UNIVERSITÉ DU LUXEMBOURG

85th Workshop on General Algebra
85. Arbeitstagung Allgemeine Algebra

January 31 – February 2, 2013

University of Luxembourg

ABSTRACTS

Supported by the Fonds National de la Recherche, Luxembourg

Supported by the University of Luxembourg, Mathematics Research Unit and Computer Science and Communications Research Unit

Fonds National de la Recherche Luxembourg

UNIVERSITÉ DU LUXEMBOURG

85th Workshop on General Algebra
85. Arbeitstagung Allgemeine Algebra

January 31 – February 2, 2013
University of Luxembourg

The *Workshop on General Algebra* (*Arbeitstagung Allgemeine Algebra*, AAA) is a series of conferences with a long tradition. It was founded in 1970 by Rudolf Wille in Darmstadt, and it is held twice a year in different locations in Europe. The purpose of these meetings is to bring researchers together to exchange their views and results in algebra and related fields. Young researchers, in particular undergraduate and graduate students, are encouraged to participate and contribute. Further information on the AAA series is available at `http://www.tu-dresden.de/math/aaaseries`.

The 85th Workshop on General Algebra (AAA85) takes place in Luxembourg during the period of January 30 through February 2, 2013, hosted by the University of Luxembourg. The topics of the conference include, but are not limited to universal algebra and its applications in computer science, classical algebra, lattices and other ordered algebraic structures, connections of algebra to logic, model theory and combinatorics, and applications of algebra.

The scientific programme of the conference consists of five plenary talks given by Jean Dhombres, Antti Kuusisto, Jaroslav Nešetřil, Tamás Waldhauser, and Gabor Wiese, and of over forty contributed talks given by the participants.

We gratefully acknowledge the financial support of the Fonds National de la Recherche, Luxembourg.

We wish all participants a most interesting and rewarding conference and a pleasant stay in Luxembourg.

Luxembourg, January 2013

Erkko Lehtonen
Karsten Schölzel

ABSTRACTS

Congruence lattices that force nilpotence

ERHARD AICHINGER

Johannes Kepler University Linz, Austria

It is well known that an algebra with permuting congruences and $\mathbf{M}_3$ as its congruence lattice is abelian. We present a condition on the congruence lattice that forces a finite algebra with a Mal'cev term to be nilpotent. For expanded groups, we prove that if this condition fails, then the algebra has a non-nilpotent expansion with the same congruence lattice.

Another condition on the congruence lattice tells when the expansion of the algebra with all its congruence preserving functions is supernilpotent.

A generalization of a theorem of Clifford

PHAM NGOC ÁNH

MTA Rényi Institute, Hungary

Roughly speaking, by a result of Clifford divisibility theories of valuation rings, i.e., monoids of principal ideals partially ordered by reverse inclusion are precisely Rees factors of positive cones of totally ordered abelian groups. We show that divisibility theories of Bezout rings having one minimal prime ideal such that their localization at this ideal is not a field, are lattice factors of positive cones of lattice-ordered abelian groups. Divisibility theory of Bezout rings, i.e., rings whose finitely generated ideals are principal, can be axiomatized as Bezout monoids.

A characterisation of categorical equivalence of finite algebras via nonrefinable covers

MIKE BEHRISCH

TU Dresden, Germany

Relational Structure Theory (RST) is a localisation theory for algebras in locally finite varieties founded on ideas by K. Kearnes and Á. Szendrei on how to extend concepts and constructions from Tame Congruence Theory (TCT) beyond congruences and polynomial operations to arbitrary compatible relations and term operations. Within this framework, algebras are localised to special subsets, called neighbourhoods, which are the images of unary idempotent operations in their clone, i.e. in the local closure of the clone of term operations. The localisation process is the familiar one, known from TCT, and corresponds to a natural restriction $\restriction$ of invariant relations. A neighbourhood U of an algebra $\mathbf{A}$ separates a pair of invariant relations S, T of identical arity if $S\restriction_U \neq T\restriction_U$. A collection $\mathcal{U}$ of neighbourhoods of $\mathbf{A}$ is a cover of $\mathbf{A}$ if every pair of distinct compatible relations is separated by one member of $\mathcal{U}$. Up to local term equivalence, covers allow representations of

A as a local retract of a product of restricted relational structures, and thus a reconstruction from these local pieces. Furthermore, one may define a quasiorder, called refinement, on all covers of an algebra. A cover is nonrefinable if it has no proper refinement and is irredundant, i.e. minimal w.r.t. inclusion among all covers. One of the main results of RST is that finite algebras have nonrefinable covers and that these are unique, up to a canonical notion of isomorphism.

Two algebras **A** and **B** are categorically equivalent ($\mathbf{A} \equiv_{\mathrm{cat}} \mathbf{B}$) if their generated varieties, viewed as categories, are equivalent in the sense of category theory.

For finite algebras we characterise this in the following way: finite algebras **A** and **B** fulfil $\mathbf{A} \equiv_{\mathrm{cat}} \mathbf{B}$ if and only if their nonrefinable covers have the same cardinality and may be enumerated in such a way that corresponding neighbourhoods yield isomorphic restricted relational structures w.r.t. some common relational signature that is in accordance with the natural indexing of relations via restriction of the invariants of **A** and **B**, respectively.

The modularity conjecture holds for linear idempotent varieties

WOLFRAM BENTZ

Centro de Algebra da Universidade de Lisboa, Portugal

(Joint work with Luis Sequeira)

In 1984, Garcia and Taylor formulated the "Modularity Conjecture", which states that the join of two non-congruence modular varieties is non-congruence modular. We show that the conjecture holds for the special case of linear idempotent varieties. Our proofs rely on Dent, Kearnes, and Szendrei's recently introduced concept of the derivative of an equational theory. We will give an overview of the notions and of our results.

On the extensions of Di Nola's Theorem

MICHAL BOTUR

Palacky University, Czech Republic

(Joint work with Jan Paseka)

The main aim of this talk is to present a direct proof of Di Nola's representation Theorem for MV-algebras and to extend his results to the restriction of the standard MV-algebra on rational numbers. The results are based on a direct proof of the theorem which says that any finite partial subalgebra of a linearly ordered MV-algebra can be embedded into $\mathbb{Q} \cap [0, 1]$.

Cyclic and symmetric polymorphisms and CSPs

CATARINA CARVALHO

University of Hertfordshire, UK

(Joint work with A. Krokhin)

It is known that a relational structure $\mathcal{B}$ is preserved by totally symmetric polymorphisms of all arities if and only if the constraint satisfaction problem $CSP(\mathcal{B})$ is solved by arc-consistency. Recently, Kun et al. showed that the existence of symmetric polymorphisms of all arities is equivalent to the existence of totally symmetric polymorphisms of all arities. We give an example of a relational structure with cyclic polymorphisms of all arities but no symmetric operations of all arities, and discuss its implications on the complexity of CSPs and related problems.

Dualities induced by canonical extensions: The Boolean case

RUKIYE CAVUS

University of Liege, Belgium

Whenever a concept of canonical extension makes sense, it can usually be obtained as a double dual process. Using a duality in one direction and coming back with a discretized version of it. In fact the existence of the canonical extension is often shown to be equivalent to the existence of duality.

So the maxim "under each canonical extension is hidden a duality" may come as triviality. However it may be considered as interesting in certain circumstances.

Here we want to work out details of this process in the case of Boolean algebras with operations (mind you: not operators) (and it can be easily extended to all lattice expansions). As there exists an upper and lower extension for these non-normal operations, we are confronted to two dualities, an upper and lower one. The comparison of these two non-natural dualities gives us a better understanding of canonical extensions.

Combinations of congruence properties in varieties of algebras

IVAN CHAJDA

Palacky University, Czech Republic

In a certain study of non-classical logics, the following problem was formulated: find a variety of algebras which is congruence modular but neither congruence distributive nor congruence 3-permutable. We will present a general method based on the so-called independent varieties which enable us to combine Maltsev conditions and hence infer a variety with prescribed congruence properties.

Expressibility of digraph homomorphisms in the logic LFP+Rank

DEJAN DELIC

Ryerson University, Canada

One of the fundamental problems in finite model theory is the quest for the logic which captures polynomial time on finite (di)graphs. From the algebraic point of view, an interesting restriction of this problem asks whether there is a logic L strong enough to capture, given a finite digraph G, the class $\neg$HOM(G) of all finite digraphs not homomorphic to G and such that the truth of L-sentences on finite digraphs can be decided in polynomial time. In 2009, Atserias, Bulatov, and Dawar showed that the LFP+C cannot capture the homomorphism problem on digraphs, where C is the counting operator. Recently, with Bulin, Jackson, and Niven, we refined the original method of Feder and Vardi of translating the constraint satisfaction problem for general relational structures to digraphs in such a way that it preserves the algebraic reasons for polynomial time solvability. In this talk, we present a very recent result, obtained with F. McInerney, which shows that, under the aforementioned transformation, if $\neg$ HOM($\mathbb{A}$) is definable by a LFP+Rank sentence, then $\neg$ HOM($D_{\mathbb{A}}$) is definable in the same logic, where $D_{\mathbb{A}}$ is the digraph obtained from the relational template $\mathbb{A}$. In conclusion, we discuss some related conjectures.

What amounts to algebra and what is due to analysis in the invention and uses of complex numbers, from the fundamental theorem of algebra to Heisenberg relations

JEAN DHOMBRES

École des hautes études en sciences sociales, France

According to a structural point of view, the fact that there are only three Banach fields, and the complex field among the three, proves the fundamental algebraic character of complex numbers. But an opposite point of view recalls that the terminology for the fundamental theorem of algebra has been ironically coined by Gauss to show that there is essentially the need for some result from analysis. The purpose of my talk is not to discuss a priori these two opposite points of view, but to look at the ways proofs were invented for complex numbers from the moment, around 1750, when the complex exponential was understood, and so I would like to study in the same line the requirement for complex Hilbert spaces and the proofs of Heisenberg relations in quantum mechanics.

On quantifiers on pocrims

RADOMÍR HALAŠ

Palacky University, Czech Republic

(Joint work with M. Botur)

Monadic MV-algebras (MMV-algebras, in short) were introduced and studied by J. Rutledge [7] as an algebraic model of the predicate calculus of the Łukasiewicz infinite valued logic in which only a single individual variable occurs. MMV-algebras were also studied as polyadic algebras by D. Schwarz [8], [9]. Recently, the theory of MMV-algebras has been developed in [1], [2] and [3].

The results have been recently extended in [6] for GMV-algebras (pseudo-MV-algebras), which form a non-commutative generalization of MV-algebras.

Recall that monadic, polyadic and cylindric algebras, as algebraic structures corresponding to classical predicate logic, have been investigated by Halmos in 60's and by Henkin, Monk and Tarski. Similar algebraic structures have been considered for various logics in [4] and [5].

The aim of our talk is to built up the theory monadic operators in a more general setting, namely for bounded pocrims. Bounded pocrims form a large class of algebras containing as proper subclasses the class of BL-algebras (an algebraic semantics of Hájek's BL-logic) as well as the class of Heyting algebras (algebras of intuitionistic logic).

We show that for so-called normal pocrims, i.e. those satisfying the identity

$$\neg\neg(x \odot y) = \neg\neg x \odot \neg\neg y$$

(where $\neg x = x \to 0$), there is a mutual correspondence between existential and universal quantifiers. Further, the correspondence of existencial quantifiers with the m-relatively complete substructures will be discussed.

REFERENCES

[1] L.P. Belluce, R. Grigoglia, A. Lettieri: *Representation of monadic MV-algebras*, Stud. Logica, **14**(2005), 123-144.
[2] A. DiNola, R. Grigoglia: *On monadic MV-algebras*, Ann. of Pure and Appl. Logic, **128** (2004), 125-139.
[3] G. Georgescu, A. Iorgulescu, I. Leustean: *Monadic and closure MV-algebras*, Mult. Val. Logic, **3** (1998), 235-257.
[4] I. Neméti: *Algebraization of quantifier logics*, Stud. Logica, **50** (1991), 485-569.
[5] D.Pigozzi, A. Salibra: *Polyadic algebras over nonclassical logics*, In: Algebraic Methods in Logic and Computer Science, Banach Center Publ., **28** (1993), 51-66.
[6] J. Rachůnek, D. Šalounová: *Monadic GMV-algebras*, Arch. Math. Logic, **128** (2004), 125-139.
[7] J.D. Rutledge: *A preliminary investigation of the infinitely many-valued predicate calculus*, Ph.D. Thesis, Cornell University (1959).
[8] D. Schwarz: *Theorie der polyadischen MV-algebren endlicher Ordnung*, Math. Nachr., **78** (1977), 131-138.
[9] D. Schwarz: *Polyadic MV-algebras*, Zeit. Math. Logik und Grundlagen d. Math., **26** (1980), 561-564.

Modal logic of primitive spaces

GEORGES HANSOUL

University of Liege, Belgium

Classically, a modal formula is necessarily true at a point of a topological space if it is true in some neighborhood of the space. This means that the possibility operator is interpreted as closure. If we interpret it as the derived set (set of accumulation points) we obtain the so-called derivational logic of the space.

A space is said to be finitary if it is a Boolean space with finitely many orbits under the action of its automorphism group. We determine the derivational logic of all finitary spaces.

Coherent monoids

MIKLOS HARTMANN

University of York, United Kingdom

There are several ways to generalise the notion of coherency from rings to monoids. In this talk we are interested in the way which arises from Wheeler's notion of a coherent theory of a first-order language. The theory of right S-acts over a monoid S is coherent in Wheeler's sense if and only if every finitely generated S-subact of every finitely presented right S-act is finitely presented. We call such monoids right coherent.

We are going to investigate several classes of monoids with respect to right coherency – namely free monoids, free inverse monoids and left ample monoids.

A general framework for island systems

ESZTER K. HORVÁTH

University of Szeged, Hungary

(Joint work with Stephan Foldes, Sándor Radeleczki and Tamás Waldhauser)

The notion of an island defined on a rectangular board is an elementary combinatorial concept that occurred first in a paper of G. Czédli. This paper was a starting point for many investigations exploring several variations and various aspects of islands.

In this talk, a general framework for islands will be introduced that subsumes all earlier studied concepts of islands on finite boards, moreover it will be shown that the prime implicants of a Boolean function, the formal concepts of a formal context and convex subgraphs of a simple graph, and some particular subsets of a projective plane fit into this framework.

Axiomatizations of those cases will be presented where islands have the comparable or disjoint property, or they are distant, introducing the notion of a connective island domain and of a proximity domain, respectively. In the general case the maximal systems of islands are characterized by using the concept of an admissible system. Characterization of all possible island systems in the case of connective island domains and proximity domains will be also given.

On a class of ordered involutive monoids

KALLE KAARLI

University of Tartu, Estonia

We consider a class of finite ordered involutive monoids (briefly OIM) that arose in our study of a certain categorical equivalence problem (see [1, 2, 3, 4]). It consists of OIMs $\mathbf{S}$ satisfying the following conditions:

- **(C)** completeness, that is, the elements $s, t \in S$ have a join in $\langle S; \leq \rangle$ if and only if $s^{-1}t \leq 1$;
- **(D)** distributivity, that is, for every $r, s, t \in S$, if the join $s \vee t$ exists, then $rs \vee rt$ also exists and $r(s \vee t) = rs \vee rt$;
- **(O)** for every maximal element $m \neq 1$ of $\mathbf{S}$, we have $mm^{-1} \not\leq m$.

In the present work we study the behaviour of OIMs with respect to some natural constructions (direct and semidirect product, ordinal sum, taking substructures). Also the full description of small OIMs (up to size 5) satisfying **(C)**, **(D)** and **(O)** is given.

REFERENCES

[1] K. Kaarli, Arithmetical affine complete varieties and inverse monoids, *Studia Sci. Math. Hungar.* **45** (2008), 13–28.

[2] K. Kaarli, Subalgebras of the squares of weakly diagonal majority algebras, *Studia Sci. Math. Hungar.* **49 (4)** (2012), 509–524.

[3] K. Kaarli, Subalgebras of the squares of finite minimal majority algebras. Submitted.

[4] K. Kaarli and L. Márki, A characterization of the inverse monoid of bicongruences of certain algebras, *International J. Algebra and Computation* **6** (2009), 791–808.

The free spectra of semigroups (an overview)

KAMILLA KÁTAI-URBÁN

University of Szeged, Hungary

(Joint work with Gábor Horváth, Péter Pál Pach, Gabriella Pluhár, András Pongrácz and Csaba Szabó)

The free spectrum of a variety is the sequence of cardinalities of the size of the free algebras. For locally finite varieties there are strong connections between the structural property and the free spectrum. We investigated the free spectra of several semigroup varieties: the varieties of completely 0-simple semigroups, the varieties of iterated semidirect product of semilattices and the varieties of k-piecewise testable languages. In this talk we summarize the results on the free spectra of semigroup varieties.

A tropical version of the Gelfand representation

SEBASTIAN KERKHOFF

TU Dresden, Germany

(Joint work with Friedrich Martin Schneider)

In the talk, we outline a tropical version of the Gelfand representation for compact topological spaces and extend this to a similar representation for Tychonoff spaces.

More precisely, we show that the Stone-Cech compactification of any topological space is naturally homeomorphic to the canonical Hausdorff reflection of the prime ideal spectrum of the semiring of upwards bounded tropical-valued continuous functions. Extending this idea, we show that the Tychonoff reflection of any topological space is naturally homeomorphic to the canonical Hausdorff reflection of the closed prime ideal spectrum of the suitably topologized semiring of tropical-valued continuous functions.

Every skew effect algebra can be extended into a total algebra

MIROSLAV KOLARIK

Palacky University Olomouc, Czech Republic

(Joint work with Ivan Chajda, Palacky University Olomouc)

Skew effect algebras were already introduced as a non-associative modification of the so-called effect algebras which serve as an algebraic axiomatization of the propositional logic of quantum mechanics. Since skew effect algebras have a partial binary operation, we search for an algebra with a total binary operation which extends a given skew effect algebra and such that the underlying posets coincide. It turns out that the suitable candidate is a skew basic algebra. Algebraic properties of skew basic algebras are described and they are compared with the so-called pseudo basic algebras introduced by the authors recently.

States on residuated lattices

MICHIRO KONDO

Tokyo Denki University, Japan

We define states on non-commutative bounded residuated lattices and consider their property. We show that, for a non-commutative bounded residuated lattice X,

(1) If s is a state, then $X/\ker(s)$ is an MV-algebra.
(2) If s is a state-morphism, then $X/\ker(s)$ is a locally finite linearly ordered MV-algebra.

P-categorical equivalence of algebras

OLEG KOŠIK

University of Tartu, Estonia

A variety of algebras is considered as a category: the objects are the algebras in the variety and the morphisms are the homomorphisms between them.

Two algebras A and B are called *categorically equivalent*, if there is a categorical equivalence between the varieties they generate that maps A to B.

We denote by A^+ the algebra obtained from an algebra A by adding to its fundamental operations all nullary operations on A. We will call algebras A and B *p-categorically equivalent* if A^+ and B^+ are categorically equivalent.

It appears that categorically equivalent algebras are also p-categorically equivalent. The converse is however generally not true. One of the simplest examples are finite simple nonabelian groups: while any two simple nonabelian groups are p-categorically equivalent, they are categorically equivalent iff they are isomorphic.

We have shown that also the extensions with trivial center of finite simple nonabelian groups by some finite abelian group are p-categorically equivalent. In particular, symmetric groups of order at least five are p-categorically equivalent.

On distributive "basic algebras"

JAN KÜHR

Palacky University, Czech Republic

(Joint work with Michal Botur)

We present a few results on distributive bounded lattices whose principal ideals are equipped with antitone involutions.

Second-order logic and definability

ANTTI KUUSISTO

University of Tampere, Finland

In addition to algebraic characterizations, regular languages can be characterized logically as languages definable in (so-called) Monadic Second-Order Logic (MSO). The well-known correspondence between regular languages and MSO can be regarded as an early result in Descriptive Complexity Theory. The objective of Descriptive Complexity Theory is to characterize complexity classes in terms of related logical formalisms. Another early result in Descriptive Complexity is the characterization of NP as exactly the collection of classes of relational structures definable in Existential Second-Order Logic (Fagin 1973). This talk is an accessible survey of Descriptive Complexity Theory. We first survey the classical results of the field. We then take a look at some of the more recent developments. These include the uses of linear algebra in descriptive complexity, logical characterizations of PTIME over restricted classes of structures, and characterizations of complexity classes of distributed computing.

On modular dimension of distributive lattices

LEONARD KWUIDA

Bern University of Applied Sciences, Switzerland

(Joint work with Stefan Schmidt, TU Dresden)

Let L be a lattice. A function $f : L \to \mathbb{R}$ is submodular if $f(x \wedge y) + f(x \vee y) \leq f(x) + f(y)$, supermodular if $f(x \wedge y) + f(x \vee y) \geq f(x) + f(y)$, and modular if it is both submodular and supermodular. Modular functions on a finite lattice form a finite dimensional vector space. For finite distributive lattices, we compute this (modular) dimension. This turns out to be another characterization of distributivity.

Some remarks on essentially minimal clones

HAJIME MACHIDA

Tokyo, Japan

(Joint work with I. G. Rosenberg)

For a non-empty set A, a clone C on A is essentially minimal if C is minimal among all clones on A containing essential functions and is not a minimal clone. For a finite A with $|A| = k\,(> 2)$, we show that the minimum arity of generators of C is no greater than k for any essentially minimal clone C on A. We also discuss some properties of essentially minimal groupoids on a three-element set and determine their conjugate classes. There are 16 conjugate classes.

L-groups and systems of inequalities in residuated lattices

ROZÁLIA MADARÁSZ

University of Novi Sad, Serbia

(Joint work with Milanka Bradić (PhD student, University of Novi Sad))

L-algebras, where L is some structure of truth values, are structures which have two parts: the functional part, which is an ordinary algebra (the "skeleton") and the relational part, which is an L-valued relation compatible with all the fundamental operations of the skeleton. We investigated L-algebras where L is a residuated lattice and the skeleton is a group. The problem of characterization of all L-groups with a given skeleton reduces to the problem of solving some non-trivial systems of inequalities in residuated lattices.

Operations on p-groups are basically homomorphisms

PETER MAYR

JKU Linz, Austria

I will explain the statement in the title and give a proof using some basic facts from the representation theory of groups. Then I will apply it to obtain efficient algorithms for several computational problems (closure, membership, size,...) on subpowers of expansions of finite p-groups.

A Gumm type result for ternary commutators in Mal'cev algebras

NEBOJŠA MUDRINSKI

Novi Sad, JKU Linz, Serbia

We prove that a Mal'cev algebra is 2-supernilpotent ([1,1,1]=0) if and only if it is polynomially equivalent to a special expanded group. This can be seen as a generalization of a theorem by H.P.Gumm that a Mal'cev algebra is abelian if and only if it is polynomially equivalent to a module over a ring.

Universality and homogeneity

JAROSLAV NEŠETŘIL

Charles University, Czech Republic

Countable universal posets arising from particular finite structures are a classical algebraic and combinatorial theme. Particular attention was devoted to highly symmetric – homogeneous – structures and this became very actual recently in the context of Ramsey theory and combinatorial characterization of amenability and dynamical systems. We survey recent activity in this area with particular focus on universality and Ramsey aspects.

Hereditarily absorption-free algebras

JAKUB OPRŠAL

Charles University, Czech Republic

A subuniverse B of algebra A is absorbing if there is a term t, such that $t(a_1, \ldots, a_n)$ lies in B whenever all a_i's except at most one lie in B. An algebra is absorption-free if it has no non-trivial absorbing subuniverse and hereditarily absorption-free (or HAF) if each of its subalgebras is absorption-free. We aim to characterize all finite HAF algebras. As an easy corollary to Barto-Kozik's Absorption theorem, these algebras have to have a Maltsev term.

A partial result is that all finite groups are HAF and there is no one-element absorbing subuniverse in idempotent reduct of the variety of groups (i.e., a variety whose basic operations are all idempotent terms of groups).

On realization of partially ordered abelian groups

JAN PASEKA

Masaryk University, Czech Republic

The paper is devoted to algebraic structures connected with the logic of quantum mechanics. Since every (generalized) effect algebra with an order determining set of (generalized) states can be represented by means of an abelian partially ordered group and events in quantum mechanics can be described by positive operators in a suitable Hilbert space, we are focused in a representation of partially ordered abelian groups by means of sets of suitable linear operators.

We show that there is a set of points separating $\mathbb{R}$-maps on a given partially ordered abelian group G if and only if there is an injective non-trivial homomorphism of G to the symmetric operators on a dense set in a complex Hilbert space $\mathcal{H}$ which is equivalent to an existence of an injective non-trivial homomorphism of G into a certain power of $\mathbb{R}$. A similar characterization is derived for an order determining set of $\mathbb{R}$-maps and symmetric operators on a dense set in a complex Hilbert space $\mathcal{H}$. We also characterize effect algebras with an order determining set of states as interval operator effect algebras in groups of self-adjoint bounded linear operators.

On the Bergman property for clones

CHRISTIAN PECH

TU Dresden, Germany

(Joint work with Maja Pech)

An infinite group has the Bergman property if each of its connected Cayley graphs has a finite diameter. In this talk we define the Bergman property for clones and study its connection with cofinality questions. For a large class of countable homogeneous structures we show that their clones of polymorphisms have uncountable cofinality and the Bergman property.

On generating sets of polymorphism clones of homogeneous structures

MAJA PECH

TU Dresden, University of Novi Sad, Germany/Serbia

(Joint work with Christian Pech)

We show for a large class of countable homogeneous structures that their polymorphism clones are generated by the monoid of homomorphic self-embeddings together with one further endomorphism and one further binary polymorphism. Our results generalize a classical theorem by Sierpiński, that the clone of all functions on an arbitrary set is generated by its binary part.

Semilattice ordered algebras – congruences, closure operators and subvarieties II

AGATA PILITOWSKA

Warsaw University of Technology, Poland

(Joint work with A. Zamojska-Dzienio)

In general, finding all "admissible" closure operators on the free algebra in the variety $\mathcal{U}$ of all Ω-algebras may be very difficult and laborious task. We simplify the problem restricting it to some chosen subvarieties $\mathcal{V} \subseteq \mathcal{U}$.

For example, we study some closure operators on n-semigroups as well as on idempotent entropic algebras (*modes*). This allows to describe some subvarieties of semilattice ordered n-semigroups and some subvarieties of semilattice ordered modes (*modals*). (This talk is a continuation of the talk "Semilattice ordered algebras – congruences, closure operators and subvarieties I").

The lattice of quasiorder lattices of algebras

REINHARD PÖSCHEL

Technische Universität Dresden, Germany

(Joint work with D. Jakubíková-Studenovská and S. Radeleczki)

The lattice of all quasiorder lattices of algebras on a fixed set is investigated. In particular, meet- and join-irreducible elements in this lattice are characterized.

State operators on monotone basic algebras

JIŘÍ RACHŮNEK

Palacký University, Czech Republic

(Joint work with Jan Kühr)

Basic algebras form a large class which contains certain classes of algebras of many valued and quantum logics. For example, MV-algebras, orthomodular lattices and lattice effect algebras can be viewed as particular cases of basic algebras.

States on basic algebras, which are mappings into the real interval [0,1], constitute analogues of measures on basic algebras. State operators (or internal states) on basic algebras are unary operations satisfying some properties of states.

We study state operators on monotone basic algebras and, moreover, we give connections between states and state operators on them.

The algebra of mode homomorphisms

ANNA ROMANOWSKA

Warsaw University of Technology, Poland

There are two important and typical ways of constructing new modes (idempotent and entropic algebras) from given ones. One is based on the fact that subalgebras of a mode form a mode again, providing so-called modes of submodes. The other is based on the fact that homomorphisms of a mode also have the structure of a mode, providing so-called (homo)morphism algebras or homomorphism modes. While there is already quite a number of results concerning mode of subalgebras and their generalizations, there are only a few results concerning algebras of mode homomorphisms. The talk will provide preliminary results concerning algebras in the latter class. We will describe the structure of homomorphism modes and the structure of the sets of homomorphisms from subalgebras of one mode to subalgebras of another mode of the same type.

Rough approximations in commutative basic algebras

DANA ŠALOUNOVÁ

VSB-Technical University Ostrava, Czech Republic

In the classical rough set theory, subsets are approximated by means of pairs of ordinary sets, so-called lower and upper approximations, which are e.g. composed by some classes of given equivalences. It is known that MV-algebras are related to probability through so-called states and their kernels are formed by some of ideals of MV-algebras. This led to study of rough sets which are closely related to the structure of MV-algebras from the algebraic point of view.

Commutative basic algebras are non-associative generalizations of MV-algebras, more precisely, MV-algebras coincide with associative commutative basic algebras. Their ideals are in a one-to-one correspondence with the congruences. Therefore, similarly as in the case of MV-algebras, we study classical approximation spaces based on their ideals.

Simplex design spaces

MAIC SASSO-SANT

Wallerfangen, Germany

Special nearrings constructed from automorphism groups or fixed point free automorphism groups can be applied to get geometries and block designs. Simplex Design Space (SDS) is the unification space of both mathematical structures geometry and block design. I get new designs (NPIBD, NBIBD) as well as a hierarchy of all the designs (NPBIBD, PBIBD, NBIBD, BIBD) in this space (especially when applied to nearrings) together with their corresponding geometric spaces. Any finite SDS has a matrix representation and all the design parameters can be calculated by a computer. In this context we also have a relation to fibered geometric design spaces. A special class of matrices coming from certain nearrings lead us to projective planes. If there are planes of order 12,15,18.....still must be checked by a computer! Finally my intention is to arouse interest for fruitful interdisciplinary research.

A construction of the Tychonoff reflection via topological lattices of continuous functions

FRIEDRICH MARTIN SCHNEIDER

TU Dresden, Germany

(Joint work with Sebastian Kerkhoff)

In the talk, we give a description of Tychonoff reflections in terms of spectra of topological lattices of continuous functions.

More precisely, we show that the Tychonoff reflection of any topological space is naturally homeomorphic to the canonical Hausdorff reflection of the closed prime ideal spectrum of the suitably topologized lattice of its continuous functions ranging in the closed unit interval.

Representation of lattices by lattice valued weak congruence relations

BRANIMIR SESELJA

University of Novi Sad, Serbia

(Joint work with Andreja Tepavčević and Vanja Stepanović)

It is known that the lattice of all weak congruences of an algebra is algebraic; congruence lattices of all its subalgebras, as well as, up to an isomorphism, the lattice of its subalgebras, come out to be its sublattices. An open problem in universal algebra is a representation of algebraic lattices by weak congruences. Here we investigate the analogue topic in the lattice valued framework. We introduce lattice valued weak congruences of an algebra and connect them with congruences on its lattice valued subalgebras. We prove that the lattice of lattice valued weak congruences is complete and fully describe its structure in lattice-theoretic terms. In addition, we deal with the representation problem of lattices by lattice valued weak congruences. It turns out that in this framework it is possible to represent complete lattices which are not algebraic. We also prove representation theorems for several classes of lattices which are known to be non-representable in the classical way.

Isometrical lattice embeddings

BENEDEK SKUBLICS

University of Szeged, Hungary

Based on Finkbeiners embedding results, Grätzer and Kiss focused on embeddings of lattices that have a pseudorank function. They proved that every finite lattice with a pseudorank function is isometrically embeddable into a finite geometric lattice. We extended their results for a special class of algebraic lattices that we call finite height generated lattices.

Directional algebras

JONATHAN SMITH

Iowa State University, USA

Directional algebras are generalizations of dimonoids, which may themselves be regarded as directional semigroups. Given a constant-free type, a directional type is obtained by pointing to each of the arguments of the original, undirected type. For each axiomatization of a variety of algebras of constant-free type, a corresponding directional variety is determined. Dimonoids and digroups are shown to arise from the general procedure. For quasigroups, various choices of equational bases lead to various varieties of directional quasigroups. Under one natural axiomatization, the variety of quasigroups is shown to be directionally complete, in the sense that the corresponding directional variety is again the variety of quasigroups. Another axiomatization yields (4+2)-quasigroups. Digroups are equivalent to a certain class of (4+2)-quasigroups.

On monoidal characterization of closed maps

SERGEJS SOLOVJOVS

Masaryk University, Czech Republic

There exist two different characterizations of the same class of continuous maps between topological spaces. Given a continuous map $f : X \to Y$, on the one hand, f is said to be *proper* provided that the product map $f \times 1_Z : X \times Z \to Y \times Z$ is closed for every topological space Z. On the other hand, f is said to be *perfect* provided that it is closed, and the fibre $f^{-1}(y)$ is compact for every $y \in Y$. N. Bourbaki showed the equivalence of these two concepts, and emphasized the importance of proper maps in general topology. The result of N. Bourbaki generalizes the famous Kuratowski-Mrówka theorem, the latter stating that for every topological space X, the unique map $X \to 1$ is perfect if and only if it is proper. Recently, M. M. Clementino and W. Tholen [1] extended the above two results to *monoidal topology* [2], which is a new approach to categorical topology, based in monads and quantales. Their machinery, however, relies on a monoidal analogue of the notion of closed map, which requires the involved quantales to be constructively completely distributive.

Following the quantale independent definition of proper map of [1], in this talk, we propose an alternative concept of closed map, which, firstly, is not dependent on the involved quantales; secondly, coincides with the notion of M. M. Clementino and W. Tholen in their proposed framework of complete distributivity; and, thirdly, makes the relationships between monoidal versions of proper and closed maps easier to follow (every proper map is then trivially closed). We also show that the analogues of the Kuratowski-Mrówka and Bourbaki theorems are valid in the new setting as well.

Acknowledgements

This research was supported by the ESF Project No. CZ.1.07/2.3.00/20.0051 "Algebraic methods in Quantum Logic" of the Masaryk University in Brno, Czech Republic.

References

[1] M. M. Clementino and W. Tholen, *Proper maps for lax algebras and the Kuratowski-Mrówka theorem*, available at `http://www.math.yorku.ca/~tholen/Kuratowski_Mrowka.pdf`.

[2] G. J. Seal and W. Tholen (eds.), *Monoidal Topology: A Categorical Approach to Order, Metric and Topology*, in preparation.

Lattice valued Boolean functions and threshold functions

ANDREJA TEPAVCEVIC

University of Novi Sad, Serbia

(Joint work with Eszter K. Horváth and Branimir Šešelja)

A lattice valued Boolean function is a function $f : \{0,1\}^n \to L$, where L is a complete lattice. We investigate permutation groups representable under such functions, in the sense that the invariance group of the function is the given group. We prove that a function is invariant under a permutation if and only if it is invariant under all the cuts of this lattice valued function. Furthermore, for any $n \in \mathbb{N}$, there is a lattice L and a lattice valued Boolean function $F : \{0,1\}^n \to L$ such that every group $G \subseteq S_n$ which is 2-representable, is also 2-representable by a cut of F. This representation is not unique. Furthermore, we prove that the analogous property is true also on the k-element set for $k \geq n$ as well as some further representations.

In the second part, we present our investigations concerning isotone lattice valued Boolean functions and lattice valued threshold functions. Every isotone Boolean function is a lattice valued threshold function and vice versa. We give connection of obtained results with lattice valued sets and cuts.

Nullstellen and subdirect representation

WALTER THOLEN

York University, Canada

David Hilbert's solvability criterion of the 1890s for polynomial systems in n variables was linked by Emmy Noether in the 1920s to the decomposition of ideals in commutative rings which, in turn, led Garret Birkhoff in the 1940s to his subdirect representation theorem for general algebras. The Hilbert-Noether-Birkhoff linkage was emphasized in the late 1990s in talks by Bill Lawvere. The aim of this talk is to analyze this linkage in the most elementary terms and then, based on our work of the 1980s, to present a general categorical framework for Birkhoff's theorem.

Invariance groups of finite functions

TAMÁS WALDHAUSER

University of Szeged, Hungary

(Joint work with Eszter K. Horváth, Géza Makay and Reinhard Pöschel)

The *invariance group* of an n-variable function is the group of permutations of its variables that leave the function invariant. It is easy to see that every subgroup of the symmetric group S_n is the invariance group of some n-ary function with a sufficiently large domain and codomain. However, representability of permutation groups as invariance groups of functions $f\colon A^n \to B$ with given sets A and B is a nontrivial problem. This problem has been studied previously mainly in the Boolean case $A = \{0,1\}$; here we propose a natural generalization. Let us say that a group $G \leq S_n$ is (k,m)-*representable* if there exists a function $f\colon A^n \to B$ with $|A| = k$ and $|B| = m$ such that the invariance group of f is G. Furthermore, we call a group (k,∞)-*representable* if it is (k,m)-representable for some natural number m.

Clote and Kranakis investigated $(2,m)$-representability of groups, and they applied it in the study of circuit complexity of Boolean functions and languages. It has been claimed that the intersection of $(2,2)$-representable groups is again $(2,2)$-representable. This claim has been disproved by Kisielewicz, by showing that the Klein four-group is the intersection of two $(2,2)$-representable groups, but it is not $(2,2)$-representable.

On the other hand, the class of (k,∞)-representable subgroups of S_n for any given k is easily seen to be closed under intersection; in fact, a group $G \leq S_n$ is (k,∞)-representable if and only if it is the intersection of invariance groups of operations $f\colon A^n \to A$ on a k-element set A. We introduce a Galois connection between permutations of $\{1,\dots,n\}$ and n-ary operations on $\{1,\dots,k\}$ such that a subgroup G of S_n is Galois closed if and only if it is (k,∞)-representable. The study of the Galois closures yields the explicit description of (k,∞)-representable groups for $k = n-1$ and $k = n-2$, and we also obtain a characterization in the general case $k = n-d$ under the additional assumption that n is much larger than d. (Note that the case $k \geq n$ is trivial, as in this case all subgroups of S_n are (k,∞)-representable.)

Our Galois connection is closely related to orbit equivalence of permutation groups. From the description of orbit equivalent pairs of primitive groups obtained by Seress we deduce that all primitive groups except the alternating groups are $(3, \infty)$-representable.

A manuscript is available at `arxiv.org/abs/1210.1015`.

Modular forms and the inverse Galois problem

GABOR WIESE

University of Luxembourg, Luxembourg

Modular forms which are eigenfunctions for all Hecke operators give rise to two-dimensional modular representations of the absolute Galois group of the rational numbers. By varying the modular form, the attached Galois representations lead to the existence of "many" of the simple groups $\mathrm{PSL}_2(\mathbb{F}_{p^d})$ as Galois groups of $\mathbb{Q}$.

In the talk I will describe and explain this strategy, possible generalisations and some results that were obtained partially in collaboration with Luis Dieulefait and Sara Arias-de-Reyna.

Semilattice ordered algebras – congruences, closure operators and subvarieties I

ANNA ZAMOJSKA-DZIENIO

Warsaw University of Technology, Poland

(Joint work with A. Pilitowska)

A semilattice ordered $\mathcal{V}$-algebra is an algebra of the form $(A, \Omega, +)$, where $+$ is a join-semilattice operation, (A, Ω) is an algebra from some given variety $\mathcal{V}$ and operations from the set Ω distribute over the operation $+$.

In 2005 M. Kuřil and L. Polák introduced the notion of "admissible" closure operators and used it to study the subvarieties of semilattice ordered semigroups.

We generalize their approach and provide the connection between fully invariant congruences on the free semilattice ordered Ω-algebra and some closure operators satisfying additional conditions on the free Ω-algebra. This allows to give a new description of the lattice of all subvarieties of semilattice ordered algebras.

List of participants

Erhard Aichinger, *Johannes Kepler University Linz, Austria*
Pham Ngoc Ánh, *MTA Rényi Institute, Hungary*
Mike Behrisch, *TU Dresden, Germany*
Wolfram Bentz, *Centro de Algebra da Universidade de Lisboa, Portugal*
Michal Botur, *Palacky University, Czech Republic*
Catarina Carvalho, *University of Hertfordshire, UK*
Rukiye Cavus, *University of Liege, Belgium*
Ivan Chajda, *Palacky University, Czech Republic*
Miguel Couceiro, *Université Paris Dauphine, France*
Dejan Delic, *Ryerson University, Canada*
Jean Dhombres, *École des hautes études en sciences sociales, France*
Radomír Halaš, *Palacky University, Czech Republic*
Georges Hansoul, *University of Liege, Belgium*
Miklos Hartmann, *University of York, United Kingdom*
Eszter K. Horváth, *University of Szeged, Hungary*
Kalle Kaarli, *University of Tartu, Estonia*
Kamilla Kátai-Urbán, *University of Szeged, Hungary*
Sebastian Kerkhoff, *TU Dresden, Germany*
Miroslav Kolarik, *Palacky University Olomouc, Czech Republic*
Michiro Kondo, *Tokyo Denki University, Japan*
Oleg Košik, *University of Tartu, Estonia*
Jan Krňávek, *Palacky University, Czech Republic*
Jan Kühr, *Palacky University, Czech Republic*
Antti Kuusisto, *University of Tampere, Finland*
Leonard Kwuida, *Bern University of Applied Sciences, Switzerland*
Erkko Lehtonen, *University of Luxembourg, Luxembourg*
Hajime Machida, *Tokyo, Japan*
Rozália Madarász, *University of Novi Sad, Serbia*
László Márki, *Rényi Institute, Hungary*
Peter Mayr, *JKU Linz, Austria*
Nebojša Mudrinski, *Novi Sad, JKU Linz, Serbia*
Jaroslav Nešetřil, *Charles University, Czech Republic*
Jakub Opršal, *Charles University, Czech Republic*
Patrice Ossona de Mendez, *EHESS/CNRS, France*
Péter P. Pálfy, *MTA Rényi Institute, Hungary*
Jan Paseka, *Masaryk University, Czech Republic*
Christian Pech, *TU Dresden, Germany*
Maja Pech, *TU Dresden, University of Novi Sad, Germany/Serbia*
Agata Pilitowska, *Warsaw University of Technology, Poland*
Reinhard Pöschel, *Technische Universität Dresden, Germany*
Jiří Rachůnek, *Palacký University, Czech Republic*
Julien Raskin, *University of Liege, Belgium*
Anna Romanowska, *Warsaw University of Technology, Poland*
Dana Šalounová, *VSB-Technical University Ostrava, Czech Republic*

Maic Sasso-Sant, *Wallerfangen, Germany*
Fabienne Schmitz, *University of Luxembourg, Luxembourg*
Friedrich Martin Schneider, *TU Dresden, Germany*
Karsten Schölzel, *University of Luxembourg, Luxembourg*
Branimir Seselja, *University of Novi Sad, Serbia*
Benedek Skublics, *University of Szeged, Hungary*
Jonathan Smith, *Iowa State University, USA*
Sergejs Solovjovs, *Masaryk University, Czech Republic*
Bruno Teheux, *University of Luxembourg, Luxembourg*
Andreja Tepavcevic, *University of Novi Sad, Serbia*
Walter Tholen, *York University, Canada*
Jörg Tomaschek, *University of Luxembourg, Luxembourg*
Tamás Waldhauser, *University of Szeged, Hungary*
Gabor Wiese, *University of Luxembourg, Luxembourg*
Anna Zamojska-Dzienio, *Warsaw University of Technology, Poland*