71st Workshop on General Algebra

together with

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University of Zielona Góra (Poland) and Potsdam University (Germany)

ABSTRACTS

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Types of polynomial completeness for expanded groups

Erhard Aichinger

with
Nebojša Mudrinski

From results of Maurer, Rhodes, and Fröhlich, we know that every function on a finite simple non-abelian group is a polynomial function; these groups are called *polynomially complete*. Later, it was studied when every congruence preserving function on an algebra is a polynomial function; such algebras were called *affine complete*. In 2001, P. Idziak and K. Słomczyńska introduced the concept of *polynomial richness*. In general, it seems hard to characterize when a single algebra is affine complete or polynomially rich. Characterizations of affine complete algebras have been given for abelian groups (Nöbauer, Kaarli) and for finite algebras with a Mal'cev polynomial that have a distributive congruence lattice (Hagemann, Herrmann, Kaarli). It is not known if there is an algorithm that decides whether a given finite algebra (of finite type, with given operation tables for all operations) is affine complete. We will investigate finite modular lattices that satisfy a condition that is more general than distributivity. Based on work by Idziak and Slomczynska, we can characterize affine complete and polynomially rich algebras among those finite algebras that have a group operation among their binary polynomial functions, and whose congruence lattice satisfies this condition on the congruence lattice. This is joint work with Nebojša Mudrinski (Novi Sad).

Address: Johannes Kepler University Linz
E-mail: erhard@algebra.uni-linz.ac.at

Algebraic universality of set endofunctors

Libor Barto

In this talk, we will survey some results concerning various forms of universality: group-universality, monoid-universality, alg-universality, universality and hyper-universality. A new result (alg-universality of set endofunctors) will be mentioned together with its connection with the category of clones and clone homomorphisms.

Address: Charles University, Prague
E-mail: jetel@matfyz.cz

Some subloops of the loop of norm 1 elements in the split Cayley - Dickson algebra

Evgenii Bashkirov

Let F be a field. By $O(F)$ we denote the split Cayley–Dickson algebra over F. Let $S(F)$ be the set of all elements x in $O(F)$ whose norm $N(x)$ is equal to 1. Since the equality $N(xy) = N(x)N(y)$ satisfies for any x, y in $O(F)$, the set $S(F)$ forms a non-associative Moufang loop with respect to the multiplication defined in $O(F)$.

Theorem. Let F be a field of characteristic $\neq 2$ and k be a subfield of F such that the extension F/k is algebraic. Assume k contains more than three elements. If G is a subloop of the loop $S(F)$ such that G contains $S(k)$ and G is generated by unipotent elements, then $G = S(L)$ where L is a subfield of F containing k.

Address: Belorussian State University of Informatics and Radioelectronics
E-mail: bashkirov57@mail.ru

The table of characters of some quasigroups

Grzegorz Bińczak

with
Joanna Kaleta

In my talk I describe the table of characters of quasigroups $(Z_n, -_n)$.

Address: warsaw University of Technology
E-mail: binczak@mini.pw.edu.pl

The locally closed clones over a countable set that contain all permutations

Manuel Bodirsky

with
Hubie Chen

The clones over a countable set that contain all the permutations were first studied by Heindorf, and subsequently by Pinsker. Locally closed clones containing all permutations arise as the polymorphism clones of structures with a highly transitive automorphism group. These structures were studied in theoretical computer science, where they are used as templates for constraint satisfaction problems. In previous work, Kara and the first author showed that there are two minimal such clones, which correspond to two maximally tractable constraint languages. We present a full description of the lattice of locally closed clones on a countable set that contain all permutations. This has applications for the complexity of quantified infinite-valued constraint satisfaction problems.

Address: Humboldt-Universität zu Berlin
E-mail: bodirsky@informatik.hu-berlin.de

Results on Category of Hyper BCK-algebras

R. A. Borzooei

with
H. Harizavi

In this paper we first define the category of hyper BCK-algebras. After that we show that the category of hyper BCK-algebras is connected, factorisable and has equalizers, coequalizers, products, coproducts, intersection and kernel. It is a consequence that this category is complete and cocomplete and hence has pullbacks and pushouts.

Address: Department of Mathematics, Islamic Azad University of Zahedan, Zahedan, Iran
E-mail: borzooei@hamoon.usb.ac.ir

Coalgebras as models of systems.

Tomasz Brengos

This talk is devoted to a presentation of some aspects of the theory of coalgebras. A certain algorithm for finding greatest bisimulation between two coalgebras is introduced. Interesting properties are proved. Crucial property concerning the number of steps performed by the "algorithm" is given and its connection with some preservation properties of the Set endofunctor and boundedness of the functor is shown.

Address: Warsaw University of Technology, Faculty of Mathematics and Information Sciences
E-mail: t.brengos@gmail.com

Periodicity of Bi-ideals

Jānis Buls

We investigate so called finetely generated bi-ideals. Our terminology is more or less standard (cf.[1]). Let A be a finite non-empty set and A^* be a free monoid generated by A. The identity element of A^* is denoted by λ. We set $A^+ = A^* \setminus \{\lambda\}$; $w^0 = \lambda$, $w^{n+1} = w^n w$, $w^* = \bigcup_{n=0}^{\infty}\{w^n\}$ for any $w \in A^*$. The word u (respectively v) is called a *prefix* (respectively a *suffix*) of $w \in A^*$ if $w = uv'v$. We denote respectively by $\mathrm{Pref}(w)$ and $\mathrm{Suff}(w)$ the sets of w prefixes and suffixes. Any total map $x : \mathbb{N} \to A$ is called ω-*word* on the alphabet A. We represent it as sequence $x = x_0x_1 \dots x_n \dots$ We denote by w^ω the ω-word $w^\omega = ww \dots w \dots$ for any $w \in A^+$ and call it *periodic*. **Definition**. Let $u_0, u_1, \dots, u_n, \dots$ be an infinite sequence, where $u_0 \neq \lambda$ and $\forall i u_i \in A^*$. We set $v_0 = u_0$, $v_{n+1} = v_n u_{n+1} v_n$. The word $x = \lim\limits_{n\to\infty} v_n$ is called a *bi-ideal*. The bi-ideal x is called *finitely generated* if $\exists m \forall i \forall j (i \equiv j (\mathrm{mod} m) \Rightarrow u_i = u_j)$. We say in this situation m–tuple $(u_0, u_1, \dots, u_{m-1})$ generates the bi-ideal x. **Theorem**. If $\bigcup_{i=0}^{m-1} \mathrm{Pref}(u_i)$ or $\bigcup_{i=0}^{m-1} \mathrm{Suff}(u_i)$ has at least two words with one and the same length then the bi-ideal generated by $(u_0, u_1, \dots, u_{m-1})$ is not periodic. **Theorem**. The bi-ideal generated by the tuple $(u_0, u_1, \dots, u_{m-1})$ is periodic if and only if then

$$\exists w \forall i \in \overline{0, m-1} u_i \in w^*.$$

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Address: University of Latvia
E-mail: buls@fmf.lu.lv

Unsolid and Fluid Strong Varieties of Partial Algebras

Saofee Busaman

with
Klaus Denecke

A partial algebra A is a pair consisting of a set A and an indexed set of partial operations. A pair of terms (p, q) over the partial algebra A is said to be a strong regular identity in A if in p and q the same variables occur and if the right hand side is defined whenever the left hand side is defined and both are equal. A strong regular identity (p, q) is called a strong regular hyperidentity if after replacing of operation symbols occurring in p and q by terms of the same arity, the arising identity is satisfied as a strong regular identity. If every strong regular identity in a strong variety of partial algebras is satisfied as a strong regular hyperidentity, the strong variety is called solid. We consider the other extreme case when the set of all strong regular identities of a strong variety of partial algebras is invariant only under the identical replacement of operation symbols by terms. This leads to the concepts of unsolid and fluid varieties and some generalizations.

Address: Potsdam University, Germany
E-mail: bsaofee@bunga.pn.psu.ac.th

Equational Characterization of Boolean clones

Runglawan Butkote

with
K. Denecke

Let $C \subseteq O^{(n)}(A)$ be a set of n-ary operations defined on A. The operation $f^A \in C$ satisfies an equation $s \approx t$ where $s, t \in W_{(n)}(X)$, if $s \approx t$ is an identity in the algebra (A, f^A). This relation defines a Galois connection between classes of n-ary operation and sets of equations of type $\tau = (n)$. We denote the Galois-closed set of operations by $FMod^{(n)}\Sigma$ for $\Sigma \in W_{(n)}(X)^2$. Finally we prove that the Galois-closed sets of operations are unions of n-ary clones iff every equation is satisfied by every operation as hyperidentity.

Address: University of Potsdam, Department of Mathematics,Postdam, Germany
E-mail: unique1st@hotmail.com

Constructions on P-choice algebras

Ivan Chajda

with
Shelly L.Wismath

Consider algebras of a given type. Let P be a partition of the set of operation symbols of this type. An identity s = t of this type is called P-compatible if eithe s and t are the same variable or s and t start with operation symbols which belong to the same class of the partition P. This concept was introduced by J.Plonka in 1990. We introduce the concept of a P-choice algebra which is a generalization of the choice algebra introduced by the author recently. We will deal with algebraic constructions of homomorphic immages, subalgebras and products of P-choice algebras and we will present isomorphism theorems for these algebras.

Address: Palacky University Olomouc
E-mail: CHAJDA@INF.UPOL.CZ

Retracts and Q-independence

Anna Chwastyk

A non-empty set X of the carrier A of an algebra $\mathbf{A}$ is called Q-*independent* (for a given set Q of mappings) if
an equality of two term operations f and g of the algebra $\mathbf{A}$ on any finite system of elements $a_1, a_2, ..., a_n$ of X implies
$f(p(a_1), p(a_2), ..., p(a_n)) = g(p(a_1), p(a_2), ..., p(a_n))$ for any mapping $p \in Q$. An algebra $\mathbf{A}$ is a *retract* of $\mathbf{B}$ if $\mathbf{A}$ is the image of a *retraction* (i.e. of an idempotent endomorphism of $\mathbf{B}$). We investigate Q-independent subsets of algebras which have a retraction in their set of term operations.

Address: Technical University of Opole
E-mail: ach@po.opole.pl

On Stone semirings

Jānis Cīrulis

An ordered semiring $(S, +, \cdot, 0. \leq)$ is said to be

sum-ordered if $x \leq y$ iff $y = x + u$ for some u,
pseudocomplemented if every element a of S has the pseudocomplement $a^* := \max\{x\colon xa = 0 = ax\}$,
a *Glivenko semiring* if it is psedocomplemented and $x = 0$ whenever $x^2 = 0$,
a *Stone semiring*, if it is a Glivenko semiring in which $x^* + x^{**} = 0^*$.

A distributive pseudocomplemented lattice is an example of a Glivenko semiring. It follows from Theorem 1 of [1] that the set of all pseudocomplements in a Glivenko semiring is a Boolean lattice $(B, \curlyvee, \curlywedge)$ w.r.t. the original ordering of S. Moreover, $(B, \curlywedge)$ is a subalgebra of $(S, \cdot)$ iff $\cdot$ is idempotent on B. The following theorem generalises Theorems 4 and 5 of [1].
Theorem. Suppose that a Glivenko semiring S is sum-ordered. Then

(a) S is a Stone semiring if and only if $x^* + y^* = (xy)^*$,
(b) S is a Stone semiring only if $(B, \curlyvee)$ is a subalgebra of $(S, +)$,
(b) if S is a Stone semiring with $\cdot$ idempotent on B, then $(B, \curlyvee)$ is a subalgebra of $(S, +)$.

These three assertions were proved in [1] under a stronger assumption that the additive semigroup $(S, +)$ in the Glivenko semiring S is an upper semilattice w.r.t. $\leq$.
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Address: University of Latvia
E-mail: jc@lanet.lv

Implication in Equational Logic and the Commutator

Janusz Czelakowski

The focus of the talk is on applying a general notion of an implication viewed as a set of quaternary equations that jointly possess the property of detachment relative to a given equational system. A significant role of a Day implication system is shown in the context of the commutator theory for equational logics. Let $\vdash^{eq}$ be a finitary and structural equational consequence relation defined on the set $Eq(\tau)$ of equations of a given signature τ. A finite set of equations in four variables $P = \{p_i(x, y, z, w) \approx q_i(x, y, z, w) : i \in I\}$, more suggestively denoted by $x \approx y \Rightarrow_P z \approx w$, is called an *implication system* for $\vdash^{eq}$ if the following condition holds

(iD1) $\quad x \approx y, x \approx y \Rightarrow_D z \approx w \vdash^{eq} z \approx w,$

i.e., the set $x \approx y \Rightarrow_D z \approx w$ has the detachment property relative to the system $\vdash^{eq}$. This means that the equation $z \approx w$ follows from the set of equations $\{x \approx y\} \cup \{p_i(x, y, z, w) \approx q_i(x, y, z, w) : i \in I\}$ in the system $\vdash^{eq}$. Since every finitary and structural consequence $\vdash^{eq}$ is semantically determined by a unique quasivariety $\mathbf{Q}$ of algebras, (iD1) means that the implication $x \approx y \wedge \bigwedge_{i\in I} p_i(x, y, z, w) \approx q_i(x, y, z, w) \Rightarrow z \approx w$ is valid in $\mathbf{Q}$. A set of equations in four variables $D = \{p_i(x, y, z, w) \approx q_i(x, y, z, w) : i \in I\}$, denoted by $x \approx y \Rightarrow_D z \approx w$, is called a *Day implication system* for $\vdash^{eq}$ if it satisfies (iD1) together with two other conditions:

(iD2) $\quad \vdash^{eq} x \approx y \Rightarrow_D x \approx y,$

i.e., $\Rightarrow_D$ has the identity property relative to $\vdash^{eq}$, and

(iD3) $\quad \vdash^{eq} x \approx x \Rightarrow_D y \approx y.$

(Condition (iD2) thus means that for every $p_i(x, y, z, w) \approx q_i(x, y, z, w)$ in D, the equation $p_i(x, y, x, y) \approx q_i(x, y, x, y)$ is $\vdash^{eq}$-valid. Similarly, (iD3) states that for every $p_i(x, y, z, w) \approx q_i(x, y, z, w)$ in D, the equation $p_i(x, x, y, y) \approx q_i(x, x, y, y)$ is $\vdash^{eq}$-valid.) The significance of Day implications for equational logics systems from the fact that it characterizes congruence-modular varieties of algebras and, more widely, in the context of quasi-varieties of algebras, it characterizes the relative shifting property. The notion of a Day implication, although not explicitly defined by Day himself, can be shown to be equivalent to a Malcev's type characterization of congruence-modular *varieties* provided by Day [1969]. The main result shows that in the presence of a Day implication, various definitions of centralizators of relative congruences in quasivarieties of algebras are equivalent. (Centralizators are certain congruences defined on the algebras of the quasivariety corresponding to $\vdash^{eq}$.) Some further refinements of relevant notions in this context are also discussed.. The presence of Day implication systems in equational systems makes the theory of centralizators and of the commutator itself more digestible and transparent. However, when one is concerned with the additivity and the correspondence properties of the commutator for quasivarieties, the above implication systems are generally too weak to yield a workable commutator theory. A stronger property is needed, viz. modularity. The theory of the commutator for congruence modular varieties of algebras is presented in Freese and McKenzie [1987]. Kearnes and McKenzie [1992] have subsequently extended this theory onto relative congruence-modular quasivarieties. Our contribution consists mainly in an attempt to disentangle various intricate (often syntactic) characterizations of the commutator and relevant notions and to render them in a more transparent logical form provided by the conceptual framework of the contemporary Abstract Algebraic Logic.

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Address: Opole University & Technical University of Opole
E-mail: jczel@math.uni.opole.pl

Condition of Abelity and Linear Representations of Algebras

S.S.Davidov

The abelian algebras were studied by various authors (in particular Smith, Romanovska, Kepka and Movsisyan [1-4]) and have been given various names in literature: entropic, medial, alternation, bi-commutative, bisymmetric, surcommutative. There is a connection with the information-theoretic concept of entropy [3]. A binary algebra (Q, Σ) is caaed abelian if it satisfies the abelian hyperidentity X(Y(x,y), Y(u,v)) = Y(X(x,u), X(y,v)). Our finaly aim is to find a simple algorithm deciding for any pairs (u,v) of terms if the identity u = v is satisfied in all commutative abelian algebras. For which, we study the linear representations of abelian algebras. 1. A.B.Romanovska, J.D.H.Smith, Modes, World Scientific, Singapore, 2002. 2. Yu.M.Movsisyan, Hyperidentities in algebras and varieties, Uspekhi Mat. Nauk. 53(1998), 61-114. English transl. in Russ. Math. Surveys 53(1998). 3. J.D.H.Smith, Entropy, character theory and centrality of finite quasigroups, Math. Proc. Cambridge Philos. Soc., 108, 1990, 435-443. 4. J.Jezek, T.Kepka, Medial groupoids, Praha, 1983.

Address: Yerevan State University, Department of Mathematics
E-mail: davidov@ysu.am

ND-solid varieties

K. Denecke

with
P. Glubudom

A non-deterministic hypersubstitution maps any operation symbol of a tree language of type τ to a tree language. Non-deterministic hypersubstitutions can be extended to mappings which map tree languages to tree languages preserving the arities. We define the application of a non-deterministic hypersubstitution to an algebra of type τ and obtain a class of derived algebras. Non-deterministic hypersubstitutions can also be applied to equations of type τ. Formally, we obtain two closure operators which turn out to form a conjugate pair of completely additive closure operators. This allows us to use the theory of conjugate pairs of additive closure operators for a characterization of M-solid non-deterministic varieties of algebras. As an application we consider M-solid non-deterministic varieties of semigroups. *Key Words: Non-deterministic hypersubstitution, conjugate pair of additive closure operators, M-solid non-deterministic variety. AMS Mathematical Subject Classification: 08A35, 08A40, 08A70.*

Address: University of Potsdam
E-mail: kdenecke@rz.uni-potsdam.de

Maximal submonoids of the monoid of all hypersubstitutions of type $\tau = (n)$

Ilinka Dimitrova

with
Jörg Koppitz

For all transformations α of the full transformation semigroup $\mathcal{T}_n$ on the chain $X_n = \{1, \ldots, n\}$, we denote by σ_α the hypersubstitution σ with $\sigma(f) = f(y_{(1\alpha)}, \ldots, y_{(n\alpha)})$, where $Y_n = \{y_1, y_2, \ldots, y_n\}$ is a finite alphabet of variables. We consider the set $T_n^h := \{\sigma_\alpha : \alpha \in \mathcal{T}_n\}$ and the set $O_n^h := \{\sigma_\alpha : \alpha \in O_n^*\}$, where $O_n(O_n^*)$ is the semigroup of all isotone transformations (with the identity mapping ε on the set X_n). The sets T_n^h and O_n^h form submonoids of the monoid $Hyp(n)$ of all hypersubstitutions of type $\tau = (n)$. We give the connection between maximal subsemigroups of a given transformation semigroup $S \subseteq \mathcal{T}_n$ and maximal submonoids of the monoid $S^h \subseteq T_n^h$. Since we know all maximal subsemigroups of the ideals I_k $(1 \leq k \leq n)$ of $\mathcal{T}_n$ (see [3]) and all maximal subsemigroups of the ideals $\hat{I}_s$ $(s = n-2, n-1)$ of O_n (see [4] and [5]) we determine all maximal submonoids of $I_k^h \cup \{\sigma_\varepsilon\}$ and $\hat{I}_s^h \cup \{\sigma_\varepsilon\}$. Finally, we characterize all maximal submonoids M of $Hyp(n)$ with $T_n^h \setminus M \neq \emptyset$.

Address: Potsdam University, Germany
E-mail: ilinka_dimitrova@yahoo.com

Quasi-distributive implication groupoids

Petr Emanovský

with
Radomír Halaš

Distributive implication groupoids as an essential generalization of the implication reduct of intuitionistic logic were introduced and studied by the coauthor and I. Chajda. It has been proved that for these algebras ideals, deductive systems and congruence cernels coincide. In the paper the same connection is shown even if the implication groupoid is quasi-distributive.

Address: Palacky University Olomouc
E-mail: emanovsky@inf.upol.cz

Tensor products for closure spaces and Galois connections

Marcel Erné

We introduce three kinds of tensor products for an arbitrary (possibly infinite) family of closure spaces; all three tensor products coincide in case the number of factors is finite. In particular, the tensor product of two closure spaces $\underline{X}_1$ and $\underline{X}_2$ consist of all subsets T of $X_1 \times X_2$ (called tensors) such that $Y_1 \times Y_2 \subseteq T$ implies $cl(Y_1) \times cl(Y_2) \subseteq T$. However simple the definition may look like, the structure of tensor products is rather complicated even for very small spaces or posets. We characterize the tensor products by universal properties with respect to certain classes of (separately or jointly) continuous maps. Under mild restrictions, tensor products satisfy the expected (finite and infinite) associative and distributive laws, but the proofs are rather involved. Binary tensor products carry a natural multiplication (obtained by taking the closure of the relation product). It turns out that $\underline{X} \otimes \underline{X}$ becomes a quantale with that multiplication if and only if $\underline{X}$ (respectively, the corresponding closure system) is pseudocomplemented (here, for simplicity, one has to assume the empty set to be closed). Surprisingly, that quantale has a unit element only in very special cases, namely, when the closure system is an atomic Boolean algebra (hence isomorphic to a powerset). This part is closely related to the theory of Galois connections and was obtained in collaboration with Jorge Picado (Coimbra). Our results have interesting applications in various fields of algebra, order and lattice theory, topology and formal concept analysis—and the approach via closure spaces unifies and facilitates the arguments considerably.

Address: Faculty for Mathematics and Physics, University of Hannover, Welfengarten 1, D 30167 Hannover, Germany
E-mail: erne@math.uni-hannover.de

Latin squares

Jan Gałuszka

Connections between non-regular varieties of groupoids and classes of right(left)-quasigroups are described. Moreover a characterization of column and row Latin squares with respect to some non-regular identities is done.

Address: Silesian University of Technology, Gliwice, Poland
E-mail: Jan.Galuszka@polsl.pl

Matrix Graphs and Matrix Networks: Algebra and Combinatorics

Armenak Gasparyan

Connection and hybridization of graph, hypergraph and matrix conceptions brough to new type algebraic objects and structures enabling us to draw a new look on many famous facts and to new settings, approaches and solutions. The matrix graphs, matrix hypergraphs and matrix networks arised instantly in our research devoted to enumerative problems in graph theory. But formally we introduce them as special type formulas within a partial algebra of multidimensional matrices, directly associated with graphs, hypergraphs and networks. Decomposition of newly defined objects seems as highly important both for algebraic and combinatorial tasks. Particularly, we give an essential extension of chromatic structures of graphs and hypergraphs. Another advantage of matrix network formalism we will illustrate on sample applications to graphical substructural combinatorics.

Address: Pereslavl-Zalesskii, Russia
E-mail: armen@armen.pereslavl.ru

M-hyperquasivarieties

Ewa Graczyńska

with
Dietmar Schweigert, in memoriam

In our common paper [2] we considered the notion of M-hyperquasi-identity and M-hyperquasivariety, as a common generalization of the concept of hyper-quasi- identity and hyperquasivariety of [1]. We presented some natural examples of such notion. Results are mainly connected with some natural operators on classes of algebras, invented by Mal'cev in [3] and many others. One of the main operator is the operator D of taking so called derived algebras of a given type. We pointed the role of this operator and its interplay with the well known ones.

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Address: University of Technology, Opole, Poland
E-mail: egracz@po.opole.pl

Three-weight problem and Combinatorial Nullstellensatz

Jarosław Grytczuk

with
Tomasz Bartnicki and Stanisław Niwczyk

Let $\mathbb{F}$ be a field and let S be a subset of $\mathbb{F}$. We say that a simple graph G is *weight colorable* by S over $\mathbb{F}$ if there is a function (*edge weighting*) $w : E \to S$ such that for any two adjacent vertices $u, v \in V(G)$, the sum of weights around u is different than the sum of weights around v. This notion was introduced by Karoński, Łuczak and Thomason, who proved that every connected graph G (except a single edge) is weight colorable by any subset $S \subset \mathbb{R}$, $|S| \geq 183$, which is independent over the rational field $\mathbb{Q}$. They also conjectured that any such graph is weight colorable by the set $\{1, 2, 3\}$ over $\mathbb{Q}$, which is still open.

Our approach to the problem uses the algebraic method of Alon, known as Combinatorial Nullstellensatz. The basic idea goes as follows. We associate a multivariable polynomial P_G with a graph G, so that a non-zero substitution for variables of P_G gives a desired weighting of G. Then we look at the exponents of variables in the expansion of P_G into a linear combination of monomials. If there is a non-vanishing monomial with the highest exponent of a variable less than 3, then we are done by Combinatorial Nullstellensatz. We conjecture that this is the case for every graph G without an isolated edge. If true this implies the "123-conjecture" in a stronger, *list version*. We prove our conjecture for several classes of graphs, including cliques, complete bipartite graphs, and trees, by providing general recursive constructions preserving the desired algebraic properties of the polynomial P_G.

Address: University of Zielona Góra
E-mail: j.grytczuk@wmie.uz.zgora.pl

Algebra labeled transition systems

H. Peter Gumm

In previous work, we have described the coalgebraic properties of state based systems whose transitions were labeled either by the elements of a complete lattice $\mathcal{L}$ or by a commutative monoid $\mathcal{M}$. It was shown that the corresponding coalgebraic signature functors weakly preserve kernel pairs (congruences) iff $\mathcal{L}$ is *join-infinitely-distributive*, resp. iff $\mathcal{M}$ is *refinable*. Here we first show that the mentioned coalgebraic type functors can both be seen as instances of a more general construction of a *copower functor* . We can instantiate this functor with any variety $\mathcal{V}$ and any algebra $\mathcal{A} \in \mathcal{V}$ so that its coalgebras become $\mathcal{A}$-labeled transition systems whose structure theory further depends on $\mathcal{V}$. As an example, we consider the case where $\mathcal{S}$ is the variety of all semigroups and $\mathcal{M} \in \mathcal{S}$. We show that the corresponding signature functor weakly preserves kernel pairs if and only if $\mathcal{M}$ is *equidivisible*.

Address: Philipps University Marburg
E-mail: gumm@mathematik.uni-marburg.de

Structure of varieties determined by their numerical invariants

Pawel M. Idziak

with
Joel Berman, Petar Markovic, Ralph McKenzie and Matt Valeriote

We address the questions how the numerical invariants influence the structure of algebras in a finitely generated (or locally finite) variety $\mathcal{V}$. We answer these questions for the following invariants (as functions of n):
- number of nonisomorphic n-generated algebras in $\mathcal{V}$,
- number of subalgebras of $\mathbf{A}^n$, for $\mathbf{A} \in \mathcal{V}$,
- number of congruences in subalgebras of $\mathbf{A}^n$, for $\mathbf{A} \in \mathcal{V}$,
- number of generators needed for subalgebras of $\mathbf{A}^n$, for $\mathbf{A} \in \mathcal{V}$,

under the assumption that those numbers are small (polynomially or singly exponentially bounded).
Also, a connection with the existence of PTIME-algorithms for Constraints Satisfaction Problems is discussed.

Address: Jagiellonian University
E-mail: idziak@ii.uj.edu.pl

Green's relations on Menger semigroups of terms

Prakit Jampachon

with
Klaus Denecke

Defining an $(n+1)$-ary superposition operation S^n on the set $W_\tau(X_n)$ of all n-ary terms of type τ, one obtains an algebra n-clone $\tau := (W_\tau(X_n); S^n, x_1, \ldots, x_n)$ of type $(n+1, 0, \ldots, 0)$. The algebra n-clone τ is free in the variety of all Menger algebras. Using the operation S^n there are different possibilities to define binary associative operations on the set $W_\tau(X_n)$. In this paper we study Green's relations in semigroups of terms with these binary associative operations as fundamental operations.

Address: Potsdam University, Potsdam, Germany
E-mail: prajam@kku.ac.th

Semidirect product of lattices

Přemysl Jedlička

The semidirect product of lattices is the lattice analogue of the semidirect product in the group theory. Both exist in the "inner version", *i.e.* as a decomposition of the structure and in the "outer version", *i.e.* as a construction made from two substructures.

The analogy is best to see in the case of semilattices: Let K, H be two join-semilattices and let $\varphi : K^2 \to \mathrm{End}(H)$ be a mapping satisfying some associativity condition. Then the semidirect product $K \ltimes_\varphi H$ is defined as the set $K \times H$ equipped with the operation

$$(k_1, h_1) \vee (k_2, h_2) = (k_1 \vee k_2, \varphi_{k_1,k_2}(h_1) \vee \varphi_{k_2,k_1}(h_2)).$$

On the other hand, nearly every semilattice with a congruence having all classes of the same shape (thus isomorphic), can be expressed as the above construction.

Address: Czech Agricultural University, Prague
E-mail: vincenzo@artax.karlin.mff.cuni.cz

Recognizing collapsing words

Andrzej Kisielewicz

with
Alessandra Cherubini (Politecnico di Milano)

I will tell about problems of collapsing words arising in automata theory, and present the main ideas of the proof that the problem of recognizing collapsing words is co-NP-complete. A special focus is on interesting connections with combinatorial problems concernig systems of permutation conditions and coloring binary trees with distinguihed vertices.

Address: University of Wrocław, Dept.artment of Mathematics and Computer Science
E-mail: kisiel@math.uni.wroc.pl

Number of solutions to systems of equations over finite semigroups

Ondrej Klima

with
Pascal Tesson, Benoit Larose

We study the computational complexity of counting the number of solutions to systems of equations over a fixed semigroup. We extend the work of Nordh and Jonsson who obtained a dichotomy theorem in the case of monoids, to prove a dichotomy theorem for semigroups. These results use strong connections between our problem and the counting constraint satisfaction problem.

Address: Masaryk University, Brno, Czech Republic
E-mail: klima@math.muni.cz

Sectional pseudocomplemented nearlattices

Miroslav Kolarik

with
Ivan Chajda (Palacky University Olomouc)

By a nearlattice is meant a join-semilattice where every principal filter is a lattice with respect to the induced order. Alternatively, a nearlattice can be described as an algebra with one ternary operation satisfying eight simple identities. Hence, the class of nearlattices is a variety. We characterize nearlattices every sublattice of which is distributive. Then we introduce the so-called section pseudocomplementation on nearlattices which can also be characterized by identities.

Address: Department of Algebra and Geometry, Palacky University Olomouc
E-mail: kolarik@inf.upol.cz

Green's relation and maximal monoids of hypersubstitutions

Jörg Koppitz

For a natural number $n \geq 1$, we consider the set $W(X_n)$ of all terms of type $\tau = (n)$ over the alphabet $X_n := \{x_1, \ldots, x_n\}$. On this set one can define an associative operation $\circ$ by setting $s \circ t$ is the term that we get by the application of the hypersubstitution σ_s (which maps the n-ary operation symbol to the n-ary term s) to the term t. The set $W^f(X_n)$ of all terms, in which all variables of X_n are occurring, forms a subsemigroup of $(W(X_n); \circ)$. We determine all maximal subsemigroups of $(W^f(X_n); \circ)$. Using these results we can describe a class of maximal submonoids of the monoid of all hypersubstitutions of type $\tau = (n)$. This is of particular interest for the case $n = 2$, where we know already the atoms.

Address: University Potsdam
E-mail: koppitz@rz.uni-potsdam.de

A non-associative generalization of MV-algebras

Jan Kühr

with
Ivan Chajda

We generalize MV-algebras omitting the associativity of the addition. These structures, NMV-algebras, still can be partially ordered in a natural way, but they are not lattices. We discuss the relations between NMV-algebras and so-called λ-lattices with involutions.

Address: Palacký University Olomouc
E-mail: kuhr@inf.upol.cz

An algebraic generalization of omega-regular and omega-context-free languages

Werner Kuich

Büchi and Cohen-Gold have introduced a theory of regular and context-free languages over infinite words, respectively. The purpose of this lecture is to generalize these theories by an algebraic approach independent of any alphabets and languages. For this algebraic approach we need semiring-semimodule pairs, which are equivalent to the quemirings of Elgot. The semiring models languages over finite words, the semimodule models languages over infinite words. We then consider finite automata over quemirings and achieve as a main result Kleene Theorems for these finite automata, thus generalizing Büchi's results. Moreover, we consider algebraic finite automata and connect them to algebraic systems over quemirings, thus generalizing Cohen-Gold's results.

Address: Technische Universität Wien
E-mail: kuich@tuwien.ac.at

Prime ideal theorem for double Boolean algebras.

Leonard Kwuida

Double Boolean algebras are algebras $(D, \sqcap, \sqcup, \neg, ^{\lrcorner}, \bot, \top)$ of type (2,2,1,1,0,0). They have been introduced by Rudolf Wille to capture the equational theory of protoconcept algebras. Each double Boolean algebra D contains two Boolean algebras: $D_{\sqcap}$ and $D_{\sqcup}$. He constructed a context having as objects filters F of D whose intersections with $D_{\sqcap}$ are prime filters and as attributes ideals I of D whose intersections with $D_{\sqcup}$ are prime ideals and quasi-embedded D into the protoconcept algebra of that context. Between the lines we can read the idea of a "prime ideal theorem". For Boolean algebras, it is well known that prime filters can be defined as filters F satisfying $(*)$ $x \in F$ or $x' \in F$ for all x. With a similar definition we got the "prime ideal theorem" for weakly dicomplemented lattices, introduced to capture the equational theory of concept algebras. In the search of a common definition for such filters I was asking myself whether the prime ideal theorem can be proved for double Boolean algebras using $(*)$ as definition. The answer is yes, and will be presented in this talk.

Address: Universität Bern
E-mail: kwuida@math-stat.unibe.ch

On structure and primitive permutation representations of finite groups

Xianhua Li

In group theory and its application, the primitive permutation representation and the quantitative characterization of finite groups are interesting and important subjects. In my talk, I want to give a survey on my last results in this area.

Address: Suzhou University and Potsdam University
E-mail: xhli@suda.edu.cn

Universal aspects of general radical theory

László Márki

with
George Janelidze

In this talk we shall discuss how radicals can be defined from Galois connections. These considerations bring us to short exact sequences. Then we show that factorization systems in categories and radicals in varieties of multioperator groups can both be obtained as special cases of the same construction: simplicial sets of short exact sequences.

Address: Rényi Institute, Hungarian Academy of Sciences
E-mail: marki@renyi.hu

On polynomial functions on squarefree expanded groups

Peter Mayr

We show that on a finite expanded group whose order is squarefree and whose congruence lattice forms a chain every commutator preserving function is polynomial. This generalizes a previous result by E. Aichinger and P. Mayr that characterizes the polynomially inequivalent expansions of groups whose orders are a product of 2 distinct primes. Still we do not have a proof for P. M. Idziak's conjecture that each squarefree group has only finitely many polynomially inequivalent expansions.

Address: Johannes Kepler University Linz, Austria
E-mail: peter.mayr@jku.at

On the representations of boolean bisemigroups

Yuri Movsisyan

with
J.D.H. Smith, A. Romanowska

An algebra Q(+,·,—,$^-$,0,1) with two binary, two unary and two nullary operations is called a Boolean bisemigroup, if Q(+,·,—,0,1) is a DeMorgan algebra, Q(+,·,',0,1) is a Boolean algebra and unary operations are commute. Every two element Boolean bisemigroup is a Boolean algebra and every Boolean algebra is a Boolean bisemigroup with equal unary operations. We shall call an element a of a given Boolean bisemigroup a fixed point if $a^- = a$. A Boolean bisemigroup with four elements and two fixed points is uniquely determined up to isomorphism and will be denoted by 4+. We give a characterization of Boolean bisemigroups by fuzzy sets $X \to 4+$.

Address: Department of Mathematics, Yerevan State University
E-mail: yurimovsisyan@yahoo.com

Subdirectly irreducible fibered automata

Anna Mućka

An automaton is a fibered automaton if and only if in the transition diagram of automaton each vertex has indegree one. Based on it I will describe graphical representation of all fibered automata. Later I will show which graphs are transition diagrams of subdirectly ireducible fibered automata.

Address: Warsaw University of Technology, Warsaw, Poland
E-mail: aniam@b4.pl

Some Congruences on Inverse Semirings

P. Mukhopadhyay

with
S. K. Maity

In the first part of this paper we introduce kernels and traces of congruences and the concepts of congruence-pairs and k-systems for full ideals, and obtain two different presentations for any congruence on an inversive semiring with commutative addition. Further the class of semirings which are subdirect products of a ring and an additively idempotent semiring has been characterized. Then the least lattice congruence on such a semiring is obtained. In the second part, shunning the additive commutativity, the concepts of normal ideal and normal congruence on an additive inverse semiring is introduced. Also the concepts of congruence pair and kernel normal system for an additive inverse semiring are introduced. Finally the least skew-ring congruence, Clifford congruence and generalized Clifford congruence on additive inverse semirings are described.

Address: R. K. Mission Residential College, affiliated to University of Calcutta, India
E-mail: dugumita@vsnl.net

Semigroups defined by automaton extension mappings

Miroslaw Osys

Arbitrary transformation over free monoid can be extended to an automaton mapping, called automaton extension [1], that uniquely determines the initial transformation. We study semigroups generated by the restrictions of automaton extensions and give a characterization of automaton extensions that generate finite semigroups. Keywords: automaton mapping, Mealy automaton, semigroup Mathematics Subject Classification: 68Q70, 68Q45 1. Glushkov V. M., *Abstract Theory of Automata*, (Russian)
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Address: Silesian University of Technology, Gliwice, Poland
E-mail: Miroslaw.Osys@polsl.pl

Maximal clones and maximal permutation groups

Peter P. Palfy

A fundamental result in universal algebra is the theorem of Rosenberg describing the maximal subclones in the clone of all operations over a finite set. In group theory, the maximal subgroups in the symmetric group are classified by the celebrated O'Nan-Scott Theorem. We shall explore the similarities and the differences between these two analogous major results.

Address: Renyi Institute, Budapest
E-mail: ppp@renyi.hu

Characterization of Completely Simple Semigroup Digraphs.

Sayan Panma

with
Prof. Ulrich Knauer and Prof. Srichan Arworn

We characterize directed graphs which are Cayley graphs of completely simple semigroup, i.e. of completely regular semigroups which are also called completely simple semigroups.

Address: Carl von Ossietzky Universität
E-mail: panmayan@yahoo.com

The minimal clones above the permutations

Michael Pinsker

with
K. Kong, H. Machida

Consider the interval of the clone lattice consisting of those clones which contain all permutations of the base set: Recently, we obtained a complete list of the atoms of this interval. In the talk, we are going to mention 11 conditions necessary and sufficient for a function to generate such an atom, as well as 7 conditions n&s for two functions to generate the same atom, but we promise not to read all the conditions to the audience.

Address: Technische Universität Wien
E-mail: marula@gmx.at

On the universal automaton of a regular language

Libor Polák

A well-known structure for a regular language L is the so-called minimal complete deterministic automaton for L. A less-known structure is the universal automaton for L. We will discuss its construction and basic properties from an algebraic point of view. In particular, we will use it for a minimalization of non-deterministic finite automata.

Address: Masaryk University, Brno, Czech Republic
E-mail: polak@math.muni.cz

Classifications and decompositions

Sándor Radeleczki

Classification systems were defined in concept lattices, but they can be introduced as particular system of elements in any complete lattice. The decomposition systems are defined as partitions whose blocks are closed sets in a closure system. We show that in the case of an atomistic closure system, its decomposition systems and the classification systems of the lattice of its closed sets are the same. A particular case of decomposition systems constitute the so called interval decompositions, which can be generated by a particular algebraic closure system. We discuss the raport of the above two approaches and prove necessary and sufficient conditions for the semimodularity and the distributivity of the lattice of all decompositions of an algebraic atomistic closure system.

Address: Mathematical Institute, University of Miskolc, Hungary
E-mail: matradi@gold.uni-miskolc.hu

On subalgebra lattices

Vladimir Repnitskiĭ

For an arbitrary algebra A, we denote by $Sub\ A$ the subalgebra lattice of A (if it is necessary, e.g. in case of semigroups, $Sub\ A$ needs to contain the empty set as a subalgebra). Given a class $\mathbf{K}$ of algebras, we call a lattice L representable by the subalgebra lattice of an algebra from $\mathbf{K}$ if L is embeddable in $Sub\ A$ for some algebra $A \in \mathbf{K}$. $Lat\ \mathbf{K}$ denotes the class of all lattices which are representable by subalgebra lattices of algebras from $\mathbf{K}$. In our talk we present some results concerning various connections between classes $\mathbf{K}$ and $Lat\ \mathbf{K}$. Here $\mathbf{K}$ means certain important classes of groups, semigroups, rings etc.

Address: Ural State University, Ekaterinburg
E-mail: vladimir.repnitskii@usu.ru

Idempotence, entropicity and linearity

Anna Romanowska

The talk will survey some background and results concerning the possibility of representing modes (idempotent and entropic algebras) as (sub)reducts of certain "linear spaces". In this case. the appropriate linear spaces are semimodules over commutative semirings.

Address: Warsaw University of Technology, Warsaw, Poland
E-mail: aroman@alpha.mini.pw.edu.pl

Separation of Boolean clones of Cooperations by hyperidentities

Kittisak Saengsura

with
Klaus Denecke

A Boolean clone of cooperations is a clone of cooperations on $\{0,1\}$. An equation $s \approx t$ of coterms of type τ is an identity in a Boolean clone C writen as $C \vDash s \approx t$ if for any replacement of an cooperation in $s \approx t$ by a cooperation from C of the same arity we get equality. For each pair (G, H) of Boolean clones of cooperations, the equation $s \approx t$ separates G from H if $G \vDash s \approx t$, but $H \nvDash s \approx t$.

Address: Thailand
E-mail: saengsur@rz.uni-potsdam.de

The dimension of a variety

Dietmar Schweigert

with
Ewa Graczyńska, University of Technology, Opole, Poland

This is an abstract written by Professor Dietmar Schweigert in the hospital this year. He has passed away on 23 of January 2006. I will always remember him as an excellent mathematician, a very good friend and a big scientific authority for me. Let F be the set of all fundamental operations of a type τ, $F = \{f_i : i \in I\}$.

Let $\sigma = \{t_i : i \in I\}$ be a fixed choise of terms of type τ of the variety V. For every algebra $\mathbf{A} = (A, \Omega)$ of type τ the algebra $\mathbf{A}_\sigma = (A, \{t_i : i \in I\})$ of type τ is called a *derived algebra* of $\mathbf{A}$. The variety generated by the class of all derived algebras $\mathbf{A}_\sigma$ of algebras $\mathbf{A} \in V$ is called the *derived variety* of V using σ and will be denoted by V_σ. A derived variety V_σ is *proper* if it is not equal to V, i.e. $V \neq V_\sigma$. A variety V is called *solid* if the variety V contains all derived varieties V_σ for every choise of σ of type τ. A variety V of type τ is called *fluid*, if it contains no proper derived varieties V_σ for any choise σ of type τ. The *dimension* of a variety V is the cardinality κ of all proper derived varieties of V which are subvarieties of V. First we concentrate on varieties of a finite dimension. We present some results on semigroups, semilattices, lattices and some other algebraic structures.

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Address: Technische Universität Kaiserslautern, Germany
E-mail: schweige@mathematik.uni-kl.de

Some new results concerning cardinal powers

Branimir Seselja

We investigate cardinal power P^X, i.e., the collection of all isotone functions from a poset X into a poset P. Particular subsets of the domain X, so called cut sets are used as a tool for representation of functions in P^X. These cut sets are up sets in X. Necessary and sufficient conditions are given under which all up sets appear in such a representation. We investigate cases with P being a lattice, and also give some classification of the power according to equality of collections of cut sets.

Address: University of Novi Sad
E-mail: seselja@im.ns.ac.yu

Presentations of inverse semigroups generated by two-state partially defined Mealy automata over two-symbol alphabet.

Janusz Słupik

We use results from [1]. We present two descriptions of inverse semigroups generated by two-state partially defined invertible Mealy automata over two-symbol alphabet: in terms of generators and relations, and in terms of partial permutations on natural numbers.

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Address: Institute of Mathematics, Silesian University of Technology, Gliwice, Poland
E-mail: Janusz.Slupik@polsl.pl

The lattice of varieties of entropic differential modals

Karolina Ślusarska

A differential modal is an algebra with two binary operations such that one of the reducts is a differential mode and the other is a semilattice, satisfying certain distribution law. We will describe the lattice of varieties of entropic differential modals.

Address: Warsaw University of Technology
E-mail: k.slusarska@mini.pw.edu.pl

Some remarks about duality for convex sets

Piotr Ślusarski

Convex sets can be considered as barycentric algebras (a barycentric algebra is a convex set if it is cancelative). In this talk we will present the way that duality for various categories of convex sets can be established.

Address: Warsaw University of Technology
E-mail: P.Slusarski@mini.pw.edu.pl

Quasigroups and Quasisymmetry.

Jonathan D.H. Smith

The talk presents two recent developments in the theory of quasigroups: new foundations, and new applications. On the foundational side, quasigroups were originally defined by a combinatorial condition that did not lend itself to algebraic treatment. Trevor Evans redefined quasigroups as algebras with three binary operations, satisfying four identities. In the new, more symmetrical definition, quasigroups are defined as two-sorted algebras. One sort is the underlying set of the quasigroup, while the other is a group with two special elements. Two hyperidentities are satsified: hypercommutativity and hypercancellativity. The new formulation greatly simplifies rewriting systems for various varieties of quasigroups. On the applications side, quasigroups are used to address an issue arisng from the study of biological and other complex systems: the question of approximate symmetry. Recall that group homogeneous spaces are used to model exact symmetry. It is then shown how certain quasigroup homogeneous spaces may be used to give exact models of a particular form of approximate symmetry, in which a two-level hierarchical system exhibits exact symmetry on the macroscopic level, but not necessarily at the microscopic level.

Address: Iowa State University
E-mail: jdhsmith@math.iastate.edu

Completion of partially ordered sets

Sergejs Solovjovs

Let **Pos** be the category of partially ordered sets and order-preserving maps and let **JCPos** be its subcategory consisting of complete lattices and join-preserving maps. It is known that the category **JCPos** is reflective in **Pos**. The completion of a poset goes through the collection of all its lower-sets.

Given a quantale Q one can consider the category Q-**Mod** of modules over Q. Since the categories **2-Mod** and **JCPos** are isomorphic one could ask about the generalization of the aforesaid result for an arbitrary quantale Q. We answer the question in two ways using the generalization of the category **Pos** in the latter one.

Address: Department of Mathematics, University of Latvia
E-mail: sergejs@lu.lv

N-Fluid Varieties

Ratana Srithus

with
K. Denecke, J. Koppitz

Let $(f_i)_{i\in I}$ be an indexed set of operation symbols, let $W_\tau(X_n)$ be the set of all n-ary terms of type τ and let $W_\tau(X_n)$ be the set of all terms of type τ. A mapping $\sigma = \{f_i \mid i \in I\} \longrightarrow W_\tau(X)$ such that $\sigma(f_i) \in W_\tau(X_{n_i})$ for all $i \in I$ called a *hypersubstitution* of type τ. If $\mathcal{A} = (A; (f_i^A)_{i\in I})$ is an algebra, then $\sigma(\mathcal{A}) = (A; (\sigma(f_i)^A)_{i\in I})$ is said to be an algebra *derived* from $\mathcal{A}$ by using the hypersubstitution σ. If a variety V contains all derived algebras, then it is called *solid*. In the opposite case we speak of a fluid variety. We generalize this concept to the concepts of n-fluid, weakly fluid and $\aleph_0$-fluid varieties, prove some general properties and determine all n-fluid varieties of bands. *Key Words: hypersubstitution, derived algebra,n-fluid variety, weakly fluid variety, $\aleph_0$-fluid variety.*

Address: University of Potsdam, Germany
E-mail: srithus@uni-potsdam.de

Commutative idempotent residuated lattices

David Stanovský

Residuated lattices are lattice ordered monoids with residuated multiplication. We investigate the variety CIRL of residuated lattices with a commutative and idempotent monoid reduct. Tsinakis et al. proved that any variety of residuated lattices relatively based by identities in the language of lattices satisfies no non-trivial monoid equation. We prove an opposite statement: any variety of residuated lattices relatively based by identities in the language of monoids contains CIRL and it satisfies no non-trivial lattice equation. We touch the problem of representation of lattices as reducts of CIRLs. We show that the lattice of subvarieties is uncountable and it has two atoms.

Address: Charles University in Prague, Czech Republic
E-mail: stanovsk@karlin.mff.cuni.cz

Embedding entropic algebras into polyquasigroups

Michał Stronkowski

A groupoid is cancellative if all left and right translations are injective, and is a quasigroup if all these mappings are bijective. We generalize these concepts by considering algebras of any type and more general sets of translations. For certain sets M of translations, we define corresponding concepts of M-cancellativity and M-polyquasigroup. We show that, for a "big" set M of translations, an entropic M-cancellative algebra embeds into an M-polyquasigroup. Moreover this embedding preserves all identities, and in the idempotent case, even more general formulae.

Address: Warsaw University of Technology, Warsaw, Poland
E-mail: m.stronkowski@mini.pw.edu.pl

Infinite semidistributivity and weak congruences

Andreja Tepavcevic

Semidistributive elements in weak congruence lattices are considered. In particular, connections between infinite join-semidistributivity and congruence intersection property are investigated via equivalence classes $x \to x \vee \Delta$ in weak congruence lattices. Semidistributivity of weak congruences are characterized by means of geometrical methods.

Address: University of Novi Sad
E-mail: etepavce@eunet.yu

Mal'cev Conditions and Idempotent Algebras

Matt Valeriote

with
Ralph Freese

In general it appears difficult to determine whether a given finite algebra $\mathbf{A}$ generates a variety that is congruence modular or distributive or permutable. A straightforward but computationally inefficient algorithm is to generate all 3 variable terms of the variety generated by $\mathbf{A}$ and then search for appropriate sequences of Gumm, Jonsson, or Mal'cev terms. In contrast with the general case, we exhibit algorithms that run in polynomial time that accept as input a finite idempotent algebra $\mathbf{A}$ and determine whether the variety generated by $\mathbf{A}$ is congruence modular, congruence distributive or congruence permutable. We also exhibit a polynomial time algorithm that determines whether a given finite idempotent algebra has a majority term. Note that an algebra $\mathbf{A}$ is idempotent if each basic operation $f(x_1, x_2, \ldots, x_n)$ of $\mathbf{A}$ satisfies the equation $f(x, x, x, \ldots, x) = x$.

Address: McMaster University
E-mail: matt@math.mcmaster.ca

Symmetric groupoids and cores

Alena Vanžurová

The talk will survey equational theory of Bol loops, properties of their cores, question of mediality for cores, some properties of the variety SID (of left symmetric idempotent left distributive groupoids) generated by cores, and its generalization SD: normal forms for terms, multiplication for normal form hypersubstitutions, idempotents, proper n.f.h.

Address: Palacký University Olomouc, Czech Republic
E-mail: vanzurov@inf.upol.cz

Locally Strong Endomorphisms of Cycles

Apirat Wanichsombat

with
Prof. Ulrich Knauer

In this topic, we want to study the structure of cycles. But when n is positive odd number, the cycle of n vertices n edges is not interesting to study. Then we will study the structure of cycle when n is positive even number more than 4. First, we find the formula of the cardinal number of the set contains all locally strong endomorphisms are not automorphism of n-cycle. And next we will continue to study any algebraic structure of this set.

Address: Carl Von Ossietzky Universität
E-mail: apirat589@yahoo.com

Radical and Torsion Theory of Acts

Richard Wiegandt

Although each torsionfree class as well as torsion assignment determines a torsion class, *a torsion class may belong to several torsionfree classes and torsion assigments.* A (Kurosh-Amitsur) radical class is in a one-to-one correspondence with its semisimple class and with the determined radical assignment $r(A)$, which latter is always a Rees congruence for every S-act A. *A hereditary radical assigmnent* $r(A)$ *is just a torsion assignment (i.e. a hereditary Hoehnke radical) which designates a Rees congruence to every* S*-act* A. Also torsion and torsionfree classes as well as radical and semisimple clases (belonging to hereditary radical assignments) are characterized.

Address: A. Rényi Institute of Mathematics, Budapest
E-mail: wiegandt@renyi.hu

The basic theorem on preconcept lattices

Rudolf Wille

with
Christian Burgmann

Preconcept lattices are identified to be (up to isomorphism) the Complete distributive lattices in which the supremum of all atoms is equal or greater than the infimum of all coatoms. This is a consequence of the Basic Theorem on Preconcept Lattices, which also offers means or checking line diagrams of preconcept lattices.

Address: Darmstadt University of Technology
E-mail: wille@mathematik.tu-darmstadt.de

Partially ordered of Clifford Semigroups

Somnuek Worawiset

with
Professor Knauer

We describe structure of partially ordered Clifford semigroups.

Address: Carl von Ossietky University
E-mail: nuek_w@yahoo.com

Functorial properties of Cayley constructions.

Xia Zhang

with
Prof. Ulrich Knauer and Prof. Yanming Wang

We describe the constructions of the Cayley graph of a semigroup as a functor and investigate certain reflections and presevations on properties of this functor. In the final part we investigate it with respect to several product constructions including pullbacks.

Address: Carl von Ossietzky Universität
E-mail: zhangxia_1@yahoo.com.cn

Weak injectivity on Clifford semigroups

Xia Zhang

with
Ulrich Knauer, Yanming Wang

We study various weak injectivity on Clifford semigroups. We characterize an ideal of a Clifford semigroup to be principal weakly injective,finitely weakly injective and weakly injective, respectively. In addition, we obtain the homological classification by properties of ideals of a Clifford semigroup.

Address: Carl Von Ossietzky University
E-mail: zhangxia_1@yahoo.com.cn

Distributive lattices and topologies modulo an ideal

Irina Zvina

It is well-known that every topology forms a complete infinitely distributive (cid) lattice, while the converse may not hold. We prove that every cid lattice of subsets of a set with the natural partial order (notice that every lattice is isomorphic to at least one lattice of that kind) is an i-topology (or a topology modulo an ideal), i.e., that every cid lattice is closed under arbitrary i-unions and finite i-intersections. We study some properties of i-topologies with compatible ideals. We show under what conditions the ideal of an i-topology may be extended to a compatible ideal. We prove that the compatibility is necessary for the existence of a topological space that is i-homeomorphic to a given i-topological space.

References

[1] D. Janković and T.R. Hamlett, *Compatible Extensions of Ideals*, Bollettino U.M.I., **7** 1992, 453 – 465

Address: University of Latvia
E-mail: IrinaZvina@inbox.lv