even with the impressive gains in food production of the last 25 years, much of the increase in per capita food consumption was supplied by imports from developed countries. Most developing countries are becoming steadily less self-sufficient in supplying their people with food, as population growth and income growth and urbanization expand the demand for food faster than their farmers can produce it.
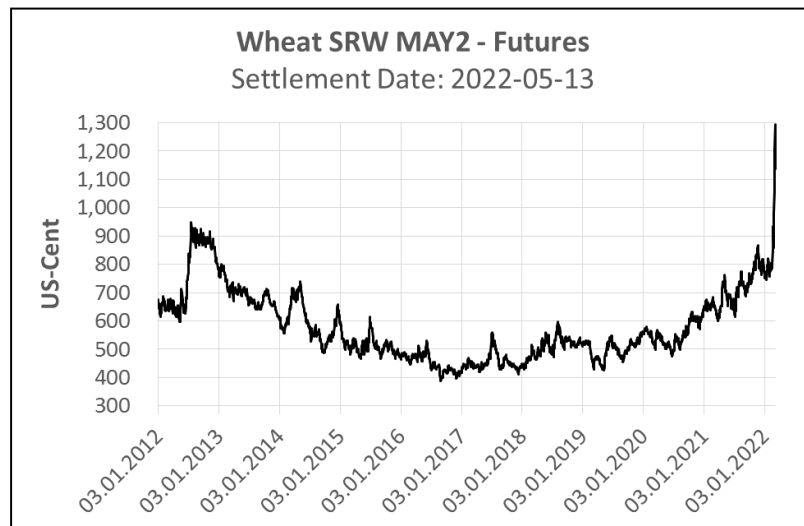

Figure 1: Wheat Futures (SWR May 2), Stettlement Date: 13.05.2022, Contract Size: 5.000 BSH, Chicago CBOT (closing price), Data Source: Refinitiv, 10.03.2022

Developing countries increasingly are recognizing, after decades of neglect in many cases, the benefits of risk sharing markets. However, risk sharing markets in the real world are far from complete. For example, not all agricultural goods have futures markets. This is especially prominent in less developed countries and economies in transition where risk sharing markets are embryonic and markets are heavily controlled. Farmers that expose to commodity price uncertainty, thus have to rely on commodity futures contracts on related goods to indirectly hedge against their price risk exposure (see figure 1). Such a risk management technique is referred to as cross-hedging (see, e.g., Newbery and Stiglitz (1981), Anderson and Danthine

(1981), Saha (1994), Broll et al. (1999), Frechette (2000), Haigh and Holt (2000), Chang and Wong (2003), Broll et al. (2010), Broll and Wong (2015) to name just a few).

The purpose of this note is to provide theoretical insights into the optimal cross-hedging strategies in farmers contracts usage. To this end, we consider a risk-averse farmer who sells its output to two markets. Only one of these two markets has a futures market to which the farmer has access. We show that the farmer's optimal future position depends on the bivariate dependence of the random commodity prices. To derive concrete results, we propose the concepts of strong correlation. We show that over-hedging, full-hedging, or under-hedging is optimal, depending on whether the two random prices are strongly positively correlated, uncorrelated, or negatively uncorrelated, respectively.

The rest of this note is organized as follows. The next section develops the model of a farmer facing price risk and cross-hedging opportunities. Section 3 characterizes the farmer's optimal hedging strategy in commodity futures. The final section concludes.

2. The Model

We consider a farmer who produces two final outputs, indexed by $i = 1$ and 2. Let x_i and p_i be the amount of outputs and the per-unit selling commodity price in market i, where $i = 1$ and 2. Profit risk comes from two sources, $\tilde{p}_1$ and $\tilde{p}_2$, that denote the random goods price in the market 1 and 2, respectively.[2] Cross-hedging is modeled by allowing the farmer to trade infinitely divisible futures contracts in the first goods market at the forward rate, p_1^f. There are no direct hedging instruments for the random goods price, $\tilde{p}_2$.

The farmer's profits are given by

$$\tilde{\Pi} = \tilde{p}_1 x_1 + \tilde{p}_2 x_2 + (p_1^f - \tilde{p}_1)h, \tag{1}$$

where h is the number of the futures contracts sold (purchased if negative). The farmer is risk-averse and possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its profits, Π, with $U'(\Pi) > 0$ and $U''(\Pi) < 0$. For given production the farmer's decision problem is to choose its futures market position, h, so as to maximize the expected utility of its profits

$$\max_{h} \; E[U(\tilde{\Pi})] \tag{2}$$

where $E(\cdot)$ is the expectation operator. The first-order condition for program (2) is given by

$$E[U'(\tilde{\Pi}^{*})(p_1^f - \tilde{p}_1)] = 0, \tag{3}$$

where an asterisk ($_*$) indicates an optimal level. The second-order condition for program (2) is satisfied given the assumed properties of utility function $U(\Pi)$.

3. Optimal hedging policy

To examine the farmer's optimal future position, h^{*}, we write equation (3) as[3]

$$E[U'(\tilde{\Pi}^{*})][p_1^f - E(\tilde{p}_1)] - \text{Cov}[U'(\tilde{\Pi}^{*}), \tilde{p}_1] = 0, \tag{4}$$

where $\text{Cov}(\cdot,\cdot)$ is the covariance operator. Evaluating the left-hand side of equation (4) at $h^{*} = x_1$ yields

$$E\{U'[p_1^f x_1 + \tilde{p}_2 x_2)]\}[p_1^f - E(\tilde{p}_1)] - \text{Cov}\{U'[p_1^f x_1 + \tilde{p}_2 x_2], \tilde{p}_1\}. \tag{5}$$

If the above expression is positive, zero, or negative, equation (4) and the strict concavity of $E[U'(\tilde{\Pi})]$ imply that h^{*} is greater than, equal to, or less than x_1, respectively.

Without imposing some concepts of bivariate dependence upon $\tilde{p}_1$ and $\tilde{p}_2$, it is impossible to determine the sign of expression (5). As such, we offer the following definition.

Definition: *The random variable, $\tilde{x}$, is said to be strongly positively correlated, uncorrelated, or negatively correlated to the random variable, $\tilde{y}$, if, and only if,* $\text{Cov}[\tilde{x}, f(\tilde{y})]$ *is positive, zero, or negative, respectively, for all strictly increasing functions, $f(\cdot)$.*

This definition is motivated by similarly ordered random variables in Hardy, Littlewood, and Pólya (1937) and Ingersoll (1987). An example of strongly correlated random variables is the linear specification:

$$\tilde{p}_2 = \alpha + \beta \tilde{p}_1 + \tilde{\varepsilon},$$

where α and β are scalars, and $\tilde{\varepsilon}$ is a zero-mean random variable independent of $\tilde{p}_1$. This linear specification is widely used in the hedging literature.

Propositon 1. *Given that the farmer is allowed to hedge with futures contracts. If $\tilde{p}_1$ and $\tilde{p}_2$ are strongly uncorrelated, then the farmer's optimal future position, h^*, is greater than, equal to, or less than x_1, depending on whether p_1^f is greater than, equal to, or less than $E(\tilde{p}_1)$, respectively. If $\tilde{p}_1$ and $\tilde{p}_2$ are strongly positively (negatively) correlated, then the farmer's optimal future position, h^*, is greater (less) than x_1 when $p_1^f \geq (\leq) E(\tilde{p}_1)$.*

Proof. If $\tilde{p}_1$ and $\tilde{p}_2$ are strongly uncorrelated, then the covariance term of expression (5) vanishes. Thus, expression (5) is positive, zero, or negative, depending on whether p_1^f is greater than, equal to, or less than $E(\tilde{p}_1)$, which implies that h^* is greater than, equal to, or less than x_1, respectively.

If $\tilde{p}_1$ and $\tilde{p}_2$ are strongly positively (negatively) correlated, then the covariance term of expression (5) is positive (negative). Thus, expression (5) is positive (negative) when $p_1^f \geq (\leq) E(\tilde{p}_1)$ so that $h^* > (<) x_1$.

The intuition of Proposition 1 is as follows. Taking variance on both sides of equation (1), we have

$$
\mathrm{Var}(\tilde{\Pi}) = \mathrm{Var}(\tilde{p}_1)(x_1 - h)^2 + \mathrm{Var}(\tilde{p}_2)x_2^2
$$
$$
+ 2\mathrm{Cov}(\tilde{p}_1, \tilde{p}_2)(x_1 - h)x_2, \tag{6}
$$

where $\mathrm{Var}(\cdot)$ is the variance operator.

Partially differentiating equation (6) with respect to h and evaluating the resulting derivative at $h = x_1$ yields

$$
\frac{\partial}{\partial h}\mathrm{Var}(\tilde{\Pi})|_{h=x_1} = -2\mathrm{Cov}(\tilde{p}_1, \tilde{p}_2)x_2. \tag{7}
$$

If $\tilde{p}_1$ and $\tilde{p}_2$ are strongly positively (negatively) correlated, we have $\mathrm{Cov}(\tilde{p}_1, \tilde{p}_2) > (<) 0$. To reduce the variability of its profits, the farmer finds it optimal to set $h > (<) x_1$ according to equation (7). When $p_1^f > (<) E(\tilde{p}_1)$, there is a speculative motive that induces the farmer to sell (purchase) the forward contracts. Thus, the over-hedging (under-hedging) incentive for risk minimization is reinforced by the speculative motive when $p_1^f \geq (\leq) E(\tilde{p}_1)$.

If $\tilde{p}_1$ and $\tilde{p}_2$ are strongly uncorrelated, we have $\mathrm{Cov}\,(\tilde{p}_1, \tilde{p}_2) = 0$. Thus, equation (7) implies that $h = x_1$ minimizes the variability of the farmer's domestic profits. The farmer deviates from this full-hedge only when $p_1^f \neq E(\tilde{p}_1)$. If $p_1^f > (<) E(\tilde{p}_1)$, the speculative motive induces the farmer sell (purchase) the forward contracts, thereby making over-hedging (under-hedging) optimal.

4. Conclusion

In this note, we have examined the optimal hedging decisions of a risk averse farmer facing multiple sources of commodity price uncertainty. The farmer sells commodities to two markets, only one of which has a forward market. We have shown that the farmer's optimal forward position is an over-hedge, a full-hedge, or an under-hedge, depending on whether the two random commodity prices are strongly positively correlated, uncorrelated, or negatively correlated, respectively.

Notes

1. A tilde denotes a random variable.

2. For any two random variables, $\tilde{x}$ and $\tilde{y}$, we have $\mathrm{Cov}\,(\tilde{x}, \tilde{y}) = E(\tilde{x}\tilde{y}) - E(\tilde{x})E(\tilde{y})$.

References

Ahlers, C., Broll, U. and B. Eckwert (2013). Information and Output in Agricultural Markets: The Role of Marktet Transparency, *Agricultural and Food Economics* 1, 1-10.

Anderson, R.W. and J. P. Danthine (1981). Cross Hedging, *Journal of Political Economy* 89, 1182-1196.

Broll, U., Clark, E. and E. Lukas (2010). Hedging Mean-Reverting Commodities, *IMA Managemant Mathematics* 21, 19-26.

Broll, U. and K.P. Wong (2015). Trade and Cross Hedging Exchange Rate Risk, *International Economics and Economic Policy* 12, 509-520.

Broll, U., Wong, K.P. and I. Zilcha (1999). Multiple Currencies and Hedging, *Economica* 66, 421-432.

Chang, E.C. and K.P. Wong (2003). Cross-Hedging with Currency Options and Futures, *Journal of Financial and Quantitative Analysis* 38, 555-574.

Frechette, D. (2000). The Demand for Hedging and the Value of Hedging Opportunities, *American Journal of Agricultural Economics* 82, 897-907.

Haigh M.S. and M.T. Holt (2000). Hedging Multiple Price Uncertainty in International Grain Trade, *American Agricultural Journal of Economics* 82, 881-896.

Hardy, G.H., J.E. (1934). Littlewood and G. Pólya, *Inequalities*, London: Cambridge University Press.

Ingersoll, J.E. (1987). Jr., *Theory of Financial Decision Making*, Savage, MD: Rowman and Littlefield.

Newbery, D.M.G., J.E. Stiglitz, *The Theory of Commodity Price Stabilization*, Oxford: Clarendon Press 1981.

Saha, A. (1994). A Two-Season Agricultural Household Model of Output and Price Uncertainty, *Journal of Development Economics* 45, 245-269.