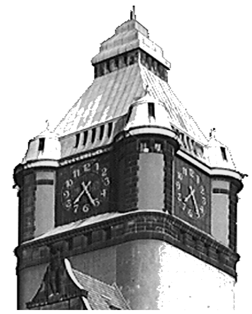
TECHNISCHE UNIVERSITÄT DRESDEN

TU Dresden
Faculty of Business and Economics

Dresden Discussion Paper Series in Economics



The Firm under Uncertainty: Capital Structure and Background Risk

UDO BROLL

KEITH K.P. WONG

Dresden Discussion Paper in Economics No. 04/10

ISSN 0945-4829

Address of the author(s):

Udo Broll
Technische Universität Dresden
Faculty of Business and Economics
01062 Dresden
Germany

e-mail : udo.broll@tu-dresden.de

Keith K.P. Wong
The University of Hong Kong
School of Economics and Finance
Hong Kong

e-mail : kpwong@econ.hku.hk

Editors:

Faculty of Business and Economics

Internet:

An electronic version of the paper may be downloaded from the homepage:
http://rcswww.urz.tu-dresden.de/wpeconomics/index.htm

English papers are also available from the SSRN website:
http://www.ssrn.com

Working paper coordinator:

Andreas Buehn
e-mail: wpeconomics@mailbox.tu-dresden.de

The Firm under Uncertainty: Capital Structure and Background Risk

Udo Broll
Technische Universität Dresden
Faculty of Business and Economics
1062 Dresden
udo.broll@tu-dresden.de

Keith K.P. Wong
The University of Hong Kong
School of Economics and Finance
Hong Kong
kpwong@econ.hku.hk

Abstract:

This paper examines the interplay between the real and financial decisions of the competitive firm under output price uncertainty. The firm faces additional sources of uncertainty that are aggregated into a background risk. We show that the firm always chooses its optimal debt-equity ratio to minimize the weighted average cost of capital, irrespective of the risk attitude of the firm and the incidence of the underlying uncertainty. We further show that the firm's optimal input mix depends on its optimal debt-equity ratio, thereby rendering the interdependence of the real and financial decisions of the firm. When the background risk is either additive or multiplicative, we provide reasonable restrictions on the firm's preferences so as to ensure that the firm's optimal output is adversely affected upon the introduction of the background risk.

JEL-Classification: D21; D81; G32

Keywords: Background risk; Capital structure; Price uncertainty

1 Introduction

The seminal work of Sandmo (1971) has inspired a great number of papers examining the theory of the competitive firm under output price uncertainty (see, e.g., Turnovsky, 1973; Batra and Ullah, 1974; Hartman, 1976; Chavas, 1985; Wong, 1996; to name just a few). In all these studies, it is implicitly assumed that the competitive firm is all-equity financed. This assumption may be innocuous in a perfect world in which Modigliani and Miller (1958) assert that the choice of capital structure (i.e., the mix of debt and equity) is a matter of irrelevance to the firm. A corollary to this irrelevance theorem is that the real and financial decisions of the firm are independent and therefore can be made separately.

In the real world, imperfections such as corporate and personal taxation, bankruptcy costs, information asymmetries, and agency costs are a fact of life, thereby making the celebrated Modigliani-Miller theorem fragile.[1] The purpose of this paper is to reconsider the theory of the competitive firm under output price uncertainty when the real and financial decisions of the firm are *de facto* interdependent. To this end, we modify the tax-adjusted valuation model of Modigliani and Miller (1963) and DeAngelo and Masulis (1980), and place it in the context of the competitive firm under output price uncertainty *à la* Sandmo (1971) and Batra and Ullah (1974). The model is further complicated to shed light on how additional sources of uncertainty, aggregated into a background risk, affect the behavior of the firm.

Irrespective of the risk attitude of the firm and the incidence of the underlying uncertainty, we show that the firm always chooses its optimal debt-equity ratio to minimize the weighted average cost of capital. We further show that the firm's optimal input mix depends on its optimal debt-equity ratio, thereby rendering the interdependence of the real and financial decisions of the firm. Myers (1974), Hite (1977), Cooper and Franks (1983), Dotan and Ravid (1985), and Dammon and Senbet (1988) establish similar interactions between corporate investment and financing decisions, albeit without considering the risk attitudes of firms.

Even though the introduction of the background risk has no effect on the firm's optimal debt-

[1] The effects of market imperfections on the Modigliani-Miller theorem have been studied by numerous papers. Notable examples are Modigliani and Miller (1963), Miller (1977), and DeAngelo and Masulis (1980) on corporate and personal taxation; Kraus and Litzenberger (1973), Scott (1976), and Brennan and Schwartz (1978) on bankruptcy costs; Myers and Majluf (1984), Narayanan (1988), and Noe (1988) on information asymmetries; Jensen and Meckling (1976), and Barnea et al. (1981) on agency costs.

equity ratio and the marginal rate of technical substitution, it does affect the input mix chosen, and the amounts of debt and equity issued, by the firm. When the background risk is either additive or multiplicative, we show that the firm optimally produces less in the presence of the background risk if its preferences exhibit risk vulnerability in the sense of Gollier and Pratt (1996) or multiplicative risk vulnerability in the sense of Franke et al. (2006), respectively. Furthermore, if capital is a normal input, the firm acquires less capital by issuing less debt and equity upon the introduction of the background risk.[2]

The rest of the paper is organized as follows. In the next section, we develop a model of the competitive firm under output price uncertainty, which fully integrates the production and financing decisions of the firm. Section 3 characterizes the firm's optimal input mix and financing mix when additional sources of uncertainty, aggregated into a background risk, are present. Section 4 examines the economic implications of the background risk on the production and financing decisions of the firm. The final section concludes.

2 The Model

Consider the one-period model of the competitive firm under output price uncertainty *à la* Sandmo (1971) and Batra and Ullah (1974). The firm produces a single commodity according to a known production function, $Q(K, L)$, where K is the amount of capital stock and L is the quantity of labor input. We assume that the production function, $Q(K, L)$, is strictly concave in K and L, i.e., $Q_K(K, L) > 0$, $Q_L(K, L) > 0$, $Q_{KK}(K, L) < 0$, $Q_{LL}(K, L) < 0$, and $Q_{KK}(K, L)Q_{LL}(K, L) - Q_{KL}(K, L)^2 > 0$, where subscripts denotes partial derivatives. We further assume that isoquants of $Q(K, L)$ are convex to the origin so that $Q_L(K, L)^2 Q_{KK}(K, L) - 2Q_K(K, L)Q_L(K, L)Q_{KL}(K, L) + Q_K(K, L)^2 Q_{LL}(K, L) < 0$ (see Silberberg and Suen, 2001). At the end of the period, the firm sells its entire output, $Q(K, L)$, at a per-unit price, $\tilde{P}$. When the firm makes its production decision at the beginning of the period, it regards $\tilde{P}$ as a positive random variable.[3]

To finance the acquisition of capital, K, at the beginning of the period, the firm issues debt and equity to raise the amounts, D and E, respectively. The firm's initial balance sheet is, therefore,

[2] Bear (1965) defines a normal (an inferior) input as one for which an increase in output price results in increased (decreased) utilization of that input.

[3] Throughout the paper, a tilde (~) always signifies a random variable.

given by

$$K = D + E, \tag{1}$$

where we have normalized the price of capital to unity for simplicity. We assume that the economic rate of capital depreciation equals one, thereby yielding zero salvage value of capital at the end of the period. Labor, L, is hired at a known wage rate, w, at the beginning of the period. The total labor costs, wL, will be paid out of the firm's revenues, $\tilde{P}Q(K, L)$, at the end of the period.

We assume that the cost of debt comprises a default risk premium that is positively related to the firm's debt-equity ratio. Throughout the paper, we consider only the case that the firm never defaults on its debt (i.e., D is sufficiently small). However, due to a lack of bargaining power, the firm has to encounter a pre-specified schedule of interest rate, $r_d(\lambda)$, where $\lambda = D/E$ is the firm's debt-equity ratio, $r_d'(\lambda) > 0$, and $r_d''(\lambda) > 0$. Since shareholders are residual claimants, the cost of equity also contains a default risk premium that is positively related to the firm's debt-equity ratio (see Scott, 1976), and is higher than the cost of debt. Let $1 + r_e(\lambda)$ be the cost of equity such that $r_e(\lambda) > r_d(\lambda)$, $r_e'(\lambda) > 0$, and $r_e''(\lambda) > 0$.

Interest costs of debt are fully tax-deductible so that the firm's tax liability at the end of the period is given by

$$\tilde{T} = t[\tilde{P}Q(K, L) - wL - \delta K - r_d(\lambda)D], \tag{2}$$

where $t \in (0, 1)$ is a constant corporate income tax rate, and $\delta \in [0, 1)$ is the firm-specific rate of capital depreciation for tax purposes. Thus, the firm's end-of-period cash flow that is accrued to existing shareholders is given by

$$\tilde{W} = \tilde{P}Q(K, L) - wL - \tilde{T} - [1 + r_d(\lambda)]D - [1 + r_e(\lambda)]E, \tag{3}$$

where the first two terms on the right-hand side of Eq. (3) give the firm's operating profits, the third term is the corporate income taxes, the fourth term is the total costs of debt, and the last term is the total costs of equity.

The firm possesses a von Neumann-Morgenstern utility function, $U(W, Z)$, defined over its end-of-period cash flow that is accrued to existing shareholders, W, and other sources of uncertainty that are aggregated into a single random variable, $\tilde{Z}$, hereafter referred to as background risk. The firm is risk averse so that $U_W(W, Z) > 0$ and $U_{WW}(W, Z) < 0$, where subscripts denote partial derivatives.

Before any uncertainty is resolved, the firm chooses an input mix, (K, L), and a financing mix, (D, E), so as to maximize its expected utility:

$$\max_{K,L,D,E} \ \mathrm{E}[U(\tilde{W}, \tilde{Z})] \ \text{ s.t. } \ K = D + E, \tag{4}$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the joint probability distribution function of $\tilde{P}$ and $\tilde{Z}$, and $\tilde{W}$ is defined in Eq. (3).

3 Solution to the Model

Substituting Eqs. (1) and (2) into Eq. (3) yields

$$\tilde{W} = (1-t)[\tilde{P}Q(K, L) - wL] - [1 + r_k(\lambda) - t\delta]K, \tag{5}$$

where $1 + r_k(\lambda)$ is the firm's weighted average cost of capital (WACC) and

$$r_k(\lambda) = (1-t)r_d(\lambda)\left(\frac{D}{K}\right) + r_e(\lambda)\left(\frac{E}{K}\right) = (1-t)r_d(\lambda)\left(\frac{\lambda}{\lambda+1}\right) + r_e(\lambda)\left(\frac{1}{\lambda+1}\right). \tag{6}$$

The second equality in Eq. (6) follows from Eq. (1). Inspection of Eq. (6) reveals that the firm's WACC is indeed a function of λ only.

We can equivalently state Program (4) as

$$\max_{K,L,\lambda} \ \mathrm{E}[U(\tilde{W}, \tilde{Z})], \tag{7}$$

where $\tilde{W}$ is defined in Eq. (5).[4] The first-order necessary conditions for Program (7) are given by

$$\mathrm{E}\{U_W(\tilde{W}^*, \tilde{Z})[(1-t)\tilde{P}Q_K(K^*, L^*) - 1 - r_k(\lambda^*) + t\delta]\} = 0, \tag{8}$$

$$\mathrm{E}\{U_W(\tilde{W}^*, \tilde{Z})(1-t)[\tilde{P}Q_L(K^*, L^*) - w]\} = 0, \tag{9}$$

and

$$-\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})]r_k'(\lambda^*) = 0, \tag{10}$$

[4]The balance sheet identity, Eq. (1), has been substituted into $\tilde{W}$ by means of λ, as is evident from Eq. (5), where $D = \lambda K/(\lambda + 1)$ and $E = K/(\lambda + 1)$ so that $D + E = K$.

where an asterisk (*) denotes an optimal level.

Since $\mathrm{E}[U_W(\tilde{W},\tilde{Z})] > 0$, Eq. (10) reduces to $r_k'(\lambda^*) = 0$, thereby invoking our first proposition.[5]

Proposition 1: The firm's optimal debt-equity ratio, λ^*, is the one that minimizes the firm's WACC.

Proposition 1 states that the firm's optimal debt-equity ratio, λ^*, depends neither on the firm's risk attitude nor on the firm's input mix, (K^*, L^*). This optimal debt-equity ratio is governed solely by the firm's WACC, $r_k(\lambda)$, which is a function of the after-tax cost of debt, $(1-t)r_d(\lambda)$, and the cost of equity, $r_e(\lambda)$. However, the firm's optimal amount of debt, $D^* = \lambda^* K^*/(\lambda^* + 1)$, and that of equity, $E^* = K^*/(\lambda^* + 1)$, do depend on the amount of capital, K^*, optimally chosen by the firm.

From Proposition 1, we know that λ^* is the one that minimizes $r_k(\lambda)$. The firm's optimal input mix, (K^*, L^*), is thus determined by solving Eqs. (8) and (9) simultaneously. Rearranging terms of Eq. (8), we have

$$\mathrm{E}[U_W(\tilde{W}^*,\tilde{Z})\tilde{P}](1-t)Q_K(K^*,L^*) = \mathrm{E}[U_W(\tilde{W}^*,\tilde{Z})][1 + r_k(\lambda^*) - t\delta]. \tag{11}$$

Likewise, rearranging terms of Eq. (9) yields

$$\mathrm{E}[U_W(\tilde{W}^*,\tilde{Z})\tilde{P}](1-t)Q_L(K^*,L^*) = \mathrm{E}[U_W(\tilde{W}^*,\tilde{Z})](1-t)w. \tag{12}$$

Dividing Eq. (11) by Eq. (12) yields

$$\frac{Q_K(K^*,L^*)}{Q_L(K^*,L^*)} = \frac{1 + r_k(\lambda^*) - t\delta}{(1-t)w}, \tag{13}$$

where the left-hand side of Eq. (13) is the marginal rate of technical substitution. Hence, we establish the following proposition.

Proposition 2: The real and financial decisions of the firm are integrated in that the marginal rate of technical substitution equals the ratio of the marginal cost of capital and the tax-adjusted wage rate at the optimum.

Eq. (13) states that the firm equates the marginal rate of technical substitution, which is the ratio of the marginal product of capital and the marginal product of labor, to the ratio of the marginal cost of capital and the tax-adjusted wage rate at the optimum. Since the marginal cost of

[5] All proofs of propositions are relegated to the appendix.

capital depends on the optimal debt-equity ratio, λ^*, the real and financial decisions of the firm are indeed integrated.

4 The Effect of Background Risk on Firm Behavior

In this section, we examine how the presence of background risk affects the behavior of the firm. To this end, we restrict our attention to two special cases: (1) additive background risk in that the firm's utility function is given by $U(W + Z)$, and (2) multiplicative background risk in that the firm's utility function is given by $U[W(1 + Z)]$. The additive background risk can be interpreted as random initial wealth (see, e.g., Kihlstrom et al., 1981; Chavas, 1985; Wong, 1996; Battermann et al., 2008), whereas the multiplicative background risk can be interpreted as inflation risk (see, e.g., Adam-Müller, 2000, 2002). In either case, the background risk, $\tilde{Z}$, has zero mean and is independent of the output price risk, $\tilde{P}$.

Define the following derived utility function:

$$V(W) = \mathrm{E}_Z[U(W + \tilde{Z})], \tag{14}$$

in the case of additive background risk, or

$$V(W) = \mathrm{E}_Z\{U[W(1 + \tilde{Z})]\}, \tag{15}$$

in the case of multiplicative background risk, where $\mathrm{E}_Z(\cdot)$ is the expectation operator with respect to the probability distribution function of $\tilde{Z}$. Using either Eq. (14) or Eq. (15), and applying the law of iterated expectations, we can state Program (7) as

$$\max_{K,L,\lambda} \ \mathrm{E}[V(\tilde{W})], \tag{16}$$

where $\tilde{W}$ is defined in Eq. (5). If the background risk is absent, i.e., $\tilde{Z} \equiv 0$, Program (16) reduces to

$$\max_{K,L,\lambda} \ \mathrm{E}[U(\tilde{W})], \tag{17}$$

where $\tilde{W}$ is defined in Eq. (5). Inspection of Programs (16) and (17) reveals that the effect of introducing $\tilde{Z}$ on the behavior of the firm is equivalent to that of replacing the utility function, $U(W)$, by the derived utility function, $V(W)$.

It is reasonable to believe that the two utility functions, $U(W)$ and $V(W)$, are closely related. However, the theory of risk aversion developed by Arrow (1965) and Pratt (1964) is too weak to offer an intuitive linkage between $U(W)$ and $V(W)$. To resolve this problem, Gollier and Pratt (1996) introduce the concept of "risk vulnerability" for the case of additive background risk, while Franke et al. (2006) introduce the concept of "multiplicative risk vulnerability" for the other case of multiplicative background risk, both of which describe preferences under which the derived utility function, $V(W)$, is more risk averse than the original utility function, $U(W)$, in the usual Arrow-Pratt sense, i.e., $-V''(W)/V'(W) > -U''(W)/U'(W)$ for all W. Gollier and Pratt (1996) show that $U(W)$ is risk vulnerable if the Arrow-Pratt measure of absolute risk aversion, $-U''(W)/U'(W)$, is decreasing and convex in W. On the other hand, Franke et al. (2006) show that $U(W)$ is multiplicatively risk vulnerable if the Arrow-Pratt measure of relative risk aversion, $-WU''(W)/U'(W)$, is increasing and convex in W, and is everywhere less than unity.

Equipped with the concepts of risk vulnerability and multiplicative risk vulnerability, we recognize that the effect of the presence of additive background risk or multiplicative background risk on the behavior of the firm is qualitatively equivalent to that of increased risk aversion. Following Diamond and Stiglitz (1974), we work with a differentiable family of utility functions, $U(W, \rho)$, where ρ is an ordinal index of risk aversion. Given this notation, Diamond and Stiglitz (1974) show that an increase in ρ represents an increase in risk aversion if, and only if, the Arrow-Pratt measure of absolute risk aversion increases with ρ:

$$\frac{\partial}{\partial \rho}\left[-\frac{U_{WW}(W,\rho)}{U_W(W,\rho)}\right] = \frac{U_{WW}(W,\rho)U_{W\rho}(W,\rho) - U_W(W,\rho)U_{WW\rho}(W,\rho)}{U_W(W,\rho)^2} > 0. \tag{18}$$

We perform the comparative static exercise with respect to ρ, and report the results in the following proposition.

Proposition 3: If the firm's preferences are risk vulnerable (multiplicatively risk vulnerable), introducing the additive (multiplicative) background risk induces the firm to produce less. Furthermore, if capital is a normal input, the firm acquires less capital by issuing less debt and equity in the presence of the additive (multiplicative) background risk.

According to Bear (1965), capital is said to be a normal input if an increase in the output price increases the utilization of capital. It is the case when the production function, $Q(K, L)$, satisfies that $Q_L(K,L)Q_{KL}(K,L) - Q_K(K,L)Q_{LL}(K,L) > 0$. Proposition 3 is consistent with the consensus in the literature that uncertainty is output-reducing (see, e.g., Sandmo, 1971; Batra and Ullah, 1974;

Chavas, 1985; and Wong, 1996).

5 Conclusion

This paper has investigated the interaction between the production and financing decisions of the competitive firm under output price uncertainty. The firm faces additional sources of uncertainty that are aggregated into a background risk. We have shown that the firm always chooses its optimal debt-equity ratio to minimize the weighted average cost of capital, irrespective of the risk attitude of the firm and the incidence of the underlying uncertainty. Even though the introduction of the background risk has no effects on the firm's optimal debt-equity ratio and the marginal rate of technical substitution, it does affect the input mix chosen, and the amounts of debt and equity issued, by the firm. When the background risk is either additive or multiplicative, we have shown that the firm optimally produces less in the presence of the background risk if its preferences are risk vulnerable (Gollier and Pratt, 1996) or multiplicatively risk vulnerable (Franke et al., 2006), respectively. Furthermore, if capital is a normal input, we have shown that the firm optimally acquires less capital by issuing less debt and equity upon the introduction of the background risk.

From the work on monotone comparative statics (see Milgrom and Shannon, 1994; and Athey, 2002), there is a general result that any comparative statics that hold for the portfolio problem of a risk-averse investor will automatically hold for the production problem of a risk-averse firm. In light of this result, the method advanced in this paper should be applicable to many other choice problems under multiple sources of uncertainty.

Appendix

Proof of Proposition 1: Differentiating Eq. (6) with respect to λ twice and evaluating the resulting derivative at $\lambda = \lambda^*$ yields

$$r_k''(\lambda^*) = \left(\frac{1}{\lambda^* + 1}\right)\{(1-t)[2r_d'(\lambda^*) + r_d''(\lambda^*)\lambda^*] + r_e''(\lambda^*)\} > 0, \tag{A.1}$$

where we have used the fact that $r'(\lambda^*) = 0$, and the inequality follows from $r_d'(\lambda) > 0$, $r_d''(\lambda) > 0$, and $r_e''(\lambda) > 0$. As is evident from Eq. (A.1), the second order condition that λ^* minimizes $r_k(\lambda)$ is satisfied. □

Proof of Proposition 2: It remains to show that the solution, (K^*, L^*), satisfies the second-order sufficient conditions for Program (7). Note that

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial K^2} = \mathrm{E}\{U_{WW}(\tilde{W}^*, \tilde{Z})[(1-t)\tilde{P}Q_K(K^*, L^*) - 1 - r_k(\lambda^*) + t\delta]^2\}$$

$$+\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})(1-t)\tilde{P}Q_{KK}(K^*, L^*)] < 0, \tag{A.2}$$

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial L^2} = \mathrm{E}\{U_{WW}(\tilde{W}^*, \tilde{Z})(1-t)^2[\tilde{P}Q_L(K^*, L^*) - w]^2\}$$

$$+\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})(1-t)\tilde{P}Q_{LL}(K^*, L^*)] < 0, \tag{A.3}$$

and

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial K \partial L} = \mathrm{E}\{U_{WW}(\tilde{W}^*, \tilde{Z})[(1-t)\tilde{P}Q_K(K^*, L^*) - 1 - r_k(\lambda^*) + t\delta]$$

$$\times(1-t)[\tilde{P}Q_L(K^*, L^*) - w]\} + \mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})(1-t)\tilde{P}Q_{KL}(K^*, L^*)], \tag{A.4}$$

where the inequalities follow from the assumptions on $U(W, Z)$ and $Q(K, L)$. Using Eq. (13), we can write Eqs. (A.2) and (A.4) as

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial K^2} = \mathrm{E}\{U_{WW}(\tilde{W}^*, \tilde{Z})(1-t)^2[\tilde{P}Q_L(K^*, L^*) - w]^2\}\left[\frac{Q_K(K^*, L^*)}{Q_L(K^*, L^*)}\right]^2$$

$$+\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})(1-t)\tilde{P}Q_{KK}(K^*, L^*)], \tag{A.5}$$

and

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial K \partial L} = \mathrm{E}\{U_{WW}(\tilde{W}^*, \tilde{Z})(1-t)^2[\tilde{P}Q_L(K^*, L^*) - w]^2\}\left[\frac{Q_K(K^*, L^*)}{Q_L(K^*, L^*)}\right]$$

$$+\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})(1-t)\tilde{P}Q_{KL}(K^*, L^*)]. \tag{A.6}$$

Using Eqs. (A.3), (A.5), and (A.6), we have

$$\Delta = \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial K^2} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial L^2} - \left\{\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \tilde{Z})]}{\partial K \partial L}\right\}^2$$

$$= \mathrm{E}\{U_{WW}(\tilde{W}^*, \tilde{Z})[\tilde{P}Q_L(K^*, L^*) - w]^2\}\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})\tilde{P}](1-t)^3$$

$$\times\left\{Q_{KK}(K^*, L^*) - 2\left[\frac{Q_K(K^*, L^*)}{Q_L(K^*, L^*)}\right]Q_{KL}(K^*, L^*) + \left[\frac{Q_K(K^*, L^*)}{Q_L(K^*, L^*)}\right]^2 Q_{LL}(K^*, L^*)\right\}$$

$$+\mathrm{E}[U_W(\tilde{W}^*, \tilde{Z})\tilde{P}](1-t)^2[Q_{KK}(K^*, L^*)Q_{LL}(K^*, L^*) - Q_{KL}(K^*, L^*)^2] > 0, \tag{A.7}$$

where the inequality follows from the assumptions on $U(W, Z)$ and $Q(K, L)$. As is evident from Eqs. (A.2), (A.3), and (A.7), the solution, (K^*, L^*) satisfies the second-order sufficient conditions for Program (7). □

Proof of Proposition 3: We replace the utility function in program (7) with the differentiable family of utility functions, $U(W, \rho)$. From Proposition 1, the optimal debt-equity ratio, λ^*, does not depend on the firm's preferences. The firm's optimal input mix, (K^*, L^*), is governed by solving Eqs. (8) and (9) simultaneously, where $U(W, Z)$ is replaced by $U(W, \rho)$. Totally differentiating Eqs. (8) and (9) with respect to ρ and using Cramer's rule yields

$$\frac{\mathrm{d}K^*}{\mathrm{d}\rho} = \frac{1}{\Delta}\left\{\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K \partial L} - \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K \partial \rho} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L^2}\right\}, \tag{A.8}$$

and

$$\frac{\mathrm{d}L^*}{\mathrm{d}\rho} = \frac{1}{\Delta}\left\{\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K \partial \rho} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K \partial L} - \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K^2}\right\}, \tag{A.9}$$

where $\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]/\partial L^2 < 0$, $\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]/\partial K^2 < 0$, $\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]/\partial K \partial L$, and $\Delta > 0$ are given by Eqs. (A.3), (A.5), (A.6), and (A.7), respectively, with $U(W, Z)$ replaced by $U(W, \rho)$,

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K \partial \rho} = \mathrm{E}\{U_{W\rho}(\tilde{W}^*, \rho)[(1-t)\tilde{P}Q_K(K^*, L^*) - 1 - r_k(\lambda^*) + t\delta]\}, \tag{A.10}$$

and

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho} = \mathrm{E}[U_{W\rho}(\tilde{W}^*, \rho)(1-t)(\tilde{P}Q_L(K^*, L^*) - w)]. \tag{A.11}$$

Substituting Eqs. (13) and (A.11) into Eq. (A.10) yields

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial K \partial \rho} = \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho} \times \frac{Q_K(K^*, L^*)}{Q_L(K^*, L^*)}. \tag{A.12}$$

Let $J(W, \rho) = U_{W\rho}(W, \rho)/U_W(W, \rho)$. Note first that

$$J_W(W, \rho) = \frac{U_W(W, \rho)U_{WW\rho}(W, \rho) - U_{W\rho}(W, \rho)U_{WW}(W, \rho)}{U_W(W, \rho)^2} < 0, \tag{A.13}$$

where the inequality follows from Eq. (18). Define $\bar{W}^*$ as $\tilde{W}^*$ evaluated at $\tilde{P} = w/Q_L(K^*, L^*)$. Using $J(W, \rho)$ and Eq. (9), we can write Eq. (A.11) as

$$\frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho} = \mathrm{E}\{[J(\tilde{W}^*, \rho) - J(\bar{W}^*, \rho)]U_W(\tilde{W}^*, \rho)(1-t)[\tilde{P}Q_L(K^*, L^*) - w]\}. \tag{A.14}$$

Since $J(W, \rho)$ is decreasing in W from Eq. (A.13) and $\tilde{W}^*$ is increasing in P, the sign of $J(\tilde{W}^*, \rho) - J(\bar{W}^*, \rho)$ must be opposite to that of $\tilde{P}Q_L(K^*, L^*) - w$. Thus, the right-hand side of Eq. (A.14) is unambiguously negative so that $\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]/\partial L \partial \rho < 0$.

Substituting Eqs. (A.3), (A.6), and (A.12) into Eq. (A.8) yields

$$\frac{\mathrm{d}K^*}{\mathrm{d}\rho} = \frac{\mathrm{E}[U_W(\tilde{W}^*, \rho)\tilde{P}](1-t)}{\Delta Q_L(K^*, L^*)} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho}$$
$$\times [Q_L(K^*, L^*)Q_{KL}(K^*, L^*) - Q_K(K^*, L^*)Q_{LL}(K^*, L^*)]. \tag{A.15}$$

Likewise, substituting Eqs. (A.5), (A.6), and (A.12) into Eq. (A.9) yields

$$\frac{\mathrm{d}L^*}{\mathrm{d}\rho} = \frac{\mathrm{E}[U_W(\tilde{W}^*, \rho)\tilde{P}](1-t)}{\Delta Q_L(K^*, L^*)} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho}$$
$$\times [Q_K(K^*, L^*)Q_{KL}(K^*, L^*) - Q_L(K^*, L^*)Q_{KK}(K^*, L^*)]. \tag{A.16}$$

Totally differentiating $Q(K^*, L^*)$ with respect to ρ yields

$$\frac{\mathrm{d}Q(K^*, L^*)}{\mathrm{d}\rho} = Q_K(K^*, L^*) \times \frac{\mathrm{d}K^*}{\mathrm{d}\rho} + Q_L(K^*, L^*) \times \frac{\mathrm{d}L^*}{\mathrm{d}\rho}$$
$$= \frac{\mathrm{E}[U_W(\tilde{W}^*, \rho)\tilde{P}](1-t)}{\Delta Q_L(K^*, L^*)} \times \frac{\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]}{\partial L \partial \rho}$$
$$\times \Big\{ 2Q_K(K^*, L^*)Q_L(K^*, L^*)Q_{KL}(K^*, L^*)$$
$$- Q_L(K^*, L^*)^2 Q_{KK}(K^*, L^*) - Q_K(K^*, L^*)^2 Q_{LL}(K^*, L^*) \Big\}, \tag{A.17}$$

where the second equality follows from Eqs. (A.15) and (A.16). Since $\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]/\partial L \partial \rho < 0$, it follows from the assumptions on $Q(K, L)$ and Eq. (A.17) that $\mathrm{d}Q(K^*, L^*)/\mathrm{d}\rho < 0$.

If capital is a normal input, we have $Q_L(K, L)Q_{KL}(K, L) - Q_K(K, L)Q_{LL}(K, L) > 0$ (see Bear, 1965). Since $\partial^2 \mathrm{E}[U(\tilde{W}^*, \rho)]/\partial L \partial \rho < 0$, Eq. (A.15) implies that $\mathrm{d}K^*/\mathrm{d}\rho < 0$. Since $D^* = \lambda^* K^*/(\lambda^* + 1)$ and $E^* = K^*/(\lambda^* + 1)$, it follows from Proposition 1 and $\mathrm{d}K^*/\mathrm{d}\rho < 0$ that $\mathrm{d}D^*/\mathrm{d}\rho < 0$ and $\mathrm{d}E^*/\mathrm{d}\rho < 0$. □

References

Adam-Müller, A. F. A. (2000): "Hedging Price Risk When Real Wealth Matters." *Journal of International Money and Finance* 19: 549–560.

Adam-Müller, A. F. A. (2002): "What to Do If a Dollar Is Not a Dollar? The Impact of Inflation Risk on Production and Risk Management." *Journal of Futures Markets* 22: 371–386.

Arrow, K.J. (1965): *Aspects of the Theory of Risk-Bearing.* Helsinki: Yrjo Jahnsson Saatio.

Athey, S. (2002): "Monotone Comparative Statics under Uncertainty." *Quarterly Journal of Economics* 62: 187–223.

Barnea, A., Haugen, R. A., and Senbet, L. W. (1981): "An Equilibrium Analysis of Debt Financing under Costly Tax Arbitrage and Agency Problems." *Journal of Finance* 36: 569–581.

Batra, R. N., and Ullah, A. (1974): "Competitive Firm and the Theory of Input Demand under Price Uncertainty." *Journal of Political Economy* 82: 537–548.

Battermann, H. L., Broll, U. and Wahl, J. E. (2008): "Utility Functions of Equivalent Form and the Effect of Parameter Changes on Optimum Decision Making." *Economic Theory* 34: 401–414.

Bear, D. V. T. (1965): "Inferior Inputs and the Theory of the Firm." *Journal of Political Economy* 73: 287–289.

Brennan, M. J., and Schwartz, E. S. (1978): "Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure." *Journal of Business* 51: 103–114.

Chavas, J.-P. (1985): "On the Theory of the Competitive Firm under Uncertainty When Initial Wealth Is Random." *Southern Economic Journal* 51: 818–827.

Cooper, I., and Franks, J. R. (1983): "The Interaction of Financing and Investment Decisions When the Firm Has Unused Tax Credits." *Journal of Finance* 38: 571–583.

Dammon, R. M., and Senbet, L. W. (1988): "The Effect of Taxes and Depreciation on Corporate Investment and Financial Leverage." *Journal of Finance* 43: 357–373.

DeAngelo, H., and Masulis, R. W. (1980): "Optimal Capital Structure under Corporate and Personal Taxation." *Journal of Financial Economics* 8: 3–29.

Diamond, P. A., and Stiglitz, J. E. (1974): "Increases in Risk and in Risk Aversion." *Journal of Economic Theory* 8: 337–360.

Dotan, A., and Ravid, S. A. (1985): "On the Interaction of Real and Financial Decisions of the Firm under Uncertainty." *Journal of Finance* 40: 501–517.

Franke, G., Schlesinger, H., and Stapleton, R. C. (2006): "Multiplicative Background Risk." *Man-*

agement Science 52: 146–153.

Gollier, C., and Pratt, J. W. (1996): "Risk Vulnerability and the Tempering Effect of Background Risk." *Econometrica* 64: 1109–1123.

Hartman, R. (1976): "Factor Demand with Output Price Uncertainty." *American Economic Review* 66: 675–681.

Hite, G. L. (1977): "Leverage, Output Effects, and the M-M Theorem." *Journal of Financial Economics* 4: 177–202.

Jensen, M. C., and Meckling, W. H. (1976): "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* 3: 305–360.

Kihlstrom, R. E., Romer, D., and Williams, S. (1981): "Risk Aversion with Random Initial Wealth." *Econometrica* 49: 911–920.

Kraus, A., and Litzenberger, R. H. (1973): "A State Preference Model of Optimal Financial Leverage." *Journal of Finance* 28: 911–921.

Miller, M. H. (1977): "Debt and Taxes." *Journal of Finance* 32: 261–275.

Milgrom, P., and Shannon, C. (1994): "Monotone Comparative Statics." *Econometrica* 62: 157–180.

Modigliani, F., and Miller, M. H. (1958): "The Cost of Capital, Corporation Finance, and the Theory of Investment." *American Economic Review* 48: 261–297.

Modigliani, F., and Miller, M. H. (1963): "Corporate Income Taxes and the Cost of Capital: A Correction." *American Economic Review* 53: 433–443.

Myers, S. C. (1974): "Interactions of Corporate Financing and Investment Decisions—Implications for Capital Budgeting." *Journal of Finance* 29: 1–25.

Myers, S. C., and Majluf, N. S. (1984): "Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have." *Journal of Financial Economics* 13: 187–221.

Narayanan, M. P. (1988): "Debt Versus Equity under Asymmetric Information." *Journal of Financial and Quantitative Analysis* 23: 39–51.

Noe, T. H. (1988): "Capital Structure and Signaling Game Equilibria." *Review of Financial Studies* 1: 331–355.

Pratt, J. W. (1964): "Risk Aversion in the Small and in the Large." *Econometrica* 32: 122–136.

Sandmo, A. (1971): "On the Theory of the Competitive Firm under Price Uncertainty." *American Economic Review* 61: 65–73.

Scott, J. H., Jr. (1976): "A Theory of Optimal Capital Structure." *Bell Journal of Economics and Management Science* 7: 33–54.

Silberberg, E., and Suen, W. (2001): *The Structure of Economics: A Mathematical Analysis*, 3rd ed. New York: McGraw-Hill.

Turnovsky, S. J. (1973): "Production Flexibility, Price Uncertainty and the Behavior of the Competitive Firm." *International Economic Review* 14: 395–413.

Wong, K. P. (1996): "Background Risk and the Theory of the Competitive Firm under Uncertainty." *Bulletin of Economic Research* 48: 241–251.

Dresden Discussion Paper Series in Economics

10/08 **Lehmann-Waffenschmidt, B. Cornelia:** An Evolutionary Agent-Based Simulation Model for the Industry Lifecycle

11/08 **Mukherjee, Vivekananda / Ramani, Shyama V.:** R&D Cooperation, Asymmetrical Technological Capabilities and Rationale for Technology Parks

12/08 **Lehmann-Waffenschmidt, Marco / Roth, Gerhard / Thießen, Friedrich:** Die (innere) Logik des Entscheidens – Zur neurobiologischen Fundierung ökonomischer Entscheidungen

13/08 **Dopfer, Kurt:** Zur Neubestimmung von Geschichte und Theoriegeschichte in den Wirtschaftswissenschaften – Ko-evolution von Wirtschaftsbeobachtung und Wirtschaftswirklichkeit

14/08 **Lessmann, Christian / Markwardt, Gunther:** One Size fits all? Decentralization, Corruption, and the Monitoring of Bureaucrats

15/08 **Farzanegan, Mohammad Reza / Markwardt, Gunther:** The Effects of Oil Price Shocks on the Iranian Economy

16/08 **Fuchs, Michaela / Wohlrabe, Klaus:** Instituitions, Trade and Integration: What can be expected within the enlarged EU?

01/09 **Broll, Udo / Egozcue, Martin / Wong, Wing-Keung:** Prospect Theory and Two Moment Model: the Firm under Price Uncertainty

02/09 **Broll, Udo / Eckwert, Berhard:** Modelling Information and Hedging: the Exporting Firm

03/09 **Binswanger, Hans Christoph:** Die Wachstumsspirale in der Krise – Ansätze zu einem nachhaltigem Wachstum

04/09 **Brunow, Stefan / Hirte, Georg:** Regional Age Structure and Economic Growth: An Econometric Study for German Regions

05/09 **Broll, Udo / Kemnitz, Alexander / Mukherjee, Vivekananda:** Globalization and a Welfare Program for the Marginalized

06/09 **Tscharaktschiew, Stefan / Hirte, Georg:** An Urban General Equilibrium Model with Multiple Household Structures and Travel Mode Choice

07/09 **Tscharaktschiew, Stefan / Hirte, Georg:** How does the Household Structure Shape the Urban Economy?

08/09 **Lessmann, Christian:** Fiscal Decentralization and Regional Disparity: Evidence from Cross-section and Panel Data

09/09 **Lessmann, Christian / Markwardt, Gunther:** Aid, Growth and Decentralization

10/09 **Broll, Udo / Wahl, Jack E. / Wessel, Christoph:** Export and Benefits of Hedging in Emerging Economies

11/09 **Rudolph, Stephan:** The Gravity Equation with Micro-Founded Trade Costs

12/09 **Biswas, Amit K.:** Import Tariff Led Export Under-invoicing: A Paradox

13/09 **Broll, Udo / Wahl, Jack E.:** Mitigation of Foreign Direct Investment – Risk and Hedging

14/09 **Broll, Udo / Wahl, Jack E.:** Güterwirtschaftliches Risikomanagement: - Ein Entscheidungsmodell zur Lagerpolitik bei Unsicherheit

15/09 **Lukas, Daniel:** Efficiency Effects of Cross-Border Medical Demand

16/09 **Broll, Udo / Bieta, Volker / Milde, Hellmuth / Siebe, Wilfried:** Strategic Pricing of Financial Options

16/09 **Broll, Udo / Bieta, Volker / Milde, Hellmuth / Siebe, Wilfried:** Strategic Pricing of Financial Options

17/09 **Broll, Udo / Wahl, Jack E.:** Liquidity Constrained Exporters: Trade and Futures Hedging

01/10 **Rudolph, Stephan:** Estimating Gravity Equations with Endogenous Trade Costs

02/10 **Lukas, Daniel / Werblow, Andreas:** Grenzen der Spezialisierung grenzüberschreitender Gesundheitsversorgung im Rahmen des Heckscher-Ohlin Modells

03/10 **Broll, Udo / Roldán-Ponce, Antonio / Wahl, Jack E.:** Spatial Allocation of Capital: The Role of Risk Preferences