77th Workshop on General Algebra, 24th Conference for Young Algebraists

University of Potsdam, Institute of Mathematics
Potsdam, Germany

March 20–22, 2009

Contents

Properties of (x(yz))z with Loop Graph Varieties of Type (2,0)

Amp. Anantpinitwatna
Mahasarakham University, Thailand
amporn.a@msu.ac.th
Coauthors: T. Poomsa-ard

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra $\underline{A(G)}$ satisfies $s \approx t$. A class of graph algebras V is called a graph variety if $\overline{V = Mod_g\Sigma}$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $V' = Mod_g\Sigma^{'}$ is called an $(x(yz))z$ with loop graph variety if $\Sigma^{'}$ is a set of term $(x(yz))z$ with at least one loop term equations.

In this paper we characterize all $(x(yz))z$ with loop graph varieties.

Special M-hyperidentities in Biregular Leftmost Graph Varieties of Type (2,0

A.. Anantpiniwatna
Mahasarakham University, Thailand
apinant.a@msu.ac.th
Coauthors: T. Poomsa-ard

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra $\underline{A(G)}$ satisfies $s \approx t$. A class of graph algebras V is called a graph variety if $\overline{V = Mod_g\Sigma}$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $V' = Mod_g\Sigma^{'}$ is called a biregular leftmost graph variety if $\Sigma^{'}$ is a set of biregular leftmost term equations. A term equation $s \approx t$ is called an identity in a variety V if $\underline{A(G)}$ satisfies $s \approx t$ for all $G \in V$. An identity $s \approx t$ of a variety V is called a $\overline{\text{hyper}}$identity of a graph algebra $\underline{A(G)}$, $G \in V$ whenever the operation symbols occurring in s and t are replaced by any term operations of $\underline{A(G)}$ of the appropriate arity, the resulting identities hold in $\underline{A(G)}$. An identity $s \approx t$ of a variety V is called an M-hyperidentity of a graph algebra $\underline{A(G)}$, $G \in V$ whenever the operation symbols occurring in s and t are replaced by any term operations in a subgroupoid M of term operations of $\underline{A(G)}$ of the appropriate arity, the resulting identities hold in $\underline{A(G)}$. An identity $s \approx t$ of a variety V is called an M-hyperidentity of V if it is an M-hyperidentity of $\underline{A(G)}$ for all $G \in V$. In this paper we characterize all special M-hyperidentities in each biregular leftmost graph variety.

On Verbal Subgroups of Finitely Generated Nilpotent Groups

A. Bier
Silesian University of Technology, Poland
agnieszka.bier@polsl.pl

For a set of words F, the *verbal subgroup* $V_F(G)$ of group G is the subgroup generated by all values of the words from F in group G [1]. In any group G the terms of its lower central series $G = \gamma_1(G) \geq \gamma_2(G) \geq ... \geq \gamma_n(G) = \{\mathbf{1}\}$ are verbal subgroups generated by the following commutator words:

$$c_1 = x_1, \quad c_{i+1} = [x_{i+1}, c_i(x_1, ..., x_i)],$$

satisfying the equality $\gamma_i(G) = V_{c_i}(G)$.

We will say that the group G is *verbally poor* if it has no verbal subgroups but the terms of its lower central series. In the talk we are interested in conditions for finitely generated nilpotent groups to be verbally poor. We will show that every verbally poor finitely generated nilpotent group is torsion and therefore finite. Moreover, it will be shown that

Theorem. *Every verbally poor finitely generated nilpotent group is a p-group.*

The talk will be concluded with few examples of verbally poor p-groups.

References

1. Neumann H., "Varieties of groups", Springer-Verlag New York, 1967.

Basic Algebras with vt-operators

M. Botur
Dept. of Algebra and Geometry, Palack University Olomouc, Czech Republic
botur@inf.upol.cz

The aim of our talk is to introduce and study very true-like operators on basic algebras. Basic algebras are a common generalization of algebras of logic which are lattices with section antitone involutions.

Prime n-Clones and the Representation of $O^n(A)$

R. Butkote
Potsdam University, Germany
unique1st@hotmail.com
Coauthors: K. Denecke (Potsdam University)

A clone is a set of operations defined on a base set A which is closed under composition and contains all the projections. In this paper we study n-ary parts of Boolean clones, for short n-clones, and define the concept of prime n-clone. Moreover, we prove that $O^n(A)$ can be represented as a sum of two prime n-clones.

When Can we do Computations with Infinitary Linear Combinations Without Worrying About Convergence?

R. Börger
Fernuniversität Hagen
Reinhard.Boerger@FernUni-Hagen.de
Coauthors: R. Kemper (Frankfurt)

When can one define infinitary linear combinations satisfying the usual rules in a module over a complete valuation ring? Convergence to zero plays only a role in the definition of the operations; the operations themselves are totally defines and have countable arity. It turns out that this is possible exactly for reduced Matlis cotorsion modules. Moreover; these infinitary operations are uniquely detemined by the module structure, and they are preserved by all linear maps.

On Semigroups of Relations with the Descriptor of Fixed Points

D. A. Bredikhin
Lermontova 7-22, Saratov, Russia, 410002
bredikhin@mail.ru

A set of binary relations closed with respect to some collection of operations forms an algebra which is called an algebra of relations. Any algebra of relations can be considered as partial ordered by set-theoretical inclution.

For any set Ω of operations on relations, denote by $R\{\Omega\}$ ($R\{\Omega, \leq\}$) the class of algebras (partial ordered algebras) isomorphic to ones whose elements are binary relations and whose operations are members of Ω. Let $Var\{\Omega\}$ ($Var\{\Omega, \leq\}$) be the variety generated by $R\{\Omega\}$ ($R\{\Omega, \leq\}$).

We shall concentrate our attention on the operations of relation product $\circ$, union U and the unary operations Δ defined as follows:

$$\Delta(\rho) = \{(x, x) : \text{(there exists } z\text{) such that } (z, z) \text{ belongs to } \rho\}.$$

Note that $\Delta(\rho)$ is equal to the identical relation if ρ contains a fixed point, and $\Delta(\rho)$ is equal to the empty relation otherwise. For these reasons, the operation Δ can be considerred as the descriptor of fixed points.

The main result are formulated in the following theorems.

Theorem 1. An algebra $(A, \cdot, ^*)$ of the type $(2,1)$ belongs to the variety $Var\{\circ, \Delta\}$ if and only if it satisfies the identities:

$$(xy)z = x(yz) \ (1), \quad (x^*)^2 = x^* \ (2), \ \ xy^* = y^*x \ (3),$$
$$(xy)^* = (yx)^* \ (4), \ \ (xy^*)^* = x^*y^* \ (5),$$
$$x^*(x^k)^* = x^* \ (6) \ \text{ for any natural number } k.$$

Theorem 2. A partial ordered algebra $(A, \cdot, ^*, \leq)$ of the type $(2,1)$ belongs to the variety $Var\{\circ, \Delta, \leq\}$ if and only if it satisfies the identities (1) - (6) and the identity $xy^* \leq x$ (7).

Theorem 3. The varieties $Var\{\circ, \Delta\}$ and $Var\{\circ, \Delta, \leq\}$ are not finitely based.

Theorem 4. An algebra $(A, \cdot, +, ^*)$ of the type $(2,2,1)$ belongs to the variety $Var\{\circ, U, \Delta\}$ if and only if $(A, +)$ is a semilattice and the following identities hold: (1) - (6) and

$$x + xy^* = x, \quad (x+y)z = xz + yz, \quad x(y+z) = xy + xz, \quad (x+y)^* = x^* + y^*.$$

Discriminator Order Algebras

I. Chajda
Palacky University Olomouc, Czech Republic
`chajda@inf.upol.cz`

The concept of order algebra was introduced recently by J.Berman and W. J. Blok. By an order algebra is meant an algebra defined on an ordered set whose operations are derived by means of the order relation and, conversely, the partial order is determined by these operations. We study order algebras with least and greatest element and with a unary operation which is an involution. We answer to the question when these algebras contain the ternary discriminator as a term operation.

Units in a Ternary Algebra

R. Chandra
3/3, Government Girls Normal School Campus, Civil Lines, Faiabad-224001, (UP), INDIA
rameshchandra51@gmail.com
Coauthors: S. K. Gantayat

Firstly we have established a ternary system and then we find the number of units in the ternary algebra.

On Semigroups of Regular Hypersubstitutions

T. Changphas
Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand
thacha@kku.ac.th
Coauthors: W. Hemvong, T. Changphas, K. Denecke

Hypersubstitutions were introduced as a way of making precise the concept of hyperidentities and generalization to M-hyperidentities. Semigroup's properties of hypersubstitutions have been studied by many authors. In this paper, we characterize Green's relations of every subsemigroup of the semigroup of regular hypersubstitutions. Moreover, we give a partial solution concerning $\mathcal{G}$-subsemigroup of this semigroup.

The Vertex Arboricity of Regular Graphs

A. Chantasartrassmee
The University of the Thai Chamber of Commerce, Bangkok, Thailand
avapa_cha@utcc.ac.th
Coauthors: N. Punnim

The *vertex arboricity* of a graph G, denoted by $\mathrm{a}(G)$, is the minimum integer k in which there exists a partition $V_1 \cup V_2 \cup \cdots \cup V_k$ of $V(G)$ such that $\langle V_i \rangle$ is acyclic for all $i = 1, 2, \ldots, k$.

We prove that if G runs over the set of graphs with a fixed degree sequence d, then the values $\mathrm{a}(G)$ completely cover a line segment $[a, b]$ of positive integers. Thus for an arbitrary graphical sequence d, two invariants $a := \min(\mathrm{a}, \mathrm{d})$ and $b := \max(\mathrm{a}, \mathrm{d}$ naturally arise. For a regular graphical sequence $\mathrm{d} = r^n := (r, r, \ldots, r)$ where r is the degree and n is the number of vertices, the exact values of $\min(\mathrm{a}, \mathrm{d})$ are found in all situations and $\max(\mathrm{a}, \mathrm{d})$ for all $n \geq 2r + 2$.

Freeoids

J. Cīrulis
Department of Computer Science, University of Latvia, Riga
jc@lanet.lv

Let X be a fixed infinite set. A freeoid is defined to be a pair (W, E), where W is a superset of X and E is a submonoid of W^W that contains just one extension of every mapping $X \to W$. For example, if $\mathbf{W}$ is a relatively free algebra with X the set of free generators, then $F(\mathbf{W}) := (W, End(\mathbf{W}))$ is a freeoid. Let $\mathcal{W}_X$ and $\mathcal{F}_X$ stand for the class of all relatively free algebras of various signatures and for that of all freeoids, respectively. We characterise the kernel equivalence and the range of the transformation $F\colon \mathcal{W}_X \to \mathcal{F}_X$. The classes $\mathcal{W}_X$ and $\mathcal{F}_X$ naturally expand to categories, and F, to a full functor that is constant on morphisms. The category $\mathcal{W}_X$ is equivalent to the category of all varieties. We also show that it is equivalent to $F(\mathcal{W}_X)$.

Algebras, Coalgebras and State-based Systems

K. Denecke
University of Potsdam, Germany
kdenecke@rz.uni-potsdam.de

Most mathematicians first encounter algebraic structures in the classical examples of groups, rings, fields and vector spaces. In each of these areas, common themes arise: we have sets of objects which are closed under one or more operations performed on the objects, and we are interested in subsets which inherit the structure (subgroups, subspaces, etc.), in mappings which preserve the structure (group homomorphisms, linear transformations, etc.) and construction of new structures from old, for instance by Cartesian products or quotients. We can also classify our structures according to the laws or *identities* they satisfy, as for instance with commutative groups or groups of order four.

In universal algebra we abstract and generalize from these examples to a core structure of *an algebra*: a set A of objects, with one or more operations defined on the set. We study substructures, homomorphisms and product algebras, and we classify algebras according to the identities they satisfy. To study such algebras we also need to know how many operation symbols our algebra has, and the arity of each one. This information is called the *type* or *signature* of the algebra. In general we assume a type indexed by some set I: for each $i \in I$ we have an operation symbol f_i, of arity $n_i \geq 0$, and we write the type as $\tau = (n_i)_{i \in I}$.

While universal algebras can be used to model most algebraic structures, they are not as useful in modelling state-based systems. The main reason for this is the following. An n_i-ary operation on set A is a mapping $f^A : A^{n_i} \mapsto A$, which combines n_i "input" elements of A into one output element. In a state-based system however, we often have the opposite situation: we need to map a single state to an output which carries several pieces of information, for instance to a state-output-symbol pair. That is, we need mappings from set A to some more complex set involving A.

In order to model dynamic state-based systems, the codomain might be a product of the form $A \times \Sigma$ where Σ is an input or output language of the machine. In general, we use some functor F to describe this structure, and consider mappings $f : A \mapsto F(A)$. This leads to the definition of an F-coalgebra, for a functor F, as a structure with a base set A together with one or more mappings from A to $F(A)$. There is an algebraic dual of this concept too: an F-algebra is a set A with one or more mappings from $F(A)$ to A. Any algebra of type τ can be transformed into an F-algebra.

We take a further step in abstraction to a single structure which encompasses both F-algebras and F-coalgebras. This can be done by using two functors F_1 and F_2, instead of a single functor F. A functorial system or (F_1, F_2)-structure, for functors F_1 and F_2, consists of a set A and mappings $f : F_1(A) \mapsto F_2(A)$. This expresses both F-algebras, by taking $F_1 = F$ and F_2 to be the identity functor, and F-coalgebras, when $F_2 = F$ and F_1 is the identity functor. But this concept also models other interesting algebraic structures as well, such as power algebras (also called hyperstructures) and power coalgebras, and tree automata. This structure thus allows us to unify results from the two different areas of algebra and theoretical computer science.

The Maximal Subsemigroups of the Semigroup of all Monotone Partial Injections

I. Dimitrova
South-West University "Neofit Rilski", Blagoevgrad, Bulgaria
ilinka_dimitrova@yahoo.com
Coauthors: J. Koppitz

We study the structure of the semigroup of all monotone, i.e. order-preserving or order-reversing partial injections on an n-element set. The main result is the characterization of the maximal subsemigroups of this semigroup. There are exactly $2^{n+1} - 3$ such semigroups.

Random Constructions Imply Symmetry

M. Droste
Leipzig University, Institute of Computer Science, Leipzig, Germany
droste@informatik.uni-leipzig.de
Coauthors: D. Kuske; Guo-Qiang Zhang

We will argue for the claim of the title in the areas of algebra, theoretical computer science, and theoretical physics. In algebra, we will consider the random graph. For theoretical computer science, we will give a probabilistic construction of locally finite and of Scott domains and show that with probability 1 our construction produces a universal homogeneous domain. Finally, we

consider causal sets which have been used as basic models for discrete space-time in quantum gravity.

De Morgan Quasirings and De Morgan Algebras

G. Eigenthaler
TU Vienna
`G.Eigenthaler@tuwien.ac.at`
Coauthors: I. Chajda

The concept of a De Morgan quasiring was introduced by the authors in a recent paper. The aim of the talk is twofold. At first we find an axiom system which determines both De Morgan algebras and De Morgan quasirings (in dependence of the value of the algebraic constant $1 + 1$). The second aim is to show when an interval of a De Morgan quasiring A can be equipped by operations such that the resulting algebra is a De Morgan quasiring again and the operations are polynomials over A. Finally, we present a certain kind of representation of De Morgan algebras by algebras of binary functions.

Polygroupoids, Polyringoids and Polyalgebroids: from Strings to Nets

A. Gasparyan
19/43, 50 Let Komsomola, 152026 Pereslavl Zalesskii, Russia
`armen@armen.pereslavl.ru`

Most of all type classical algebras involve mainly binary operations (multiplication, addition, composition etc.), and this was key precondition for mathematics to be writtable means of linear strings — expressions, formulas, equations, together with textual explanations. However, recent development of science and, particularly, mathematics, determined new requirements concerning theoretical and technical base of algebraic constructions in cases if we are interested in solving problems where the presented relations, interactions, maps and operations are sufficiently multiary, multityped and even multisorted. We meet with such situations, for example, if we get to description or modelling systems with complex interacting components — namely networks or network-like ones. It becomes clear: the algebraic instrumentary that may be appropriate for studying network- like complex objects, same should necesarily contain network-like elements and constructs. It is obvious that also resulting mathematical expressions and even the texts should necessarily contain network-like pices (graphs, nets, schemes etc.).
Roughly speaking, our observation is that todays string- mathematics have tendency to become network-mathematics. and one can suggest another name for the mathematics of tomorrow — "polymathematics" — with appropriate terms: polyalgebras, polystructures, polygeometry, polyfunctions, polyspaces, polymorphisms, polycategories . . .

In the talk we will introduce several algebraic systems generalizing corresponding fundamental structures of classical general algebra but in some time allowing to compose different network-like algebraic expressions. The elements of algebraic systems, we introduce and study, usually have one, two or more inputs and/or outputs with a method of their numeration and some composition rules. Combining the elements and compositions one can obtain a wealth of expressions that we name algebraic networks, schemes or graphs. If we require the holding of additional appropriate conditions, we define algebraic systems of more concrete type, that only in much particular cases are the classical algebraic systems with chain-like expressions.

Critical Points of Pairs of Varieties of Algebras

P. Gillibert
LMNO, Universit de Caen, France
`pierre.gillibert@math.unicaen.fr`

For a class V of algebras, denote by $\mathrm{Con_c}(V)$ the class of all semilattices isomorphic to the semilattice $\mathrm{Con_c}(A)$ of all compact congruences of A, for some A in V. For classes V and W of algebras, we denote by $\mathrm{crit}(V, W)$ the smallest cardinality of a semilattice in $\mathrm{Con_c}(V)$ which is not in $\mathrm{Con_c}(W)$ if it exists, infinity otherwise. We prove a general theorem, with categorical flavor, that implies that for all finitely generated congruence-distributive varieties V and W, $\mathrm{crit}(V, W)$ is either finite, or $\aleph_n$ for some natural number n, or infinity. We also give some examples of critical points with varieties of lattices.

Regular Elements and Green's Relations on Languages of Generalized n-ary Terms

P. Glubudom
Department of Mathematics, Chiangmai University, Thailand 50200
`puprisana@yahoo.com`
Coauthors: K. Denecke

Tree languages are sets of terms of a given type. The power set of the set of all tree languages forms an algebra with respect to superposition operations and with respect to infinitely many nullary operations. These algebras are called unitary Menger algebras of infinite rank. Unitary Menger algebras can be extended to generalized power Menger algebras. We are looking for regular elemnts and Green's relations on the generalized power Menger algebra. This generalizes and extends results of n-ary terms. This generalization is useful since operations in tree languages are based on the generalized superposition on sets of terms. The idea of a near endomorphism was used to solve the problem.

Generalized E-Rings

R. Goebel
Universität Duisburg
ruediger.goebel@uni-due.de

In the paper (joint work with Daniel Herden and Saharon Shelah; submitted to the European Journal of Math. in 2008) basic for this talk we solve an exactly fifty year old problem on R-algebras A over cotorsion-free commutative rings R with 1.

For simplicity I will assume that $R = \mathbb{Z}$ is just the ring of integers. Thus A is an ordinary ring. It is called a generalized E-ring if the ring of endomorphisms $EndA+$ of its additive group $A+$ is isomorphic to A as a ring.

Subrings of $\mathbb{Q}$ are the first obvious examples. Properties, including the existence of many such rings are derived in various papers. The study was stimulated by Fuchs first edition of his book on "Abelian Groups" from 1958, and specially by the PhD-thesis of Phil Schultz from 1973. But due to Schultz' work the investigations concentrated on ordinary thus commutative E-rings. A substantial part of problem 45 of the 1958-Fuchs-monograph (repeated in later publications by Vinsonhaler and others) remained open:

Can we find non-commutative generalized E-rings?

I will indicate the proof about the existence of such rings, thus of proper generalized E-rings.

The new strategy should be interesting and useful for other problems as well: We will first translate the heart of the algebraic question on the existence of certain monoids via model theory into geometric structures leading to a special class of

finite (decorated) trees and solve this problem introducing products of trees etc. This can be compared with the well-known, but different process translating group problems to small cancelations in groups via the van Kampen lemma used for answering famous problems in group theory. By small cancelation of trees we are able to find a suitable monoid and thus a non-commutative ring A with an important non-canonical embedding $A \hookrightarrow EndA+$, our $*$-scalar multiplication. In a second part of this paper we must enlarge A to get rid of all undesired endomorphisms and getting the desired ring. This can be done more easily. I will make plausible how to get rid of all unwanted endomorphisms. This follows roads of work in the last twenty years using our useful Black Box predictions principle as outlined in the recent book by Trlifaj and Göbel, Approximation Theory and Endomorphism Algebras, Walter de Gruyter, Berlin (2006).

Dependences spaces

E. Graczyńska
Opole University of Technology, Poland
e.graczynska@po.opole.pl

N. J. S. Hughes proved Steinitz' Exchange Theorem for infinite bases in [1] under the following assumptions: "Given a system in which a suitable relation of dependence is defined, we give a construction (assuming well ordering), by which some of the elements of any basis may be replaced, in a one-one manner, by all the elenents of any independent subset to give a new basis". His construction includes the classical examples of the theorem. In fact, the author used a natural ordering in his proof. Therefore we propose a modifiation of the proof of Steintz' theorem, assuming Zorn's Maximum principle [2,3].

References

[1] N.J.S. Hughes, Steinitz' Exchange Theorem for Infinite Bases, University College, Cathaya Park, Cardiff. 30-3-62.
[2] K. Kuratowski, Une méthode d'élimination des nombres transfinis des raisonnements mathématiques, Fund. Math. 3 (1922), p. 89.
[3] K. Kuratowski, A. Mostowski, Teoria Mnogośći, PWN, Warszawa 1966, p. 242.

Quantum Polynomials
Ashish Gupta
University of Melbourne, Australia
a0gupt@gmail.com

Quantum polynomials play an important role in mathematical Physics. They are multiplicative analogues of the Weyl algebras. We shall introduce the quantum polynomials and describe the basic properties and also discuss some module theory.

On Very True Operators on Pocrims
Radomir Halas
Dept. of Algebra and Geometry, Palack University Olomouc, Czech Republic
halas@inf.upol.cz
Coauthors: M. Botur

P. Hajek introduced the logic BLvt enriching the logic BL by a unary connective vt which is a formalization of Zadehs fuzzy truth value very true. BLvt algebras, i.e. BL-algebras with unary operations, called vt-operators, which are among others subdiagonal, are an algebraic counterpart of BLvt. Partially ordered commutative integral residuated monoids (pocrims) are common generalizations of both BL-algebras and Heyting algebras. The aim of our talk is to present algebraic properties of pocrims endowed by "very-true" and "very-false"-like operators.

Regularity of Weak Projection Hypersubstitutions I

W. Hemvong
Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand.
hwonlop@hotmail.com
Coauthors: T. Changphas

Let f and g be the binary operation symbols of type $\tau = (2, 2)$. For binary terms a and b of type τ, the hypersubstitution which maps the operation symbol f to the term a and the operation symbol g to the term b will be denoted by $\sigma_{a,b}$. Using the fact that any $\sigma_{a,b}$ can be inductively extended to a map $\widehat{\sigma}_{a,b}$ on the set of all terms of type τ, the set $Hyp(2, 2)$ of all hypersubstitutions of type τ forms a semigroup. In this paper, we give sufficient and neccessary conditions for a weak projection hypersubstitution of type τ (that is a hypersubstitution $\sigma_{a,b}$ which either a or b is a variable) to be regular.

Some New Aspects of Islands

E. K. Horváth
University of Szeged, Bolyai Institute, Hungary
horeszt@math.u-szeged.hu
Coauthors: P. Hajnal (University of Szeged, Bolyai Institute), B. Šešelja (University of Novi Sad), A. Tepavčević (University of Novi Sad)

Given a square grid in a big rectangle, where each cell is filled with a real number, its height. A rectangle on a grid is called a rectangular island, iff there is a possible water level such that the rectangle is an island in the usual sense ([1]). The notion comes from information theory ([2]). The talk starts with a summary about the history of islands (since 2007). Then a surprising exact formula and its proof will be presented for the maximum number of hypercubic islands in a big hypercube. The set of cells – the board – now consists of all vertices of a hypercube, in other words the elements of a Boolean algebra $\{0, 1\}^n$. We consider two cells neighbouring if they are neighbouring in the usual sense, i.e. if their Hamming distance is 1. We present the exact formula for the maximum number of hypercubic islands, i.e. the sub-Boolean algebras that are intervals. In the remaining part of the talk, first the definition of fuzzy rectangular relation will be given. This definition is based on the original rectangular island definition of G. Czédli, but uses it only in implicite way. Some basic properties of fuzzy rectangular relations will be reported.

References

[1] G. Czédli, The number of rectangular islands by means of distributive lattices, European Journal of Combinatorics, 30 (2009), 208-215.
[2] S. Földes, N. Singhi, On instantaneous codes, J. of Combinatorics, Information and System Sci., 31 (2006), 307-316.

[3] E. K. Horváth, G. Horváth, Z. Németh, Cs. Szabó, The number of square islands on a rectangular sea, Acta Sci. Math., submitted.
[4] E. K. Horváth, Z. Németh, G. Pluhár, The number of triangular islands on a triangular grid, Periodica Mathematica Hungarica, to appear.
[5] Zs. Lengvárszky, The mimimum cardinality of maximal systems of rectangular islands, European Journal of Combinatorics, to appear.
[6] G. Pluhár, The number of brick islands by means of distributive lattices, Acta Sci. Math. to appear.

Bounded Boolean Powers of Pseudo MV-algebras and Related Structures

M. Hyčko
Mathematical Institute, Slovak Academy of Sciences, Štefánikova 49, SK-81473, Bratislava, Slovakia
`hycko@mat.savba.sk`
Coauthors: A. Dvurečenskij

Algebraic contruction of Boolean powers and bounded Boolean powers were investigated for orthomodular posets by Pták ([Pta]), for orthoalgebras by Foulis and Pták ([FoPt]) and for difference posets by Dvurečenskij and Pulmannová ([DvPu]). We extend definition of bounded Boolean power for pseudo MV-algebras and also for more general structures. We show that bounded Boolean power of a finite Boolean algebra and a pseudo MV-algebras is isomorphic to their free product. There is a topological construction of bounded Boolean powers for arbitrary universal algebras ([Fos1], [Fos2]). We show that the algebraic construction is dual to the topological one.

References

[DvPu] A. Dvurečenskij, S. Pulmannová, *Difference posets, effects, and quantum measurements*, Inter. J. Theor. Phys. **33** (1994), 819–850.
[Fos1] A. L. Foster, *Generalized "Boolean" theory of universal algebras. Part I: Subdirect sums and normal representation theorem*, Math. Z. **58** (1953), 306–336.
[Fos2] A. L. Foster, *Generalized "Boolean" theory of universal algebras. Part II: Identities and subdirect sums in functionally complete algebras*, Math. Z. **59** (1953), 191–199.
[FoPt] D. J. Foulis, P. Pták, *On the tensor product of a Boolean algebra and an orthoalgebra*, Czechoslovak Math. J. **45 (120)** (1995), 117–126.
[Pta] P. Pták, *Summing of Boolean algebras and logics*, Demostratio Math. **19** (1986), 349–357.

Symmetry Groups of Boolean Functions

P. Jasionowski
Silesian University of Technology, Poland
`pawel.jasionowski@polsl.pl`

Mapping of the type $f : \{0,1\}^n \to \{0,1,...,k-1\}$ is called k-values boolean function of n variables. Let $F_{k,n}$ be the set of all such function. The symmetric group S_n act on the set $F_{k,n}$ according the rule:

$$f(x_1, x_2, ..., x_n) = f(x_{\sigma(1)}, x_{\sigma(2)}, ..., x_{\sigma(n)}) = f^{\sigma}(x)$$

for $f \in F_{n,k}$, $\sigma \in S_n$.
Subgroup $S(f) = \{\sigma \in S_n | f^{\sigma} = f\}$ is called the symmetry group of the function f. Permutation group $G < S_n$ is called k-representable if $G = S(f)$ for some $f \in F_{k,n}$. In [1] was a statement that every k-representable subgroup of S_n is 2-representable, for all $k \geq 2$. In [2] was proposed a counter - example to this statement:
Counterexample [by A. Kisielewicz] Let $G \subseteq S_4$ is a group generate by $\sigma_1 = (1,2)(3,4)$, $\sigma_2 = (1,3)(2,4)$. Then G is 3-representable , but is not 2 - representable.
We propose the following generalisation of the Kisielewicz example:
Theorem 1
For every positive integer number $n \geq 3$ the symmetric group S_{2^n} contains a regular representation of elementary abelian 2-group Z_2^n, which is s-representable, but is not a $s-1$ - representable for some s which hold condition

$$2^n \leq s \leq \binom{2^n}{2^{n-1}} - 2^n$$

References

1. P. Clote, E. Krenakis, *Boolean function invariance groups, and parallel complexity*, J. Comput. , Vol. 20, (1991), 553-590.
2. A. Kisielewicz, *Symmetry groups of Boolean functions and constructions of permutation groups* , Journal of Algebra. , 199 , (1998), 379-403.

Some (known) Resutls on Free-by-Cyclic Groups

C. Kocapinar
Balkesir University
`cnyilmaz@gmail.com`
Coauthors: F. Ates

Let P and Q be algebraic properties. A group G is a P-by-Q group if G has a normal subgroup N such that N has P and G/N has Q. There are various studies about P-by-Q groups and its connections between the other group structures such as decision problems (word, conjugacy, isomorphism problems), subgroup separability, polynomial growth, some group extensions,etc. As an one of the important application, in this talk, I would like to present some results on free-by-cyclic groups, especially, connection between free-by-cyclic groups and subgroup separability, and also present examples of free-by-cyclic groups by using topological tecniques and group extensions.

References

[1] Baumslag, G., A Non-Cyclic, Locally Free, Free-by-Cyclic Groups all of Whose Finite Factor Groups are Cyclic, Bull. of the Austral. Math. Soc., 6 (1972), 313-314.
[2] Baumslag, G., Troeger, D., Virtually Free-by-Cyclic One-Relator Groups.I., Algebra and Discrete Mathematics, 1 (2008), 9-25.
[3] Bogopolski, O., Martino, A., Maslakova, O., Ventura, E., Free-by-Cyclic Groups have Solvable Conjugacy Problem, Bull. of the London Math. Soc., 38(5) (2006), 787-794.
[4] Leary, I. J., Niblo, G. A., Wise, D. T., Some Free-by-Cyclic Groups, Groups St. Andrews 1997 in Bath II, Cambridge University Press, Cambridge, (1999), 512 516.

The Word and Generalized Word Problem for Semigroups under Wreath Products

E. G. Karpuz
Balikesir University, Turkey
eguzel@balikesir.edu.tr
Coauthors: A. S. Cevik

Algorithmic problems such as the "word, conjugacy and isomorphism problems" have played an important role in group theory since the work of M. Dehn in early 1900's. These problems are called "decision problems" which ask for a yes or no answer to a specific question. In this paper, we investigate the solvability of the word problem for the wreath product SwrT, where S and T are infinite and finite semigroups, respectively and then present a result giving the solvability of the generalized word (membership) problem for the same wreath product by using the normal form constructions of words.

On Congruence Lattices of Commutative Unary Algebras

A. Kartashova
Volgograd State Pedagogical University, Russia
kartashovaanna@mail.ru

A unary algebra is called *commutative* if every two operations of this algebra commute with each other. We consider the variety $\mathbf{M}$ consisting of all commutative unary algebras $< A, f, g, h >$ that satisfy the identity $f(g(h(x))) = x$. We describe the class of all distributive lattices each of which is isomorphic to a congruence lattice of some algebra from the variety $\mathbf{M}$. We also characterize algebras of this variety whose congruence lattices are linearly ordered.

Rings Over Which all Modules are Strongly Gorenstein Projective

Ou. Khalid
Department of Mathematics, Faculty of Science and Technology of Fez, Box 2202, University S. M. Ben Abdellah Fez, Morocco
ouarghi.khalid@hotmail.fr
Coauthors: D. Bennis and N. Mahdou

One of the main results of this paper is the characterization of the rings over which all modules are strongly Gorenstein projective. We show that these kinds of rings are very particular cases of the well-known quasi-Frobenius rings. We give examples of rings over which all modules are Gorenstein projective but not necessarily strongly Gorenstein projective.

Commutative Directoids with Sectionally Antitone Bijections

M. Kolařík
Palacký University Olomouc, Czech Republic
kolarik@inf.upol.cz
Coauthors: I. Chajda (Palacký University Olomouc); S. Radeleczki (University of Miskolc)

We study commutative directoids with a greatest element, which can be equiped with antitone bijections in every principal filter. These can be axiomatized as algebras with two binary operations satisfying four identities. A minimal subvariety of this variety is described.

On the Classification of Simple Finite Jordan Pseudo-Algebras
P. Kolesnikov
Sobolev Institute of Mathematics, Novosibirsk, Russia
pavelsk@math.nsc.ru

The notion of a pseudo-algebra is an appropriate generalization of ordinary and conformal algebras. By definition, a pseudo-algebra is a module over a Hopf algebra H endowed with a family of binary multiplications indexed by the dual space H^*. The natural categorical approach leads to what is called associative (commutative, Lie, etc.) pseudo-algebras. Our aim is to consider those Jordan pseudo-algebras finite over H, and classify simple objects in this class. We will also consider a variety of (ordinary) algebras which naturally arises from Jordan pseudo-algebras; these algebras relate to Jordan algebras as Leibniz algebras to Lie algebras.

Preclones
J. Koppitz
Universität Potsdam, Institut für Mathematik
koppitz@rz.uni-potsdam.de

The concept of a preclone was introduced by Ésik and Weil in order to describe recognizable tree languages from the algebraic point of view. We will determine the free preclone and introduce the notation of the preclone of an algebra of a given type in a natural way. It is known that the identities of the clone of an algebra correspond to the hyperidentities of this algebra, that the hypersubstitutions are endomorphims on the free clone and that eqivalent varieties generate isomorphic clones. We want to discuss these facts for preclones. Although, preclones are very close to clones there are essential and interesting differences.

On Prime Deductive Systems in Pseudo-BCK-Algebras
J Kühr
Palacky University in Olomouc, Czech Republic
kuhr@inf.upol.cz

Pseudo-BCK-algebras or biresiduation algebras are the residuation subreducts of non-commutative integral residuated lattices. We describe the meet prime elements of the lattice of deductive systems and the lattice of compatible deductive systems (= congruence kernels) and characterize those pseudo-BCK-algebras in which the primes in the two lattices coincide.

Folding Theory of some Types of Ideals in BCI-algebras

C. Lele
University of Dschang, Cameroon
lele_clele@yahoo.com

In 1,6,7,8, some types of ideals in BCI-algebras have been studied as well as various relations between them. The main purpose of our work is to investigate the folding of other types of ideals and the relation diagram between them similar as in 6,7. We have already managed with the folding theory in BCK-algebras in 3, the folding theory of quasi-associative ideals (namely q-ideals) in 2, the n-folds of H-ideals in 4 and n-folds P-ideals in 5. The expecting results will be a generalization of the results that appear in 1,6,7,8,9.

References

1.Y. B. Jun, Fuzzy Sub-Implicative Ideals in BCI-algebras, Bull. Korean Math. Soc., 39 (2002) pp. 185-198.
2.Y. B. Jun, S. Z. Song and C. Lele, Foldness of Quasi-associative Ideals in BCI-algebras, Scientiae Mathematicae Japonicae, (2002) pp. 227-231.
3.C. Lele and S. Moutari, On some computational algorithms for n-folds ideals in BCK-algebras , J. Appl. Math. Computing, 1-2(2007)pp. 369-383 .
4.C. Lele and S. Moutari, Computational methods for study of foldness of H-ideals in BCI-algebras , Soft computing,12 (2008) pp. 403-407.
5.C. LELE ,S. Moutari and M. L. Mbah Algorithms and computations for foldenes of P-ideals in BCI-algebras, Journal of applied Logic 6(2008),pp. 580-588.
6. Y. L. Liu and J. Meng, Fuzzy Ideals in BCI-algebras, Fuzzy Sets and Systems 123 (2001) pp. 227-237.
7. Y. L. Liu, J. Meng, X. H. Zhang and Z. C. Yue , q-ideals and a-ideals in BCI-algebras, Southeast Asian Bulletin of Mathematics, (2000) pp. 243-253.
8. Y. L. Liu and J. Meng Sub-implicative Ideals and sub-commutative Ideals in BCI-algebras, Soochow Journal of Mathematics, (2000) pp. 441-453.
9. Y. L. Liu , Yang Xu and J. Meng, BCI- Implication ideals of BCI-algebras, Information sciences (2008) (online).

The Order of Generalized Hypersubstitutions of Type (3)

S. Leeratanavalee
Deptm. of Mathematics Chiang Mai University
scislrtt@chiangmai.ac.th
Coauthors: S. Sudsanit

In this paper we characterize all idempotent generalized hypersubstitutions of type (3) and determine the order of each generalized hypersubstitutions of this type.

On the Finite Index Property of Clones

Erkko Lehtonen
University of Luxembourg
erkko.lehtonen@uni.lu
Coauthors: . Szendrei

For each clone $\mathcal{C}$ on a set A, there is an associated equivalence relation, called $\mathcal{C}$-equivalence, on the set $\mathcal{O}_A$ of all operations on A, which relates two operations if and only if each one is a substitution instance of the other using operations from $\mathcal{C}$. The set of clones $\mathcal{C}$ on A that have the property that the associated $\mathcal{C}$-equivalence relation has finite index in $\mathcal{O}_A$, i.e., a finite number of equivalence classes, constitutes an order filter in the lattice of clones on A. In this talk, we present some recent results towards understanding the structure of this filter.

Hypersubstitutions of $Clone\tau$

S. Lekkoksung
University of Potsdam
lekkoksung_somsak@hotmail.com
Coauthors: K. Denecke

Defining superposition operations on the set $W_\tau(X)$ of all terms of type τ one obtains a many-sorted algebra of a particular type:

$$clone\ \tau := ((W_\tau(X_n))_{n\geq 1}; (S_n^m)_{m,n\geq 1}, (x_i)_{1\leq i\leq n}).$$

If one wants to study identities and hyperidentities of *clone* τ one needs to define terms over *clone* τ, i.e., terms over these particular many-sorted algebras. On the set of all those terms superposition operations (of second level) are definable and one gets a clone (of second level) where the universes are the sets of n-ary many-sorted clone terms and the operations are these superposition operations. As on the first level one can now define hypersubstitutions of second level. If turns out that their extensions are endomorphisms of clones of second level. These results can be generalized to the non-arity preserving case.

Some Endoprimal Monoids over a Three-Element Set

H. Machida
Hitotsubashi University, Tokio, Japan
machida@math.hit-u.ac.jp
Coauthors: Ivo G. Rosenberg (University of Montreal)

For a set S of multi-variable functions on a finite set A, the centralizer S^* of S is the set of functions which commute with all functions in S. For a monoid M of unary functions on A, M is called *endoprimal* if the unary part of the bicentralizer $(M^*)^*$ coincides with M itself. Endoprimal monoids have been studied in universal algebra. In this talk we show some examples of endoprimal monoids on a 3-element set A. We also show some applications of Kuznetsov Criterion.

Quasi-Exact Sequences

A. Madanshekaf
Math. Department, Faculty of Science, Semnan University, Semnan, Iran
a_madanshekaf@yahoo.co.uk

All rings in this lecture are assumed to be commutative with non-zero identity and all modules are unitary. Exact sequences have been used intensively in many discipline of mathematics such as commutative algebras. Let R be a ring and $A \xrightarrow{f} B \xrightarrow{g} C$ an exact sequence of R-modules. Then $imf = \ker g (= g^{-1}(\{0\}))$. It is raising a natural question:
What does happen if we substitute a submodule U of C instead of the trivial submodule $\{0\}$ above? In [b:ano], Davvaz and Parniam-Gramaleky introduced the concept of quasi-exact sequences and answered the above question. They generalized some results from the standard case to the modified case. In [b:ago] Davvaz and Shabani-Solt introduced a generalization of some notions in homological algebra. They defined the concepts of chain U-complex, U-homology, chain (U, U')-map, chain (U, U')-homotopy and U-functor. They gave a generalization of the Lambek lemma, snake lemma, connecting homomorphism, exact triangle and established new basic properties of the U-homological algebra (See for example [H:aci]). In [s:use], Anvariyeh and Davvaz studied U-split exact sequences and established several connections between U-split sequences and projective modules.

In this talk we investigate further this notion. In particular, some interesting results concerning this concept and torsion functor are given.

References

[s:use] S. M. Anvariyeh and B. Davvaz, U-split exact Sequences, Far East J. Math. Sci. 4 (2002), no. 2, 209-219.
[b:ago] B. Davvaz amd H. Shabani-Solt, A generalization of homological algebra, J. Korean Math. Soc. 39 (2002), no. 6, 881-898.

[b:ano] B. Davvaz and Y. A. Parnian-gramaleky, A note on exact sequences, Bull. Malaysian Math. Soc. (2) 22 (1999), no. 1, 53-56.
[s:oqe] S. M. Anvariyeh and B. Davvaz, On quasi-exact sequences, Bull. Korean Math. Soc. 42(2005), no. 1, 149-155.
[H:aci] P. J. Hilton and U. Stammbach, A Course in Homological Algebra, Second Edition, Springer-Verlag, 1996.
[M:crt] H. Matsumara, Commutative Ring Theory, Cambridge University Press, Cambridge, 1986.

Endomorphal Multiplication Modules

K. Mecam
Maejo University Sansai Chiangmai, Thailand
`kamonthep@mju.ac.th`
Coauthors: J. Sanwong

In this paper we prove that if is a multiplication module with is cyclic for all , then is endomorphal. As an application, we get that every semisimple multiplication is endomorphal. Also, we list some rings in which all multiplication modules over them are endomorphal.

The Fibonacci and Lucas Subsequences as Principal Minors of Quasi-Pascal Matrices

A. R. Moghaddamfar
Department of Mathematics, Faculty of Science, K. N. Toosi University of Technology, P. O. Box 16315-1618, Tehran, Iran
`moghadam@kntu.ac.ir`

In the literature one may encounter certain infinite tridiagonal matrices the principal minors of which constitute the Fibonacci or Lucas sequence (see [1-3]). The major purpose of this lecture is to find some new infinite matrices the principal minors of which again form the Fibonacci or Lucas sequence. In particular, we obtain families of quasi-Pascal matrices whose principal minors generate any arbitrary linear subsequences F(nr+s) or L(nr+s), (n=1,2,3,...) of Fibonacci or Lucas sequence.

References

[1] N. D. Cahill, J. R. D'Errico, D. A. Narayan, and J. Y. Narayan, "Fibonacci determinants", The College Math. J., 33(3)(2002), 221-225.
[2] N. D. Cahill and D. A. Narayan,"Fibonacci and Lucas numbers as tridiagonal matrix determinants", Fibonacci Quart., 42(3)(2004), 216-221.
[3] K. Griffin, J. L. Stuart and M. J. Tsatsomeros, "Noncirculant Toeplitz matrices all of whose powers are Toeplitz", Czechoslovak Mathematical Journal, 58(4) 2008, 1185-1193.

On Automorphism Groups of Universal Hypergraphical Automata

V. Molchanov
Saratov State Socio-Economic University, Russia
`v.molchanov@inbox.ru`
Coauthors: A. Molchanov

A hypergraph is a system $H = (X, R)$, where X is a non-empty set and R is a family of arbitrary subsets of X. The elements of X and R are called vertices and edges.

By automaton we mean a system $A = (X, S, Y, \delta, \lambda)$ consisting of a state set X, a semigroup S of input signals, a set Y of output signals, a transition function $\delta : X \times S \to X$ and an exit function $\lambda : X \times S \to Y$ such that $\delta(x, s_1 s_2) = \delta(\delta(x, s_1), s_2)$ and $\lambda(x, s_1 s_2) = \lambda(\delta(x, s_1), s_2)$ for any $x \in X$, $s_1, s_2 \in S$. An automaton A is said to be hypergraphical if its sets X and Y endowed with a structure of hypergraphs $H = (X, R)$ and $H' = (Y, R')$ such that, for any $s \in S$, the mappings $\delta_s(x) = \delta(x, s)$, $\lambda_s(x) = \lambda(x, s)$ $(x \in X)$ are homomorphisms of the corresponding hypergraphs. In this case we denote the automaton $A = (H, S, H', \delta, \lambda)$.

For hypergraphs H, H', the universal hypergraphical automaton is the hypergraphical automaton $\mathrm{Atm}(H, H') = (H, S(H, H'), H', \delta, \lambda)$, where $S(H, H') = \mathrm{End}(H) \times \mathrm{Hom}(H, H')$ and, for every $x \in X$, $(\varphi, \psi) \in S(H, H')$, $\delta(x, (\varphi, \psi)) = \varphi(x)$, $\lambda(x, (\varphi, \psi)) = \psi(x)$.

In this talk we investigate a connection between the automorphism groups of hypergraphs H, H' and the automorphism group of the universal hypergraphical automaton $\mathrm{Atm}(H, H')$.

Lattice Ordered Polynomial Algebras–Cayley Theorem for DeMorgan Algebras

Y. Movsisyan
Department of Mathematics and Mechanics, Yerevan State University,Armenia
`yurimovsisyan@yahoo.com`

Let $\mathcal{A} = (A; F)$ be an algebra, and let $P^{(2)}(\mathcal{A})$ be the set of all binary polynomials in the operations of F. The algebra $\mathcal{B}_{\mathcal{A}} = (P^{(2)}(\mathcal{A}); +, \cdot, ', 0, 1)$ is defined as follows:

$$(f + g)(x, y) = f(x, g(x, y)),$$
$$(f \cdot g)(x, y) = f(g(x, y), y),$$
$$f'(x, y) = f(y, x),$$
$$1(x, y) = x,$$
$$0(x, y) = y.$$

In this talk we find necessary and sufficient conditions for $\mathcal{B}_{\mathcal{A}}$ to be a DeMorgan algebra. We also prove a "Cayley theorem" for DeMorgan algebras, using hyperidentities.

Finitely Presented Monomial Algebras

Jan Okninski
Warsaw University, Poland
okninski@mimuw.edu.pl

A structural approach to the study of finitely presented monomial algebras is discussed. As an application, it is shown that a monomial algebra $K[X]/J$ over a field K, where J is a prime ideal of a finitely generated free algebra $K[X]$ generated by finitely many elements of the free monoid X, is primitive whenever it does not satisfy a polynomial identity. This yields a proof of the trichotomy conjecture of Bell and Smoktunowicz, in the finitely presented case, proved independently also by Bell and Pekcagliyan. Applications to the class of algebras defined by permutation relations, studied recently in a joint work with F.Cedo and E.Jespers, are discussed.

On Representations of Permutation Groups as Isometry Groups of Finite Metric Spaces.

B. Oliynyk
National University "Kyiv-Mohyla Academy", Kyiv, Ukraine
bogdana.oliynyk@gmail.com

Let (X, d_x) be a finite metric space. We can consider its isometry group IsX as a permutation group (IsX, X). We intend to discuss which permutation groups can be realized as the isometry group of some finite metric space. It easy to see that there exists a permutation group (for example, the regular cyclic group of order $n \geq 3$) that is not an isometry group of any finite metric space. We consider some constructions of finite permutation groups that we can represent as isometry groups of finite metric spaces.

Generalized Derivations and Commutativity of Rings with Involution

L. Oukhtite
Department of Mathematics, Faculty of Science and Technology of Errachidia, Box 509-Boutalamine, University My Ismal Errachidia , Morocco
oukhtitel@hotmail.com
Coauthors: S. Salhi and L. Taoufiq

Let $(R, \star)$ be a 2 -torsion free ring with involution and F a generalized derivation, associated to a derivation d, satisfying one of the following conditions:
1) for each $x, y \in R$ either $d(x)oF(y) = 0$ or $d(x)oF(y) = xoy$.
2) for each $x, y \in R$ either $[d(x), F(y)] = 0$ or $d(x)oF(y) = [x, y]$.
In this paper it is shown that if R is $\star$-prime, then R is commutative. Moreover, examples proving the necessity of the $\star$-primeness condition for R are given.

Group Divisible Designs with two Associate Classes and $\lambda_2 = 1$

N. Pabhapote
University of the Thai Chamber of Commerce, Bangkok 10400, Thailand
nittiya_pab@utcc.ac.th;
Coauthors: N. Punnim

The original classification of PBIBDs defined group divisible designs with $\lambda_1 \neq 0$. Keeping up with that tradition, we study the group divisible designs with two groups of unequal sizes and block size three. The necessary conditions have already been proved to be sufficient for the case of two groups of equal sizes and block size three. Here we obtain the necessary conditions and prove that the conditions are sufficient for some infinite families.

Locally finite M-solid Varieties of Semigroups

B.ä Pibaljommee
The Department of Mathematics Khonkaen University, Thailand
banpib@kku.ac.th
Coauthors: K. Denecke

We use the theory of M-solid varieties to prove that a type (2) M-solid variety of the form $V = H_M Mod\{F(x_1, F(x_2, x_3)) \approx F(F(x_1, x_2), x_3)\}$, which consists precisely of all algebras which satisfy the associative law as an *M-hyperidentity* is locally finite iff the hypersubstitution which maps F to the word $x_1x_2x_1$ or to the word $x_2x_1x_2$ belongs to M and that V is finitely based if it is locally finite.

Unityped Algebras

B. Plotkin
Hebrew University, Jerusalem, Israel
plotkin@macs.biu.ac.il

In the talk we give an extension of the ideas developed in B. Plotkin, G. Zhitomirski, "Some logical invariants of algebras and logical relations between algebras", St.Peterburg Math. J., 19:5, (2008) 859 – 879, whose main notion is that of logic-geometrical equivalence of algebras (LG-equivalence of algebras). This equivalence of algebras is more strict than elementary equivalence. We introduce the notion of unityped algebras and relate it to LG-equivalence. We show that these notions coincide. The idea of the type is one of the central ideas in Model Theory. The correspondence between types and LG-equivalence stimulates a bunch of problems which connect universal algebraic geometry and Model Theory. We touch the following topics: 1. General look 2. Logical noetherianity 3. Unitypeness and isomorphism 4. Logically perfect algebras 5. Some facts from algebraic logic. We provide a new general view on the subject, arising "on the territory" of universal algebraic geometry, which yield applications of algebraic logic and universal algebraic geometry in Model Theory. We give a list of new unsolved problems.

Automorphic Equivalence of Multi-Models Versus Graphs

T. Plotkin
Bar Ilan Ubiversity, Israel
plotkin@macs.biu.ac.il
Coauthors: M. Knyazhansky

A model is treated as a triple consisting of an algebra from a fixed variety of algebras, a set of symbols of relations and an interpretation which realizes all symbols of relations in the given algebra. Algebras in the given variety may be multi-sorted. This leads to multi-sorted operations and relations. Multi-model differs from a model by the set of interpretations instead of a single one. Definitions of a knowledge base and a category of knowledge bases rely on the notion of multi-model which is treated as a subject of knowledge. Knowledge base includes a category of knowledge description as well as categories of knowledge content for each interpretation from the given set of interpretations. There is also a functor transforming a knowledge description to its content. A notion of informational equivalence of two knowledge bases with different subjects of knowledge is defined in these terms. It has been proved that in case of finite subjects of knowledge the corresponding knowledge bases are informationally equivalent if and only if the subjects of knowledge are automorphically equivalent. We define the notion of automorphic equivalence of two multi-models and study this notion. If two multi-models are isomorphic, then they are automorphically equivalent. The opposite is not true since the notion of automorphic equivalence is wider than that of isomorphism. In the talk we consider graphs and multi-graphs as a subject of knowledge. The main problem here is an algorithm of automorphic equivalence of multi-models verification. We consider the general algorithm and its adaptation in special cases.

The class of Biregular Leftmost Graph Varieties of Type (2,0)

T. Poomsa-ard
Mahasarakham University, Thailand
tiang@kku.ac.th
Coauthors: M. Krapeedang

Graph algebras establish a connection between directed graphs without multiple edges and special universal algebras of type (2,0). We say that a graph G satisfies a term equation $s \approx t$ if the corresponding graph algebra $\underline{A(G)}$ satisfies $s \approx t$. A class of graphs V is called a graph variety if $V = Mod_g\overline{\Sigma}$ where Σ is a subset of $T(X) \times T(X)$. A graph variety $V' = Mod_g\Sigma^{'}$ is called a biregular leftmost graph varaiety if $\Sigma^{'}$ is a set of biregular leftmost term equation. A term equation $s \approx t$ is called an identity in a graph variety V if $\underline{A(G)}$ satisfies

$s \approx t$ for all $G \in V$. An identity $s \approx t$ of a graph variety V is belong to the class $\mathcal{V}$ whenever $Mod_g\{s \approx t\} = V$.

In this paper we characterize the class of all biregular leftmost graph varieties.

Lattice-Ordered Algebras of Real Continuous Functions

A. Pulgarin
University of Extremadura, Spain
aapulgar@unex.es

Throughout by algebra we mean a commutative R-algebra with identity, and by lattice-ordered algebra (briefly l-algebra) we mean both an algebra and a lattice whose order is compatible with the algebra structure. It is a standard procedure in obtaining dualities or isomorphisms in categories to deal with sets of morphisms into a significant object. In our categories such a significant object will be R - either as an l-algebra or as a topological space. For an l-algebra A consider X the set consisting in l-algebra morphisms from A to R equipped with the initial topology defined by A. Hence we have an l-algebra morphism from A to C(X).

The classical results appearing in the literature on characterizing C(X) regard the structure of A-l-algebra, that is, Archimedean l-algebras whose unity is a weak order unit. This work deals with obtaining a characterization as an l-algebra non necessary Archimedean nor with weak order unit. Our procedure has two steps:

The first step consists of finding inner conditions under which our morphism is 1-1. To this aim we define the class of real l-ideals of A as those convex ideals I which satisfy that for any f in A there exists a real number r such that f-r is in I, and we proof that it is is injective iff A is semisimple (the intersection of all the real l-ideals is 0).

In the second task we must find conditions under which it is a surjection. We found inner conditions on A in the spirit of the Urysohn lema's proof that allow A to separate disjoint zero-sets of X. Lastly by using inverse-closeness or the existence of special suprema sufficient for generating all continuous functions on X, our main result succeed.

Green's Relations on $Hyp_G(2)$

W. Puninagool
Deptm. of Mathematics Chiang Mai University, Thailand
wattapong1p@yahoo.com
Coauthors: S. Leeratanavalee

A generalized hypersubstitution of type $\tau = (2)$ is a map σ which takes the binary operation symbol f to a term $\sigma(f)$ which does not necessarily preserve the arity. Any such σ can be inductively extended to a map $\hat{\sigma}$ on the set of all terms of type $\tau = (2)$, and any two such extensions can be composed in a natural way. Thus, the set $Hyp_G(2)$ of all generalized hypersubstitutions of type $\tau = (2)$ forms a monoid. Green's relations on the monoid of all hypersubstitutions of type $\tau = (2)$ were studied by K. Denecke and Sh.L. Wismath. In this paper we use similar methods to study Green's relations on $Hyp_G(2)$.

Laudatio for Klaus Denecke

R. Pöschel
Technische Universität Dresden
reinhard.poeschel@tu-dresden.de
Coauthors: H.-J. Vogel

In 1986, Prof. Klaus Denecke founded the series *Conference for Young Algebraists* (CYA) which, since 1996, is jointly organized with *Arbeitstagung Allgemeine Algebra* (AAA) and which many times was held at Potsdam University. This year, we celebrate the 65th birthday of Prof. K. Denecke. On this occasion we want to honor his scientific activities, in particular, his contributions to the success of the conference series CYA and AAA.

On One-sided Congruences of an Idempotent Groupoid

A. Reshetnikov
Moscow Institute of Electronic Technology, Russia
Resheton@mail.ru

Let G be a groupoid. By definition, a right congruence of G is such equivalence relation s on G that (a,b)∈s implies (ac,bc)∈s for all a,b,c∈G. A left congruence can be defined by the dual way.

The groupoids were described in [1] such that each of their equivalence relations is a right congruence. We will call them R-groupoids and define an L-groupoid by the dual way. A complete description of the semigroups, whose equivalence relations are right or left congruences, was obtained also in [1]. If every equivalence relation on a groupoid is a right or a left congruence, we will call this groupoid as carbonoid. It turns out that if a semigroup is a carbonoid then either it is an R-groupoid or it is an L-groupoid. Nevertheless, there exist non-associative carbonoids, which are neither R-groupoids nor L-groupoids. An example is the set {a,b,c} with the following binary operation defined on it: aa=ba=ca=c, bb=cb=b, ab=ac=bc=cc=a. However, the existence of a carbonoid of 4 or more elements with this property is unknown for the author. He makes a hypothesis that such groupoids do not exist. Now, this hypothesis is proved for the idempotent carbonoids, i.e. the ones satisfying the identity xx=x. Namely, the following theorem is true.

Theorem. Let G be an idempotent carbonoid, and |G|≥4. Then either G is an R-groupoid or G is an L-groupoid.

The following note is important for proving this theorem: if G is an idempotent groupoid whose equivalences are right or left congruences then ab∈a,b for all a,b∈G. It means that every non-empty subject of G is a subgroupoid.

References

1. Kozhukhov I.B. Algebras whose equivalence relations are all congruences. Siberian Math.J., in press.

Flocks in Universal and Boolean Algebras.

G. Ricci
Universit di Parma, I-43100 Parma, Italy
gabriele.ricci@unipr.it

We propose the notion of flocks, which formerly were introduced only in based algebras, for any universal algebra. This generalization keeps the main properties we know from vector spaces, e.g. a closure system that extends the subalgebra one. It comes from the idempotent elementary functions, called "interpolators", that in case of vector spaces merely are linear functions with normalized coefficients.

The main example, we consider outside vector spaces, concerns Boolean algebras, where flocks form "local" algebras. Among several open problems we outline the one of generalizing the Segre transformations of based algebras, which used certain flocks, in order to approach a general transformation notion.

Endomorphism Rings of Quasi-rp-injective and Quasi-lp-injective Modules

A. Sudprasert
University of the Thai Chamber of Commerce, Bangkok 10400, Thailand
aisuriya_sud@utcc.ac.th
Coauthors: S. Sanpinij, Hoang Dinh Hai and Nguyen van Sanh

Let R be a ring. A right R-module N is called an M-p-injective module if any homomorphism from an M-cyclic submodule of M to N can be extended to an endomorphism of M. Generalizing this notion, we investigated the class of M-rp-injective modules and M-lp-injective modules, and prove that for a finitely generated Kasch module M, if M is quasi-rp-injective, then there is a bijection between the class of maximal submodules of M and the class of minimal left right ideals of its endomorphism ring S. In this paper, we give some characterizations and properties of the structure of endomorphisms ring of M-rp-injective modules and M-lp-injective modules and the relationships between them.

Hyperalgebras and Hyper-coalgebras of Type τ

K. Saengsura
University of Potsdam
saengsur@rz.uni-potsdam.de
Coauthors: K. Denecke

In this paper we consider hyperalgebras and hyper-coalgebras of type τ, as an analogue of algebras of type τ and coalgebras of type τ. We consider hyperalgebras and hyper-coalgebras as special cases of (F_1, F_2)-systems. Therefore many results in this paper will be instances of the (F_1, F_2)-system results. Nevertheless it is interesting and instructive to consider directly the theory of hyperalgebras and hyper-coalgebras of type τ.

Products of Tree Languages

N. Sarasit
University of Potsdam, Institute of Mathematics
napaporn_sarasit@hotmail.com
Coauthors: K. Denecke (University of Potsdam)

Sets of terms of type τ are called tree languages. The *tree language product* is the most important operation defined on sets of tree languages which maps recognizable tree languages to recognizable languages. This tree language product can be described as superposition of sets of terms. Based on the superposition operation we define a binary associative operation. In the theory of tree languages this operation is called the *z-product*. The aim of this paper is to describe some properties of the arising semigroup. We are especially interested in idempotent and regular elements, Green's relations $\mathcal{L}$ and $\mathcal{R}$, in constant, left-zero and right-zero subsemigroups and in rectangular bands. The iteration of this binary operation plays the role of the *Kleene-$*$-operation* of the theory of formal languages.

Weak Homomorphisms of F-Algebras

F. M. Schneider
TU Dresden
friedrich-martin.schneider@online.de

In the theory of universal algebras homomorphisms are considered only between algebras of the same similarity type. Different from that the notion of a weak homomorphism does not depend on a signature, but only on the clones of term operations generated by the examined algebras. We try to generalize this idea by defining weak homomorphisms between F- and G-algebras, where F and G denote not necessarily equal set-functors. The aim is to derive well-known results as the weak homomorphism theorem.

C-dense Injectivity and C-dense Essential Monomorphisms in the Category $Act - S$ for an Arbitrary Closure Operator C

L. Shahbaz
Department of Mathematics, Faculty of Basic Sciences, University of Maragheh, Maragheh, Iran
leilashahbaz@yahoo.com

Let C be a closure operator in the category Act-S of right S-acts. One has the usual two classes of monomorphisms (C-dense and C-closed monomorphisms) related to the notion of a closure operator. The class of sequentially dense monomorphisms resulting from an special closure operator (sequential closure operator) were first defined and studied by Giuli, Ebrahimi, and Mahmoudi for projection algebras (acts over the monoid (N^infty, min), of interest to computer scientists, as studied by Herrlich, Ehrig, and some others) and generalized to acts over arbitrary semigroups. Essentiality is an important notion closely related to injectivity. In this paper, first we study C-dense monomorphisms of acts for an arbitrary closure operator C. Then, we study injectivity and essentiality with respect to C-dense monomorphisms. We will show that the three different definitions of essentiality usually used in literature with respect to a subclass of monomorphisms are equivalent for the class of C-dense monomorphisms and, among other things, we show the existence and the explicit description of a maximal such essential extension for any given act.

References

1. Banaschewski, B., *Injectivity and essential extensions in equational classes of algebras*, Queen's Papers in Pure and Applied Mathematics, **25** (1970), 131-147.
2. Dikranjan D., W. Tholen, *Categorical structure of closure operators, with applications to topology, algebra, and discrete mathematics*; Mathematics and Its Applications, Kluwer Academic Publ., 1995.
3. Giuli, E. *On m-separated projection spaces*, Appl. Categ. Structures, **2**(1) (1994), 91-99.

4. Mahmoudi M. and Shahbaz, L., *Characterizing semigroups by sequentially dense injective acts*, Semigroup Forum, **75**(1) (2007), 116-128.

Characteristics of Quasigroups Isotopic to Some Loops

K. Shahbazpour
Dept. of Maths., Urmia University, Urmia, I.R.Iran
Shahbazpour2003@hotmail.com

A loop (Q,.) is quasigroup with identity element. In this talk we give the characteristic quasigroups which are isotopic to Bruck loops,Moufang loops, Bol loops and Alternative loops.

On the Essential Arity Gap of Finite-valued Functions

S. Shtrakov
Neofit Rilsky South-West University, Blagoevgrad, Bulgaria
shtrakov@swu.bg

We study and describe finite valued functions with given essential arity gap. This description is based on the representation of the functions in their SC-forms as sums of conjunctions. The combinatorial problem how many are finite valued functions depending essentially on all of its variables which have given essential arity gap is solved, also.

Lattice Identities and Colored Graphs Connected by Test Lattices

B. Skublics
University of Szeged, Bolyai Institute, Hungary
bskublics@math.u-szeged.hu

Czedli has recently given a pictorial approach to several properties of free lattices. Our goal is to generalize his construction and use it to prove some additional classical lattice theoretical results in a new, more visual way.

Diameter of Sylow p-Subgroups of the Symmetric Group Sp^2

A. Slupik
Institute of Mathematics, Silesian University of Technology, Poland
anna.slupik@polsl.pl

We deal only with finite groups. Let G be a group and X its set of generators. We denote by $Cay(G, X)$ a Cayley graph of the group G with respect to the set of generators X. A diameter $d_X(G)$ of $Cay(G, X)$ is the longest distance between vertices of this graph in the standard graph metric, i.e. the smallest k such that every element of G can be expressed as a word of length at most k in $X \cup X^{-1}$.

Let p be a fixed prime. In the talk we study diameters of Cayley graphs of Sylow p-subgroups of degree p^2 for different 2-element generating sets. Sylow p-subgroups of S_{p^2} are isomorphic to the wreath product $C_p \wr C_p$ of cyclic groups of degree p (see [1]). According to [2] every element of $C_p \wr C_p$ is presented in the form

$$[f_1, f_2(x)],\ f_1 \in Z_p,\ f_2(x) \in Z_p[x],\ deg(f_2) < p\ .$$

We investigate the following family W of generating sets of $C_p \wr C_p$

$$W = \{\{[a, 0], [0, b(x + c)^{p-1}]\} :\ a, b, c \in Z_p,\ a \cdot b \neq 0\}$$

Theorem
For any $X \in W$

$$d_X(G) \leq \frac{p^2 + 2p - 3}{2}$$

Using computer calculations we show that if $p = 3$ then for any 2-element generating set X the following inequality holds:

$$4 \leq d_X(C_p \wr C_p) \leq 6.$$

References

[1] John D. Dixon and Brian Mortimer. *Permutation Groups,* Springer-Verlag, 1996
[2] Kaloujnine L. A. *La structure du p-groupe de Sylow du goupe symtrique du degr p^2,* C. R. Acad. Sci. Paris 222, (1946). 1424–1425

Fuzzy Algebras as a Framework for Fuzzy Topology

S. Solovjovs

Department of Mathematics, University of Latvia, Zellu iela 8, LV - 1002 Riga, Latvia

sergejs@lu.lv

The famous adjunction of D. Papert and S. Papert between the categories **Top** (topological spaces) and $\mathbf{Frm}^{op}$ (the dual of the category of frames) [7] paved the way for considering the category **Loc** of *locales* (introduced by J. Isbell in [4]) as a substitute for **Top**. In particular, S. Vickers introduced the notion of *topological system* to get a single framework for treating both spaces and locales [15]. Later on J. T. Denniston and S. E. Rodabaugh considered functorial relationships between *lattice-valued topology* and topological systems [2]. To be more flexible they introduced the notion of *lattice-valued topological system* over **Loc** [1]. In [10, 12] we considered a generalization of the notion, replacing **Loc** by the dual of an arbitrary variety of algebras. In particular, we proved that the category of *variety-based topological spaces* of [14] (*à la* [8]) is isomorphic to a full coreflective subcategory of topological systems. During the 30th Linz Seminar on Fuzzy Set Theory J. T. Denniston, A. Melton and S. E. Rodabaugh presented an embedding of the category of lattice-valued topological systems into a variable-basis modification of the category of *fuzzy topological spaces* of T. Kubiak and A. Šostak [3, 5]. The problem of the opposite embedding remained open. This talk answers the question positively.

Start with a variety-based version of the Kubiak-Šostak approach.

Definition 1 (Varieties) Let $\Omega = (n_\lambda)_{\lambda\in\Lambda}$ be a class of cardinal numbers. An Ω-*algebra* is a pair $(A, (\omega_\lambda^A)_{\lambda\in\Lambda})$ (denoted by A), where A is a set and $(\omega_\lambda^A)_{\lambda\in\Lambda}$ is a family of maps $\omega_\lambda^A : A^{n_\lambda} \to A$. An Ω-*homomorphism* $\varphi : (A, (\omega_\lambda^A)_{\lambda\in\Lambda}) \to (B, (\omega_\lambda^B)_{\lambda\in\Lambda})$ is a map $\varphi : A \to B$ such that $\varphi \circ \omega_\lambda^A = \omega_\lambda^B \circ \varphi^{n_\lambda}$ for every $\lambda \in \Lambda$. $\mathbf{Alg}(\Omega)$ is the category of Ω-algebras and Ω-homomorphisms. A *variety of Ω-algebras* is a full subcategory of $\mathbf{Alg}(\Omega)$ closed under the formation of products, subalgebras and homomorphic images. The objects (resp. morphisms) of a variety are called *algebras* (resp. *homomorphisms*).

The categories **Frm**, **SFrm** and **SQuant** of frames, semiframes and semiquantales (popular in lattice-valued topology) are varieties. From now on we fix a variety **A**, denoting by **LoA** its dual category. We also assume that **L** is a subcategory of the category of completely distributive complete lattices and join-preserving maps.

Definition 2 (Fuzzy algebras) An **L**-*fuzzy algebra of type* **A** is a map $\mu : A \to L$ (denoted by μ) such that (A, L) is in $\mathbf{A} \times \mathbf{L}$ and $\bigwedge_{i\in n_\lambda} \mu(a_i) \leq \mu(\omega_\lambda^A(\langle a_i\rangle_{n_\lambda}))$ for every $\lambda \in \Lambda$ (cf. [6, 9, 11]). An **L**-*fuzzy homomorphism of type* **A** $(\varphi, \alpha) : \mu \to \mu'$ is an $\mathbf{A} \times \mathbf{L}$-morphism $(\varphi, \alpha) : (A, L) \to (A', L')$ such that $\alpha \circ \mu(a) \leq \mu' \circ \varphi(a)$ for every $a \in A$. The category **L-FA** comprises **L**-fuzzy algebras of type **A** and their homomorphisms.

The category **L-FA** is based on our approach to lattice-valued sets [13]. From now on we assume that **C** is a subcategory of **LoA**.

Definition 3 (Fuzzy topological spaces) Given a set X and a $\mathbf{C}$-object A, a $(\mathbf{C}, \mathbf{LoL})$-*fuzzy topology* on X is a fuzzy algebra $\tau : A^X \to L$. A $(\mathbf{C}, \mathbf{LoL})$-*fuzzy topological space* is a triple (X, A, τ). A $(\mathbf{C}, \mathbf{LoL})$-*fuzzy continuous map* $(f, \varphi, \alpha) : (X, A, \tau) \to (X', A', \tau')$ is a $\mathbf{Set} \times \mathbf{C} \times \mathbf{LoL}$-morphism $(f, \varphi, \alpha) : (X, A, L) \to (X', A', L')$ such that $((f, \varphi)^{\leftarrow}, \alpha^{op}) : \tau' \to \tau$ is in **L-FA**, where $(f, \varphi)^{\leftarrow}(p) = \varphi^{op} \circ p \circ f$ and $(-)^{op}$ stands for the actual morphism. The category $(\mathbf{C}, \mathbf{LoL})$-**FTop** comprises $(\mathbf{C}, \mathbf{LoL})$-fuzzy topological spaces and $(\mathbf{C}, \mathbf{LoL})$-fuzzy continuous maps.

The category $(\mathbf{C}, \mathbf{LoL})$-**FTop** incorporates both Kubiak-Šostak [5] and Rodabaugh [8] approaches to fuzzy topology. From now on we assume that $\mathbf{D}$ is a subcategory of **Lo**(**L-FA**).

Definition 4 (Fuzzy topological systems) Given a set X, a $\mathbf{D}$-object $\nu : B \to M$ and a $\mathbf{C}$-object A, a map $\models : X \times B \to A$ is a $(\mathbf{C} \times \mathbf{D})$-*fuzzy satisfaction relation* on (X, A, ν) provided that $\models (x, -) : B \to A$ is a homomorphism for every $x \in X$. A $(\mathbf{C} \times \mathbf{D})$-*fuzzy topological system* is a tuple $(X, A, \nu, \models)$. A $(\mathbf{C} \times \mathbf{D})$-*fuzzy continuous map* $(f, \varphi, (\psi, \beta)) : (X, A, \nu, \models) \to (X', A', \nu', \models')$ is a $\mathbf{Set} \times \mathbf{C} \times \mathbf{D}$-morphism $(f, \varphi, (\psi, \beta)) : (X, A, \nu) \to (X', A', \nu')$ such that $\models (x, \psi^{op}(b')) = \varphi^{op}(\models' (f(x), b'))$ for every $x \in X$ and every $b' \in B'$. The category $(\mathbf{C} \times \mathbf{D})$-**FTopSys** comprises $(\mathbf{C} \times \mathbf{D})$-fuzzy topological systems and $(\mathbf{C} \times \mathbf{D})$-fuzzy continuous maps.

The category $(\mathbf{C} \times \mathbf{D})$-**FTopSys** incorporates all above-mentioned concepts of topological system [1, 12, 15].

Lemma 5 *There exists a full embedding*

$$E : (\mathbf{LoA}, \mathbf{LoL})\text{-}\mathbf{FTop} \to (\mathbf{LoA} \times \mathbf{Lo}(\mathbf{L\text{-}FA}))\text{-}\mathbf{FTopSys}$$

given by $E(X, A, \tau) = (X, A, \tau, \models)$ *with* $\models (x, p) = p(x)$, *and* $E(f, \varphi, \alpha) = (f, \varphi, (((f, \varphi)^{\leftarrow})^{op}, \alpha))$.

Lemma 6 *There exists a functor*

$$\mathrm{Spat} : (\mathbf{LoA} \times \mathbf{Lo}(\mathbf{L\text{-}FA}))\text{-}\mathbf{FTopSys} \to (\mathbf{LoA}, \mathbf{LoL})\text{-}\mathbf{FTop}$$

given by $\mathrm{Spat}(X, A, \nu, \models) = (X, A, \tau)$ *with* $\tau(p) = \bigvee\{\nu(b) \mid p = \models (-, b)\}$, *and* $\mathrm{Spat}(f, \varphi, (\psi, \alpha)) = (f, \varphi, \alpha)$.

Theorem 7 Spat *is a right-adjoint-left-inverse of* E.

Spat is not an embedding and therefore differs significantly from the functor proposed by J. T. Denniston *et al.* By our opinion the obtained results provide a good starting point for developing the theory of variety-based fuzzy topological spaces and systems.

References

[1] J. T. Denniston, A. Melton, and S. E. Rodabaugh, *Lattice-valued topological systems*, Abstracts of the 30th Linz Seminar on Fuzzy Set Theory (U. Bodenhoer, B. De Baets, E. P. Klement, and S. Saminger-Platz, eds.), Johan. Kepler Universität, Linz, 2009, pp. 24–31.

[2] J. T. Denniston and S. E. Rodabaugh, *Functorial relationships between lattice-valued topology and topological systems*, to appear in Quaest. Math.

[3] U. Höhle and A. P. Šostak, *Axiomatic Foundations of Fixed-Basis Fuzzy Topology*, Mathematics of Fuzzy Sets: Logic, Topology and Measure Theory (U. Höhle and S. E. Rodabaugh, eds.), Kluwer Acad. Publ., 1999, pp. 123–272.

[4] J. R. Isbell, *Atomless parts of spaces*, Math. Scand. **31** (1972), 5–32.

[5] T. Kubiak and A Šostak, *Foundations of the theory of* (L, M)*-fuzzy topological spaces*, Abstracts of the 30th Linz Seminar on Fuzzy Set Theory (U. Bodenhoer, B. De Baets, E. P. Klement, and S. Saminger-Platz, eds.), Johan. Kepler Universität, Linz, 2009, pp. 70–73.

[6] J. N. Mordeson and D. S. Malik, *Fuzzy Commutative Algebra*, Singapore: World Scientific, 1998.

[7] D. Papert and S. Papert, *Sur les treillis des ouverts et les paratopologies*, Semin. de Topologie et de Geometrie differentielle Ch. Ehresmann 1 (1957/58), No. 1, p. 1-9, 1959.

[8] S. E. Rodabaugh, *Categorical Foundations of Variable-Basis Fuzzy Topology*, Mathematics of Fuzzy Sets: Logic, Topology and Measure Theory (U. Höhle and S. E. Rodabaugh, eds.), Kluwer Acad. Publ., 1999, pp. 273–388.

[9] A. Rosenfeld, *Fuzzy groups*, J. Math. Anal. Appl. **35** (1971), 512–517.

[10] S. Solovjovs, *Embedding topology into algebra*, Abstracts of the 30th Linz Seminar on Fuzzy Set Theory (U. Bodenhoer, B. De Baets, E. P. Klement, and S. Saminger-Platz, eds.), Johan. Kepler Universität, Linz, 2009, pp. 106–110.

[11] S. Solovyov, *On ordered categories as a framework for fuzzification of algebraic and topological structures*, to appear in Fuzzy Sets Syst.

[12] S. Solovyov, *Variable-basis topological systems versus variable-basis topological spaces*, submitted to Soft Comput.

[13] S. Solovyov, *Categories of lattice-valued sets as categories of arrows*, Fuzzy Sets Syst. **157** (2006), no. 6, 843–854.

[14] S. Solovyov, *Categorical frameworks for variable-basis sobriety and spatiality*, Math. Stud. (Tartu) **4** (2008), 89–103.

[15] S. Vickers, *Topology via Logic*, Cambridge University Press, 1989.

Syntactic Algebras of Formal Series over a Field in General Algebras

M. Steinby
Turku Center of Theoretical Computer Science, Finnland
steinby@utu.fi

The syntactic algebras of subsets of any algebra are natural common generalizations of syntactic semigroups or monoids of string languages and syntactic algebras of tree languages. It is actually convenient to study the properties of all such structures in this general setting, and this way one can obtain a generalization of Eilenberg's variety theory that also encompasses a variety theory for tree languages. In this lecture we shall discuss a similar common generalization of Reutenauer's syntactic K-algebras of string series and the syntactic $K\Sigma$-algebras of tree series studied by Bozapalidis *et al.*.

If K is a field and $\mathcal{C} = (C, \Sigma)$ is a Σ-algebra, we call any mapping $S : C \to K$ a $K\mathcal{C}$-series; it may be written as the formal sum $\sum_{c \in C}(S, c).c$, where for each $c \in C$, the coefficient $(S, c)(= S(c) \in K)$ is the weight of c in S. A $K\mathcal{C}$-series with just finitely many non-zero coefficients is called a $K\mathcal{C}$-polynomial. The $K\mathcal{C}$-polynomials form a K-vector space that can be endowed with multilinear Σ-operations. Such Σ-algebras based on a K-vector space are called $K\Sigma$-algebras. The syntactic $K\Sigma$-algebra $\mathrm{SA}(S)$ of a $K\mathcal{C}$-series S is a quotient algebra of the $K\Sigma$-algebra of $K\mathcal{C}$-polynomials. It can be shown that $\mathrm{SA}(S)$ is finite-dimensional iff the series S is recognizable. We shall also characterize the subdirectly irreducible $K\Sigma$-algebras and show that all of them are syntactic. Moreover, we show how various operations on $K\mathcal{C}$-series relate to the syntactic $K\Sigma$-algebras. The lecture is mostly based joint work with Z. Fülöp.

Automorphism Groups of Diagonal Direct Limits of Hamming spaces

V. Sushchansky
Institut of Mathematics, Silesian University of Technology, Gliwice, Poland
Vitaliy.Sushchanskyy@polsl.pl

In the paper [1] P. Cameron and S. Tarzi introdused the conception of diagonal inductive limits of Hamming spaces. Such limits form a big class of discrete metric spaces with many interesting properties. In the talk we discuss some properties of metric spaces from this class and we describe automorphism groups of diagonal inductive limits of Hamming spaces in terms of wreath products of a cyclic order two group and suitable homogeneous symmetric groups (

for definition homogeneous symmetric groups see [2]).

References

1. P.Cameron, S.Tarzi, Limits of cubes. Topology and Applications, 155(2008),1454 - 1461.
2. N.Kroshko, V.Sushchansky, Direct Limits of Symmetric and Alternating Groups with Strictly Diagonal Embedding. Arch Math.(Basel), 71(1998), 173-182.

Positivity of Third Order Linear Recurrence Sequences

P. Tangsupphathawat
Department of Mathematics, Kasetsart University, Bangkok 10900, Thailand
`pinthira12@hotmail.com`
Coauthors: V. Laohakosol

It is shown that the positivity problem for a sequence satisfying a third order linear recurrence with integer coefficients, i.e., the problem whether each element of this sequence is nonnegative, is decidable.

On the Maximal Subsemigroups of Finite Transformation Semigroups

K. Todorov
Bulgarian Academy of Sciences, South-West University Blagoevgrad, Bulgaria
`kalchot@yahoo.com`

In the paper are described: **(a)** the maximal subsemigroups of the $\mathcal{D}_k$-classes: $\mathcal{D}_k = \mathcal{J}_k = \{\alpha\epsilon\mathcal{T}_n,\ |im\alpha| = k,\ 2 \le k \le n-1\}$ and **(b)** the maximal subsemigroups of the Ideals $\mathcal{I}_k = \bigcup_{i=1}^{k} \mathcal{D}_\rangle$ of the finite singular transformation semigroup $\mathcal{T}_n$.

Clique Coverings of Glued Graphs at Complete Clone

Ch. Uiyyasathian
Chulalongkorn University, Bangkok, Thailand
`Chariya.u@Chula.ac.th`
Coauthors: W. Pimpasalee and W. Hemakul

A clique covering of a graph G is a set of cliques of G in which each edge of G is contained in at least one clique. The smallest cardinality of clique coverings of G is called the clique covering number of G. A glued graph results from combining two nontrivial vertex-disjoint graphs by identifying nontrivial connected isomorphic subgraphs of both graphs. Such subgraphs are referred to as the clones. The two nontrivial vertex-disjoint graphs are referred to the original graphs.

In this paper, we investigate bounds of clique covering numbers of glued graphs at clone which is isomorphic to K_n in terms of clique covering numbers of their original graphs, and give a characterization of a glued graph with the clique covering number of each possible value.

Keywords : clique coverings, glued graphs
2000 Mathematics Subject Classification : 05C69, 05C70, 05C99

Clausal Constraint Relations and C-clones.

E. Vargas
TU-Dresden
Edith_Mireya.Vargas_Garcia@mailbox.tu-dresden.de

A clausal constraint is a disjunction of inequalities of the form $x \geq d$ and $x \leq d$ were $x, d \in D = \{0, \ldots, n-1\}$. It was introduced by Nadia Creignou, Miki Hermann and collaborators in order to classify the complexity of constraints. We introduce *clausal constraint relations* and define C-clones via a Galois-connection (Pol-CInv) between the set of all finitary operations on D and the set of clausal constraint relations. We shall describe the lattice of all C-clones for the boolean case $D = \{0, 1\}$.

Congruences and Ideals in Lattice Effect Algebras as Basic Algebras

E Vinceková
Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovakia
vincekova@mat.savba.sk
Coauthors: S. Pulmannová

We study *effect basic algebras* which correspond to lattice ordered effect algebras. We reformulate some important results from the theory of effect algebras in the language of basic algebras, that are (in contrast to effect algebras) total algebraic structures. In particular, we characterize ideals, congruences and homomorphisms and their one-to-one correspondence by the tools and axioms of basic algebras.

Interpreting Graphs in 0-simple Semigroups with Involution with Applications to Computational Complexity and the Finite Basis Problem

M. Volkov
Ural State University, Ekaterinburg, Russia
`Mikhail.Volkov@usu.ru`
Coauthors: M. Jackson (La Trobe University, Bundoora, Australia)

We consider the varieties of unary semigroups generated by certain 'adjacency semigroups', which are combinatorial Rees matrix semigroups with unary operation $(i, j) \mapsto (j, i)$. The identities of these structures precisely capture a natural notion of equivalence modulo adjacency patterns in unary semigroup words. We establish a surprisingly close relationship between universal Horn classes of graphs and varieties generated by adjacency semigroups. For example, the lattice of subvarieties of the variety generated by adjacency semigroups that are regular unary semigroups is essentially the same as the lattice of universal Horn classes of reflexive graphs. A number of examples follow, including new examples of limit (minimal non-finitely based) varieties of unary semigroups and first examples of finite unary semigroups with NP-hard pseudovariety membership problems.

Completely Regular Endomorphisms of Split Graphs

A. Wanichsombat
Carl von Ossietzky Universitaet Oldenburg
`apirat589@yahoo.com`
Coauthors: U. Knauer

In [1], Weimin Li and Jianfei Chen studied split graphs such that the monoid of all endomorphisms is regular. Here, we extend the study of [2]. We find conditions such that regular endomorphism monoids of split graphs are completely regular. Moreover, we find completely regular subsemigroups contained in the monoid $End(G)$.

References

[1] W. Li and J. Chen, *Endomorphism - Regularity of Split Graphs*, Europ. J. Combinatoriecs, **22** (2001), 207-216.
[2] A. Wanichsombat, *Endo-Completely-regular Split Graphs*, in: V. Laan, S. Bulman-Fleming, R. Kaschek (Eds.), Semigroups, Acts and Categories with Applications to Graphs, Proceedings, Tartu 2007, Tartu 2008, 136-142.

Representation of Graph Monoids by Regular Rings

F. Wehrung
Université de Caen
wehrung@math.unicaen.fr
Coauthors: P. Ara, F. Perera

One of the most prominent open problems in the theory of (von Neumann) regular rings is the characterization of their nonstable K-theory. A significant advance was made in 2006 by Ara and Brustenga, who proved that the graph monoid of a row-finite quiver is always the nonstable K-theory of a regular ring. Hence the characterization of those commutative monoids that appear as graph monoids of a row-finite quiver was given a new importance. The aim of this talk is to give a quick overview of this topic, and the characterization, obtained with Ara and Perera, of all finitely generated antisymmetric graph monoids. The talk is intended to be self-contained. Representation of Graph Monoids by Regular Rings

Human Being and Mathematics – Logical and Mathematical Thinking

R. Wille
Darmstadt University of Technology
wille@mathematik.tu-darmstadt.de

Logical thinking as as an expression of human reason grasps the actual reality by the basic forms of thinking: concept, judgment, and conclusion. *Mathematical thinking* abstracts from logical thinking to disclose a cosmos of forms of potential realities hypothetically. *Mathematics* as a form of mathematical thinking can therefore support *humans* within their logical thinking about realities which, in particular, promotes sensible actions. This train of thought has been convincingly differentiated by *Peirce's philisophical pragmatism* and recently concretized by a "*contextual logic*" invented by members of the mathematics department at the TU Darmstadt.

Contents

1. Logical Thinking
2. Mathematical Thinking
3. Human Being, Mathematics, and Reality

Idempotent-closed Endomorphism Monoids of Strong Semilattices of Semigroups.

S. Worawiset
Carl-von-Ossietzky University of Oldenburg, Germany.
nuek_w@yahoo.com
Coauthors: Ulrich Knauer

In this paper, we study the endomorphism monoids of strong semilattices of semigroups which are idempotent-closed, i.e., the idempotents from a semigroup and we obtain orthodox endomorphism monoids of strong semilattices of semigroups.

Maps which are Concordant with Binary Relations

V. Yaroshevich
Moscow Institute of Electronic Technology, Moscow, Russia
v-yaroshevich@ya.ru
Coauthors: I. Kozhuhov (Moscow Institute of Electronic Technology)

M. Bötcher and U. Knauer defined some partial kinds homomorphisms of graphs. Namely, the semi-strong, locally strong, quasi-strong and strong homomorphisms. We obtained a convenient matrix form for these partial homomorphisms. For an arbitrary set X, we shall speak that a partial map α is concordant with a binary relation σ on X if $\sigma\alpha$ is a subset of $\alpha\sigma$. The set of such α forms a semigroup. We got an exhaustive description of all sets with regular semigroups of par-tial transformations which are concordant with a quasi-order on X.