Theory Seminar
July 26 2011
APCTP Pohang, Korea

Phase Transitions and Pairing Mechanisms in Open Many-Body Systems with Cold Atoms

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P. Zoller (Innsbruck)

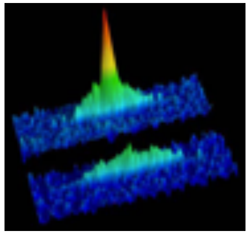
UNIVERSITY OF INNSBRUCK

IQOQI
AUSTRIAN ACADEMY OF SCIENCES

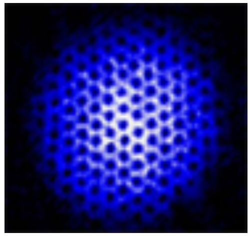
NAMEQUAM
Nanodesigning of atomic and molecular quantum matter

FWF Der Wissenschaftsfonds.

Motivation

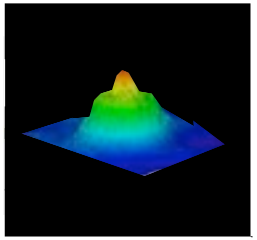


Bose-Einstein Condensate
(1995)



Vortices
(1999)

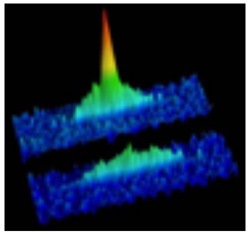
Many-body physics
with cold atoms



Mott Insulator
(2002)

Fermion superfluid
(2003)

Motivation



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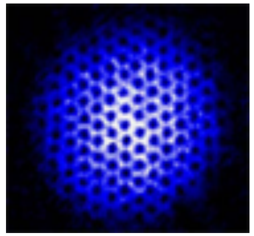

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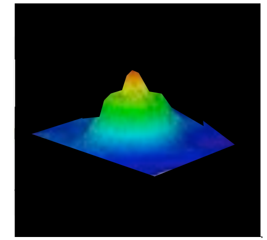

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Common theme:

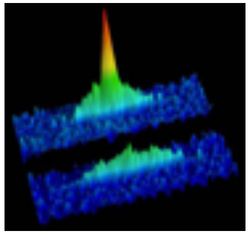
many-body
system

Temperature T,
particle number N

- closed system (isolated from environment)
- thermodynamic equilibrium

➡ Condensed matter analog systems

Motivation



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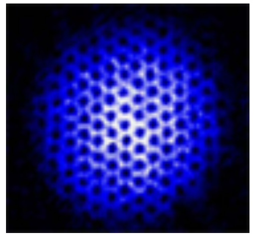

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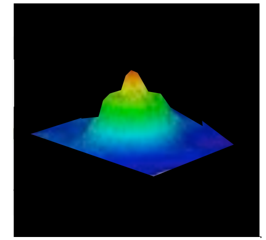

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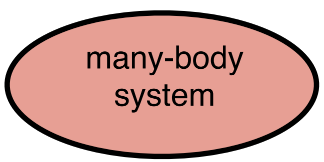

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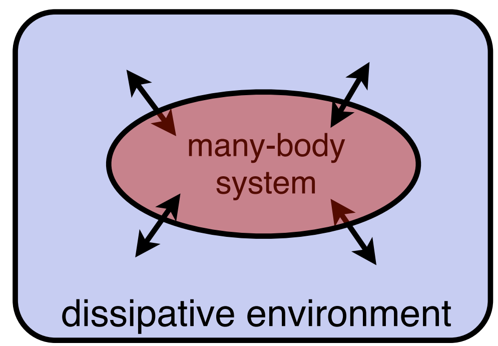
Novel Situation: Cold atoms as open many-body systems



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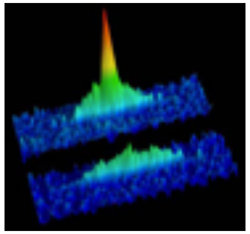

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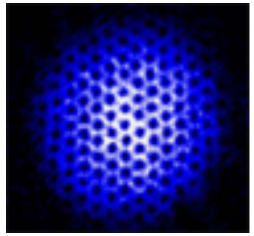

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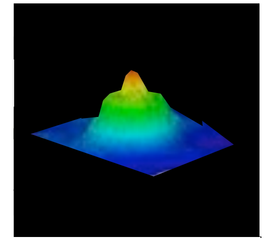

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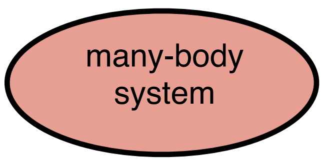

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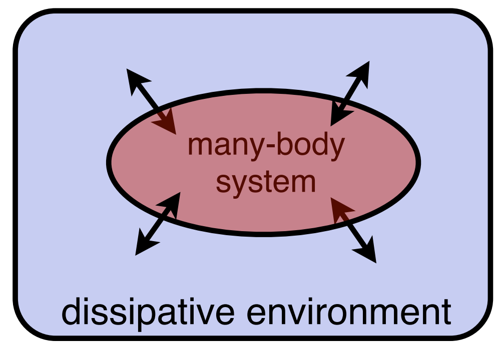
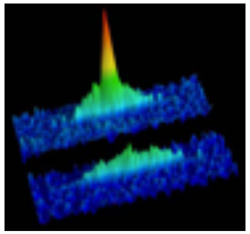


- natural occurrences of dissipation

➡ no condensed matter analog

Motivation



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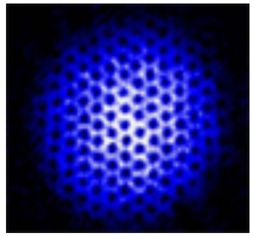

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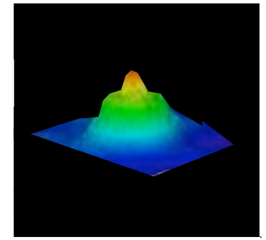

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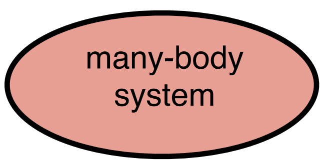

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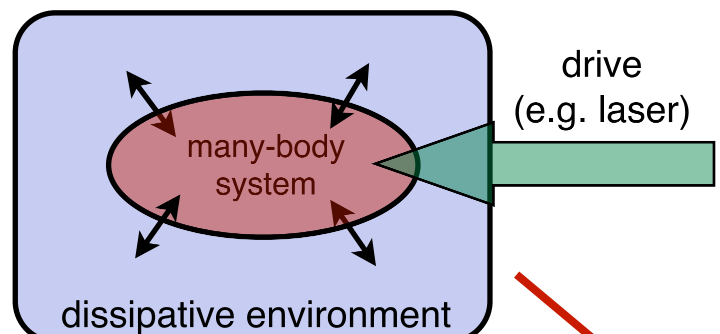


- natural occurrences of dissipation

➡ no condensed matter analog

- engineered driven/dissipative dynamics

➡ drive/dissipation as dominant resource of many-body dynamics!

➡ Think quantum optics in many-body systems!

Outline

Driven Dissipative Many-Body Dynamics

- Driven dissipative BEC

Proof of principle

Cold atomic bosons

Cold atomic fermions

Competition of Unitary and Dissipative Dynamics

- Nonequilibrium phase transitions

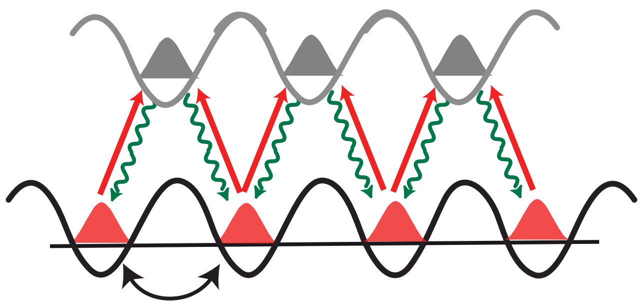
New class of interacting nonequilibrium systems

Targeting interesting many-body states

- engineered dissipative dynamics for fermions

Dissipative pairing mechanism for fermions

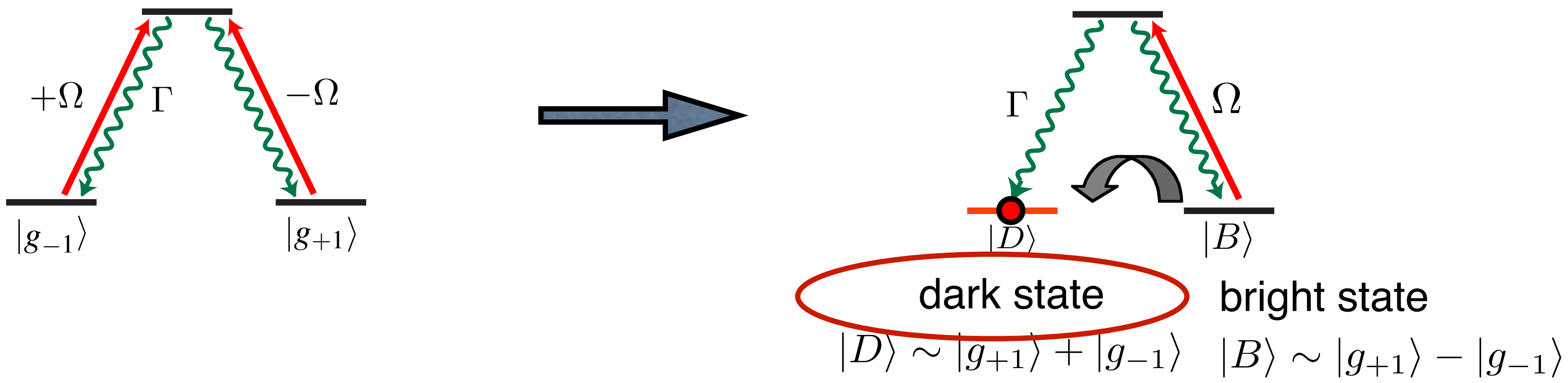
Driven Dissipative BEC



SD, A. Micheli, A. Kantian, B. Kraus, H.P. Büchler, P. Zoller, Nature Physics **4**, 878 (2008);
B. Kraus, SD, A. Micheli, A. Kantian, H.P. Büchler, P. Zoller, Phys. Rev. A **78**, 042307 (2008);

An Analogy

- Λ-system: three internal (electronic) levels (Aspect, Cohen-Tannoudji; Kasevich, Chu)



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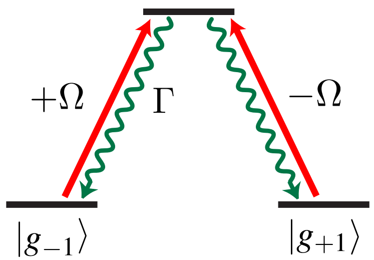
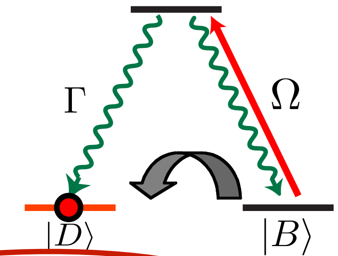




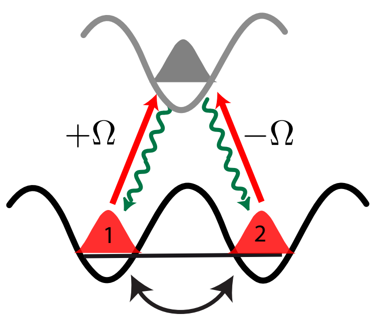
dark state

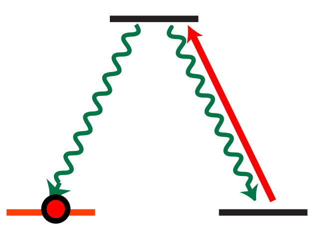
$|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle$

bright state

$|B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

- 1 atom on 2 sites: external (spatial) degrees of freedom





$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle$

symmetric

$(a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$

anti-symmetric

An Analogy

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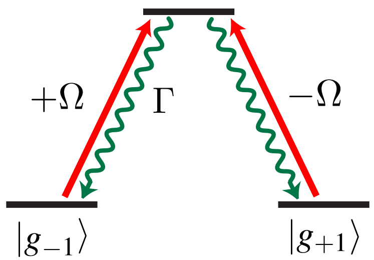

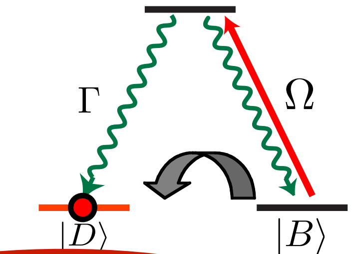

dark state — bright state

$|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle$ — $|B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

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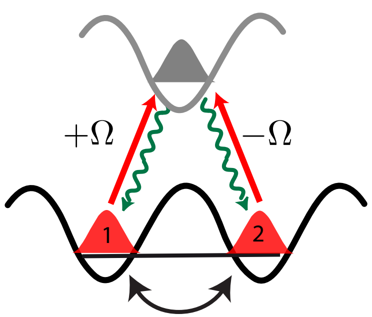
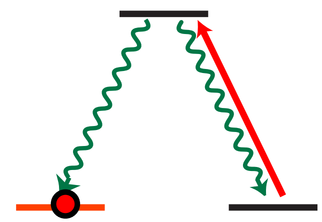
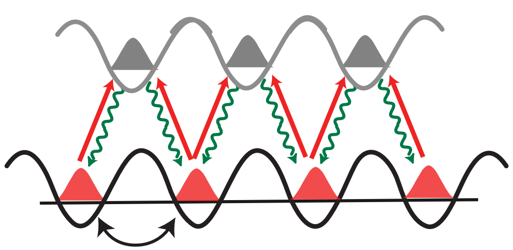




$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle$ — $(a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$

symmetric — anti-symmetric

- N atoms on M sites



$$|BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$$

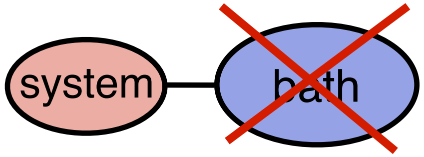
➡ “phase locking” for many external (spatial) degrees of freedom: BEC

Driven Dissipative lattice BEC

- More precisely: Master Equation evolution of density operator

Lindblad operators

$$\partial_t \rho = \underbrace{-i[H,\rho]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_i J_i \rho J_i^\dagger - \tfrac{1}{2}\{J_i^\dagger J_i, \rho\}}_{\text{dissipative evolution}}$$



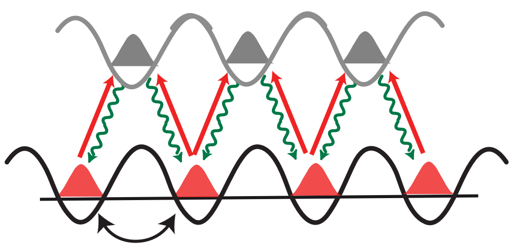
$\mathcal{L}[\rho]$ -- Liouvillian operator

- Goal: choose the Lindblad / jump operators such that

$$\rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \to \infty \qquad |BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$$

- Job is done by

$$\boxed{J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1})}$$



- Interpretation:
  - any antisymmetric component of a particle's superpositon on i, i+1 mapped onto the symmetric one
  - i.e. on each pair of sites, only the symmetric superposition persists:

$$|BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$$

➡ Long range phase coherence builds up from quasilocal dissipative operations

Uniqueness of the Steady State

(1) BEC state is the only dark state:

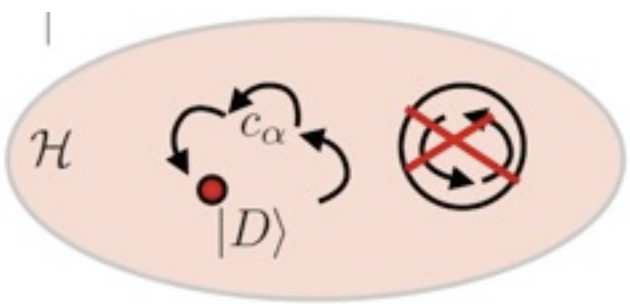
- $(a_i^\dagger + a_j^\dagger)$ has no eigenvalues (on fixed number (N-1) Hilbert space)
- $(a_i - a_j)$ has unique zero eigenvalue

$$(a_i - a_j) \ \forall i \longrightarrow \sum_\lambda (1 - e^{\mathbf{i}\mathbf{q}\mathbf{e}_\lambda}) a_\mathbf{q} \ \forall \mathbf{q}$$

➡ BEC is the unique dissipative zero mode of the jump operators

(2) |BEC> is the only stationary state (sufficient condition)

pictorially:



more precisely:

If there exists no subspace of the full Hilbert space which is left invariant under the set $\{c_\alpha\}$, then the only stationary state are the dark states

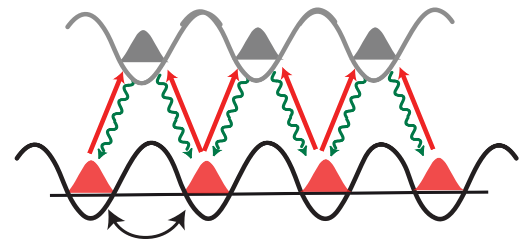
➡ Uniqueness: Final state independent of initial density matrix

➡ Therefore: pure (zero entropy) final state

Driven Dissipative lattice BEC: Physical Realization

$$\boxed{J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1})}$$

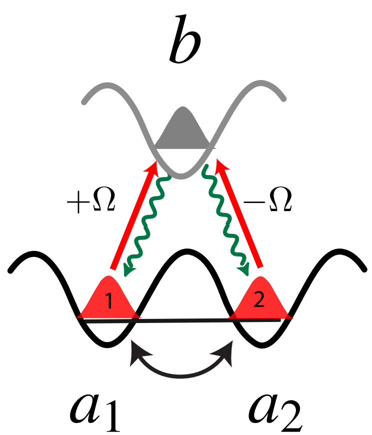
$$J_i|BEC\rangle = 0 \ \forall i$$



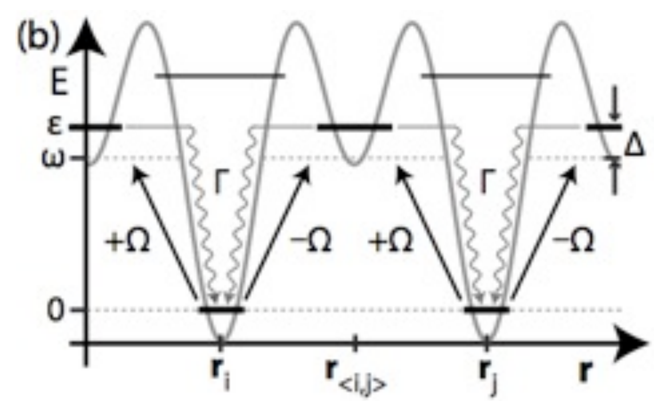
- Implementation idea for cold atoms: Immersion of driven system into superfluid reservoir

auxiliary system

system of interest



- Λ-type level structure: optical superlattice



Driven Dissipative lattice BEC: Physical Realization

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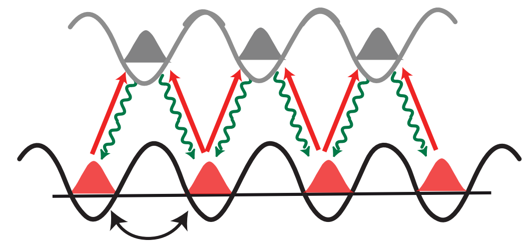

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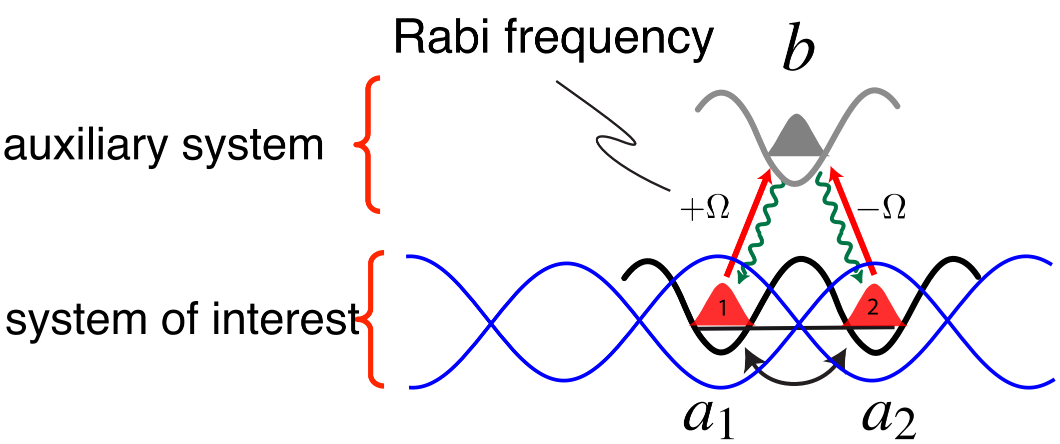

- drive: coherent coupling to auxiliary system with double wavelength Raman laser

$$\lambda_{\text{laser}} = 2\lambda_{\text{lattice}}$$

➡ The coherence of the driving laser is mapped on the matter system

Driven Dissipative lattice BEC: Physical Realization

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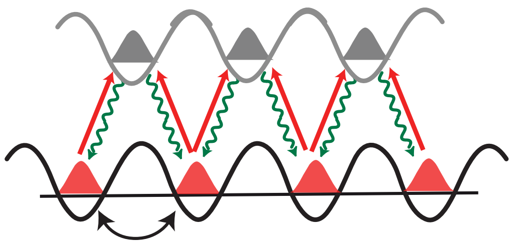

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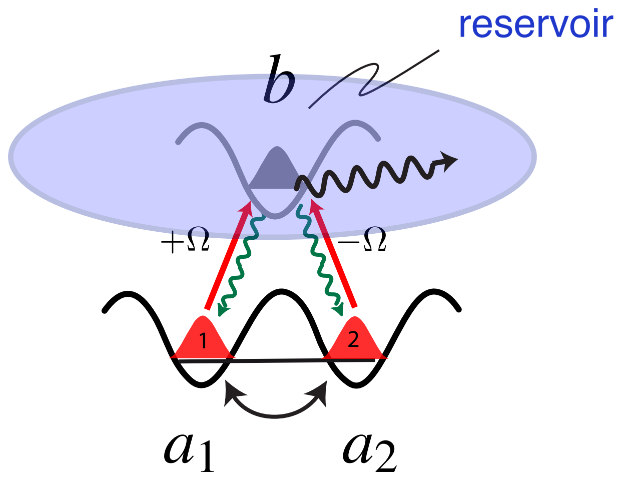
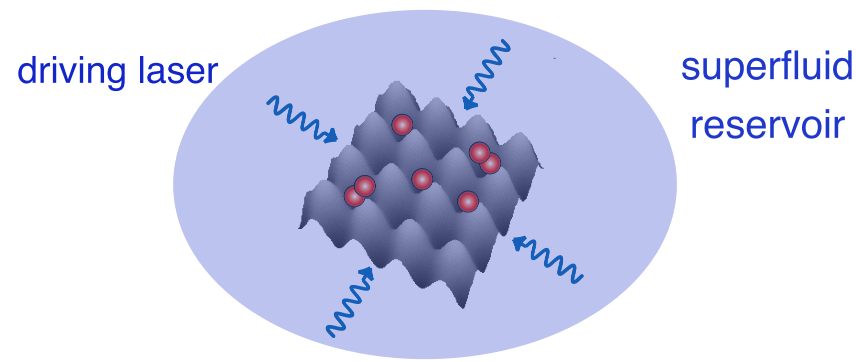


- dissipation: phonon emission into superfluid reservoir



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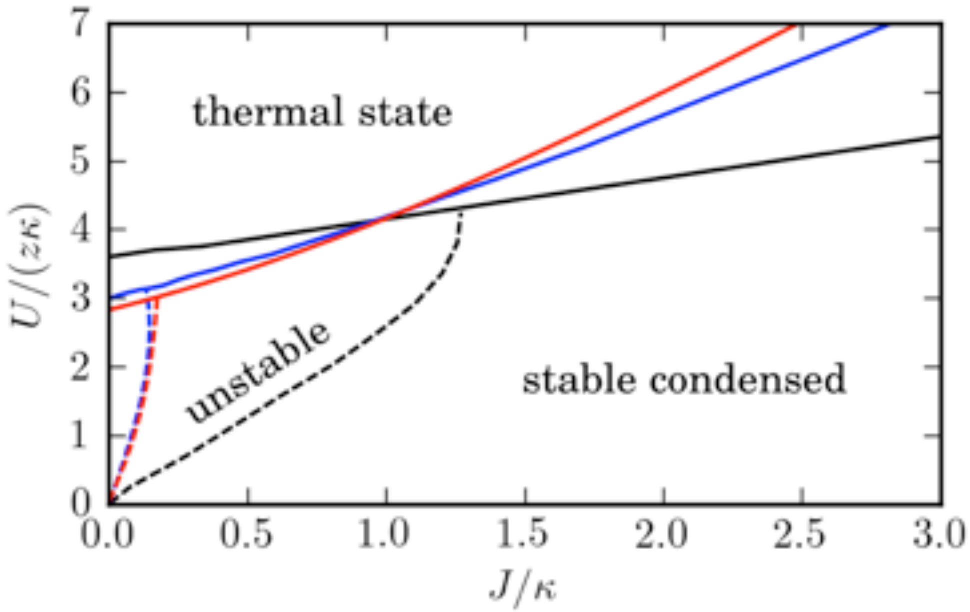
Competition of Unitary vs. Dissipative Dynamics



Many-Body Physics with Driven-Dissipative Systems

SD, A. Tomadin, A. Micheli, R. Fazio, P. Zoller, arxiv:1003.2071, Phys. Rev. Lett. **105**, 015702 (2010);
A. Tomadin, SD, P. Zoller, Phys. Rev. A **108**, 013611 (2011).

Physical Picture: Nonequilibrium Phase Transition

- Nonequilibrium master equation evolution:

drives into BEC with rate κ

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

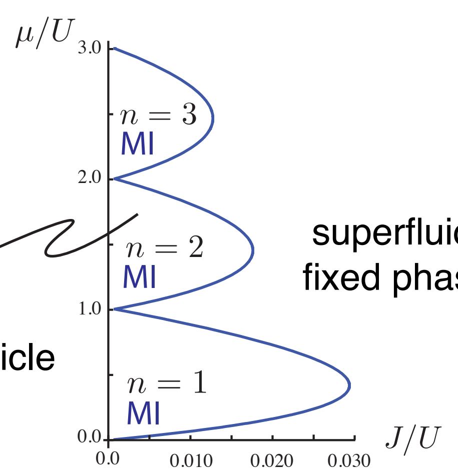
$$H = -J \sum_{<i,j>} a_i^\dagger a_j + U \sum_i a_i^{\dagger 2} a_i^2$$

Competition

- Compare to superfluid / Mott insulator quantum phase transition

kinetically dominated: superfluid

interaction dominated: Mott insulator



superfluid, fixed phase

Mott lobes, quantized particle number

Interference pattern

superfluid: Matter wave

Mott

back to superfluid

Physical Picture: Nonequilibrium Phase Transition

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drives into BEC with rate κ

$$\frac{d\rho}{dt} = -i\,[H,\rho] + \mathcal{L}\rho$$

$$H = -J \sum_{<i,j>} a_i^\dagger a_j + U \sum_i a_i^{\dagger 2} a_i^2$$

Competition

- Compare to superfluid / Mott insulator quantum phase transition

- Analogy:
  - enhancement of superfluidity: kinetic energy J driven dissipation κ
  - suppression of superfluidity: interaction U interaction U

➡ Expect phase transition as function of J/U κ/U

➡ Question: What are the true analogies and differences to equilibrium (quantum) phase transitions?

Mixed State Gutzwiller Approach

- Argumentation must be based on equation of motion

- Strategy: approximation scheme interpolating between limiting cases

$\kappa \gg U$
dissipative condensate

$\kappa \ll U$
see below!

- Implementation: Gutzwiller product ansatz for the density operator

$$\rho(t) = \prod_i \rho_i(t)$$

  - onsite (quantum) fluctuations treated exactly
  - (connected) spatial correlations neglected
  - allows to describe mixed states (unlike zero temperature Gutzwiller)

➡ Nonlinear Mean Field Master Equation for reduced density operator

- We will additionally account for a finite hopping J

From Weak to Strong Coupling

- Weak interactions: dissipative Gross-Pitaevskii equation (coherent states)

$$\partial_t \psi_\ell = -i(-J \sum_{\langle \ell' | \ell \rangle} \psi_{\ell'} + U|\psi_\ell|^2 \psi_\ell) - 2\kappa \sum_{\langle \ell' | \ell \rangle} (\psi_\ell - \psi_{\ell'} + \psi^*_{\ell'} \psi^2_\ell - |\psi_{\ell'}|^2 \psi_{\ell'})$$

- Strong interaction destroys the phase coherence:

dephasing & average out

transformation to rotating frame $V \equiv e^{iU\hat{n}(\hat{n}-1)t}$

annihilation operator in rotating frame $V\hat{b}V^{-1} = e^{-iU\hat{n}t}\hat{b} = \sum_n e^{inUt} |n\rangle\langle n|\hat{b}$

$\sim \psi$

at dark state

➡ suppression of off-diagonal order

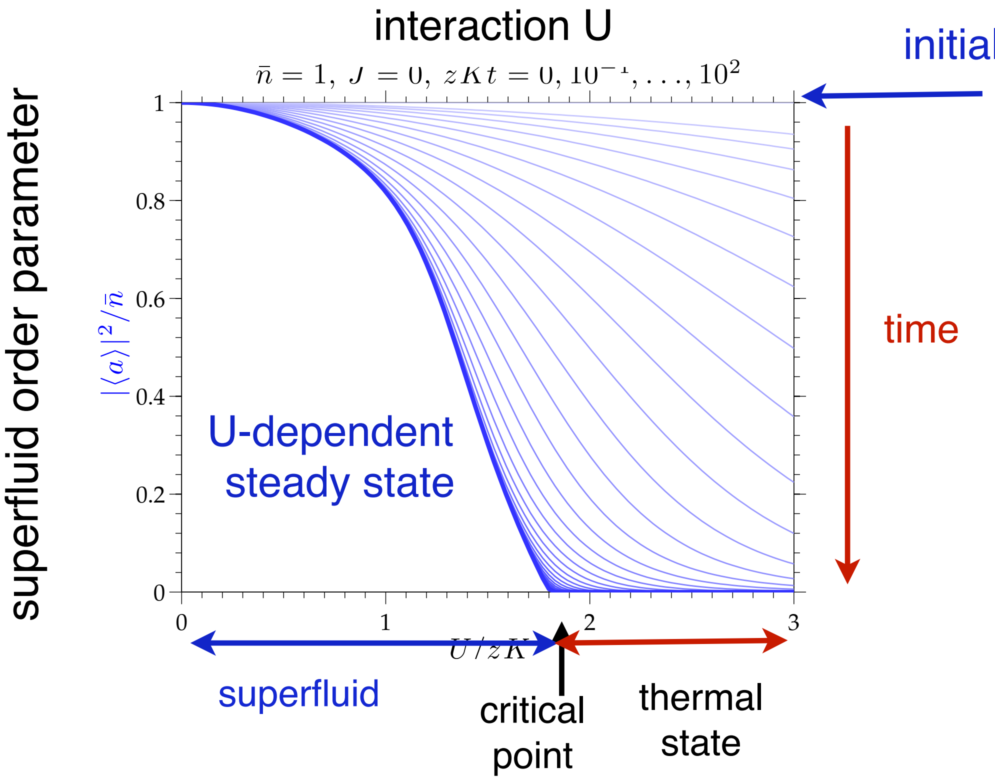
- Master equation reduces to

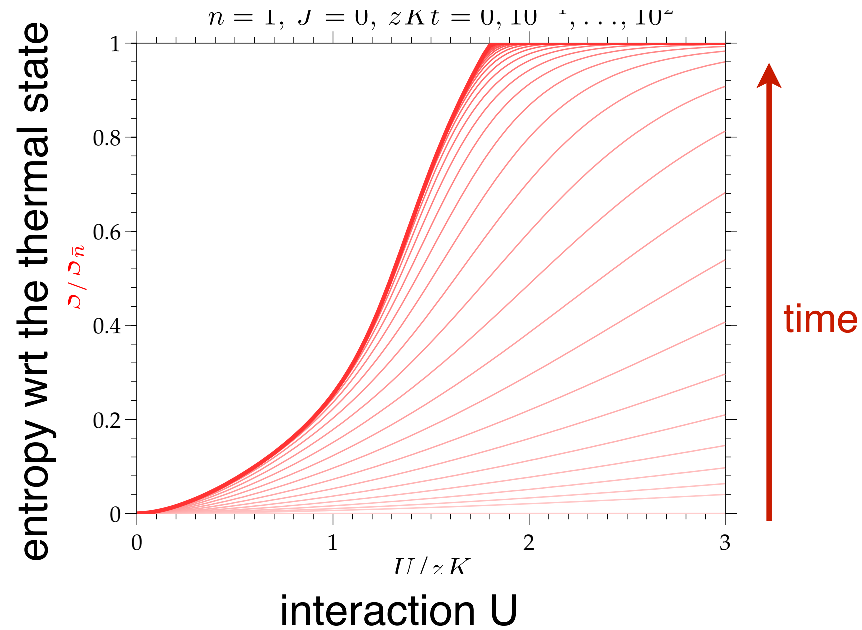
$$\partial_t \rho_\ell = \kappa[(\bar{n}+1)(2b_\ell \rho b^\dagger_\ell - \{b^\dagger_\ell b_\ell, \rho_\ell\}) + \bar{n}(2b^\dagger_\ell \rho_\ell b_\ell - \{b_\ell b^\dagger_\ell, \rho_\ell\})]$$

- Thermal equation with thermal (mixed) state solution

➡ the system acts as its own reservoir

Dependence of the Steady State on the Interaction





Nonequilibrium phase transition between pure and mixed state, driven by a competition between unitary and dissipative dynamics

- Development in time of the non-analyticity at the critical point
- Shares features of:
  - Quantum phase transition: interaction driven
  - Classical phase transition: ordered phase terminates in a thermal state
- No signature of commensurability effects (Mott) due to strong mixing of U

Analytical Approach in the Limit of Low Density

- Many-body problem: relevant information in the low order correlation functions
- Study the equations of motion of the correlation functions

$$\{\langle (b_\ell^\dagger)^n b_\ell^m \rangle\}$$

in principle: infinite and nonlocal hierarchy

- Introduce a power counting: $\boxed{b_\ell \sim \sqrt{n}, b_\ell^\dagger \sim \sqrt{n}}$
  and keep only the leading order for $n \to 0$

➡ Infinite hierarchy exhibits a closed nonlinear subset for low order correlation functions

- Can be solved exactly in special cases. E.g. hom. steady state condensate fraction

$$\frac{|\psi|^2}{n} = 1 - \frac{2u^2\left(1 + (j+u)^2\right)}{1 + u^2 + j(8u + 6j\left(1 + 2u^2\right) + 24j^2u + 8j^3)} \qquad [j = J/(4\kappa), \quad u = U/(4\kappa z)]$$

$$\frac{|\psi|^2}{n} = 1 - \left(\frac{U}{U_c}\right)^2 \qquad \begin{matrix} J = 0 \\ U_c = 4\sqrt{2}\kappa \end{matrix}$$

cf. BEC: $$\frac{|\psi|^2}{n} = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$

Critical Exponent of the Phase Transition

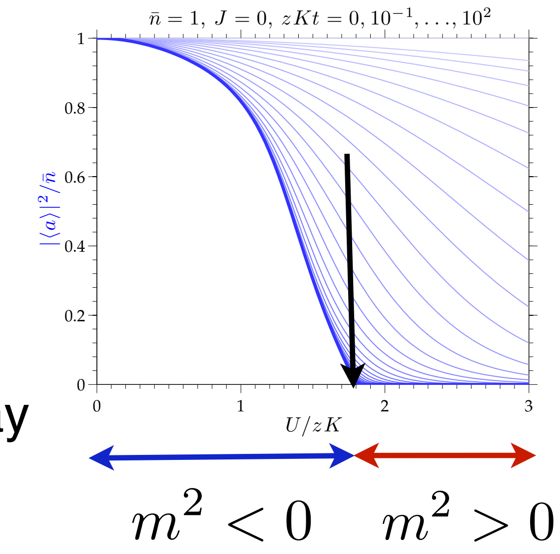
- Critical exponents can be extracted from approaching the phase transition in time

- Expect form of the order parameter evolution

$$|\psi(t)| \sim \frac{e^{-m^2 t}}{t^{\alpha}}$$

real part of lowest eigenvalue: “mass”

- At criticality: zero eigenvalue and thus dominant polynomial decay



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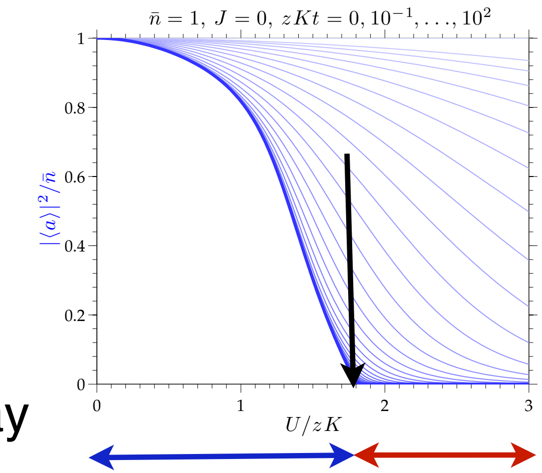
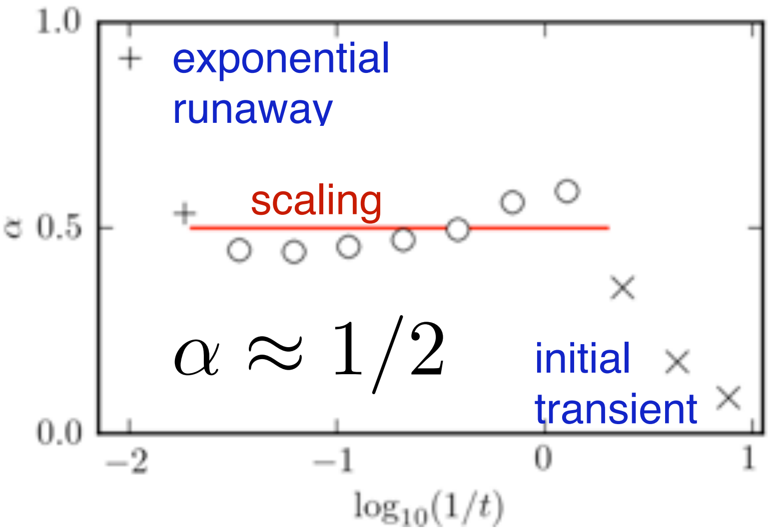


$m^2 < 0$ $m^2 > 0$

- At criticality: zero eigenvalue and thus dominant polynomial decay

- Numerical Result (high density):



- Analytical Result $(n \to 0)$:

at criticality, Landau-Ginzburg type cubic but dissipative nonlinearity

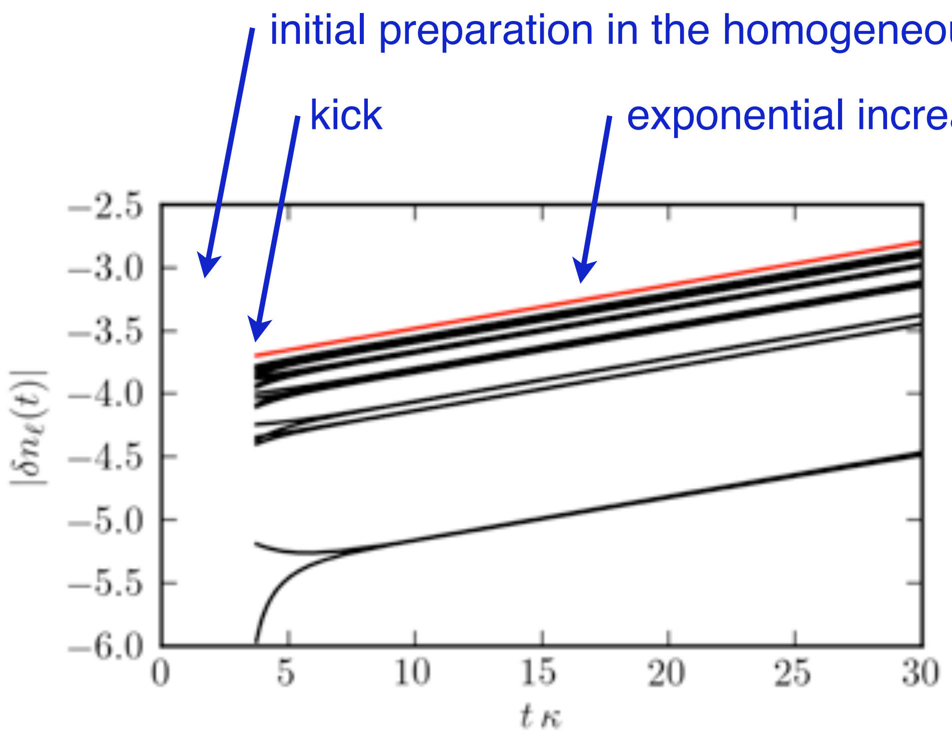
$$\boxed{|\psi(t)| \sim t^{-1/2}, \quad \alpha = 1/2}$$

Mean field value as expected.
But governs the time evolution.

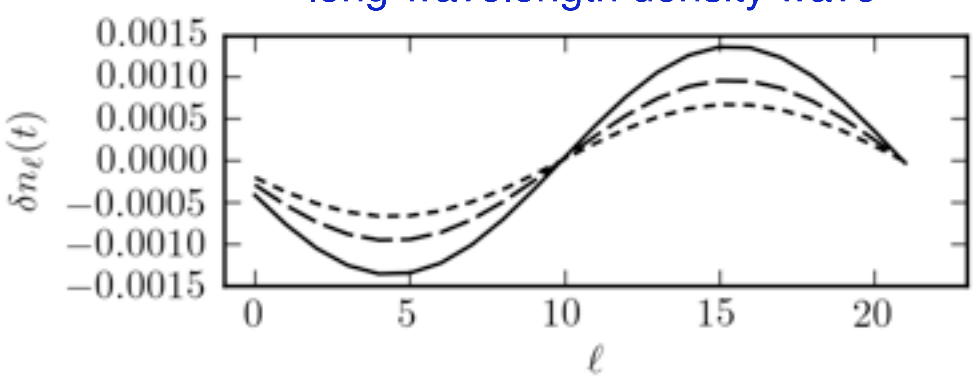
➡ Critical behavior could be studied experimentally from following the time evolution of condensate fraction

Dynamical Instability

- Numerical experiment to probe the stability: subject the inhomogeneous system to a "kick" (instantaneous perturbation of the density matrix)



$$|n_\ell(t) - n(0)| \sim e^{\gamma t} \sin q\ell$$

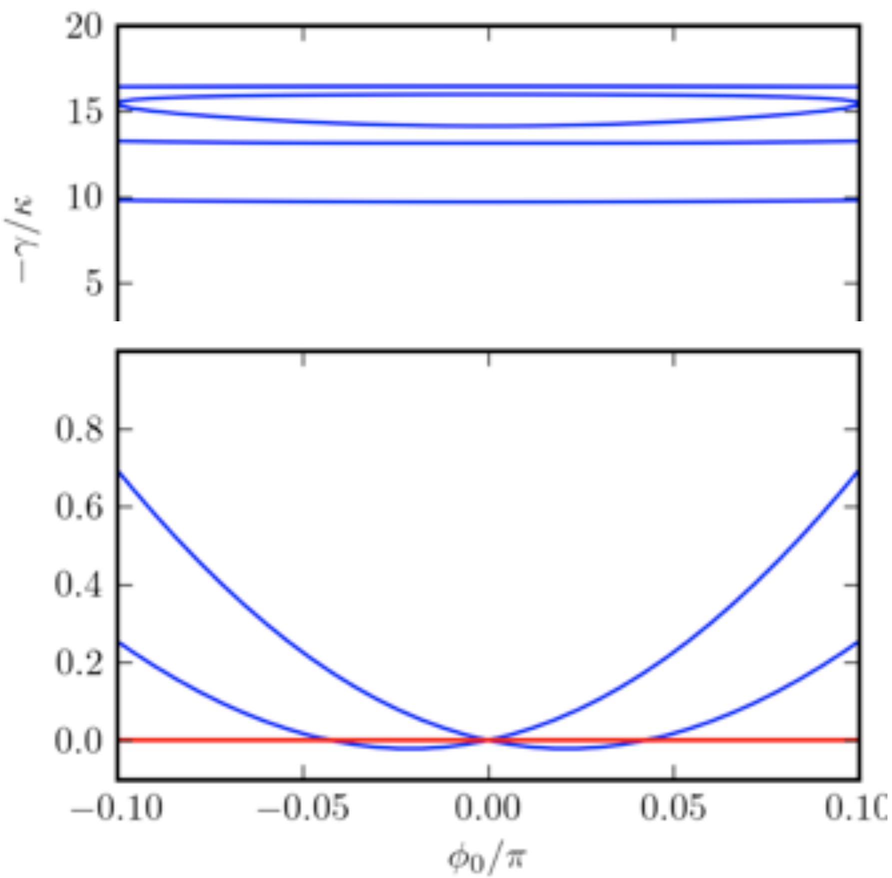


- This is a computation on 22 sites, linearization makes larger systems accessible

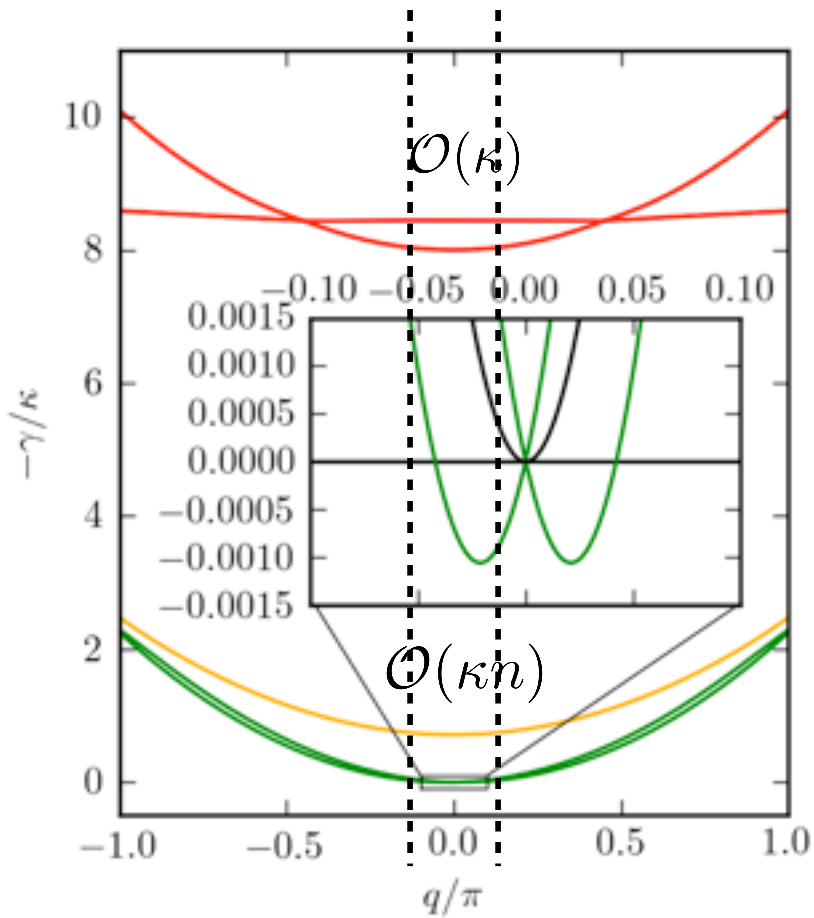
- Very slow effect: linearization of the master equation around the initial state, computation of the rate of the instability.

Linear Response around Homogeneous State

- Imaginary part of the Liouvillian as function of quasimomentum, $J \ll \kappa$



100 sites, high densities, full mean field system



Infinite system, low densities, 7x7 linear system of EoMs

- ➡ Existence of dissipatively unstable modes is a universal feature of the regime $J \ll \kappa$
- ➡ low density limit: the unstable modes belong to single particle sector

Reduction to the Low-Lying Modes

- Adiabatic elimination of the fast-decaying modes (two times)

$$\left(\begin{array}{c} \partial_t \Psi_1 \\ 0 \equiv \partial_t \Psi_2 \end{array}\right) = \left(\begin{array}{cc} M_{11} & M_{12} \\ M_{21} & M_{22} \end{array}\right) \left(\begin{array}{c} \Psi_1 \\ \Psi_2 \end{array}\right) \Bigg\}$$

collection of low density correlation functions

solve for the fast modes Ψ_2 and obtain slow modes equation only

- Low momentum equation of motion for of the condensate fluctuations only

$$\partial_t \left(\begin{array}{c} \Delta\psi_q \\ \Delta\psi^*_{-q} \end{array}\right) = \left(\begin{array}{cc} Un + \epsilon_{\mathbf{q}} - i\kappa_{\mathbf{q}} & Un + 9un\kappa_{\mathbf{q}} \\ -Un - 9un\kappa_{\mathbf{q}} & -Un - \epsilon_{\mathbf{q}} - i\kappa_{\mathbf{q}} \end{array}\right) \left(\begin{array}{c} \Delta\psi_q \\ \Delta\psi^*_{-q} \end{array}\right)$$

$$\underbrace{\epsilon_q \equiv Jq^2}_{\text{bare hopping at low momentum}}, \quad \underbrace{\kappa_q = 2(2n+1)\kappa q^2}_{\text{bare dissipative rate}}$$

➡ renormalization of the off-diagonal terms

➡ absent in the dissipative GPE

Origin of the Instability

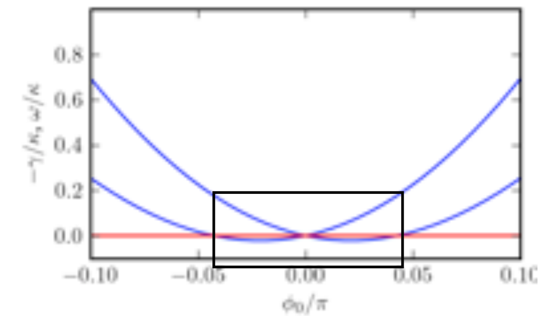
- Complex spectrum of the low-lying single particle excitations:

renormalization correction

$$\gamma_{\mathbf{q}} = \kappa_{\mathbf{q}} + ic|\mathbf{q}|, \quad c = \sqrt{2Un(J - 9Un/(2z))}$$

- Interpretation: Below a critical value

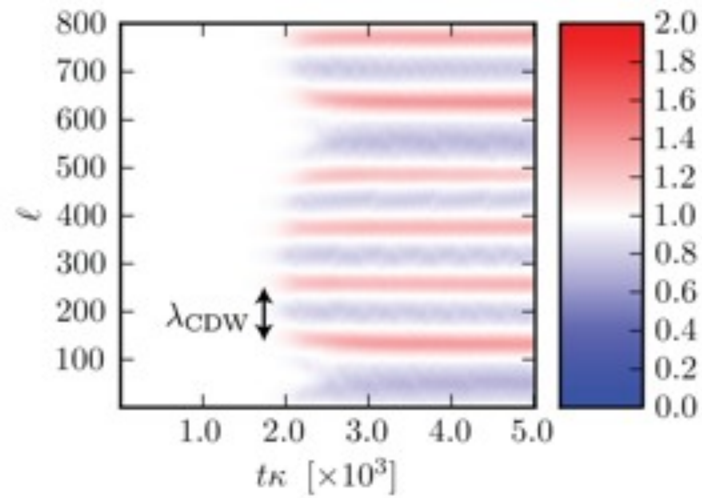
$$\boxed{J = 9Un/(2z)}$$



the speed of sound becomes imaginary.

This term always dominates at sufficiently small momenta. Its sign is opposite to $\kappa_{\mathbf{q}}$

- The fate of the system beyond linear response:

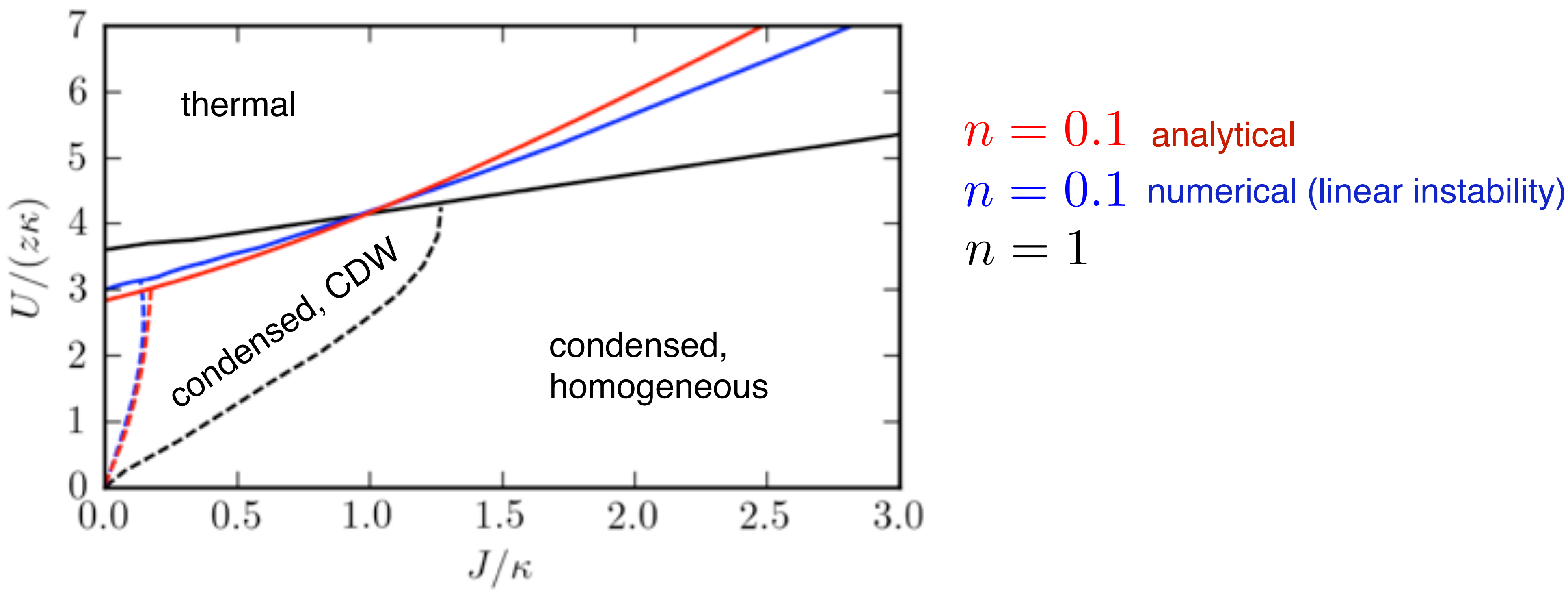


density profile signature: spontaneous breaking of translation symmetry

maximum instability momentum transmuted into CDW wavelength

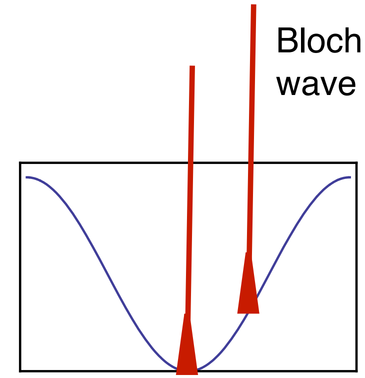
➡ The dynamical instability is fluctuation induced, a weak coupling phenomenon, and an intrinsic many-body effect

The Steady State Phase Diagram



- Strong coupling second order phase transition to a thermal-like disordered state
- Homogeneous dissipative condensate is unstable against CDW order for infinitesimal interaction
- Condensed phase and homogeneous condensate can be stabilized by finite coherent hopping

Nonlinear Dynamics in Finite Systems



- Study the response of the nonlinear dynamical system to sudden parameter changes, here: phase quench in 1D periodic chain:

$$J_i = (a_i^\dagger + a_{i+1}^\dagger)(a_i - a_{i+1}) \to (a_i^\dagger + e^{-\mathrm{i}\phi} a_{i+1}^\dagger)(a_i - e^{\mathrm{i}\phi} a_{i+1})$$

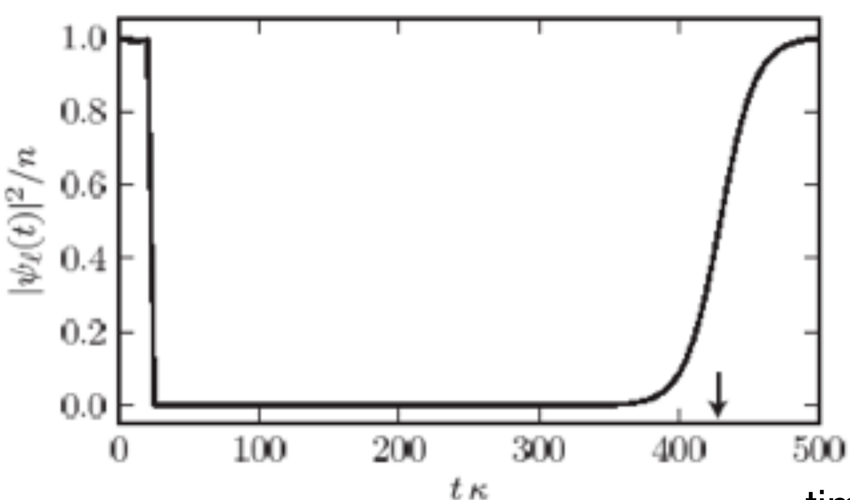
- New steady state: Bloch wave

$$|BEC\rangle(q) \sim (\sum_i e^{\mathrm{i}qx_i} a_i^\dagger |vac\rangle \qquad q = \phi a \qquad \langle x_i|BEC\rangle(q) \sim e^{\mathrm{i}qx_i}$$

lattice spacing

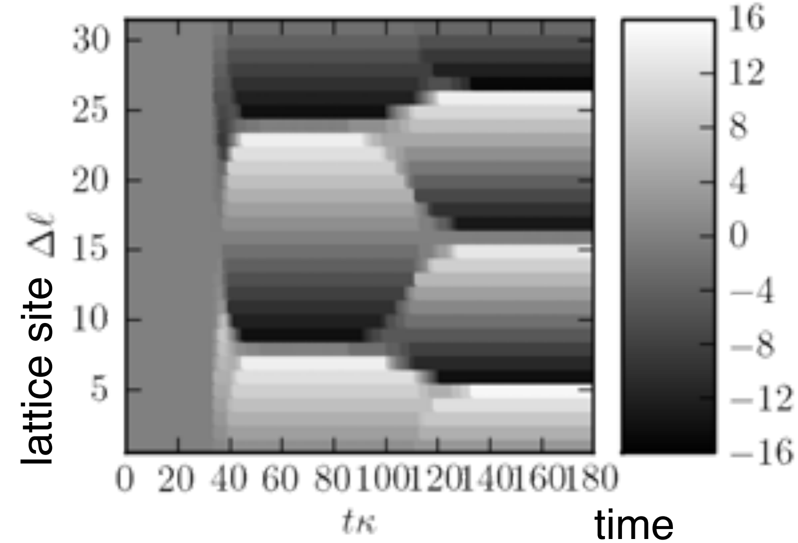
- Study equilibration dynamics:

amplitude dynamics



time

phase dynamics

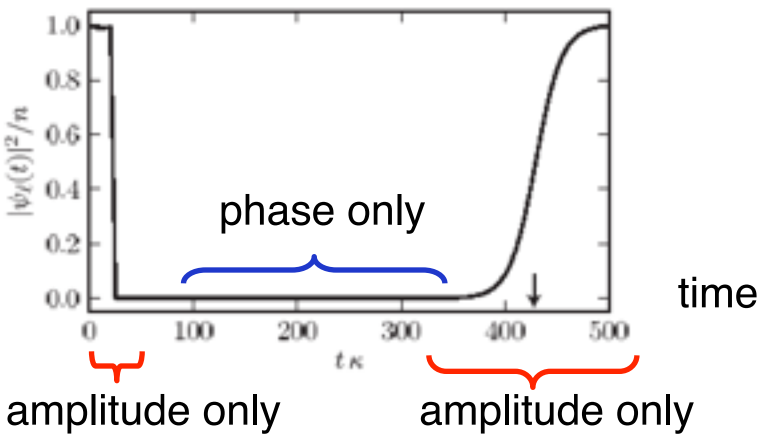


time

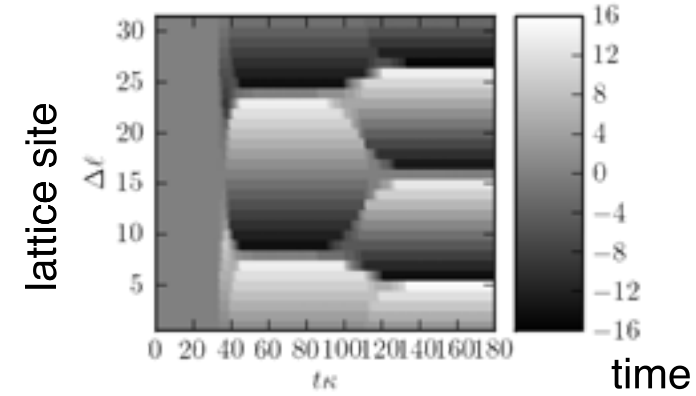
Role of Collective Variables

- Three stages in equilibration dynamics:

amplitude dynamics



phase dynamics



- Phase dynamics governed by collective variables:

external phase

$$\frac{1}{2\kappa}\partial_t\theta_\ell(t) = 2\sin\left(\frac{\Delta_\ell\theta(t)}{2}\right)\cos\left(\theta_{\ell+1}-\theta_{\ell-1}+2\phi\right)$$

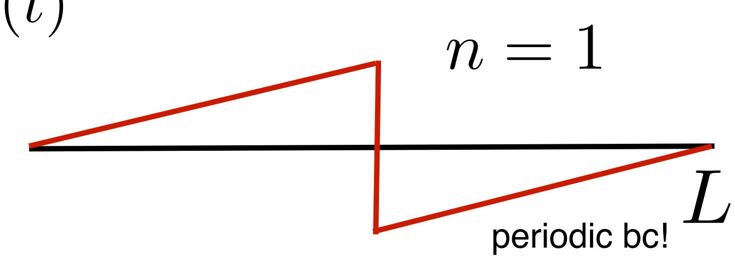
lattice Laplacian

- Ansatz with kinks/instantons

$$\theta_\ell(t) = Q(t)\,\tfrac{\ell-1}{L-1} + \delta\theta_\ell(t)$$

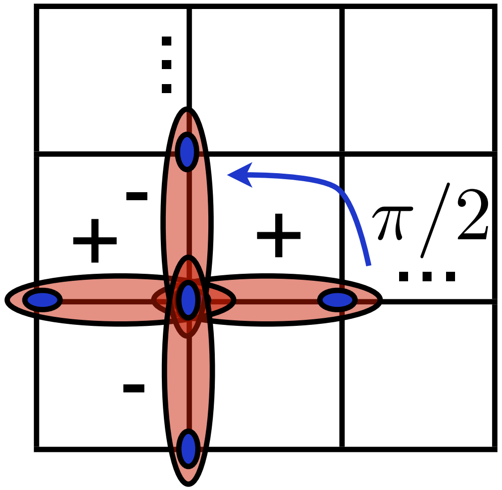
- Stable solutions:

$$Q = 2\pi n \qquad\qquad \delta\theta_\ell = 0$$



- Picture: transitions between different kink configurations driven by quantum noise

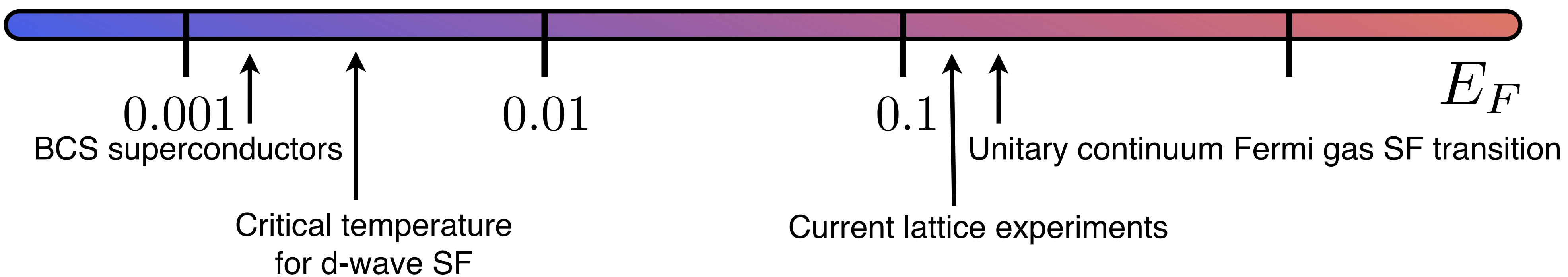
Dissipative D-Wave States of Fermions



SD, W. Yi, A. J. Daley, P. Zoller, Phys. Rev. Lett. **105**, 227001 (2010);
W. Yi, SD,A. J. Daley, P. Zoller, in preparation.

Motivation: Fermi-Hubbard model Quantum Simulation

- Clean realization of fermion Hubbard model possible
  - Detection of Fermi surface in 40K (M. Köhl et al. PRL 94, 080403 (2005))
  - Fermionic Mott Insulators (R. Jördens et al. Nature 455, 204 (2008); U. Schneider et al., Science 322, 1520 (2008))
- Cooling problematic: small d-wave gap sets tough requirements



➡ Still need to be 10-100x cooler

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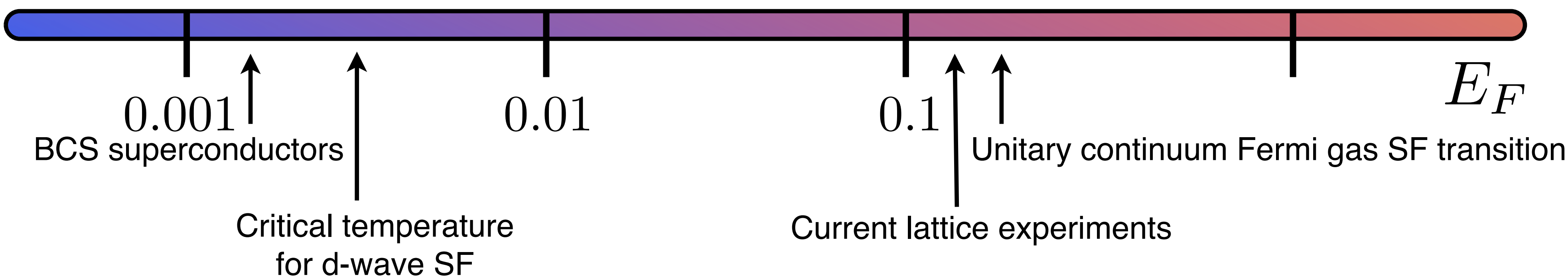


➡ Still need to be 10-100x cooler

- Roadmap via dissipative quantum state engineering approach:

(1) Dissipatively prepare pure (zero entropy) state close to the expected ground state:

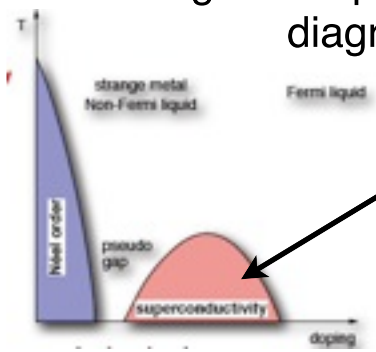
- energetically close
- symmetry-wise close

(2) Adapted adiabatic passage to the Hubbard ground state

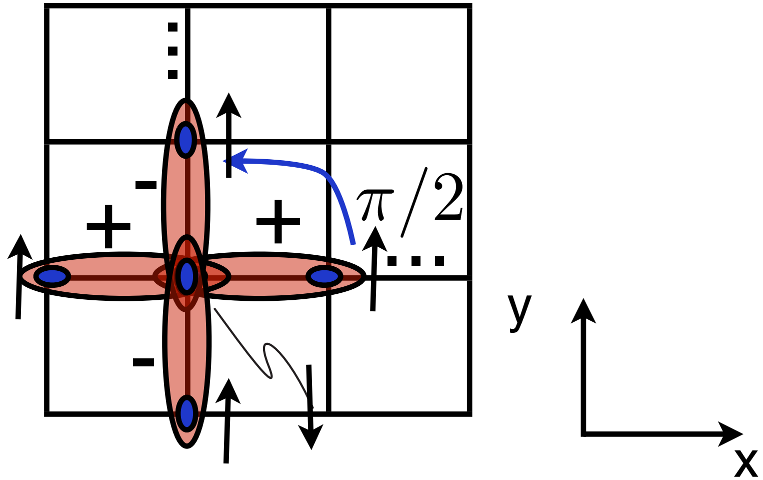
- gap protection via auxiliary Hamiltonian

The State to Be Prepared

High-Tc cuprate phase diagram



d-wave SC



product state

$$|\mathrm{BCS}_N\rangle \sim (d^\dagger)^{N/2}|\mathrm{vac}\rangle \qquad d^\dagger = \sum_i [\underbrace{c^\dagger_{i+\mathbf{e}_x,\uparrow} + c^\dagger_{i-\mathbf{e}_x,\uparrow}} - (\underbrace{c^\dagger_{i+\mathbf{e}_y,\uparrow} + c^\dagger_{i-\mathbf{e}_y,\uparrow}})]c^\dagger_{i,\downarrow}$$

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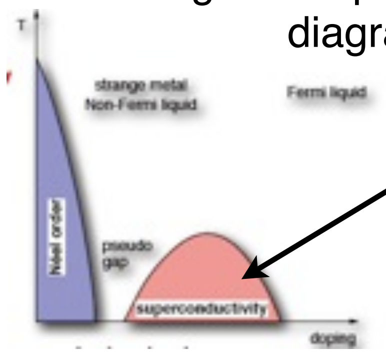

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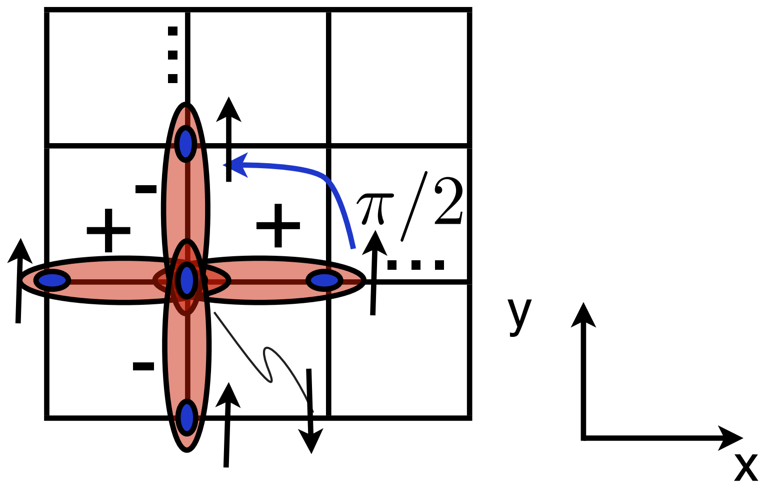

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- Features shared with expected Hubbard ground state:

(1) Quantum numbers

- pairing in the singlet channel
- transformation under spatial rotations: “d-wave”
- phase coherence: delocalization of singlet pairs

➡ State shares the symmetries of (conjectured) Hubbard GS

➡ No phase transition crossed in preparation process: gap protection

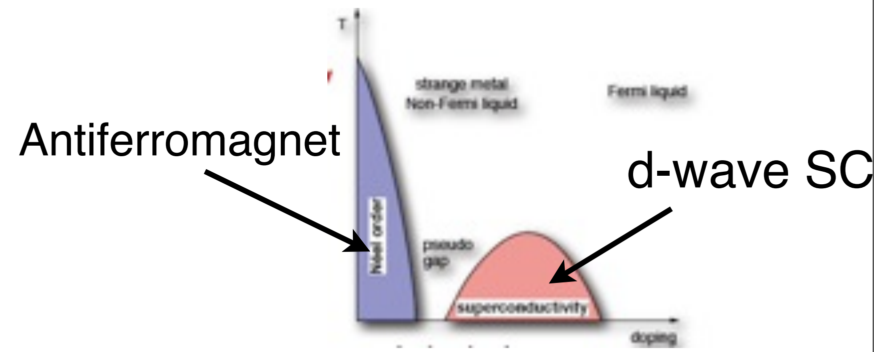
(2) Energetically close? Not known, but:

- off-site pairing avoids excessive double occupancy

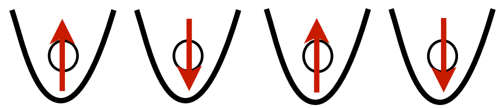
➡ Given the state, we want to find the Lindblad operators: “parent Liouvillian”

➡ “cooling” into the d-wave

Pairing mechanism



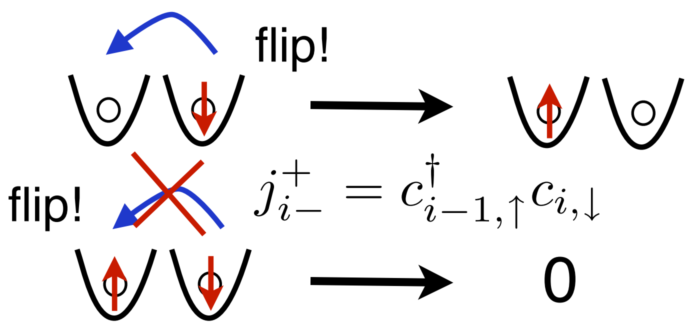
- Consider 1D cut only
- Half filling: Neel state for antiferromagnetism



- Lindblad operators (1D): e.g. $j_{i-}^{+} = c_{i-1,\uparrow}^{\dagger} c_{i,\downarrow}$

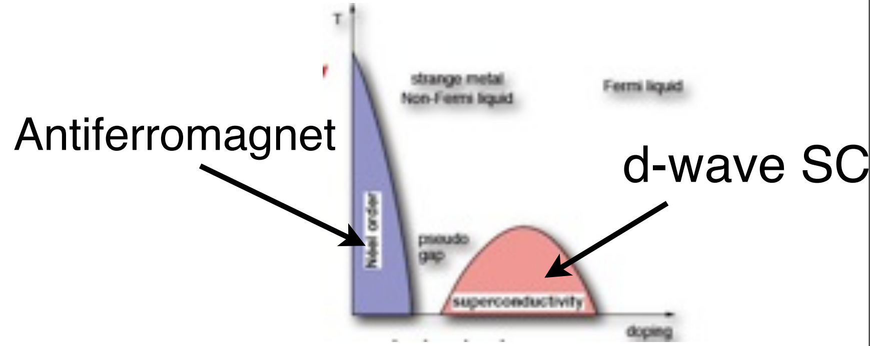
full set:

$$j_\ell = \{ j_{i\pm}^{\pm},\ j_{i\pm}^{z} \}$$



➡ Action of jump operators
- Pauli blocking
- spin transport

Pairing mechanism



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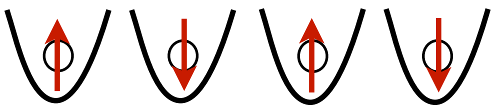

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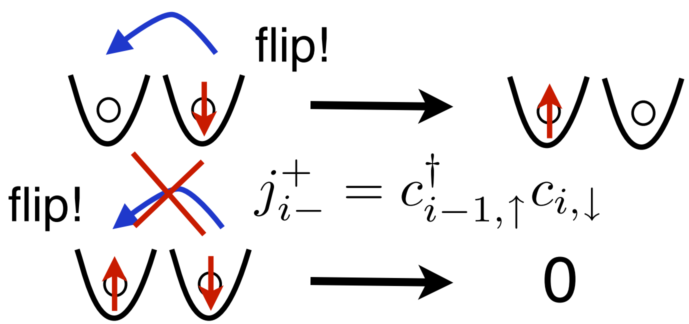
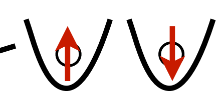


$j_{i-}^{+} = c_{i-1,\uparrow}^{\dagger} c_{i,\downarrow}$

➡ Action of jump operators
- Pauli blocking
- spin transport

- D-wave (analog) state: interpret the state as a symmetrically delocalized Neel order

$$|\mathrm{BCS}_1\rangle = (d^\dagger)^N |\mathrm{vac}\rangle, \ \ d^\dagger = \sum_i (c_{i+1,\uparrow}^{\dagger} + c_{i-1,\uparrow}^{\dagger}) c_{i,\downarrow}^{\dagger}$$



- Lindblad operators (1D): e.g. $J_i^{+} = j_{i,+}^{+} + j_{i,-}^{+} = (c_{i+1,\uparrow}^{\dagger} + c_{i-1,\uparrow}^{\dagger}) c_{i,\downarrow}$

phase locking

➡ Combine fermionic Pauli blocking with delocalization as for bosons

➡ Pauli blocking is the key for single particle nature of operators

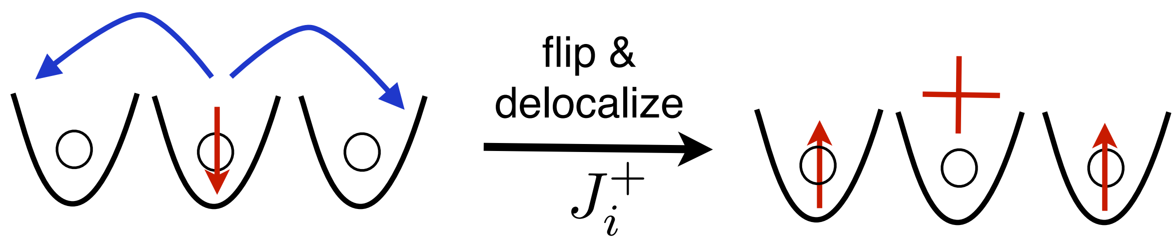
Dissipative Pairing: The d-wave jump operators

- The full set of Lindblad operators is

$$\boxed{J_i^{\pm,z} = (c_{i+1}^\dagger + c_{i-1}^\dagger)\sigma^{(\pm,z)}c_i}$$

e.g. 1D

$$J_i^+ = (c_{i+1,\uparrow}^\dagger + c_{i-1,\uparrow}^\dagger)c_{i,\downarrow}$$

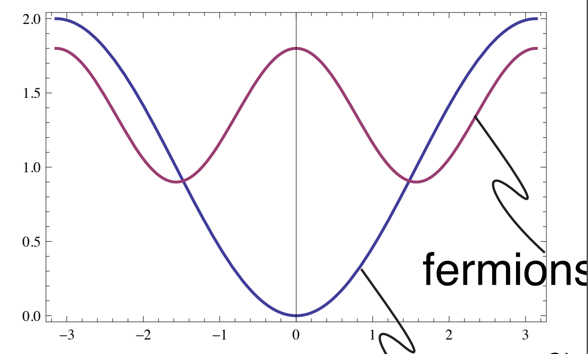


- Discussion: These operators
  - form exhaustive set: d-wave steady state unique, reached for arbitrary initial state (symmetry argument + verified in small scale simulations)
  - describe the redistribution of the superposition of a single particle
  - generate dissipatively bound pairs, which delocalize over the whole lattice
  - generalized for larger class of off-site paired fermion states: different symmetries

➡ Novel dissipative pairing mechanism, does not rely on attractive conservative forces

Late Time Dynamics

damping rates κ_q



- "near" final BCS state: Bogoliubov-type analysis: (U=0)

$$\mathcal{L}[\rho] = \sum_{\mathbf{q},\sigma} \kappa_{\mathbf{q}} [\gamma_{\mathbf{q},\sigma}\rho\gamma^\dagger_{\mathbf{q},\sigma} - \tfrac{1}{2}\{\gamma^\dagger_{\mathbf{q},\sigma}\gamma_{\mathbf{q},\sigma}, \rho\}]$$

with effective fermionic late time Lindblad operators

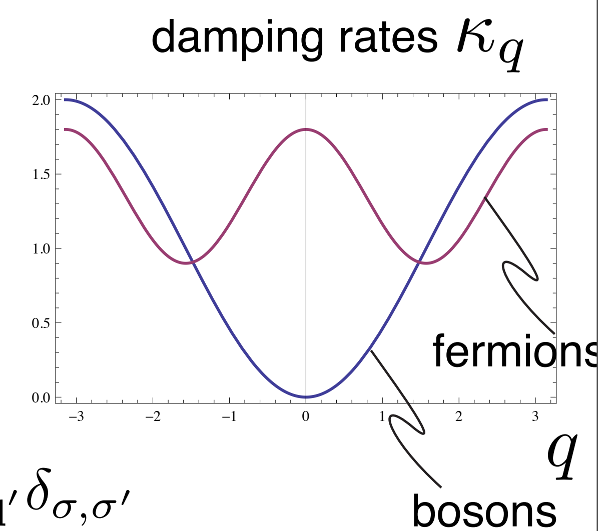
$$\gamma_{\mathbf{q},\sigma}|\text{d-BCS}_\theta\rangle = 0 \qquad \{\gamma_{\mathbf{q},\sigma}, \gamma^\dagger_{\mathbf{q}',\sigma'}\} = \delta_{\mathbf{q},\mathbf{q}'}\delta_{\sigma,\sigma'}$$

and effective damping rate

$$\kappa_{\mathbf{q}} = \kappa\tilde{n}(1 + |\varphi_{\mathbf{q}}|^2) \geq \kappa\tilde{n}$$ with a "dissipative gap"

➡ Interpretation: approach to the steady state is universal and exponentially fast

Late Time Dynamics



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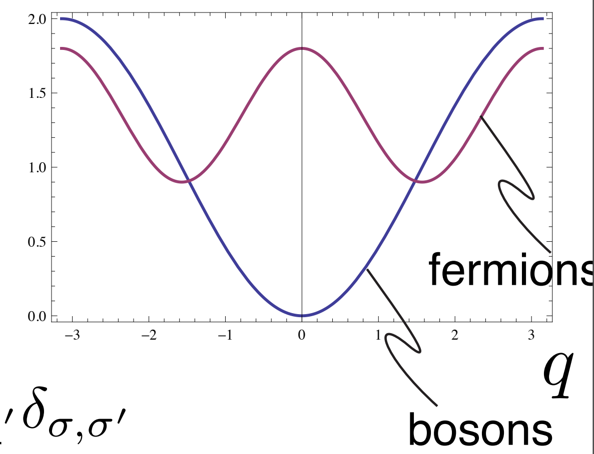

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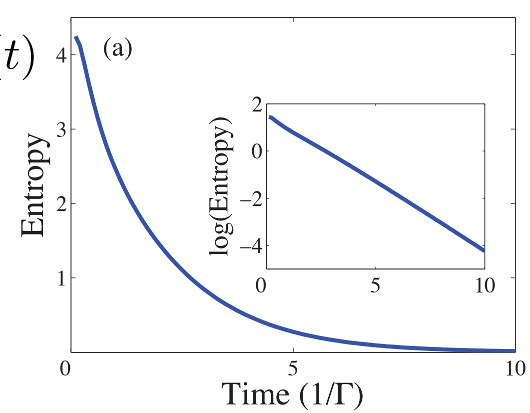
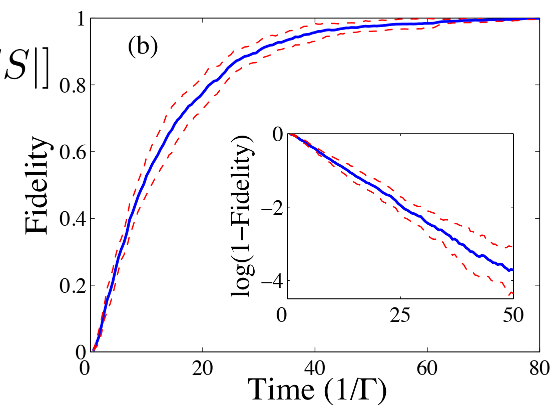
entropy

$S = \mathrm{tr}\rho(t)\log\rho(t)$



fidelity of d-wave BCS

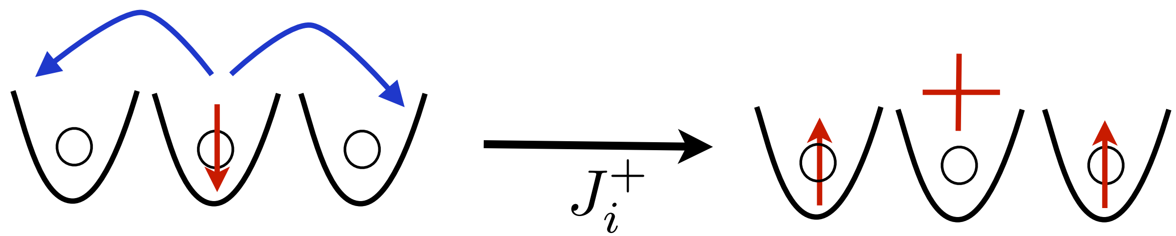
$\mathrm{tr}[\rho(t)|BCS\rangle\langle BCS|]$



Schematic implementation of d-wave jump operators

- one-dimensional analog:

$$\boxed{J_i^+ = (c^\dagger_{i-1,\uparrow} + c^\dagger_{i+1,\uparrow})c_{i,\downarrow}}$$



We need:

- spin flip
- spatial redistribution of atom over neighboring sites
- dissipative process, but coherence over several lattice sites

---

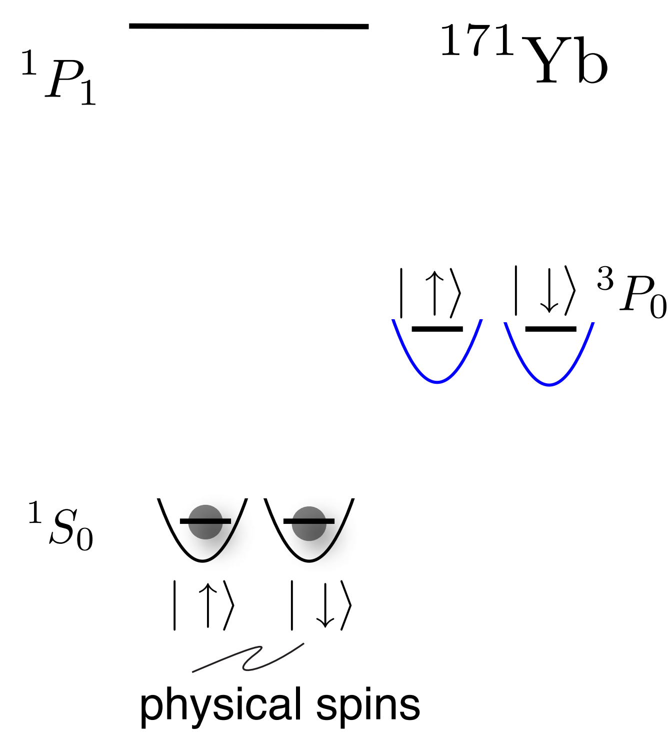
Setting:

- Earth Alkaline atoms in superlattices placed in microcavity

Earth alkaline features (Schreck, Grimm; Killian):

- metastable long-lived triplet states
- different, tunable lattice potentials for ground and excited state

- Level scheme

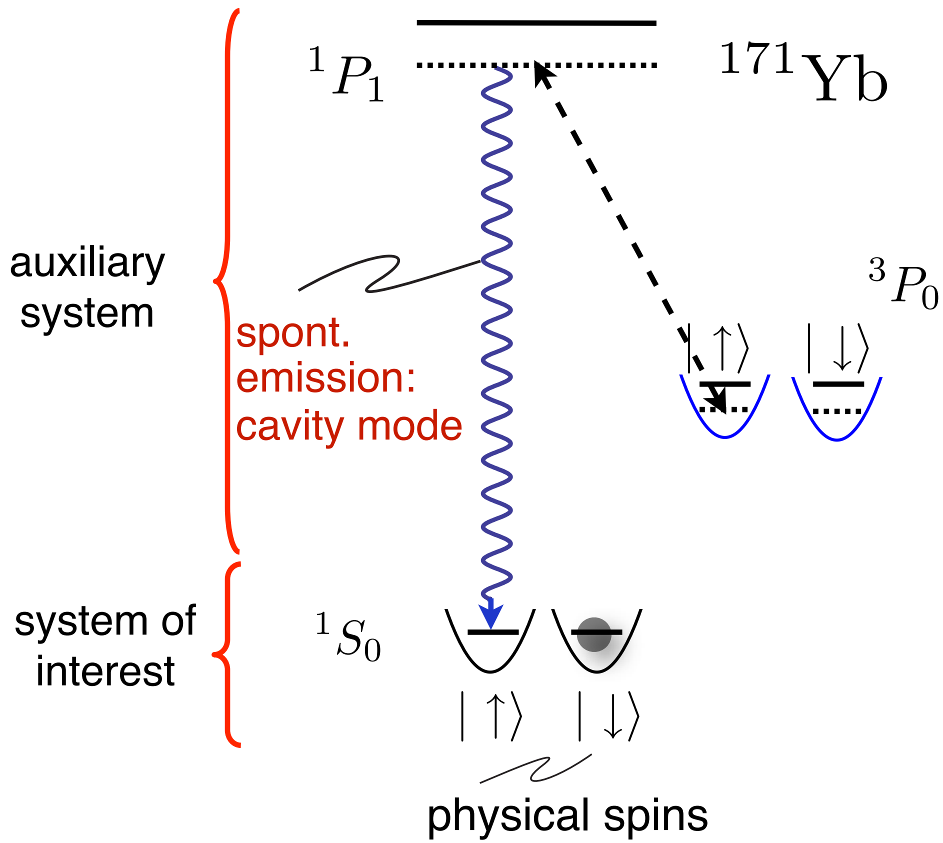


Schematic implementation of d-wave jump operators

$$\boxed{J_i^+ = (c^\dagger_{i-1,\uparrow} + c^\dagger_{i+1,\uparrow})c_{i,\downarrow}}$$

☑ spin flip

- Level scheme



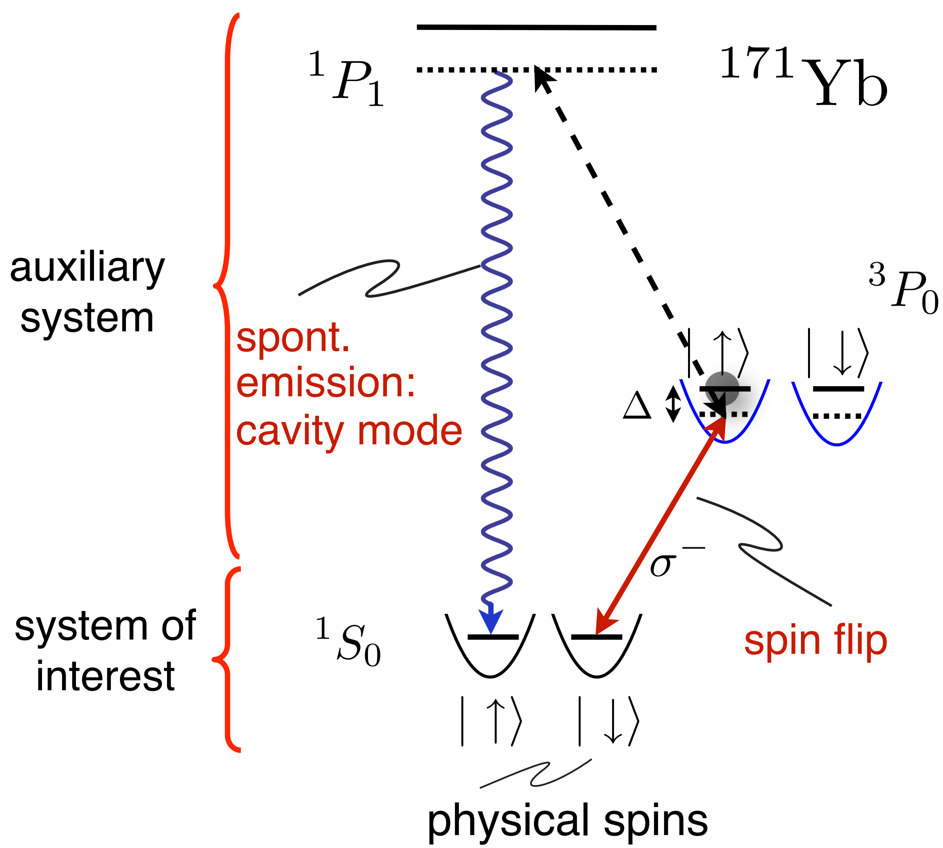
- Manipulation sequence

Schematic implementation of d-wave jump operators

$$\boxed{J_i^+ = (c_{i-1,\uparrow}^\dagger + c_{i+1,\uparrow}^\dagger)c_{i,\downarrow}}$$

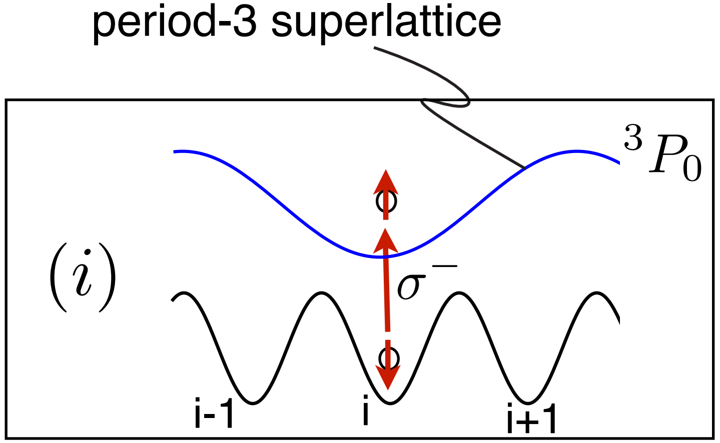
☑ spin flip

- Level scheme



- Manipulation sequence

coherent excitation with spin flip

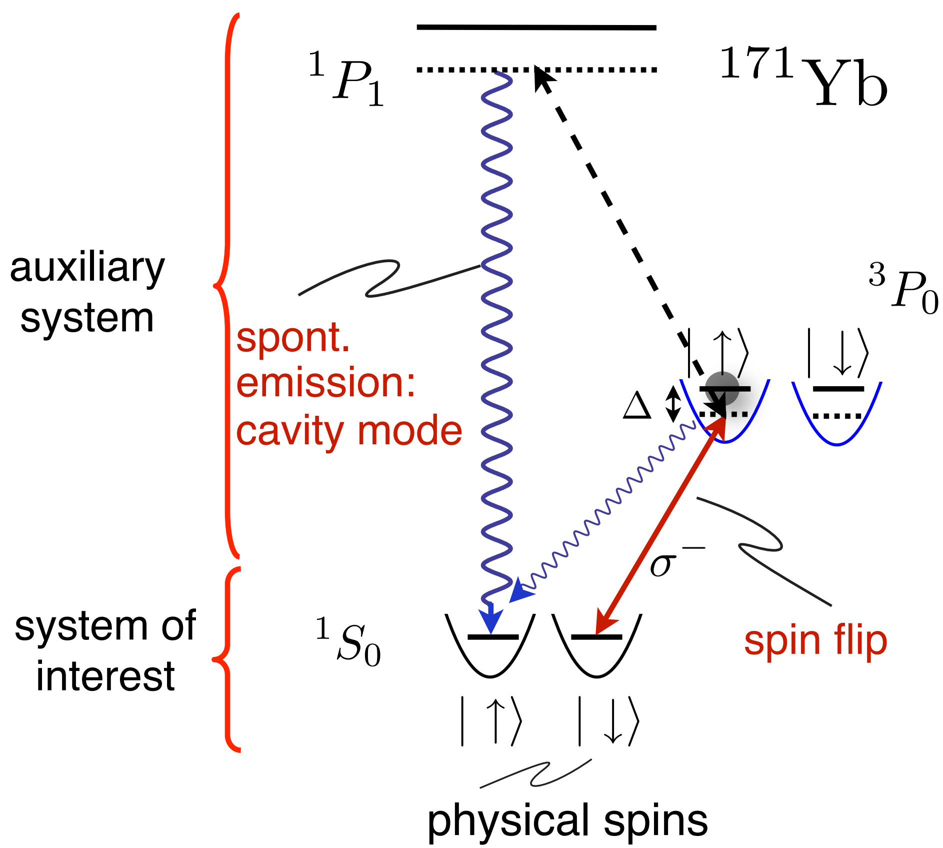


Schematic implementation of d-wave jump operators

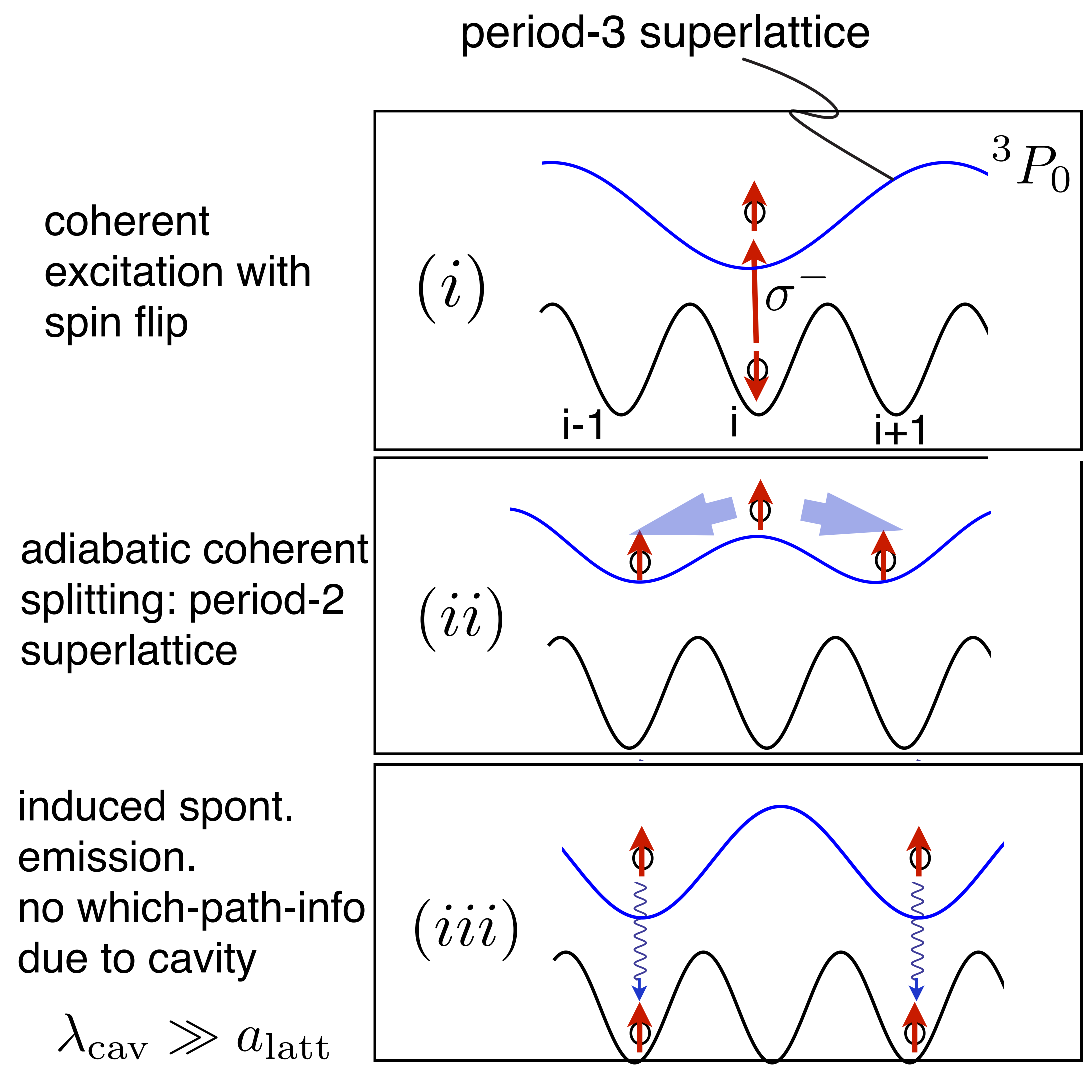
$$\boxed{J_i^+ = (c^\dagger_{i-1,\uparrow} + c^\dagger_{i+1,\uparrow})c_{i,\downarrow}}$$

- ☑ spin flip
- ☑ spatial redistribution
- ☑ coherence over several lattice sites

- Level scheme



- Manipulation sequence



repeat sequence stroboscopically for every subpartition of lattice into site triples

Conclusions and Outlook

By merging techniques from quantum optics and many-body systems:
Driven dissipation can be used as controllable tool in cold atom systems.

- Pure states with long range correlations from quasilocal dissipation
- Nonequilibrium phase transition driven via competition of unitary and dissipative dynamics
- Pairing mechanism for fermions with potential applications for quantum simulation

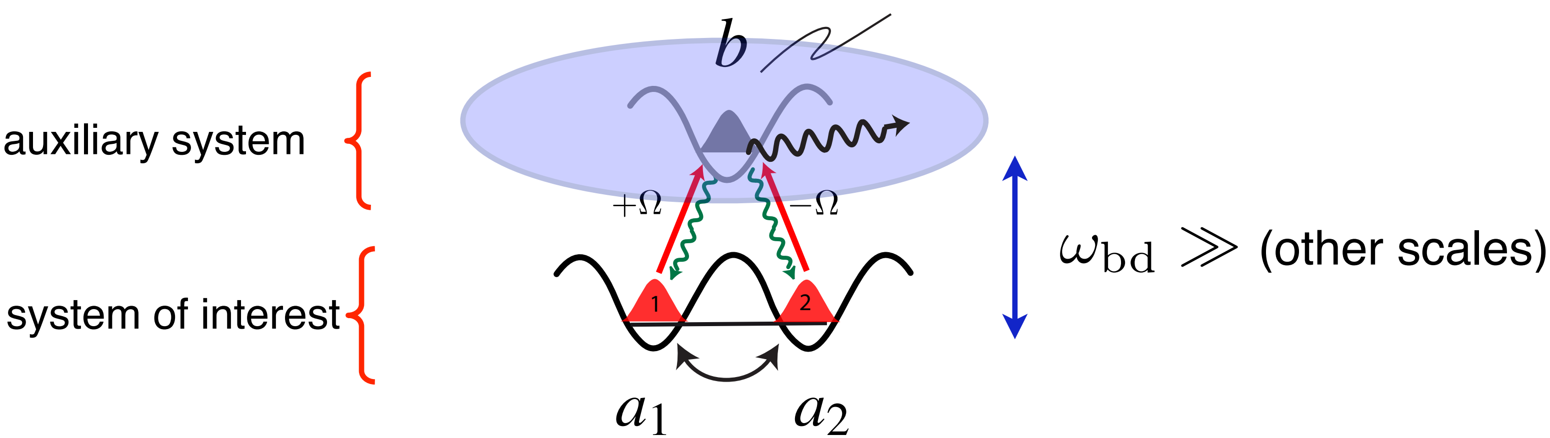
---

Questions for future research:

- Additional physical platforms for dissipation engineering: trapped ions, microcavity arrays
- Bosons: What is the nature / universality class of the dynamical phase transition?
  - Close analogies to the problem of directed percolation
  - Needs field theoretical framework: Keldysh path integral for quantum optical many-body systems
- Fermions: Cool quasi-locally into topologically ordered states (e.g. complex p-wave superconductors)? -> SD, E. Rico, M. Baranov, P. Zoller, arxiv:1105.5947

Cooling a Superfluid with a Superfluid?

- There is a large energy scale in our system-bath setting: band separation $\omega_{\rm bd}$



- In particular, under typical conditions

temperature of surrounding BEC

$$\omega_{\rm bd} \gg T_{\rm BEC}$$

- Therefore, the reservoir acts as an effective zero temperature reservoir, i.e. $\bar{n} \ll 1$

$$\partial_t \rho = -i[H,\rho] + \kappa(\bar{n}+1)\sum_i J_i \rho J_i^\dagger - \tfrac{1}{2}\{J_i^\dagger J_i, \rho\} + \kappa\bar{n}\cancel{\sum_i J_i^\dagger \rho J_i - \tfrac{1}{2}\{J_i J_i^\dagger, \rho\}}$$

- More generally, the existence of such large scale exceeding all scales relevant for many-body physics ensures validity of many-body master equation
  - dissipative dynamics temporally and spatially local
  - allows for a microscopic modelling of dissipative dynamics with similar accuracy as for Hamiltonian

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Validity of Inhomogeneous Gutzwiller Approximation

- The instability arises at weak coupling already, where the system is well described by the inhomogeneous Gutzwiller mean-field theory.
  - The instability is due to a renormalization of the single particle (complex) excitation spectrum, and thus encoded in the evolution of $(\Delta\psi_i(t), \Delta\psi_i^*(t))$
  - The exact equation of motion is a nonlinear equation, with nonlocal spatial correlations
  - The Gutzwiller approximation factorizes the correlations functions in real space, but treats onsite correlations exactly
  - The factorization is real space is justified at weak coupling (large condensate): The dominant scattering processes are those for (*-q, q)* off the macroscopically occupied condensate
  - In contrary, treating the onsite correlations properly is mandatory for the effect: Further (onsite) factorization of correlation functions (GP approximation) is insufficient

- Picture: Onsite (temporal, quantum) correlations prepare the ground for long wavelength spatial (classical) fluctuations becoming unstable

Uniqueness

- Understanding can be gained from symmetry considerations
  - Uniqueness of dark state equivalent to uniqueness of ground state (GS) of

$$H_\Delta = \sum_{i,\alpha=\pm,z} \Delta_\alpha J_i^{\alpha\,\dagger} J_i^\alpha$$

$$\left[\mathcal{L}[\rho] = \sum_{\alpha,i} \kappa_\alpha J_i^\alpha \rho J_i^{\alpha\dagger} - \tfrac{1}{2}\{\underbrace{\kappa_\alpha J_i^{\alpha\dagger} J_i^\alpha}_{\times(-\mathrm{i}^2)}, \rho\} \right]$$

-> effective Hamiltonian

- H is semi-positive
- an exact GS is the above d-wave (E=0)
- unique iff no symmetry T such that

$$THT^{-1} = H, \quad T|\mathrm{D}\rangle \neq E|\mathrm{D}\rangle$$

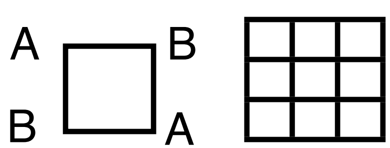
- Symmetries:
  - Translations
  - global phase rotations U(1)
  - global spin rotations SU(2) for $\Delta_z = \Delta_\pm/2$,

  d-wave is an eigenstate to these

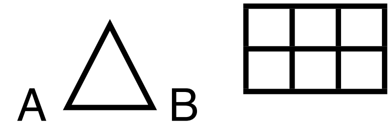
  - additional discrete symmetry on bipartite lattice for $\Delta_z = 0$ spoils uniqueness

$$T_d: \quad c_{i,\uparrow} \to -c_{i,\uparrow}; \quad c_{i,\downarrow} \to c_{i,\downarrow} \text{ for } i \in A,$$
$$c_{i,\uparrow} \to c_{i,\uparrow}; \quad c_{i,\downarrow} \to c_{i,\downarrow} \text{ for } i \in B$$

bipartite (periodic BC)



not bipartite (PBC)



SU(2) symmetry;
the jump operators are SU(2) vectors

$$[S^\alpha, J_i^\beta] = \mathrm{i}\epsilon_{\alpha\beta\gamma} J_i^\gamma \quad \forall i$$

➡ Avoid symmetries

➡ All three operators needed for uniqueness

Comments on the effective Hamiltonian

- Amusing parallel: Above Hamiltonian is a parent Hamiltonian for the d-wave state

$$H_\Delta = \sum_{i,\alpha} \Delta_\alpha J_i^{\alpha\,\dagger} J_i^\alpha = \sum_i h_i$$

  - H is semi-positive
  - an exact unique GS is the above d-wave state(E=0)
  - GS is GS for each h_i separately: projectors on GS

➡ completely analogous to e.g. AKLT model

➡ there, ground state is valence bond solid with exponentially decaying correlations

➡ different: state has long range order due to strong delocalization

➡ it has a physical role that will be important in the adiabatic passage to the Fermi-Hubbard model

➡ study excitation spectrum

Adapted Adiabatic Passage

- Naive adiabatic passage: ramp up FH Hamiltonian, switch off dissipation
- Fails: competing unitary and dissipative dynamics

➡ Adapted adiabatic passage: use auxiliary "parent Hamiltonian":

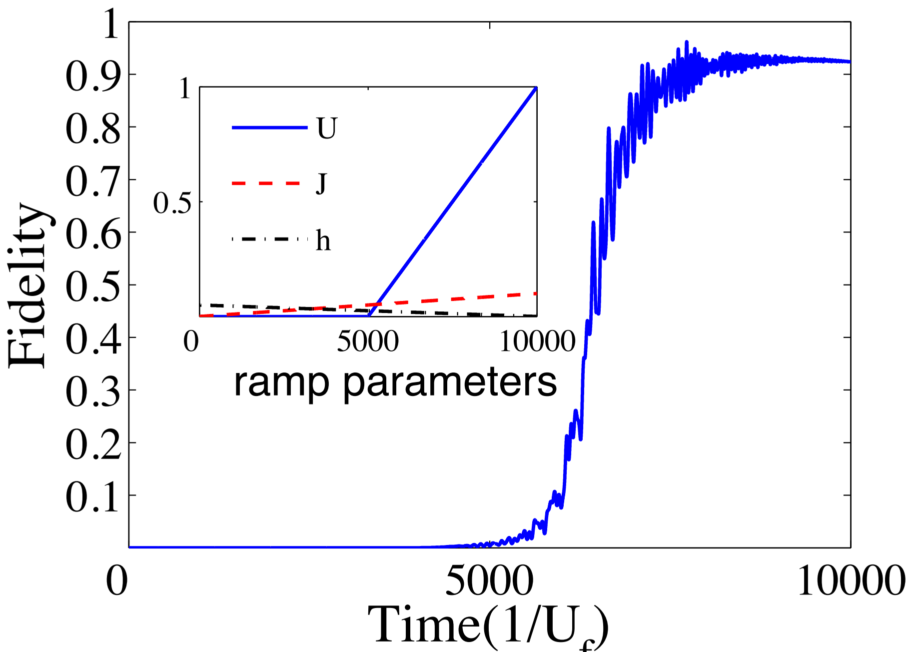
$$H_p = h \sum_{\ell=i,\{\pm,z\}} J_\ell^\dagger J_\ell$$

jump operators

- has the d-wave state as exact unique ground state
- its single particle excitation spectrum is gapped:

$$\epsilon_{\mathbf{q}} = h\,\tilde{n}\,(1+\varphi_{\mathbf{q}}^2) \geq h\,\tilde{n}, \;\; \tilde{n} \approx 0.72$$

fidelity to Hubbard GS



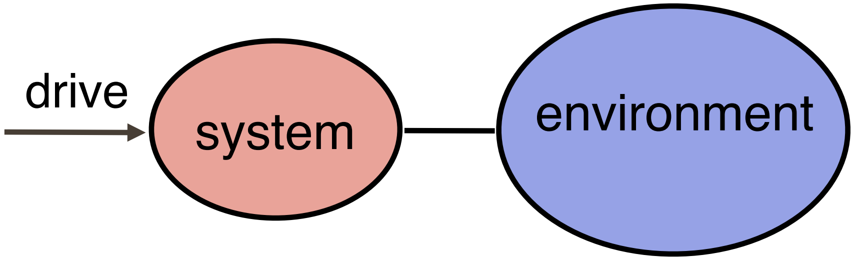
small scale numerical simulation (2x6 ladder, 4 atoms)

➡ single fermion excitations gapped

➡ d-wave state has identical symmetry and similar energy to the (conjectured) GS of the Hubbard model

➡ gap protection throughout adiabatic passage path

Open Quantum Systems



- The setting:

$$H = H_{\mathrm{S}} + H_B + H_{\mathrm{int}}$$

$$H_B = \int d\omega\, \omega b^\dagger_\omega b_\omega$$ continuum bath of harmonic oscillators

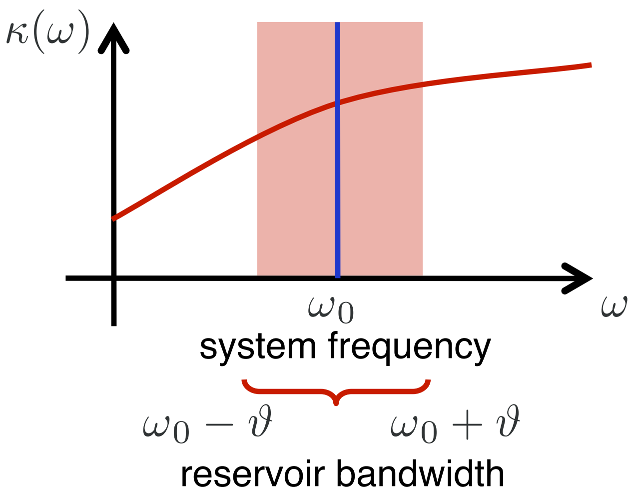
$$H_{\mathrm{int}} = i \int d\omega \kappa(\omega) \left[b^\dagger_\omega J - b_\omega J^\dagger \right]$$

Lindblad operators
polynomial in system operators

linear bath operator coupling to the system

- The quantum optical case: scale separations

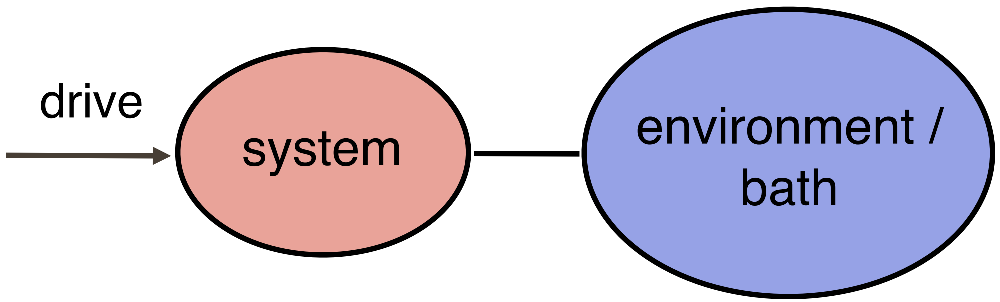
$$\kappa(\omega) \rightarrow \kappa_0$$

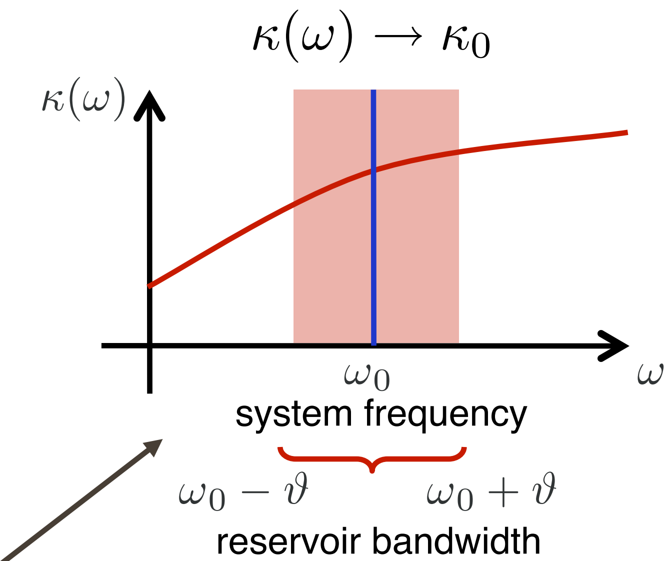


weak coupling $\kappa(\omega)/\omega_0 \ll 1$

Markovian bath $\kappa(\omega) \approx$ const.

Open Quantum Systems





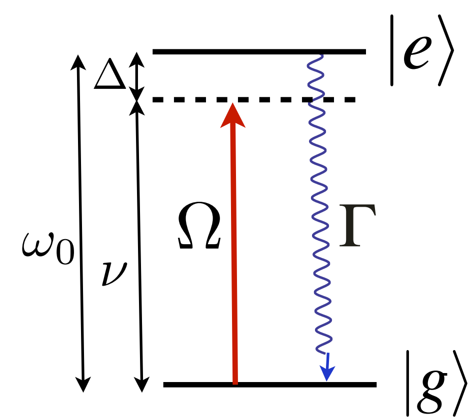
Three approximations:

(1) Born approximation: $\kappa(\omega)/\omega_0 \ll 1$

(2) Markov approximation: $\kappa(\omega) \approx$ const. $\Rightarrow \kappa(t-t') \sim \delta(t-t')$

(3) Rotating wave approximation: $\dfrac{\omega_0 - \nu}{\omega_0 + \nu} \ll 1$

$\omega_0 - \nu = \Delta$ detuning



in this example:

system Hamiltonian $H = (|e\rangle, |g\rangle) \begin{pmatrix} \Delta & \Omega \\ \Omega & 0 \end{pmatrix} \begin{pmatrix} \langle e| \\ \langle g| \end{pmatrix}$ jump operator $J_\alpha = |g\rangle\langle e| = \sigma^-$

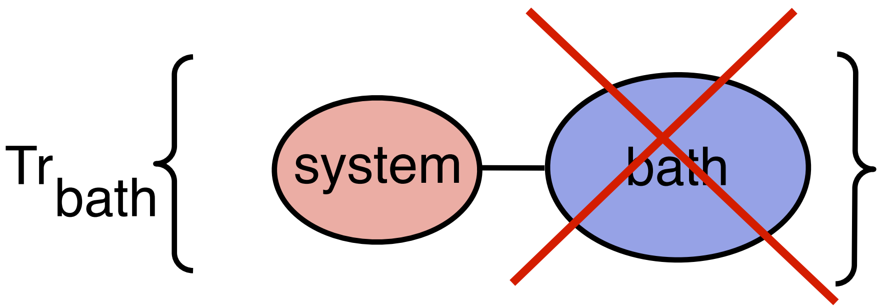
➡ These approximations give rise to well structured non-equilibrium evolution which can be implemented in cold atom systems

Open Quantum Systems

- temporally local evolution
- Structure: second order perturba
- but:

$$\partial_t \rho_{\text{tot}} = -i[H_S + H_B + H_{\text{int}}, \rho_{\text{tot}}]$$

➡ Eliminate bath degrees of freedom



quantum jump operators

effective system dynamics from Master Equation (zero temperature bath)

$$\partial_t \rho = -i[H_S, \rho] + \underbrace{\kappa \sum_\alpha J_\alpha \rho J_\alpha^\dagger - \tfrac{1}{2}\{J_\alpha^\dagger J_\alpha, \rho\}}_{\mathcal{L}[\rho]}$$

$\mathcal{L}[\rho]$ Liouvillian operator in Lindblad form

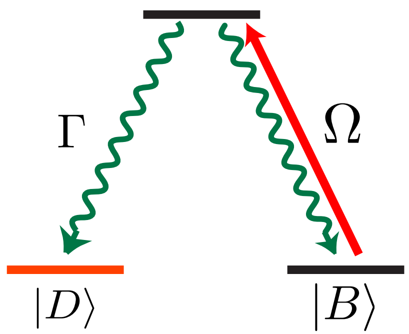
➡ the separation of scales gives rise to a temporally local well structured
➡ this gives rise to a microscopic modelling of many-body systems of similar accuracy as for the Hamiltonian

Dark States in Quantum Optics

- Goal: pure BEC as steady state solution, independent of initial density matrix:

$$\rho(t) \longrightarrow |BEC\rangle\langle BEC| \text{ for } t \to \infty$$

- Such situation is well-known quantum optics (three level system): optical pumping
  (Kastler, Aspect, Cohen-Tannoudji; Kasevich, Chu; ...)



$$\rho(t) \xrightarrow{t\to\infty} |g_+\rangle\langle g_+|$$

➡ Driven dissipative dynamics “purifies” the state

➡ $|D\rangle$ is a “dark state” decoupled from light

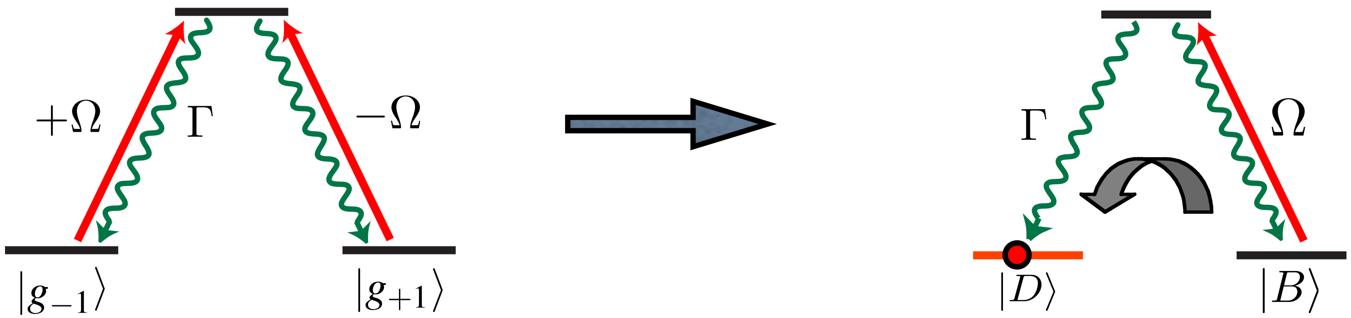
$$\boxed{J_\alpha |D\rangle = 0}$$

➡ Dark state is eigenstate of Lindblad operators with zero eigenvalue

➡ Time evolution stops when system is in DS: pure steady state

An Analogy

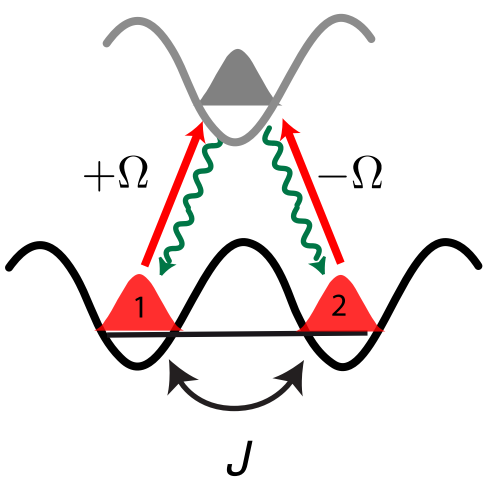
- Λ-system: three internal (electronic) levels (Aspect, Cohen-Tannoudji; Kasevich, Chu)



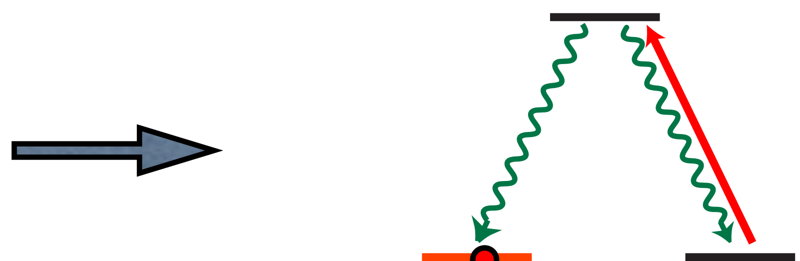
dark state $\quad$ bright state

$|D\rangle \sim |g_{+1}\rangle + |g_{-1}\rangle \quad |B\rangle \sim |g_{+1}\rangle - |g_{-1}\rangle$

- 1 atom on 2 sites



~ dissipative Josephson junction



$(a_1^\dagger + a_2^\dagger)\,|\mathrm{vac}\rangle \quad (a_1^\dagger - a_2^\dagger)\,|\mathrm{vac}\rangle$

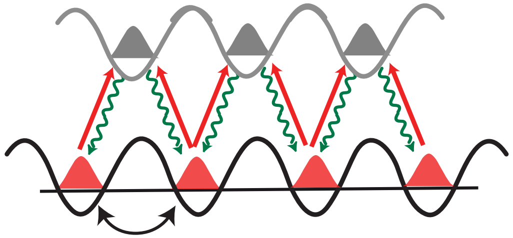
symmetric
"in-phase"

anti-symmetric
"out-of-phase"

pumping into symmetric state

➡"phase locking" for external (spatial) degrees of freedom:
like a BEC

Driven Dissipative lattice BEC



- Consider Lindblad operator:

nearest neighbours

$$J_i = (a_i^\dagger + a_j^\dagger)(a_i - a_j)$$

(1) BEC state is a dark state:

$$|BEC\rangle = \frac{1}{N!}\Big(\sum_\ell a_\ell^\dagger\Big)^N |vac\rangle$$

$$J_i|BEC\rangle = 0 \;\forall i$$

$$[(a_i - a_j), \sum_\ell a_\ell^\dagger] = \sum_\ell \delta_{i\ell} - \delta_{j\ell} = 0$$

(2) BEC state is the only dark state:

- $(a_i^\dagger + a_j^\dagger)$ has no eigenvalues (on fixed number (N-1) Hilbert space)
- $(a_i - a_j)$ has unique zero eigenvalue

$$(a_i - a_j)\;\forall i \longrightarrow (1 - e^{i\mathbf{q}\mathbf{e}_\lambda})a_{\mathbf{q}}\;\forall \mathbf{q}$$

➡ BEC is a dissipative zero mode of the jump operators

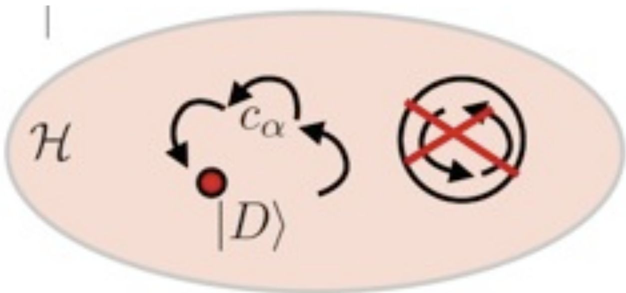
Driven Dissipative lattice BEC

(1) BEC state is a dark state

(2) BEC state is the only dark state

(3) Uniqueness: |BEC> is the only stationary state (sufficient condition)

pictorially:



more precisely:

If there exists a stationary state which is not a dark state, then there must exist a subspace of the full Hilbert space which is left invariant under the set $\{c_\alpha\}$

(4) Compatibility of unitary and dissipative dynamics

$|D\rangle$ be an eigenstate of H, $H\,|D\rangle = E\,|D\rangle$

$$\rho(t) \xrightarrow{t\to\infty} |D\rangle\,\langle D|$$