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Contents

Free profinite semigroups and symbolic dynamics

J. Almeida

There are various ways in which symbolic dynamics and the theory of profinite semigroups have interacted recently. Since the monoid of continuous endomorphisms of a finitely generated profinite semigroup is itself profinite, a meaning can be assigned to infinite iteration of such endomorphisms. This idea, applied to free profinite semigroups leads to simple descriptions of some of its elements which moreover are amenable to efficient calculation. Moreover, methods from symbolic dynamics may be employed to explore structural features of free profinite semigroups. On the other hand, consider a subshift, that is, a symbolic dynamical system consisting of a set of doubly infinite words (with a marked origin) over a finite alphabet A which is topologically closed as a subset of the product of copies of the discrete space A and which is stable under shifting the origin. The language of its nonempty finite factors has a topological closure in the free profinite semigroup over the set A from which the subshift may be recovered. In the case of an irreducible subshift, the minimal elements in this closure turn out to constitute a regular J-class and its structure group is an invariant of the subshift under the natural notion of isomorphism between time-discrete dynamical systems. These maximal subgroups of the free profinite semigroup on A turn out to be free for a wide class of minimal subshifts, including the Sturmian and Arnoux-Rauzy cases. The aim of the talk is to give a survey of these topics and to present some of the ideas and results which are currently known.

Canonical Cohomologies

D. Artamonov

In some situations in homology theory the following construction of canonical cohomologies appears. Let T be a left exact covariant functor from some abelian category A to the category of abelian groups. Suppose that an object G from A is embedded into the object $C(G)$, so that the functor $T(C(G))$ is exact. Then as in the construction of Godement's resolution we have a canonical resolution of an object G with cohomologies $H^k(G)$. In the talk it will be discussed the following questions: are objects $C(G)$ acyclic and is the theory $H^k(G)$ universal?

On Quasicrystals

V. A. Artamonov

Symmetries of crystals play a crucial role in the geometric theory of crystals, helping to classify all possible combinations for positions occupied by atoms in materials. A complete classification of symmetries of crystals was known since thirties in the last century. But in 1984 a new alloy $Al_{0,86}Mn_{0,14}$ was discovered with an icosahedral symmetry which was forbidden in the symmetry theory of crystals and which is known as *Penrose tiling*. These new metallic alloys whose diffraction patterns sharp spots with non-crystallographic symmetries are called *quasicrystals*. In the talk we shall expose a mathematical approach to the theory of quasicrystalls.

A quasicrystal Q is located in a *physical* space U of a dimensional d which is a subspace of an Eucldean *superspace* E dimension $n > d$. There is an orthogonal decomposition $E = U \oplus U^\perp$, where $U, U^\perp \neq 0$. Denote by $\mathcal{P} : E \to U$ the corresponding orthogonal projection. The space $U^\perp$ is called a *phason* space.

Let M be an a lattice (a free $\mathbb{Z}$-module) in E with an orthonormal base $e_1, \ldots, e_n$ of the superspace E such that $U^\perp \cap M = 0$ and

$$P = \left\{ \sum_{i=1}^{n} y_i e_i \mid 0 \leq y_i \leq 1 \right\} \tag{1}$$

a unit n-dimesional cube in the superspace E. Put $K = (1 - \mathcal{P})P \subset U^\perp$.

Definition A *quasicrystal* Q in U in the image of $(U \oplus K) \cap M$ under the orthogonal projection $\mathcal{P}$.

In the talk I shall explain that E, $M\, e_1, \ldots, e_n$ can be recovered from Q. The set Q is discrete. If S is the unit disk in U with the center at the origin then there exist finitely many quasicrystals Q with the given intersection $S \cap Q$. Finally I will discuss the notion of symmetry group of Q and the inverse semigroup of symmetries of Q.

The Lattice of Machine Invariant Classes

J. Buls

We investigate the lattice of machine invariant classes [1]. The design of stream ciphers motives the following constructions. A 3–sorted algebra $V = \langle Q, A, B, q_0, \circ, *\rangle$ is called *an initial Mealy machine* if Q, A, B are finite, non-empty sets, $q_0 \in Q$; $\circ : Q \times A \longrightarrow Q$ is a total function and $* : Q \times A \longrightarrow B$ is a total surjective function. *An (indexed) infinite word* x on the alphabet A is any total map $x : \mathbb{N} \longrightarrow A$. We shall set for any $i \geq 0$, $x_i = x(i)$ and write $x[n, n+k] = x_n x_{n+1} \ldots x_{n+k}$. The set of all the infinite words over A is denoted by A^ω. Let $(q_0, x, y) \in Q \times A^\omega \times B^\omega$. We write

$$y = q * x \text{ if } \forall n \; y[0,n] = q * x[0,n]$$

and say an initial machine V *transforms* x to y. Let $\mathfrak{A} = \{a_0, a_1, \ldots, a_n, \ldots\}$ be any fixed coutable alphabet and consider the set $Fin(\mathfrak{A})$ of all non-empty finite subsets of $\mathfrak{A}$. Let $\mathfrak{M} = \{\langle Q, A, B, q_0, \circ, *\rangle \mid Q \in Fin(\mathfrak{Q}) \wedge A, B \in Fin(\mathfrak{A})\}$, where $\mathfrak{Q} = \{q_1, q_2, \ldots, q_n, \ldots\}$ is any fixed countable set. We say the word $x \in A_1^\omega$ is *apt* for $\langle Q, A, B, q_0, \circ, *\rangle$ if $A_1 \subseteq A$.

Definition A set $\emptyset \neq \mathfrak{K} \subseteq \mathfrak{F} = \{x \in A^\omega \mid A \in \mathfrak{A}\}$ is called machine invariant if every initial machine $V \in \mathfrak{M}$ all apt words of $\mathfrak{K}$ transforms to words of $\mathfrak{K}$.

Corollary [1] Let $\mathfrak{L}$ be the set containing all machine invariant sets. Then $\langle \mathfrak{L}, \cup, \cap\rangle$ is a completely distributive lattice, where $\cup, \cap$ are the set union and intersection, respectively. The smallest element in this lattice is the set of all ultimately periodic words.

Let $A^n = \{v | v \in A^* \wedge |v| = n\}$, where A^* is the free monoid generated by A and $|v|$ — the length of v. The word $v \in A^*$ is a factor of $x \in A^\omega$ if there exist $u \in A^*$, $y \in A^\omega$ such that $x = uvx$. We denote by $F(x)$ the set of x factors. The map $f_x(n) = \mathrm{card}(F(x) \cap A^n)$ is called the *subword complexity* of word x. The map $g_x(n) = \sum_{i=0}^{n} f_x(i)$ is called the *growth* function of word x. Let $f, g : \mathbb{N} \longrightarrow \mathbb{R}$ be total functions. We write $g = O(f)$ if $\exists C > 0 \forall n \in \mathbb{N} |g(n)| \leq C|f(n)|$. Let $\emptyset \neq \mathfrak{K} \subseteq \mathfrak{F}$. We say the f is *complexity* of $\mathfrak{K}$ if $\forall x \in \mathfrak{K} \, f_x = O(f)$.

Proposition Let $f : \mathbb{N} \longrightarrow \mathbb{R}$ be any total function. If $\mathfrak{K}_1 = \{x \in \mathfrak{F} \mid f_x = O(f)\}$ then $\mathfrak{K}_1$ is machine invariant. If $\mathfrak{K}_2 = \{x \in \mathfrak{F} \mid g_x = O(f)\}$ then $\mathfrak{K}_2$ is machine invariant.

This comes up to our expectations that the lattice $\mathfrak{L}$ would serve as a measure of words complexity.

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On Clones of Hyperfunctions

F. Börner

A hyperfunction on the basic set A is a function $f : A^n \to P_0(A)$, i.e. the values $f(a_1, \ldots, a_n)$ are nonempty subsets of A. A hyperclone is a superposition-closed set of hyperfunctions, containing the elementary (hyper)functions. Besides hyperfunctions we also consider "hypermorphisms $g : A^n \to P_0(A^m)$. These hypermorphisms form a small category M_A. We investigate abstract properties of hyperclones and of subcategories of M_A. Moreover, we consider extension-closed and restriction-closed hyperclones and their connections with relations on A.

On Semilattice-ordered Semigroups of Binary Relations

D. A. Bredikhin

Let $Rel(X)$ be the set of all binary relations on X. We shall consider the following operation on $Rel(X)$: relation product $\circ$; intersection $\cap$; left $\triangleright$ and right $\triangleleft$ restrictive products which are defined as follows $R \triangleright Q = (pr_1 R \times X) \cap Q$, $Q \triangleleft R = Q \cap (X \times pr_2 R)$ where $pr_1 R$ and $pr_2 R$ are the first and second projections of the relation $R \in Rel(X)$ [1]. For any set Ω of operations on binary relations, denote by $R\{\Omega\}$ the class of algebras whose elements are binary relations and whose operations are members of Ω. By a semilattice ordered semigroup we mean an algebra $(A, \cdot, \wedge)$ such that $(A, \cdot)$ is a semigroup, $(A, \wedge)$ is a semilattice, and $x(y \wedge z) \leq xy \wedge xz$, $(x \wedge y)z \leq xz \wedge yz$ where $\leq$ is the natural partial order relation of the semilattice $(A, \wedge)$. As it is shown in [2], the class $R\{\circ, \cap\}$ is a variety and equal to the class of all semilattice ordered semigroups. The following theorem gives the description of the class $R\{\circ, \cap, \triangleright, \triangleleft\}$.

THEOREM. The class $R\{\circ, \cap, \triangleright, \triangleleft\}$ is a variety. An algebra $(A, \cdot, \wedge, \triangleright, \triangleleft)$ belongs to $R\{\circ, \cap, \triangleright, \triangleleft\}$ if and only if $(A, \cdot, \wedge)$ is a semilattice ordered semigroup, $(A, \triangleright)$ and $(A, \triangleleft)$ are a semigroups, and the following identities hold: $x \triangleright x = x$, $x \triangleright y \triangleright z = y \triangleright x \triangleright z$, $x \triangleleft x = x, x \triangleleft y \triangleleft z = x \triangleleft z \triangleleft y$, $(x \triangleright y) \triangleleft z = x \triangleright (y \triangleleft z)$, $x \triangleright yz = (x \triangleright y)z$, $xy \triangleleft z = x(y \triangleleft z)$, $xy \triangleright x(y \triangleright z) = x(y \triangleright z)$, $(x \triangleleft y)z \triangleleft yz = (x \triangleleft y)z$, $(x \triangleleft yz)v = (x \triangleleft y(zv \triangleright z))v$, $x(yz \triangleright v) = x((y \triangleleft xy)z \triangleright v)$, $x \triangleright (y \cap z) = (x \triangleright y) \cap z$, $(x \cap y) \triangleleft z = x \cap (y \triangleleft z)$.

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$\mathcal{O}^{SF}$-Solid Strongly Full Varieties of Partial Algebras

S. Busaman, K. Denecke

We use the concept of a weakly invariant congruence on partial algebras of type τ_n and we characterize strongly full varieties of partial algebras of type τ_n which are closed under taking of isomorphic copies of their clones of n-ary strongly full term operations. Finally we show that a strongly full variety of partial algebras of type τ_n has this property if and only if it is $\mathcal{O}^{SF}$-solid for the submonoid $\mathcal{O}^{SF}$ of strongly full hypersubstitutions which have surjective extensions.

Lattices and Semilattices Having Antitone Involution in Every Upper interval

I. Chajda

A join-semilattice S with the greatest element 1 is sectionally involutioned if for each p of S there exists an antitone involution in the interval [p,1]. A lattice L is sectionally involutioned if its semilattice reduct has this property. In every such a semilattice we can define a binary operation "." and show that the class of all sectionally involutioned semilattices (or lattices) considered in the extended type is a finitelly presented variety. We will present congruence properties of these varieties.

Hilbert Algebras as Implicative Partial Semilattices

J. Cirulis

The partial operation $\wedge$ defined on a Hilbert algebra $(A, \rightarrow, 1)$ by

$$a \wedge b := \min\{x : \ a \leq b \rightarrow x\}$$

turns the algebra into a kind of partial semilattice $(A, \wedge, 1)$. We characterise the class of all partial semilattices arising this way, and show that, in any of them, implication can be restored by

$$a \rightarrow b = \max\{x : \ b \in [a) \sqcup [x)\},$$

where $\sqcup$ stands for join in the lattice of filters of the respective partial semilattice.

Pseudoidentities and Hyper-pseudoidentities

K. Denecke, B. Pibaljommee

Pseudovarieties are classes of finite algebras which are closed under taking of subalgebras, homomorphic images and finite direct products. Pseudovarieties can be defined by sets of pseudoidentities – formal equalities of so-called *implicit operations*. We define hyper-pseudoidentities by equalities of implicit operations and we study the corresponding Galois connections and the complete lattices of all M-solid pseudovarieties of a given type.

On some Classification of the Maximal Subsemigroups of the Semigroup of all Isotone Transformations

I. Dimitrova

We denoted by $\mathbf{T_n} = \mathbf{T_X}$ the semigroup of all full transformations α of the finite set $X = \{1, \ldots, n\}$ under the operation of composition of transformations, by $D_k = J_k = \{\alpha \in T_n : |X\alpha| = k\}, (1 \leq k \leq n-1)$ the J_k-class, and by $I_k = \bigcup\limits_{i=1}^{k} J_i$ the ideal of the semigroup T_n.
In this paper we consider the finite set $X(<) = \{1 < 2 < \cdots < n\}$ – ordered in the standart way. We call the full transformation α of $X(<)$ *isotone (order-preserving)* if $i \leq j \implies i\alpha \leq j\alpha$; the full transformation α of the set $X(<)$ is *increasing (or decreasing) isotone* if for every $i \leq j \implies i\alpha \leq j\alpha \& i \leq i\alpha$ (or $i \leq j \implies i\alpha \leq j\alpha \& i \geq i\alpha$).
We calculate the number of all idempotents ϵ belonging to the same J_k-class ($1 \leq k \leq n$) with fixed kernel equivalence π_ϵ, and also with fixed range (or codomain) $X\epsilon$, i.e. $|X\epsilon| = |X/\pi_\epsilon| = k$.
We describe the maximal subsemigroups of the J_{n-i}- classes and of the I_{n-i} ideals ($i = 1, 2$) of the semigroup of all isotone transformations of the finite set $X(<) = \{1, \ldots, n\}$.
We completely obtain its classification and count its number.
Yang Xiuliang in "Communications in Algebra" (2000) considered the maximal subsemigroups of the I_{n-1} ideal of the semigroup of all isotone transformations of the finite set. Our main result is a continuation of this paper, but the proofs are obtained in different methods.

Distributors and the Power of the Ultrafilter Theorem

M. Erné

A distributor in a semilattice with a monotone multiplication is an upper set U that contains $ab \vee c$ iff it contains $a \vee c$ and $b \vee c$. Distributors are precisely the kernels of homomorphisms onto distributive lattices and form themselves an algebraic locale. The Scott-open distributors are in one-to-one correspondence to the nuclei whose ranges are Wallman locales. Using these concepts, we explain why almost all known prime ideal theorems are equivalent to the Ultrafilter Theorem.

Multidimensional Matrix Algebraic Systems and Their Applications

A. Gasparyan

In the present paper we propose a scientific program for the development of the matrix representation paradigm in an extended framework. We consider algebraic systems of multidimensional matrices with operations of different types including binary, ternary, multi-ary, schem- and network-like operations. After the preliminary study of key properties we get to abstract from matrix nature and determine contiguous algebraic systems. Some important model examples are considered to highlight the key properties and some basic problems. Then we return again to matrix algebraic systems and formulate the problems about multidimensional matrix representations of general algebraic systems, We give preliminary considerations and determine key directions. Here we give only some definitions and key notions. The p-dimensional matrix $A = \|a_{i_1 \dots i_p}\|$ is a direct generalization of a usual two-dimensional matrix to the case of many indices. The elements $a_{i_1 \dots i_p} \in K$ where K is a field, ring or a structure of other type adopting the addition(s) and(or) multiplication(s). If $i_r \in \{1, \dots, n_r\}$, so n_r is called the r-th range of A, thus A is a $n_1 \times \dots \times n_p$-matrix. By $M_{n_1,\dots,n_p}(K)$ we denote the set of all $n_1 \times \dots \times n_p$-matrices. In particular, if $n_1 = \dots = n_p = n$, we have the set $M_n^p(K)$ of $n$cubical p-dimensional matrices over K. The addition of multidimensional matrices as well as their multiples by a $\lambda \in K$ are defined as usual, but there is wide family of matrix multiplications most of which can be expressed means of three simplest operations: a)tensor multiplication, b)trace and c)diagonal selection. Combining these elementary operations in appropriate way, one can compose more and more new operations. Accordingly we can define different type matrix algebraic systems: graded semigroups, graded groups, graded rings (binary and multiary), mani-sorted algebras, tree-like algebras, network- and some other type algebraic systems.

An Introduction to Bisemirings

Sh. Ghosh

In this talk, we introduce an algebraic structure called bisemiring. An algebraic structure (S, +, . , x) is called a bisemiring if (S, +, .) and (S, . , x) are semirings. Various interesting examples show that bisemirings arise quite naturally in the literature of abstract algebra. We have studied some special classes of bisemirings with certain conditions which connect the three binary operations, especially the bisemirings which are induced by Boolean rings and lattice-ordered groups and characterize the class of bisemirings which are subdirect products of them. Next we consider distributive quasi-lattices with a third binary operation, * , that is idempotent, commutative, associative and distributive over others in all possible ways. We proved that such a bisemiring can be embedded in a subdirect product of two distributive quasi-lattices and * is uniquely determined by them.

Very Many Clones Above the Unary Clone

M. Goldstern

It is well known that on a finite base set with k elements there are exactly k+1 clones that contain all unary functions, and they form a chain in the clone lattice (Slupecki et al).
On a countable set it is known that the interval of clones above the unary clone contains exactly two coatoms (Gavrilov), and I showed recently that this interval must be uncountable.
Together with Saharon Shelah we now showed that the cardinality of this interval is the same as the cardinality of the full clone lattice: it embeds the power set of the continuum.

Solving equations over a two-element algebra

T. Gorazd, J. Krzaczkowski[*,†]

This paper presents a complete classification of the SAT problem for any two element algebra. The situation for terms and polynomials is considered. We show the duality for the class of two element algebras. The problem is either **P** or **NPC**. We propose a new definition of complexity classification for algebras.
A problem for algebra **A** will be called:

- **semantically NPC** iff for every algebra **B** such that $Clo(\mathbf{A}) = Clo(\mathbf{B})$ the problem for **B** is **NPC**
- **semantically P** iff for every algebra **B** such that $Clo(\mathbf{A}) = Clo(\mathbf{B})$ the problem for **B** is **P**
- **semantically P-NPC** iff there exist algebras **B** and **C** with $Clo(\mathbf{A}) = Clo(\mathbf{B}) = Clo(\mathbf{C})$ and the problem for **B** is **P**, and for **C** is **NPC**

We show that the problems of solving one or a system of equations over terms and polynomials are either **semantically P** or **semantically NPC**.

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L. Goracinova Ilieva

L. Goracinova Ilieva, S. Markovski

An algebra is said to have the property (k,n) if any of its subalgebras generated by k distinct elements has exactly n elements. The importance of these algebras can be seen by the fact that every such algebra represents a model of k-design. We consider examples of algebras with the property (k,n) and some varieties of such quasigroups.

Hyperquasivarieties

E. Graczyńska

Hyperidentities of a given type were invented by the authors in [1]. The notion of hyperquasivarieties as well as the notion of hyper-quasi-identity were invented in [2]. We generalize some notions invented by A. I. Mal'cev in [3]. Hyperidentities in monounary algebras, considered by T. Waldhauser [4] will be discussed. We will present various examples. Connections of considered notions with some operators on classes of algebras will be shown. Some G. Birkhoff's or Malc'ev's type theorems will be presented.

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Standard QBCC-Algebras

R. Halaš

QBCC-algebras form a natural generalization of BCC-algebras, an extension of BCK-algebras. In the talk we give a construction of QBCC-algebras from quasiordered sets with the top element, the so-called standard QBCC-algebras. Also some important properties of standard QBCC-algebras will be discussed.

Radical Polynomial of Trilinear Alterning Forms

J. Hora

Let f be a trilinear alternating form on an n-dimensional vector space V over a finite field. For every $v \in V$ denote by $r(v)$ the dimension of the subspace $\{u; f(v,u,w) = 0 \text{ for every } w \in V\}$. The polynomial $p(f) = \sum_{v \in V} x^{n-r(v)} y^{r(v)}$ is an invariant of the form and is "compatible" with direct sums of forms. Moreover, in the case of the two-element field $p(f)$ detemines the form f uniquely up to dimension 7.

Generalized Difunctionality, Pixley Categories, and a General Bourn Localization Theorem

Z. Janelidze

We introduce a notion of a *t-closed relation*, where t is an extended matrix

$$\left(\begin{array}{cccc|c} t_{11} & \dots & t_{1,m-1} & & t_{1m} \\ \dots & \dots & \dots & & \dots \\ t_{n1} & \dots & t_{n,m-1} & & t_{nm} \end{array}\right)$$

of terms t_{ij} in an algebraic theory $\mathbf{T}$. In the special case when $\mathbf{T}$ is the variety of sets and t is the matrix

$$\left(\begin{array}{ccc|c} x & y & y & x \\ x' & x' & y' & y' \end{array}\right)$$

obtained from the Maltsev identities

$$p(x, y, y) = x,$$
$$p(x', x', y') = y',$$

a t-closed relation is the same as a difunctional relation. Thus, the notion of a t-closed relation is a generalization of the notion of a difunctional relation. Moreover, the well-known theorem which states that a variety $\mathbf{V}$ of universal algebras is a Maltsev variety (i.e. contains a term p satisfying the identities above; see [S]) if and only if every binary homomorphic relation in $\mathbf{V}$ is difunctional, can be generalized as follows:

Theorem. A variety $\mathbf{V}$, whose theory is equipped with an interpretation of $\mathbf{T}$ in it, contains a term p satisfying

$$p(t_{11}(x_1, ..., x_{ar(t_{11})}), ..., t_{1,m-1}(x_1, ..., x_{ar(t_{1,m-1})})) = t_{1m}(x_1, ..., x_{ar(t_{1m})}), ...,$$
$$p(t_{n1}(x_1, ..., x_{ar(t_{n1})}), ..., t_{n,m-1}(x_1, ..., x_{ar(t_{n,m-1})})) = t_{nm}(x_1, ..., x_{ar(t_{nm})})$$

if and only if every n-ary homomorphic relation in $\mathbf{V}$ is t-closed.

Let $\mathbf{W}$ be a commutative variety and let t be a matrix of terms in the theory of $\mathbf{W}$; the class of $\mathbf{W}$-enriched varieties defined by the equational condition above can be naturally extended to a class of $\mathbf{W}$-enriched categories: take all categories enriched in $\mathbf{W}$ in which every n-ary relation is t-closed. These categories will be called *t-categories*. By extending Maltsev varieties in this way, we would obtain Maltsev categories in the sense of [CLP]. More precisely, a finitely complete category $\mathbf{C}$ is a Maltsev category if and only if $\mathbf{W}$ is a t-category, where t is the matrix corresponding to the Maltsev identities. Moreover, we show that the arithmetical categories in the sense of [Pe], the unital categories in the sense of [B], the strongly unital categories in the sense of [B] and the subtractive categories in the sense of [J3] can also be obtained from the corresponding varieties using the corresponding t-closedness. We consider another special case of the matrix t, which corresponds to the Pixley axioms $p(x,x,y) = p(x,y,x) = p(y,x,x) = x$ (see [Pi]). In this case a t-category will be called a *Pixley category.* Pixley categories (as well as Maltsev categories) have normal local projections in the sense of [J2] (see also [J1]). For varieties this was shown in [J2]. We show that Barr exact Pixley categories can be characterized by the following identity on the equivalence relations:

$$(R \circ S) \wedge T = (R \wedge T) \circ (S \wedge T);$$

this helps to transform the "equation"

(Arithmeticity) = (Maltsev) + (Pixley axiom),

known for varieties, into the "equation"

(Arithmetical [Pe]) = (Maltsev [CLP]) + (Pixley)

for Barr exact categories. Finally, we construct a map $L : M* \to M$, where M is the set of all matrices of terms in the theory of the variety of sets and $M*$ is the set of all matrices of terms in the theory of the variety of pointed sets and prove the following *general Bourn localization theorem*:

Theorem. Let $\mathbf{C}$ be a finitely complete category and let t be a matrix in $M*$. The category $\mathbf{C}$ is a $L(t)$-category if and only if $\mathbf{C}$ is *locally* a t-category, i.e. every fibre of the fibration of pointed objects of $\mathbf{C}$ is a t-category.

This theorem includes the following two theorems as special cases:

Theorem [B]. A finitely complete category $\mathbf{C}$ is a Maltsev category if and only if $\mathbf{C}$ is locally unital.

Theorem [J3]. A finitely complete category $\mathbf{C}$ is a Maltsev category if and only if $\mathbf{C}$ is locally subtractive.

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On Lattices of Topologies of Finite Unary Algebras

A. Kartashova

If $< A, \Omega >$ is an arbitrary algebra, then a topology on the set A is a *topology on the algebra* $< A, \Omega >$ if each operation from Ω is continuous with respect to this topology. The set of all topologies on $< A, \Omega >$ forms a complete lattice where the order is induced by inclusion. This lattice is the *lattice of topologies* of the algebra $< A, \Omega >$ and it is denoted by $\Im(A)$. As usual, the congruence lattice of an algebra $< A, \Omega >$ is denoted by $ConA$. R. McKenzie has shown that for every finite algebra $< A, \Omega >$ there exists a finite algebra $< B, f_1, f_2, f_3, f_4 >$ with four unary operations such that $ConA \cong ConB$ (see, e. g., [1, Theorem 4.7.2]). We prove the similar result for the class of lattices of topologies of unary algebras.
Theorem 1. *If $< A, \Omega >$ is a finite unary algebras then there exists a finite algebra $< B, f_1, f_2, f_3, f_4 >$ with four unary operations such that $\Im(A) \cong \Im(B)$.*
Further, it follows from [1, Theorem 5.6] that there is a finite algebra with two unary operations such that its congruence lattice is isomorphic neither of congruence lattices of unars (a unar is an algebra with one unary operation.) In the present paper the following theorem for the class of lattices of topologies of unary algebras is proved.

Theorem 2. *There exists an infinite set $\mathcal{K}$ of pairwise nonisomorphic lattices such that if $L \in \mathcal{K}$ then $L \cong \Im(A)$ for some finite algebra $< A, f, g >$ with two unary operations. Moreover L is isomorphic to none of the lattices of topologies of unars.*

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Unary Algebras with Hopf Property

V. K. Kartashov

Following [1], we shall say that an algebra $\mathcal{A}$ has Hopf property if every surjective endomorphism of $\mathcal{A}$ is injective. In the paper [2] it is proved that any finitely generated monounary algebra has Hopf property. In the present paper we describe more general class of unary algebras with Hopf property. Let $\mathcal{A} =< A, \Omega >$ be an arbitrary unary algebra of a signature Ω. Then $\mathcal{A}$ is called *commutative* if $fg = gf$ for all symbols $f, g \in \Omega$. The set of all finite words over Ω is denoted by Ω^*. An algebra $\mathcal{A}$ of signature Ω is *strongly connected* if for elements $a, b \in \mathcal{A}$ there exists a word $w \in \Omega^*$ such that $aw = b$. **Theorem.** *The following algebras have Hopf property: 1. any strongly connected commutative unary algebra; 2. any finitely generated commutative unary algebra of a finite signature.*

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Generalizations of MV-Algebras

J. Kühr

MV-algebras can be considered as bounded commutative lattice-ordered semigroups. We deal with (semi)lattices with sectionally antitone permutations and with dually residuated lattice-ordered semigroups which include MV-algebras as a special case and which are in general neither commutative nor bounded.

On the Structure of Generalized Rough Sets

M. Kondo

In this paper we consider some fundamental properties of generalized rough sets induced by binary relations on algebras and show that

1. Any reflexive binary relation determines a topology.
2. If θ is an equivalence relation on a set X, then $\mathcal{O} = \{A \subseteq X \mid \theta_{-}(A) = A\}$ is a toplogy such that A is open if and only if it is closed.
3. Every subalgebra is a rough subalgebra in any algebra.
4. For any pseudo ω-closed subset A of X, $\theta_{-}(A)$ is an ω-closed set if and only if $\omega(x, x, \cdots, x) \in \theta_{-}(A)$ for any $x \in X$.

Moreover we consider properties of generalized rough sets.

On Fractal Quasigroups

S. Markovski

Different kinds of quasigroup transformations of strings whose elements are from a finite set Q can be defined, using a quasigroup operation $*$ on Q. One can notice fractal structure of iteratively transformed strings in several ways. One way is to consider the distributions of the elements in the transformed strings where, for some quasigroup operations, a fractal structure appears. It follows from the realized experiments that we can define a quasigroup $(Q,*)$ to be fractal if the equality $x_{k+1}^{(k)} = x_1 * x_{k+1}$ is an identity in $(Q,*)$, where k is the cardinality of Q. Here the term $x_{k+1}^{(k)}$ with variables $x_1, x_2, \dots, x_{k+1}$ is defined inductively by: $x_i^{(0)} = x_i$, $x_{k+1}^{(k)} = x_k^{(k-1)} * x_{k+1}^{(k-1)}$. It is shown that some classes of quasigroups are fractal, like the class of groups $\mathbb{Z}_p$, for prime p, the class of (known) Steiner quasigroups, and some others.

The Lattice of Varieties of Fibred Automata

A. Mućka

The class of all fibered automata [1] is a variety of two-sorted algebras [2], [3]. The Birkhoff Type Theorem for varieties holds also in the case of many-sorted algebras. In particular, a class of many-sorted algebras is equationally definable if and only if it is a variety. In the talk I will provide a full description of the lattice of varieties of fibred automata together with an equational basis for each of them.

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Undecidability of Automorphism Groups of Countable Homogeneous Graphs

N. Mudrinski

A first-order structure M is called *homogeneous* (or *ultrahomogeneous*) if every isomorphism $\varphi_0 : F_1 \to F_2$ between its finitely generated substructures F_1, F_2 can be lifted to an automorphism φ of M, so that $\varphi|_{F_1} = \varphi_0$. In relational structures (e.g. in graphs), since every subset of the domain of M carries an induced substructure of M, in the above definition one may replace the words 'finitely generated' by 'finite'. Countable homogeneous graphs were completely classified in 1980 by Lachlan and Woodrow. These are: mK_n (m disjoint copies of complete graphs of size n), where at least one of m, n is infinite, the complements of these, the Henson graphs H_n, $n \geq 3$ (the homogeneous countable graphs universal for the class of all finite graphs omitting a clique of size n), the complements of Henson graphs, and the countable random graph R. Also, there is a number of other classes in which the homogeneous structures have been determined (including, for example, finite graphs, countable tournaments, digraphs, posets and finite groups). In this talk, we give an account on the result stating that the automorphism group of every countable homogeneous graph contains a copy of each countable group. From that fact we deduce undecidability of the (universal) theory of such a group. In passing, we show that the Henson graphs H_n are retract rigid. This is a joint research with I.Dolinka (Novi Sad).

Representations of Involutive Rings

N. Niemann

Rings of endomorphisms of vector spaces are classical examples of non-commutative rings. The question under which circumstances a given (non-commutative) ring can be considered as a subring of a ring of endomorphisms leads to a crucial result of Jacobson: Up to isomorphism, the primitive rings are exactly the subrings of endomorphism rings. If additionally, the vector space is equipped with a non-degenerate form ϕ, the set of endomorphisms having an adjoint with respect to ϕ can be considered as an involutive subring of $End(V)$. This gives rise to the term of a representation of an involutive ring: We say an involutive ring R is represented over a vector space V with form ϕ if it can be embedded in a subring of $End(V)$ such that the involution on R corresponds to the adjunction on $End(V)$ with respect to ϕ. This talk is concerned primarily with the characterisation of representable

Jordan Block Structure of Some Unipotent Elements in Modular Representations of Algebraic Groups

A. A. Osinovskaya

Let G be a simple simply connected algebraic group of rank $n > 2$ over an algebraically closed field K of characteristic $p > 0$. For a unipotent element $u \in G$ and a rational representation ϕ of G denote by $J_\phi(u)$ the set of sizes of blocks (without their multiplicities) in the canonical Jordan form of $\phi(u)$. Below $\alpha_1, \ldots, \alpha_n$ is the base of the root system of G, $\omega_1, \ldots, \omega_n$ are the fundamental weights of G, and ϕ is p-restricted with the highest weight $\omega = \sum_{i=1}^n m_i \omega_i$. Let $\mathbf{N}$ be the set of nonnegative integers and $\mathbf{N}_a^b = \{i \in \mathbf{N} \mid a \leq i \leq b,\ a, b \in \mathbf{N},\ a \leq b\}$. Root elements are assumed to be nonunity elements of root subgroups. If u is a root element for a root α, then V is the value of ω on the maximal root of the same length as α. Set $m_\phi(u) = \min(V + 1, p)$ and $c_\omega(u) = \min(m_i + 1 \mid$ α_i and α are of the same length). We assume that $p > 2$ if $G = B_n(K)$, $C_n(K)$, or $F_4(K)$. Call roots α_j and α_k linked if they are connected on the Dynkin diagram. If all roots of G have the same length, we regard them as long. **Definition 1.** The weight ω is locally p-small of type I if $m_j + m_k < p - 1$ for some linked long roots α_j and α_k or $2m_s + m_t < p - 2$ for linked long root α_s and short root α_t. The weight ω is locally p-small of type II if $m_j + m_k < p - 1$ for some linked short roots α_j and α_k, or $G = B_n(K)$ and $2m_{n-1} + m_n < p$, or $G \neq B_n(K)$ and $2m_s + m_t < p - 2$ for linked long root α_s and short root α_t. We proved the following theorem.

Theorem 1. Let $u \in G$ be a root element and u be long for $G = B_n(K)$. Assume that ω is locally p-small of type I for long u and ω is locally p-small of type II for short u. Then $J_\phi(u) = N_1^{m_\omega(u)}$. For all p-restricted representations $N_{c_\omega(u)}^{m_\omega(u)} \subseteq J_\phi(u)$.

Theorem 2. Let $p > 2$ and $G = A_n(K)$. Assume that u is a regular unipotent element of a naturally embedded subgroup of type A_2. Set $\Sigma = \sum_{i=1}^n m_i$, $g = \min(2\Sigma + 1, p)$, $I = \{k \in \mathbf{N}_1^g \mid k \equiv 1 (mod\, 2)\}$, $m = \min_i(m_i + m_{i+1})$, and $M = \min((p-1)/2, \Sigma)$. If $n > 3$ and $m_i + m_{i+1} + m_{i+2} + m_{i+3} < p - 2$ for some $i < n - 2$, then $I \subseteq Jord_\phi(u) \subseteq \mathbf{N}_1^g$. Furthermore, if in this situation $2\Sigma + 1 \leq p$, then $J_\phi(u) = I$. If $m_i + m_{i+1} \leq (p-1)/2$ for some i, then $2k + 1 \in J_\phi(u)$ for all $k \in \mathbf{N}_m^M$.

Sheaves and Presheaves of Differential Rings

A. Ovchinnikov

We construct sections of differential spectrum using only localization and projective limits. For this purpose we introduce a special form of a multiplicative system generated by one differential polynomial and call it D-localization. Due to this technique one can construct section of differential spectrum of a differential ring R without the computation of its differential spectrum diffspec R. We compare our construction with the Kovacic's structure sheaf appeared in [4, 5] and with the results obtained by Keigher in [2]. The Kovacic's approach is the generalization of the Hartshorne's point of view on commutative schemes [1]. Our way to define differential schemes uses the Shafarevich's approach to commutative schemes appeared in [6]. These two constructions are equivalent on a certain class of differential rings. We also show how to compute sections of factor-rings of rings of differential polynomials. All computations in this paper are factorization free.

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Some Aspects of Rational Arithmetic Functions

V. Laohakosol and N. Pabhapote

Rational arithmetic functions are functions of the form

$$g_1 * \cdots * g_r * h_1^{-1} \cdots * h_s^{-1}$$

where g_i, h_j are completely multiplcative functions. Three aspects of these functions, namely, characterizations, Busche-Ramanujan type identities and binomial formulas, are studied.
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Key words and phrases: *arithmetic functions, rational arithmetics functions, Busche-Ramanujan identities*

On Peircean Algebraic Logic

R. Pöschel

The existential graphs devised by Charles S. Peirce can be understood as an approach to represent and to work with relational structures long before the manifestation of relational algebras as known today in modern mathematics. It can be shown that that the expressive power of Pearcean Algebraic Logic (PAL) is equivalent to the expressive power of Krasner-algebras (which extend relational algebras). Therefore, from the mathematical point of view these graphs can be considered as a two-dimensional representation language for first-order formulas. In the talk we sketch the approach of PAL to propositional logic (existential graphs) and to first order logic (relation graphs).

Maximal Clones Containing the Permutations

M. Pinsker

We present for all base sets X of infinite regular cardinality an explicit list of all maximal clones C on X whose unary part $C^{(1)}$ is non-trivial and contains all permutations of X; this generalizes a theorem by L. Heindorf for countably infinite X. It turns out that compared to the size of the whole clone lattice, the number of such clones is relatively small. Moreover, it is a monotone function of the cardinality of the base set X, which is not trivial as for example the number of maximal clones which contain all unary functions varies heavily with certain set-theoretical properties of X. Extending a result due to G. Gavrilov for countably infinite base sets, we then determine on all infinite X all maximal submonoids ("unary clones") of $O^{(1)}$ that contain the permutations and explain their connection with the maximal clones exposed before.

On the Formulability of Derived Objects of Universal Algebras

A. G. Pinus

Let $Sub\mathcal{A}$ be the lattice of all subalgebras of some universal algebra $\mathcal{A}$. Let $P_+Sub\mathcal{A} = \{\mathcal{B} \in Sub\mathcal{A} \mid$ there exist some positive quantiferfree formula $\phi(x)$ sush that $\mathcal{B} = \{a \in \mathcal{A} \mid \mathcal{A} \models \phi(a)\}\}$. Let we define $PSub\mathcal{A}$ ($\exists^+ Sub\mathcal{A}$, $ESub\mathcal{A}$) by analogue with the definition of collection $P_+Sub\mathcal{A}$ if we replace the condition for formula $\phi(x)$ to be positive quantiferfree formula by the condition to be quantiferfree (to be positive existentional, to be elementary) formula.

THEOREM 1. For any finite algebra $\mathcal{A}$ the following conditions 1) and 1'), 2) and 2'), 3) and 3'),4) and 4') are in pairs equivalent: 1) $Sub\mathcal{A} = P_+Sub\mathcal{A}$, 1') all inner homomorphisms of algebra $\mathcal{A}$ are inner epimorphisms of algebra $\mathcal{A}$; 2) $Sub\mathcal{A} = PSub\mathcal{A}$, 2') all innere isomorphisms of algebra $\mathcal{A}$ are automorphisms of algebra $\mathcal{A}$; 3) $Sub\mathcal{A} = \exists^+ Sub\mathcal{A}$, 3') all subalgebras of algebra $\mathcal{A}$ are closed relative to any endomorphism of the algebra $\mathcal{A}$; 4) $Sub\mathcal{A} = ESub\mathcal{A}$, 4') all subalgebras of algebra $\mathcal{A}$ are closed relative any automorphism of the algebra $\mathcal{A}$.

We describe also the finite algebras $\mathcal{A}$ for which some its transpositions are conditional (elementary conditional, positive conditional, $\exists^+$-conditional) termally. For example,

THEOREM 2. For any finite algebra $\mathcal{A}$ the following conditions are equivalent: 1)$Sub\mathcal{A} = ESub\mathcal{A}$ and $Aut\mathcal{A}$ be abelian, 1') all automorphisms of algebra $\mathcal{A}$ are elementary conditional termally automorphism; 2) $Sub\mathcal{A} = \exists^+ Sub\mathcal{A}$ and $End\mathcal{A}$ be abelian, 2') all endomorphisms of algebra $\mathcal{A}$ are $\exists^+$-conditional termally endomorphism.

Meet-continuous Intervals of Subsemilattices are Algebraic

L. Ritter

We consider an interval of the closure system of all κ-subsemilattices of a given κ-semilattice. It is known that such intervals are algebraic for $\kappa = \omega$. We show that these are the only instances of algebraicity for arbitrary κ. This is derived from the fact that in weakly meet-continuous intervals of the above type κ-meets are already finite meets. In the proof we use the notion of strongness as introduced by Faigle.

Sublattices of Suborder Lattices

M. Semenova

Several results on lattices which are embeddable into lattices of suborders of partially ordered sets are presented. One of the main results is that the class of lattices embeddable into suborder lattices of posets of the height n is a finitely based variety, for any natural n.

Involutive Graph Algorithms

E. Shemyakova

In this paper, a new concept of involutive graphs is studied. The concepts of projection and completion of an involutive division are presented. Criteria of noetherity and completeness are presented. A new series of involutive divisions is constructed by applying the completion operation to known involutive divisions.. The divisions of this series are more optimal than the classical involutive divisions for Gerdt's involutive algorithms. Another series of involutive divisions is constructed yielding a solution to Gao's problem. The properties of these divisions are studied.

On the Colouration of Terms

Sl. Shtrakov

Let $\mathcal{F}$ be a finite set of operation symbols with an arity function denoted by $\tau : \mathcal{F} \to N$, X be a set of variables, and $W_\tau(X)$ the set of terms of type τ. A term is coloured if all its operation symbols are supplied with integers (called colours). Coloured terms are useful in computer science for example to describe the structure of file system, in XML technology etc. They are important when applying different hypersubstitutions, (called coloured hypersubstitutions) in universal algebra, also. We consider two ways for colouration of terms. The first one starts from the leaves (variables), and it can be realised by a tree automaton. The second one starts from the root operation symbol, and it can be realised by a Turing's machine. It is proved that these two abstract devices are equivalent.

Dualities for Polytopes

P. Ślusarski

Polytopes (finitely generated convex subsets of real affine spaces) can be considered as barycentric algebras. (And it is well known that each barycentric algebra embeds into a Plonka sum of convex sets.) In this talk, we will establish a duality between the category of such polytopes (with barycentric algebra homomorphisms as morphisms) and a certain category of representation spaces. We will also provide a characterisation of the latter category. Our result generalizes a duality for quadrilaterals given by K. Pszczo ła, A. Romanowska and J. D. H. Smith.

Totally Reflexive Totally Symmetric Pattern Algebras

E. Vármonostory

A possible approach to conservative operations is to consider them as relational pattern functions or ϱ-pattern functions. A k-ary relation ϱ on a set A induces a partion of each power A^k; into "patterns" in a natural way. An operation on A is called a ϱ-pattern operation if its restriction to each pattern is a projection. An algebra is called a ϱ-pattern algebra if its fundamental operation are ϱ-pattern functions for the same relation ϱ on A. A finite algebra A is called functionally complete if every (finitary) operation on A is a polynomial operation of A. We examine functional completenes of algebras with ϱ-pattern fundamental operations in the case when ϱ is the totally symmetric relation of A.

Hypersubstitutions in the Variety of Left Symmetric Left Distributive Groupoids

A. Vanžurová

Examples of left symmetric left distributive groupoids in low orders are given. Normal forms for terms in the variety SD are presented, and the corresponding normal form hypersubstitutions are selected. Multiplication in the groupoid of normal form hypersubstitutions is calculated, and proper normal form hypersubstitutions with respect to SD are determined. They form a monoid of order four.

On Some Properties of Characterizable Ideals in Rings of Differential Polynomials

A. Zobnin

Working in rings of differential polynomials, we study the property of a radical differential ideal to be characterizable. This property and is not stable under isomorphisms. We show that in the algebraic case almost all radical ideals are characterizable. In the differential case we represent non-characterizable ideals as homomorphic images of characterizable ones. For any natural number n, we construct a radical differential ideal that has exactly n components in its minimal characteristic decomposition.

Free Abelian Extensions of p-permutable Algebras

P. B. Zhdanovich

Let S_0, S_p be semigroups and S the free product $S_0 * S_p$ with adjoined unit element 1. Denote by V the variety of acts over S with a ternary operation $p(x, y, z)$ satisfying the two Mal'cev identities: $p(x, x, y) = p(y, x, x) = y$ and also identities of the form

$$p(s(x), s(y), s(z)) = s(p(x, y, z)),$$

for each $s \in S_p$. Algebras from V are called S_p*-permutable*. The variety V is congruence permutable due to the Mal'cev theorem. We explore the construction of free abelian extensions in V, which is applied to the construction of solvable V-algebras. The class of S_p-permutable algebras is important because the construction of free abelian extensions and free solvable algebra in an arbitrary congruence permutable variety is reduced to the case of V-algebras. The main results are the following.

1. We give a construction of a free abelian extension of an arbitrary V-algebra A in terms of A. We also construct the free solvable V algebra of degree k for each $k \in \mathbb{N}$.

2. The variety V_k of all solvable S_p-permutable algebras of degree $n \leq k$ has a solvable identity problem if and only if S has a solvable word problem.

3. A class of all solvable algebras is not contained in any proper subvariety of V.

4. Let F_q^k be the free solvable algebra of degree k and of a finite rank q. If S is finite, the algebra F_q^k is not embedded into F_r^k, provided $q > k$. This is not the case for finitely generated infinite monoids.

5. The algebra A is called *Hopfian* if each epi-endomorphism of A is in fact an automorphism.