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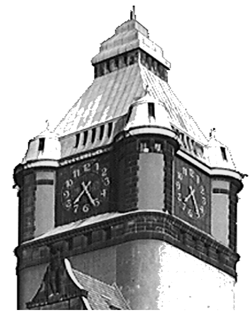

Prospect Theory and Hedging Risks

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Abstract:

The prospect theory is one of the most popular decision-making theories. It is based on the S-shaped utility function, unlike the von Neumann and Morgenstern (NM) theory, which is based on the concave utility function. The S-shape brings in mathematical challenges: simple extensions and generalizations of NM theory into the prospect theory cannot be frequently achieved. For example, the nature of monotonicity of the indifference curve depends on the underlying mean. Price hedging decisions also become more complex within the prospect theory. We discuss these topics in detail and offer a general result concerning the sign of a covariance from which we then infer desired properties of the indifference curve and also justify hedging decisions within the prospect theory. We illustrate our general considerations with a thoroughly worked out example.

JEL-Classification: D01, D03, D21, D81

Keywords: Prospect theory, mean-variance model, indifference curve, price uncertainty, hedging

1 Introduction

The pioneering work of Markowitz (1952a) on the mean-variance (MV) portfolio selection is a milestone in modern finance theory for optimal portfolio construction, asset allocation, and investment diversification. The theory is based on the assumption that investors allocate their wealth across the available assets in order to maximize their expected utilities. For details, we refer to the monograph by Markowitz (1959). The Markowitz MV portfolio theory has laid a basis for many financial economics advances, including the Sharpe-Lintner capital asset pricing model (Sharpe, 1964; Lintner, 1965) and the optimal one-fund theorem (Tobin, 1958). For retrospective and enlightening views on the theory, we refer to Markowitz (1991, 1999) and Rubinstein (2002).

According to the von Neumann and Morgenstern (NM) theory, utility functions of risk averters and risk seekers are concave and convex, respectively, and in both cases they are increasing functions. Examining the relative attractiveness of various forms of investments, Friedman and Savage (1948) note that the strictly concave functions may not be able to explain why investors buy insurance or lottery tickets. Markowitz (1952b) addresses the Friedman and Savage concern and proposes utility functions that have convex and concave regions in both the positive (i.e, gains) and the negative (i.e., losses) domains.

Kahneman and Tversky (1979), and Tversky and Kahneman (1992) put forward arguments in favour of utility functions that are concave for gains and convex for losses, thus yielding what is called the S-shaped utility function, or value function. These authors have initiated a formal theory of loss aversion, called the prospect theory, in which investors maximize the expectation of the S-shaped utility function. Throughout this paper, we call investors with S-shaped utility functions prospect investors, or investors with prospect preferences.

The prospect theory is one of the most popular decision-making theories and has gained much attention from economists and professionals in the financial sector. It has become influential in explaining a wide range of phenomena that could not be properly explained within the traditional expected utility framework. These include the disposition effect, asymmetric price elasticities, elasticities of labor supply that are inconsistent with standard models of labor supply and the excess sensitivity of consumption to income (cf., e.g., Camerer, 2000), financial anomalies (cf., e.g., Thaler, 2005).

In this paper we tackle two intertwined topics within the prospect theory: first, monotonicity of the indifference curve, and second, strategies for hedging price risks. Specifically, in Section 2 we investigate the indifference curve for investors with S-shaped utility functions: its monotonicity properties appear to be distinctly different from those of investors who are either risk-averse or risk-seeking on the entire gain-loss domain. In Section 3 we study a closely related, as elucidated by Meyer and Robison (1988), topic of hedging strategies. From the mathematical point of view, the thread connecting Sections 2 and 3 is the covariance $\mathbf{Cov}[\Pi, u'(\Pi)]$ where Π is a random variable whose meaning depends on the context and thus, in our case, on whether we deal with the indifference curve or the risk hedging. Lemma 2.1 is the main technical result from which the desired properties of $\mathbf{Cov}[\Pi, u'(\Pi)]$ follow. A proof of the lemma is given in the Appendix.

2 Indifference curves and related covariances

Here we deal with a random variable Π, which can be viewed as profit or wealth, depending on a context. We shall view positive outcomes of Π as gains and negative ones as losses. In this situation, investors with S-shaped utility functions, which imply declining sensitivity in both gains and losses, are viewed as risk averse for gains but risk seeking for losses.

2.1 Indifference curves under the NM model

We start with the location-scale family

$$\mathcal{D}_X = \big\{\sigma X + \mu : \mu \in \mathbf{R},\ \sigma > 0\big\},$$

where X is a random variable with mean zero and variance one. Consequently, for any $\Pi \in \mathcal{D}_X$, the expected utility $\mathbf{E}[u(\Pi)]$ defines a two-argument function:

$$V(\sigma, \mu) = \mathbf{E}[u(\sigma X + \mu)].$$

For any constant α, the indifference curve $\mu = \mu(\sigma)$, drawn on the (σ, μ) plane, is given by

$$C_\alpha = \big\{(\sigma, \mu) \mid V(\sigma, \mu) = \alpha\big\}\ .$$

Tobin (1958) finds that the indifference curve is convex for risk-averse investors and concave for risk-seeking investors for NM utility functions and normally distributed prospects. This theory has been further developed by Schneeweiss (1967) and Sinn (1983). In addition, Meyer (1987) and Levy

(1989) compare assets with distributions differing only by location and scale parameters while analyzing the class of general utility functions with only convexity or concavity assumptions. Sinn (1990) finds that decreasing absolute risk aversion implies that the slope of the indifference curve declines with an increase in μ, given a positive σ. Wong (2006) studies the shape of the indifference curve for risk averters, risk seekers, and risk neutral investors for generalized utility functions as stated in Meyer (1987). Wong and Ma (2008) further extend the work on the location-scale family with general multiple random seed sources and develop geometrical and topological properties of the location-scale expected utility functions.

Proceeding with our main discussion, we note the equation $V_\mu(\sigma, \mu)\, \partial\, \mu + V_\sigma(\sigma, \mu)\, \partial\, \sigma = 0$, and thus the slope of the indifference curve is given by the formula

$$S(\sigma, \mu) \equiv \frac{\partial\, \mu}{\partial\, \sigma} = \frac{-V_\sigma(\sigma, \mu)}{V_\mu(\sigma, \mu)},$$

where

$$V_\sigma(\sigma, \mu) \equiv \frac{\partial\, V(\sigma, \mu)}{\partial\, \sigma} = \mathbf{E}[X u'(\sigma X + \mu)],$$
$$V_\mu(\sigma, \mu) \equiv \frac{\partial\, V(\sigma, \mu)}{\partial\, \mu} = \mathbf{E}[u'(\sigma X + \mu)].$$

When studying the shape of the indifference curve for risk averse and risk seeking investors, a number of researchers (see Wong, 2006; Sriboonchita *et al.*, 2009; and references therein) have established the following result.

Proposition 2.1 *Let* $\Pi \in \mathcal{D}_X$. *Then for any twice continuously differentiable utility function* u*, the indifference curve can be parameterized as*

$\mu = \mu(\sigma)$ with the slope

$$S(\sigma, \mu) = \frac{-\mathbf{E}[Xu'(\sigma X + \mu)]}{\mathbf{E}[u'(\sigma X + \mu)]}.$$

Furthermore, the following statements hold:

- *For any risk-averse investor (i.e., $u''(x) \leq 0$ for all $x \in \mathbf{R}$), the indifference curve $\mu = \mu(\sigma)$ is an increasing and convex function of σ.*
- *For any risk-seeking investor (i.e., $u''(x) \geq 0$ for all $x \in \mathbf{R}$), the indifference curve $\mu = \mu(\sigma)$ is a decreasing and concave function of σ.*

Proposition 2.1 implies that for risk-averse or risk-seeking investors, the shape of their indifference curves does not change its nature depending on the value of μ, and in particular depending on the sign of μ.

2.2. Indifference curves under the prospect theory

In contrast to what we have seen in the previous subsection, and in particular in Proposition 2.1, the following theorem shows that monotonicity of the indifference curve changes depending on the sign of the mean μ when the utility function is S-shaped.

Theorem 2.1 *Let $\Pi \in \mathcal{D}_X$, where X is a symmetric around 0 random variable with unit variance. Let u be S-shaped, and let the first derivative u' be symmetric around 0, that is, $u'(x) = u'(-x)$ for all $x \in \mathbf{R}$. Then we have the following statements:*

- *If $\mu \geq 0$, then $V_\sigma \leq 0$, and so the slope $S(\mu, \sigma) \geq 0$. Furthermore, the indifference function $\mu = \mu(\sigma)$ is increasing in σ.*

- *If* $\mu \leq 0$, *then* $V_\sigma \geq 0$, *and so the slope* $S(\mu, \sigma) \leq 0$. *Furthermore, the indifference function* $\mu = \mu(\sigma)$ *is decreasing in* σ.

Theorem 2.1 follows from the following fundamental (for our paper) lemma, via the following representations

$$V_\sigma(\sigma, \mu) = \frac{\mathbf{Cov}[\Pi, u'(\Pi)]}{\sigma},$$
$$S(\sigma, \mu) = \frac{-\mathbf{Cov}[\Pi, u'(\Pi)]}{\sigma \mathbf{E}[u'(\Pi)]},$$

that hold when $\Pi \in \mathcal{D}_X$ (or equivalently X) is symmetric.

Lemma 2.1 *Let the distribution of* Π *be symmetric (around its mean* $\mu = \mathbf{E}[\Pi]$*). Furthermore, let the utility function* u *be* S*-shaped, and let the derivative* u' *be symmetric around* 0*, that is,* $u'(x) = u'(-x)$ *for all* $x \in \mathbf{R}$. *Then*

- $\mu \geq 0$ *implies* $\mathbf{Cov}[\Pi, u'(\Pi)] \leq 0$, *and*
- $\mu \leq 0$ *implies* $\mathbf{Cov}[\Pi, u'(\Pi)] \geq 0$.

Lemma 2.1 will also play a pivotal role in the next section. The proof of the lemma is relegated to the end of this paper.

3 Hedging price risk

The seminal paper by Sandmo (1971) analyzes conditions for optimal production of a competitive firm under price uncertainty. Holthausen (1979), Feder, Just and Schmitz (1980), Kawai and Zilcha (1986), Wong (2007), Broll, Clark and Lukas (2010) extend Sandmo's analysis to study firm's hedging behavior and develop what is known as separation property: in the presence of future markets, the optimal production is independent of the distribution of random prices and the firm's degree of risk aversion. Broll and Eckwert (2008) demonstrate how market transparency and information affect the production and hedging decision.

We shall next introduce and work with a model analyzed by Holthausen (1979), Feder, Just and Schmitz (1980), Hey (1981), Meyer and Robison (1988), and many others, but we shall treat it within the prospect theory. That is, we shall deal with S-shaped utility functions, which result in more complex decisions than those in the case of the classical concave utility function.

3.1 The model

Let Q be the amount of output produced by a company, and we assume that Q is known. Let $C(Q)$ be the cost of producing Q, which is also known. We assume that the output can be sold either at a random market price P or hedged in the forward market at a fixed price P_0. Let H denote the amount of hedged output. Then the firm's profit, which is a function of H, is given by

$$\Pi(H) = P(Q - H) + P_0 H - C(Q). \tag{3.1}$$

The amount $H \in \mathbf{R}$ of hedged output can be any real number: $H \in [0, Q]$ if a part of the output, or the entire output, is hedged without speculation, and $H < 0$ or $H > Q$ if speculation is involved.

The firm wants to maximize its expected profit $\mathbf{E}[u(\Pi(H))]$ with respect to H. In other words, we want to know what amount of output that needs to be hedged so that the expected utility is maximized. Hence, the firm is interested in maximizing the function

$$\rho(H) = \mathbf{E}\big[u(P(Q-H) + P_0 H - C(Q))\big].$$

Critical points of this function are solutions in hedging H of the equation $(\partial/\partial H)\rho(H,Q) = 0$, and we denote such points by H_0. The latter equation can equivalently be rewritten as follows:

$$\frac{\mathbf{E}\big[Pu'(\Pi(H_0))\big]}{\mathbf{E}\big[u'(\Pi(H_0))\big]} = P_0, \tag{3.2}$$

where we have assumed that the first derivative u' exists and the expectation $\mathbf{E}[u'(\Pi(H_0))]$ is non-zero, that is, positive.

In general, finding H_0 is a complex task. Nevertheless, equation (3.2) already tells us a remarkable story, as we shall see in the next subsection. Later, in Subsection Example, we shall have an illustrative example, where an explicit formula for H_0 is derived. Whether or not the critical point H_0 maximizes the expected utility $\rho(H)$ will be discussed in Subsection 3.4.

3.2 Speculate or not?

To begin with, we rewrite equation (3.2) as follows:

$$P_0 - \mathbf{E}[P] = \frac{\mathbf{Cov}[P, u'(\Pi(H_0))]}{\mathbf{E}[u'(\Pi(H_0))]}. \tag{3.3}$$

Since $(P - \mathbf{E}[P])(Q - H_0) = \Pi(H_0) - \mathbf{E}[\Pi(H_0)]$, we therefore have from equation (3.3) that

$$\big(P_0 - \mathbf{E}[P]\big)\big(Q - H_0\big) = \frac{\mathbf{Cov}[\Pi(H_0), u'(\Pi(H_0))]}{\mathbf{E}[u'(\Pi(H_0))]}. \tag{3.4}$$

Hence, the sign of the covariance $\mathbf{Cov}[\Pi(H_0), u'(\Pi(H_0))]$ determines the sign of the product $(P_0 - \mathbf{E}[P])(Q - H_0)$. For example, if the utility function

u is concave on the entire real line, then u' is non-increasing, and thus $\mathbf{Cov}[\Pi(H_0), u'(\Pi(H_0))] \leq 0$. This implies (cf. Hey, 1981, statement (11)) the following statements and their interpretations:

- If $P_0 < \mathbf{E}[P]$, then $H_0 \leq Q$ (speculation if $H_0 < 0$, and no speculation if $0 \leq H_0 \leq Q$). Likewise, if $H_0 < Q$, then $P_0 \leq \mathbf{E}[P]$ (normal backwardation).

- If $P_0 > \mathbf{E}[P]$, then $H_0 \geq Q$ (speculation if $H_0 > Q$). Likewise, if $H_0 > Q$, then $P_0 \geq \mathbf{E}[P]$ (contango).

When u is more complexly shaped (than just being concave), then determining the sign of the covariance is a challenging task. In the case of S-shaped utility functions, Lemma 2.1 provides an answer. In the current context, the mean μ depends on H_0 and is expressed as follows:

$$\begin{aligned}\mu(H_0) &= (\mathbf{E}[P] - P_0)(Q - H_0) + P_0 Q - C(Q) \\ &= \frac{-\mathbf{Cov}[\Pi(H_0), u'(\Pi(H_0))]}{\mathbf{E}[u'(\Pi(H_0))]} + P_0 Q - C(Q). \end{aligned} \tag{3.5}$$

Combining Lemma 2.1 with equation (3.5), we obtain the following corollary.

Corollary 3.1 *Let the distribution of P be symmetric (around its mean $\mathbf{E}[P]$). Let u be S-shaped, and let the derivative u' be symmetric around 0, that is, $u'(x) = u'(-x)$ for all real x. Then we have the following two statements:*

1. *Assuming that $\mu(H_0) \geq 0$, then:*

   (a) *If $P_0 < \mathbf{E}[P]$, then $H_0 \leq Q$, but if $P_0 > \mathbf{E}[P]$, then $H_0 \geq Q$.*

(b) *If* $H_0 < Q$*, then* $P_0 \leq \mathbf{E}[P]$*, but if* $H_0 > Q$*, then* $P_0 \geq \mathbf{E}[P]$*.*

2. *Assuming that* $\mu(H_0) \leq 0$*, then:*

   (a) *If* $P_0 < \mathbf{E}[P]$*, then* $H_0 \geq Q$*, but if* $P_0 > \mathbf{E}[P]$*, then* $H_0 \leq Q$*.*

   (b) *If* $H_0 < Q$*, then* $P_0 \geq \mathbf{E}[P]$*, but if* $H_0 > Q$*, then* $P_0 \leq \mathbf{E}[P]$*.*

The first part of Corollary 3.1 is probably the most interesting from the practical point of view, because it deals with the case when the expected profit $\mu(H_0)$ is non-negative. The conclusion of the part is of course trivial when $P_0 Q \leq C(Q)$, which implies that the cost of producing the amount Q is 'too high'. When the cost $C(Q)$ is 'normal', that is, $C(Q) < P_0 Q$, then the assumption $\mu(H_0) \geq 0$ does not trivially imply the positivity of the product $(\mathbf{E}[P] - P_0)(Q - H_0)$, thus making the conclusion of the first part of Corollary 3.1 non-trivial. Analogous considerations apply to the second part of the corollary.

3.3 An illustrative example

Assume that u is twice differentiable, and let P follow a normal distribution. Thus, $\Pi(H_0)$ also follows a normal distribution. By a classical Rubinstein-Stein's result (Rubinstein, 1973; Stein, 1973; see also Rubinstein, 1976; Stein, 1981), we have that

$$\mathbf{Cov}[\Pi(H_0), u'(\Pi(H_0))] = \mathbf{Var}[\Pi(H_0)]\mathbf{E}[u''(\Pi(H_0))].$$

Hence, equation (3.4) can be written as follows:

$$(\mathbf{E}[P] - P_0)(Q - H_0) = -\mathbf{Var}[\Pi(H_0)]\frac{\mathbf{E}[u''(\Pi(H_0))]}{\mathbf{E}[u'(\Pi(H_0))]}. \tag{3.6}$$

To proceed, we assume that the utility function u is given by the formula

$$u(x) = \Phi(x) - 1/2,$$

where Φ is the standard normal distribution function. Obviously, u is S-shaped, and we also check that $u''(x) = -xu'(x)$. Hence, we can rewrite equation (3.6) as follows:

$$(\mathbf{E}[P] - P_0)(Q - H_0) = \mathbf{Var}[\Pi(H_0)] \frac{\mathbf{E}[\Pi(H_0)u'(\Pi(H_0))]}{\mathbf{E}[u'(\Pi(H_0))]}. \tag{3.7}$$

Using formula (3.1) with $H = H_0$ and then recalling equation (3.2), we have from equation (3.7) that

$$\begin{aligned}(\mathbf{E}[P] - P_0)(Q - H_0) &= \mathbf{Var}[\Pi(H_0)] \left(\frac{\mathbf{E}[Pu'(\Pi(H_0))]}{\mathbf{E}[u'(\Pi(H_0))]}(Q - H_0) \right. \\ &= \left. + P_0 H_0 - C(Q) \right) \\ &= \mathbf{Var}[\Pi(H_0)]\big(P_0(Q - H_0) + P_0 H_0 - C(Q)\big) \\ &= \mathbf{Var}[P](Q - H_0)^2\big(P_0 Q - C(Q)\big).\end{aligned}$$

In summary, we have derived the equation $\mathbf{E}[P] - P_0 = \mathbf{Var}[P](Q - H_0)(P_0 Q - C(Q))$, whose solution in H_0 is given by the formula

$$H_0 = Q - \frac{\mathbf{E}[P] - P_0}{\mathbf{Var}[P]\big(P_0 Q - C(Q)\big)}. \tag{3.8}$$

Plugging in this H_0 into the right-hand side of the first equation in (3.5), we have the following formula for the mean:

$$\mu(H_0) = \frac{(\mathbf{E}[P] - P_0)^2}{\mathbf{Var}[P]\big(P_0 Q - C(Q)\big)} + P_0 Q - C(Q). \tag{3.9}$$

Hence, whether or not the mean $\mu(H_0)$ is positive or negative is determined by the sign of $P_0 Q - C(Q)$. It is natural to expect that any company would like to have the mean positive, which in the context of the present example implies $C(Q) < P_0Q$. Under the latter condition, equation (3.8) implies that $H_0 > Q$ (speculate) when $\mathbf{E}[P] < P_0$. Likewise, we have $H_0 < Q$ when $\mathbf{E}[P] > P_0$. In the latter case, there is speculation when $H_0 < 0$, which can equivalently be rewritten using formula (3.8) as follows:

$$Q < \frac{\mathbf{E}[P] - P_0}{\mathbf{Var}[P]\big(P_0Q - C(Q)\big)}. \tag{3.10}$$

If $\mu(H_0) > 0$, then $P_0Q - C(Q) > 0$ according to equation (3.9). Thus, condition (3.10) can be rewritten as follows:

$$\big(P_0Q - C(Q)\big)Q < \frac{\mathbf{E}[P] - P_0}{\mathbf{Var}[P]}. \tag{3.11}$$

Since $\mathbf{E}[P] > P_0$, condition (3.11) is not void. This concludes the current subsection, but we shall resume the illustrative example in the next subsection after some general preparatory notes.

3.4 Does H_0 maximize the expected utility?

As we have seen in the previous subsection, finding H_0 in closed form might be a challenging task. In practice, a quick though approximate solution to this problem can be found using statistical inferential results by first replacing the expectations in equation (3.2) by their empirical counterparts and then solving the resulting empirical equation in H. However, we need to keep in mind that critical points may not be maximums, whereas our goal is to maximize the expected utility $\rho(H)$. Assuming that u is twice differentiable, this can be achieved by checking the condition

$$\mathbf{E}[(P - P_0)^2 u''(\Pi(H_0))] < 0, \tag{3.12}$$

which we have found, in general, to be a challenging task. Nevertheless, given some information about H_0 and the support of the distribution of $\Pi(H_0)$, we may find a way to verify the condition. For example, when u is S-shaped with the reference point 0, then $u''(x) \leq 0$ for all $x \geq 0$, and if we have $\Pi(H_0) \geq 0$, then condition (3.12) holds. This argument, obviously, does not apply to our illustrative example, at least because the price P and thus the profit $\Pi(H_0)$ in the example do not follow distributions with bounded supports.

To have the illustrative example sorted out, we next present brute force arguments showing that the specified H_0 maximizes the expected utility $\rho(H)$ when $C(Q) < P_0Q$, that is, when costs of producing the amount Q do not exceed P_0Q, and minimize $\rho(H)$ when $C(Q) > P_0Q$. Recall that the utility function is $u(x) = \Phi(x) - 1/2$, and so

$$\rho(H) = \mathbf{E}\big[\Phi\big(G_1\sqrt{\mathbf{Var}[\Pi(H)]} + \mathbf{E}[\Pi(H)]\big)\big] - 1/2, \tag{3.13}$$

where G_1 is a standard normal random variable. Let G_2 be another standard normal random variable, and let G_1 and G_2 be independent. Then, continuing with equation (3.13), we have

$$\begin{aligned}\rho(H) &= \mathbf{P}\big[G_2 \leq G_1\sqrt{\mathbf{Var}[\Pi(H)]} + \mathbf{E}[\Pi(H)]\big] - 1/2\\ &= \mathbf{P}\big[G_2 - G_1\sqrt{\mathbf{Var}[\Pi(H)]} \leq \mathbf{E}[\Pi(H)]\big] - 1/2\\ &= \Phi\left(\frac{\mathbf{E}[\Pi(H)]}{\sqrt{1+\mathbf{Var}[\Pi(H)]}}\right) - 1/2.\end{aligned}$$

Consequently, we find the maximum of $\rho(H)$ by maximizing the function

$$\Upsilon(H) = \frac{\mathbf{E}[\Pi(H)]}{\sqrt{1+\mathbf{Var}[\Pi(H)]}}.$$

With the notation $\lambda = \mathbf{E}[P] - P_0$ and $\nu = P_0Q - C(Q)$ for simplicity, we have that $\mathbf{E}[\Pi(H)] = \lambda(Q-H) + \nu$ and $\mathbf{Var}[\Pi(H)] = \mathbf{Var}[P](Q-H)^2$, and thus

$$\Upsilon(H) = \frac{\lambda(Q-H)+\nu}{\sqrt{1+\mathbf{Var}[P](Q-H)^2}}.$$

The only critical point of the function $\Upsilon(H)$ is equal to $Q - \lambda/(\nu\mathbf{Var}[P])$. Using the definitions of λ and ν, we see that the critical point is equal to H_0.

In order to determine whether H_0 is the maximum or minimum of the function Υ, we evaluate the second derivative $\Upsilon''(H)$ at $H = H_0$, with the result

$$\Upsilon''(H_0) = -\nu\mathbf{Var}[P]\left(\frac{\nu^2\mathbf{Var}[P]}{\lambda^2+\nu^2\mathbf{Var}[P]}\right)^{3/2}.$$

Since $\Upsilon''(H_0)$ is negative for $\nu > 0$ and positive for $\nu < 0$, we conclude that the expected utility $\rho(H)$ achieves its maximum at $H = H_0$ when $C(Q) < P_0Q$, and minimum when $C(Q) > P_0Q$.

Appendix

Proof of Lemma 2.1.

With the notation $Z = \Pi - \mu$, the covariance $\mathbf{Cov}[\Pi, u'(\Pi)]$ is equal to the expectation $\mathbf{E}[Zu'(\mu+Z)]$. Since the distribution of Z is symmetric around 0, we have that

$$\begin{aligned}\mathbf{E}[Zu'(\mu+Z)] &= \mathbf{E}[Z\mathbf{1}\{Z>0\}u'(\mu+Z)] + \mathbf{E}[Z\mathbf{1}\{Z<0\}u'(\mu+Z)]\\ &= \mathbf{E}[Z\mathbf{1}\{Z>0\}\{u'(\mu+Z)-u'(\mu-Z)\}]. \qquad (3.14)\end{aligned}$$

We shall proceed with the proof keeping in mind that the first derivative u' is non-increasing on $[0,\infty)$ because we are dealing with the S-shaped utility function u.

Assume $\mu \geq 0$ and consider the two cases $\mu - Z \geq 0$ and $\mu - Z < 0$ separately. In the first case we have $0 \leq \mu - Z \leq \mu + Z$, and so $u'(\mu + Z) - u'(\mu - Z) \leq 0$ because u' is non-increasing on $[0, \infty)$. Consequently, the right-hand side of equation (3.14) is non-positive, and thus $\mathbf{E}[Zu'(\mu + Z)] \leq 0$. Consider now the case when $\mu - Z < 0$. Since u' is symmetric around 0 by assumption, we have that $u'(\mu - Z) = u'(Z - \mu)$. Since $0 < Z - \mu \leq Z + \mu$, we have that $u'(\mu + Z) - u'(Z - \mu) \leq 0$ and thus, in turn, $u'(\mu + Z) - u'(\mu - Z) \leq 0$. Consequently, $\mathbf{E}[Zu'(\mu + Z)] \leq 0$. This concludes the proof that $\mathbf{E}[Zu'(\mu + Z)] \leq 0$ when $\mu \geq 0$.

Assume now $\mu \leq 0$. With the notation $\mu^* = -\mu$ and $r^*(x) = u'(-x)$, we rewrite equation (3.14) as follows:

$$\mathbf{E}[Zu'(\mu + Z)] = -\mathbf{E}[Z\mathbf{1}\{Z > 0\}\{r^*(\mu^* + Z) - r^*(\mu^* - Z)\}]. \tag{3.15}$$

Since $\mu^* \geq 0$ and the function r^* is non-increasing on $[0, \infty)$, we know from the previous paragraph that the expectation on the right-hand side of equation (3.15) is non-positive, and so we have $\mathbf{E}[Zu'(\mu + Z)] \geq 0$. This concludes the proof of Lemma 2.1.

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