Statistical Interpretation of Autocorrelation Coefficients for Fields in Mode-Stirred Chambers

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Abstract*— The autocorrelation function of electrical field strengths for different boundary conditions (tuner positions) at a given spacial position is proposed in the IEC standard 61000-4-21 as a measure for the determination of the number of uncorrelated boundary conditions in mode-stirred chambers. Additionally, an upper limit for the autocorrelation coefficient is given for a fixed number N of measured tuner positions only. In this paper, we analyze an approach given in the literature that includes the treatment of different N, but still gives results that are inconsistent with the daily measurement practice in mode-stirred chambers. A slight modification of this approach is proposed that leads to consistent results. This paper gives critical values for the autocorrelation coefficients for any number of measured tuner positions based on a statistical analysis of the well known probability distribution of autocorrelation coefficients. The degree of determination and the significance level remain as free parameters that have to be established by the community. The authors propose values for these parameters that are consistent with the example given in the standard.

I. Introduction

The use of mode-stirred chambers (MSC) as an alternative test environment for EMC investigations is discussed for some time [1].

The mode-stirred chamber consists of a screened room, which possesses a mechanical tuner for the change of the electromagnetic boundary conditions. Figure 1 shows the mode-stirred chamber in Magdeburg.

Fig. 1. MSC in Magdeburg

The test environment is a cavity resonator. The lowest usable frequency of the chamber (LUF) lies well above the first resonance frequency, within a range of increased mode density. In addition, the LUF depends on the efficiency of the tuner with regard to its ability to shift the resonance frequencies. For an excitation with a fixed frequency an inhomogeneous spatial field distribution for each individual tuner position (boundary condition) is achieved. This distribution corresponds to the superposition of different solutions of Maxwell's equations for the MSC. As a result of the change of the boundary conditions the spacial field distributions are changed, too. In the case of an average of field distributions to a sufficiently large number of suitable boundary conditions, a substantial decrease of the spatial inhomogeneity of the field strength can be achieved. The number of boundary conditions is an optimization parameter regarding measuring time versus homogeneity. In particular, the different boundary conditions are suitable for decreasing the spatial inhomogeneity if they lead to statistically independent field distributions. Consequently, these boundary conditions are called "statistically independent" boundary conditions. Apart from the assumption of a very high mode density, statistical independence of boundary conditions is one of the most important assumptions in the statistical theory of mode-stirred chambers [2]–[4].

The normative part of the IEC 61000-4-21 [5] assumes statistically independent boundary conditions. Furthermore, the standard gives defaults for the number of independent tuner positions in different frequency ranges, which have to be realized. The informative part of the standard deals with the determination of the independent tuner positions. Here, the autocorrelation coefficient is used as a measure for statistical independency. Statistical independency of field distributions yields a small autocorrelation coefficient, but the reversal applies strictly only to the case of normally distributed data [6], [7]. In [5] only one value for an upper limit of the autocorrelation coefficient is given. This limit $\rho_0 = e^{-1} = 0.37$ refers to a measurement of the field strength for $N = 450$ different tuner positions. Lundén and Bäckström investigate the dependence of this limit on the number of used tuner positions (sample size) [8]. They came to the conclusion that the limit ρ_0 decreases as sample sizes increases and reaches zero for $N \to \infty$. This would imply that the number of independent boundary conditions decrease for larger N. Although the work is cited in the standard, the results are not taken into account. In the following, we will analyze the work of Lundén and Bäckström and will resolve the conflict in their conclusions.

II. Basics

A. The Autocorrelation Coefficient

The correlation coefficient r is a measure for the statistical dependency of variables. To use this quantity for the analysis of the change of spacial field distributions, measurements of field strengths versus tuner position at fixed positions in the chamber are used. The boundary conditions achieved by means of different tuner positions represent, however, only a sample of size N, out of the infinite number of possible boundary conditions. For an accurate measurement all existing boundary conditions had to be considered, which cannot be realized in praxis. From the measured data set with N values, $N-1$ further data sets are obtained by cyclic exchange of the data:

$$\begin{aligned} &\{E_1, E_2, \dots, E_{N-1}, E_N\} \to \\ &\{E_N, E_1, E_2, \dots, E_{N-1}\} \to \dots \to \\ &\{E_2, \dots, E_{N-1}, E_N, E_1\} \end{aligned} \tag{1}$$

This is similar to a variation of the zero position of the measurement with the increment $\Delta\beta^*$. Thus, a disalignment of $\beta^* = i * \Delta\beta^*$ is obtained for the i-th new result vector. The computation of the correlation coefficient, r_{ij}, is done according to equation (2).

$$\begin{aligned} r_{ij} =& \frac{\sigma_{ij}^2}{\sqrt{\sigma_i^2 \sigma_j^2}} \\ =& \frac{\sum\limits_{\nu=1}^{N} (E_{i,\nu} - \langle E_i \rangle)(E_{j,\nu} - \langle E_j \rangle)}{\sqrt{\sum\limits_{\nu=1}^{N} (E_{i,\nu} - \langle E_i \rangle)^2 \sum\limits_{\nu=1}^{N} (E_{j,\nu} - \langle E_j \rangle)^2}} \\ =& \frac{\sum\limits_{\nu=1}^{N} (E_{i,\nu} - \langle E \rangle)(E_{j,\nu} - \langle E \rangle)}{\sum\limits_{\nu=1}^{N} (E_{i,\nu} - \langle E \rangle)^2} \end{aligned} \tag{2}$$

Here, it is assumed that only a rearrangement of one data set is used. The variances σ^2 and average values $\langle E \rangle$ of the data sets are equal. Thus, the autocorrelation coefficient

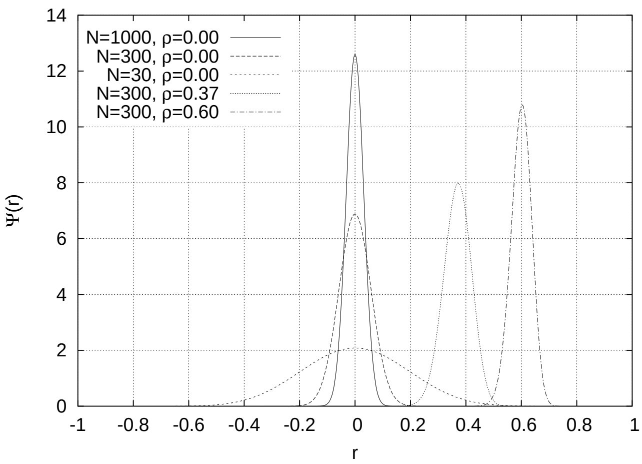

Fig. 2. Probability density function $\Psi(r)$ of the autocorrelation coefficient

r_{1j} is determined. The range of values of the autocorrelation coefficient is between -1 and 1. 1 and -1 corresponds to completely correlated, and 0 to completely uncorrelated data.

For the assessment of a limit for the autocorrelation coefficient r, the value of r^2 (degree of determination) is important. It indicates the probability for the forecast of further values by interpolation. This is $0.37^2 = 13.7\%$ for the value indicated in the standard. The arbitrary selection of the boundary conditions realized in the sample leads to statistical variations of the autocorrelation coefficient. This has to be considered when different mode-stirred chambers are compared. Therefore, an additional safety margin (critical parameter) is introduced, considering the statistical variations as a function of the sample size for given degree of determination.

B. Probability Density Function of the Correlation Coefficient

The statistical variations of r result from the probability density function (pdf), $\Psi(r)$, of the correlation coefficient r [10]:

$$\begin{aligned} \Psi(r) =& \frac{N-2}{\sqrt{2\pi}} \cdot \frac{\Gamma(N-1)}{\Gamma\left(N-\frac{1}{2}\right)} \cdot \frac{\left(1-\rho^2\right)^{\frac{(N-1)}{2}} \left(1-r^2\right)^{\frac{N-4}{2}}}{(1-\rho r)^{N-\frac{3}{2}}} \cdot \\ & \left[1 + \frac{1+\rho r}{4(2N-1)} + \dots\right] \end{aligned} \tag{3}$$

Here, ρ is the expected value of r. It represents the true value for the correlation coefficient, which would be obtained using an infinitely large sample size for its calculation. The function $\Psi(r)$ is represented in Figure 2 for different values ρ and N. With increasing sample size N the probability density for the expected value increases and the distribution becomes narrower. Thus, the probability for the calculated value r being closer to ρ is higer for increasing N. The integral

	accept H_0	reject H_0
H_0 correct	no error	type 1 error (α)
H_0 wrong	type 2 error (β)	no error

TABLE I

ERRORS OF HYPOTHESIS TESTING

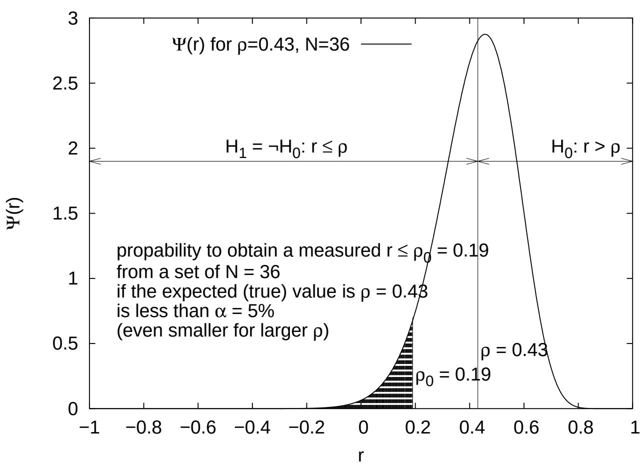

Fig. 3. Test of hypothesis.

$$\alpha = \int_{-1}^{\rho_0} \Psi(r)\, dr \tag{4}$$

gives the probability for the occurrence of values of r, which are smaller than or equal to the critical value ρ_0.

C. Test of Hypothesis

The statistical test of hypothesis is a formalism for the formulation of assured statistical statements concerning random variables [9], [11]. For a correct application it is important to know the possible errors of a test of the "null hypothesis" (H_0).

As a result of the hypothesis test, the null hypothesis H_0 can be accepted or rejected. Thus, an error occurs only, if the hypothesis H_0 is rejected, although it is correct (type 1 error, α) or accepted although it is wrong (type 2 error, β), see Table I. First we analyze the error in the the case when the hypothesis is rejected, although it is correct.

With the choice of the null hypothesis H_0 a statistical population is selected (here: selection of an expected value ρ for the correlation coefficient r and a relation, e.g., $r > \rho$). If one examines the statement of H_0 for measured data, the limit of the validity of the hypothesis has to be examined. This is done using the probability density function of the limit (here: $\Psi(r)$ for given N and ρ). As represented in Figure 3, the type 1 error α can be calculated by integration of the pdf over its tail in the limits -1 to ρ_0 (values in the tail only have small probabilities to belong to the population given by the null hypothesis). If H_0 is wrong, a type 2 error (β) arises when accepting the hypothesis. In that case, we can only conclude

	Lund´en und B¨ackstr¨om (LB)	this work (MD/BS)
ρ	0	$\rho > 0$ arbitrary, but fixed
H_0	$r = 0$ perfect uncorrelated boundary conditions	$r > \rho$ correlation is stronger than ρ
$\neg H_0 = H_1$	$\lvert r\rvert > 0$ not perfect uncorrelated boundary conditions	$r \leq \rho$ correlation weaker then ρ
limit ρ_0	$\alpha/2 = \int_{\rho_0}^{1} \Psi(r)dr$	$\alpha = \int_{-1}^{\rho_0} \Psi(r)dr$

TABLE II

COMPARISON OF THE NULL HYPOTHESISES OF LB UND MD/BS.

that the assumed pdf is not the right one, but there is an infinite number of alternatives (here: other limit ρ). Thus it is clear, that an approach according to the case of the type 1 error does not exist because of the lack of knowledge of the population's distribution. Therefore, the null hypothesis H_0 has to be formulated in such a way that it is likely to be refused in the test. Only then the error probability is limited (upwards) by α. The formalism of the hypothesis test is like follows: After the formulation of a meaningful hypothesis the error probability α (level of significance) has to be set. Usual values are 5% ($\alpha = 0.05$) and 1% ($\alpha = 0.01$). The solution of the integral from equation (4) for α results in the critical parameter ρ_0. If $r \leq \rho_0$, the probability that the sample originates from the population assumed in the null hypothesis is $\leq \alpha$, and thus, H_0 has to be rejected. Now, the correlation coefficient r of the measured values can be compared with the critical value ρ_0, in order to examine the hypothesis.

The hypotheses of Lundén and Bäckström (LB) and this work (MD/BS) are compared in Table II. The calculation of the critical value ρ_0 in the work of LB is slightly different because of the two-sided nature of their hypothesis. In the LB case $|r|$ has to become larger than ρ_0 in order to reject the null hypothesis. In our case (MD/BS), r should be smaller than ρ_0 to reject H_0.

It has to be pointed out that the formalism of hypothesis testing is correctly used in LB's work [8] and that all calculations are also correct. The difference to our approach is simply the choice of the null hypothesis.

III. CRITICAL VALUE ρ_0

From the knowledge of the probability density function $\Psi(r)$ the critical value ρ_0 of the hypothesis test can be computed. For each N the computation depends on the level of significance α and the expected value ρ. The latter is always zero in the work of LB. For our approach, it appears meaningful to select ρ such way that the (N, ρ_0)-point from the standard example (450, 0.37) is achieved (a fixpoint with the standard). The results are represented in Figure 4.

For the MD/BS approach the expected values are $\rho = 0.43$ for $\alpha = 0.05$ and $\rho = 0.456$ for $\alpha = 0.01$. These values give the statistical interpretation of the limit from the standard example, which was missing up to now. Thus, the degree

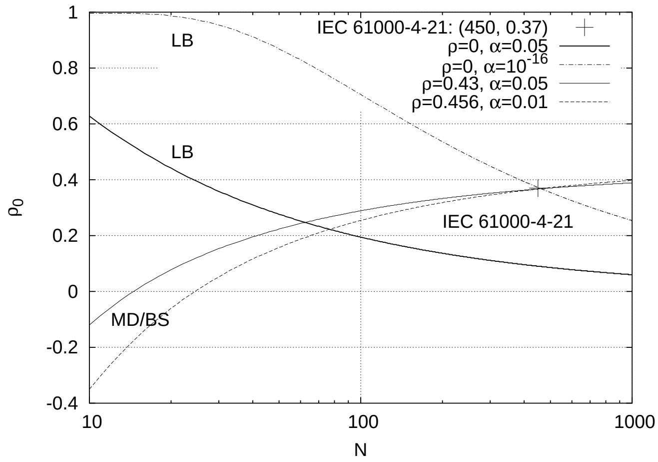

Fig. 4. Critical value ρ_0 as function of N according to LB and MD/BS.

of determination results to $\rho^2 = 18.5\%$ for $\alpha = 0.05$ and $\rho^2 = 21.6\%$ for $\alpha = 0.01$. For LB's approach, the (450, 0.37)-pair is only met with an unreasonable small selection of $\alpha = 10^{-16}$. Furthermore, it can be seen that in this case the critical value ρ_0 becomes practically one for small numbers of tuner positions. Therefore, experimental r-values will become smaller as the critical value, already for very small movements of the tuner, leading to an unreasonable large number of independent tuner positions. In particular, the decrease of ρ_0 versus N may result in the determination of a smaller number of independent tuner positions for larger N.

In contrast, our approach (MD/BS) leads to an increasing slope of ρ_0 versus N, reaching ρ in the limit of large N. This will result in an increase of the number of independent tuner positions for increasing N (increasing measurement accuracy), reaching a limit (expected value, true value) for $N \to \infty$.

IV. Experimental Results

The following results are based on measurements of the electrical field strength (individual cartesian components) at a fixed position in the active volume of the large Magdeburg mode-stirred chamber at different frequencies. Discussed are the results for $f = 200$ MHz (within the range of the LUF) and $f = 1$ GHz. The tuner was rotated over 360 degrees in steps of 1 degree ($N = 360$).

From these measured values the autocorrelation functions (r as function of the disalignment angle β^*) for $N \leq 360$ can be determined by removing data from the original dataset. For the two mentioned frequencies and different N these autocorrelation functions are represented in Figure 5 together with the critical limits ρ_0 according to LB and MD/BS which can be derived from figure 4.

The resulting numbers of independent tuner positions (#POS) for both approaches are summarized in Table III and in Figure 6. As expected, a decrease of the number of independent tuner positions for increasing N is obtained at $f = 200$ MHz using LB's approach. At $f = 1$ GHz this effect is much smaller; here, the effect of changing the pdf of the

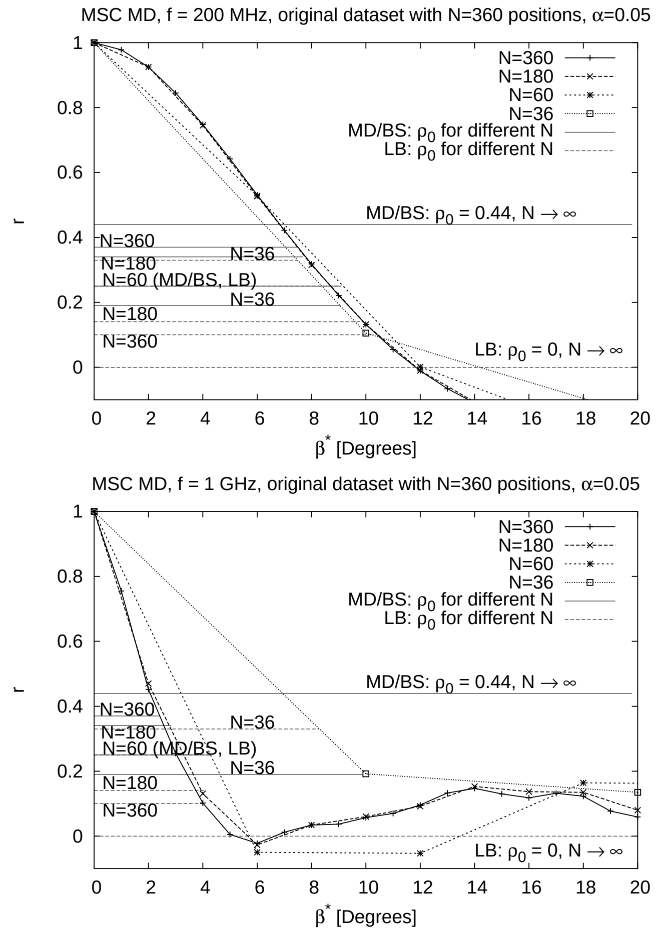

Fig. 5. Autocorrelation coefficients for differently reduced data sets with limits according to LB and MD/BS. Top: f=200 MHz; Bottom: f=1 GHz

autocorrelation coefficient by reducing N nearly compensates the variation of the critical value.

The compensation of these two effects in the LB approach is also visible in Figure 7. Here, the experimental autocorrelation coefficients are plotted versus frequency (100 MHz – 4.2 GHz) and angular displacement (0 – 40 degrees). Additionally, the critical limits according to LB and MD/BS are shown as contour lines. In the two upper graphs the upper contour line belongs to the MD/BS approach; in the lowest graph this is the lower contour line. The three graphs are obtained for $N = 360$, $N = 120$, and $N = 20$, where $N = 120$ and $N = 20$ are obtained from the original data set by reduction.

V. Conclusion

The determination of statistically independent tuner positions is of central importance for investigations of mode-stirred chambers. On one hand, the number of independent boundary conditions is of major importance for the definition of the lowest usable frequency (LUF). On the other hand, the statistical theory of of mode-stirred chambers is based on the assumption of statistically independent boundary conditions. The standard IEC 61000-4-21 treats the determination of independent boundary conditions only in the informative part

	MD/BS					LB				
		200 MHz		1 GHz			200 MHz		1 GHz	
N	ρ_0	$\beta^*[^o]$	#POS	$\beta^*[^o]$	#POS	ρ_0	$\beta^*[^o]$	#POS	$\beta^*[^o]$	#POS
36	0.19	9.05	39.8	10	36	0.33	7.49	48.1	8.29	43.4
60	0.25	9.17	39.3	4.29	83.9	0.25	9.17	39.3	4.29	83.9
180	0.34	7.76	46.4	2.77	130	0.14	9.91	36.3	3.95	91.1
360	0.37	7.5	48	2.41	149.4	0.1	10.41	34.6	4.02	89.6

TABLE III

COMPARISON OF THE NUMBER OF INDEPENDENT TUNER POSITIONS (#POS) ACCORDING TO LB AND MD/BS.

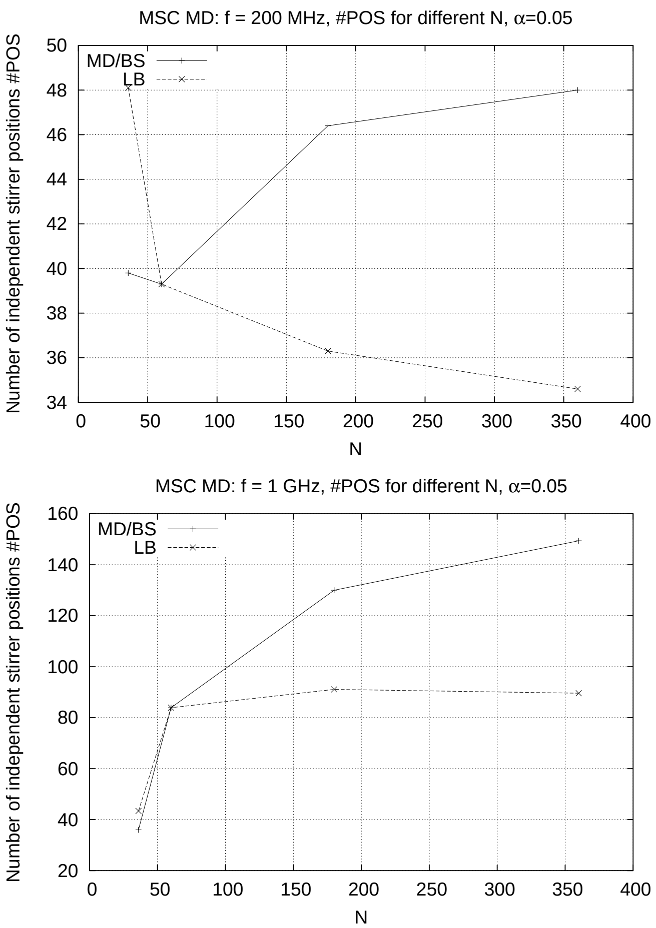

Fig. 6. Number of independent tuner positions versus N according to LB and MD/BS. Top: f=200 MHz; Bottom: f=1 GHz

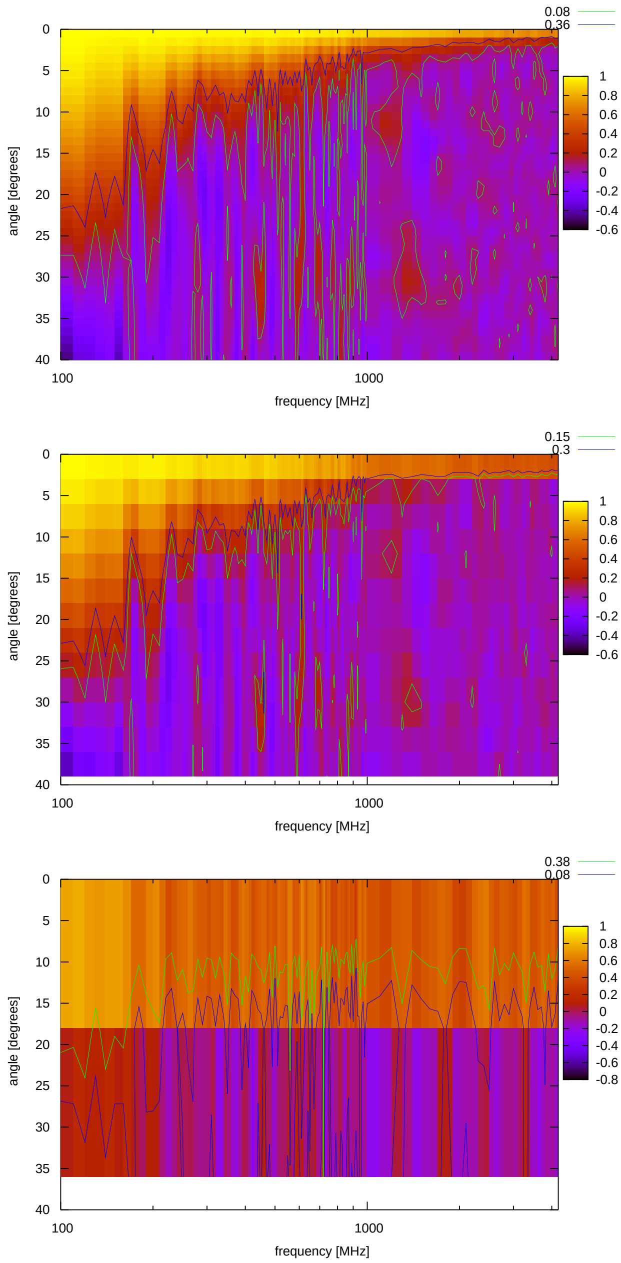

Fig. 7. Autocorrelation values ans critical limits according to LB and MD/BS for $N = 360(top)$, 120, 20.

and only on the basis of an example with a single sample size N. An approach to consider different sample sizes is given in the literature [8], but this approach leads to results — especially for low frequencies — that do not meet the daily measurement practice in mode-stirred chambers. It has been shown, that a small change of the null hypothesis can overcome that problem. This new null hypothesis implies that no perfect uncorrelation of the boundary conditions is called for, but the correlation has to be smaller than a certain limit, directly given by the degree of determination. Still meeting the $(N,\ \rho_0)$-pair from the standard's example it is now possible to give the critical limit ρ_0 for the autocorrelation coefficient

r for any number of measured tuner positions N.

The two free parameters — the degree of determination ρ^2 (ρ: expected value) and the significance level α — have to be established by the community. In Section III it is shown that $\rho^2 = 18.5\%$ ($\rho = 0.43$) and $\alpha = 0.05$ maintain the fix point given in the standard's example.

This work has been supported by the German Research Foundation (DFG) under contract number FOR417.

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