Aspects of Electromagnetic Coupling to Linear and Nonlinear Elements within a Rectangular Cavity

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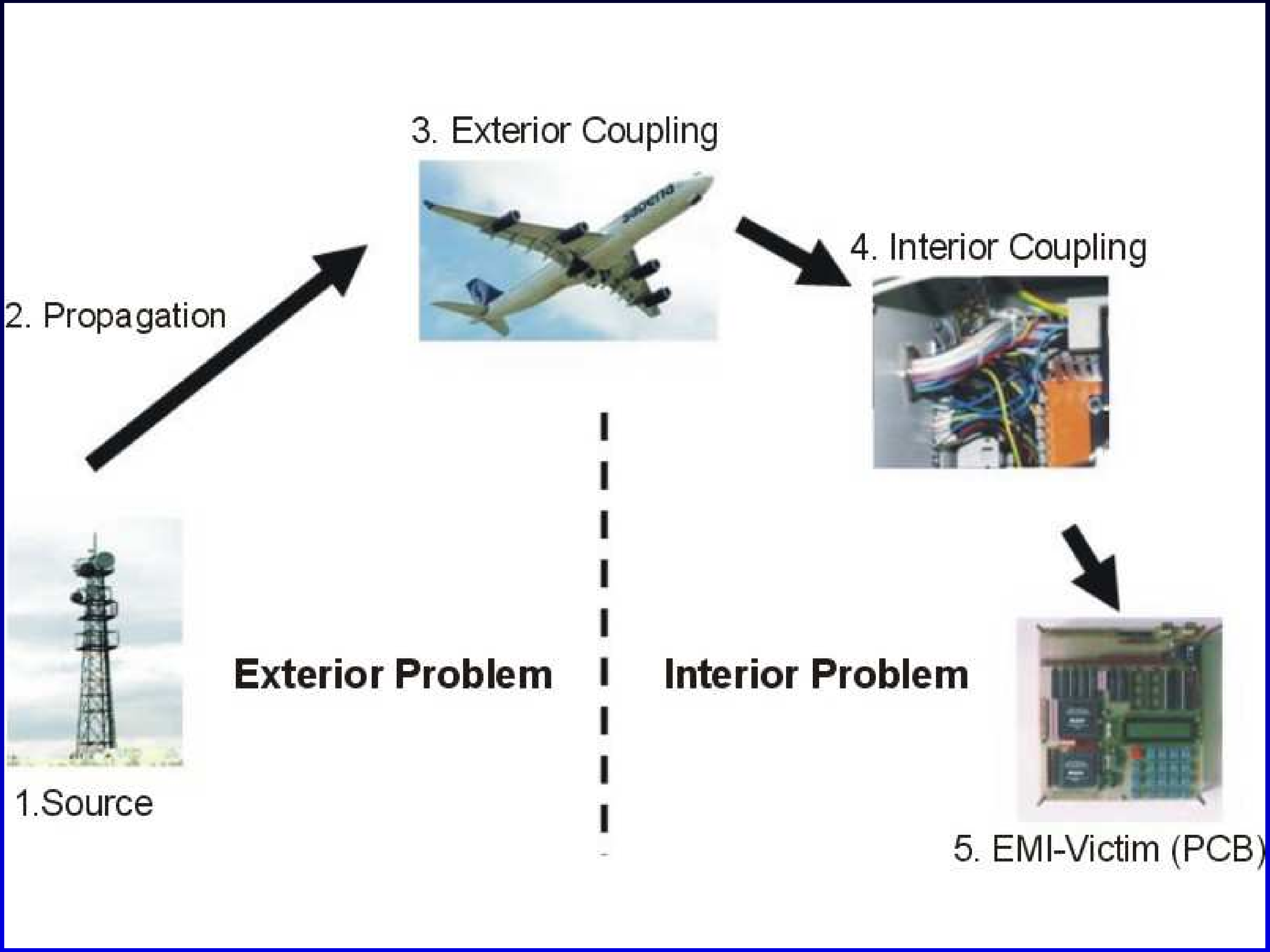
Coupling path of a common EMC problem



Problem formulation

Ingredients:

- Maxwell's equations
- Boundary conditions
- Radiation condition
- Edge conditions (finite energy)
- Material properties

But why an analytical approach?

- Yields understanding of underlying physical mechanisms
- Yields reference solutions for benchmark tests

Problems that are difficult to solve

Unsolved problems involve:

- cavities, containing metallic or dielectric scatterers
- apertures
- edges
- different media
- electrically large systems

Outline

We focus on:

Calculation of electric currents on one-dimensional structures inside of rectangular cavities

I. Analytical analysis of linear elements

- Method of Regularization

II. Numerical analysis of linear elements

- Method of Moments and effective Green’s functions

III. Experimental and analytical analysis of nonlinear elements

- Strong cavity excitation by demodulation effect

I. The method of analytical regularization

Suppose we have an equation of the first kind,

$$\boxed{Lx = b}$$

with $L = L_0 + L_1$ and L_0 contains a singular kernel. Suppose further that L_0 is analytically invertible. Then:

$$L_0^{-1}(L_0 + L_1)x = L_0^{-1}b$$

is of the form

$$\boxed{x + Hx = b'}$$

where H is compact. This is an equation of the second kind.

Advantage: better convergent and more stable!

I. General coupling to one-dimensional structures

Electric Field Integral Equation (EFIE):

$$\left(\int_{\text{1D-structure}} \underline{\underline{G}}(l,l')\underline{I}(l')\,dl'\right)_t = -E_t^{\text{inc}}(l)$$

or shorthand

$$\boxed{\hat{G}I = -E^{\text{inc}}}$$

Integral Equation of the first kind for the unknown current I.

In free space:

$$\hat{G} = \hat{G}_{\text{free}}$$

Within cavity:

$$\hat{G} = \hat{G}_{\text{cav}}$$

I. Solution by analytical regularization

Within cavity: Split integral operator in two parts,

$$\hat{G}_{\text{cav}} = \hat{G}_{\text{free}} + \hat{G}_{\text{mode}}$$

and know the inverse operator $\hat{G}_{\text{free}}^{-1}$. Then:

$$\hat{G}_{\text{free}} I_{\text{cav}} + \hat{G}_{\text{mode}} I_{\text{cav}} = -E^{\text{inc}}$$

$$\boxed{\Longrightarrow I_{\text{cav}} = \underbrace{-\hat{G}_{\text{free}}^{-1} E^{\text{inc}}}_{=: I_{\text{free}}} - \hat{G}_{\text{free}}^{-1} \hat{G}_{\text{mode}} I_{\text{cav}}}$$

Second order integral equation for the unknown current I. Solution by iteration:

$$\boxed{I_{\text{cav}} = \frac{I_{\text{free}}}{1 + \hat{G}_{\text{free}}^{-1} \hat{G}_{\text{mode}}}}$$

I. Example: Electrically small antenna

We consider first a linear wire antenna (length L, radius a, along z-axis) in free space, which is excited by an incoming wave

$$E_z^{\mathrm{inc}} = E_0 \sin\theta_i \exp(jkz\cos\theta_i)$$

Approximate solution for the induced current:

$$I_{\mathrm{free}}(z,\omega) = I_0 \Big[\Big(\cos(kz) - \exp(jkz\cos\theta_i) \Big) + \Big(\exp(jkL\cos\theta_i) - \cos(kL) \Big) \frac{\sin(kz)}{\sin(kL)} \Big]$$

where

$$I_0 := \frac{j4\pi E_0}{\eta_0 \Omega_0 k \sin\theta_i}\,, \qquad \Omega_0 := 2\ln(L/a)\,.$$

I. Example: Electrically small antenna

Low frequency approximation (kz, $kL \ll 1$):

$$\boxed{I_{\text{free}}(z,\omega) = K^{E}_{\text{free}}(\omega) f(z) E_z^{\text{inc}}}$$

where

$$K_{\text{free}}(\omega) = j\omega \frac{\pi\varepsilon_0 L}{2\ln(L/a)} \frac{L/2}{1-(kL)^2/6}$$

$$f(z) = 1 - \frac{4z^2}{L^2}$$

This approximate solution for the antenna current in free space will be used to obtain a solution for the antenna current within a rectangular cavity:

$$\boxed{I_{\text{cav}} = \frac{I_{\text{free}}}{1+\hat{G}^{-1}_{\text{free}}\hat{G}_{\text{mode}}}}$$

I. Example: Electrically small antenna within a rectangular cavity

Antenna of length L, $\lambda \gg L$, radius a, directed in z-direction:

$$\boxed{I_{\text{cav}}(z,\omega) = \frac{K_{\text{free}}(\omega) f(z)}{1 + K_{\text{free}}(\omega) \int G_{\text{mode}}(z,z') f(z') dz'} E_z^{\text{inc}}}$$

where

$$K_{\text{free}}(\omega) = j\omega \frac{\pi \varepsilon_0 L}{2 \ln(L/a)} \frac{L/2}{1 - (kL)^2/6}$$

$$f(z) = 1 - \frac{4z^2}{L^2}$$

Therefore we know: $J \rightarrow E$ and $E \rightarrow J$.

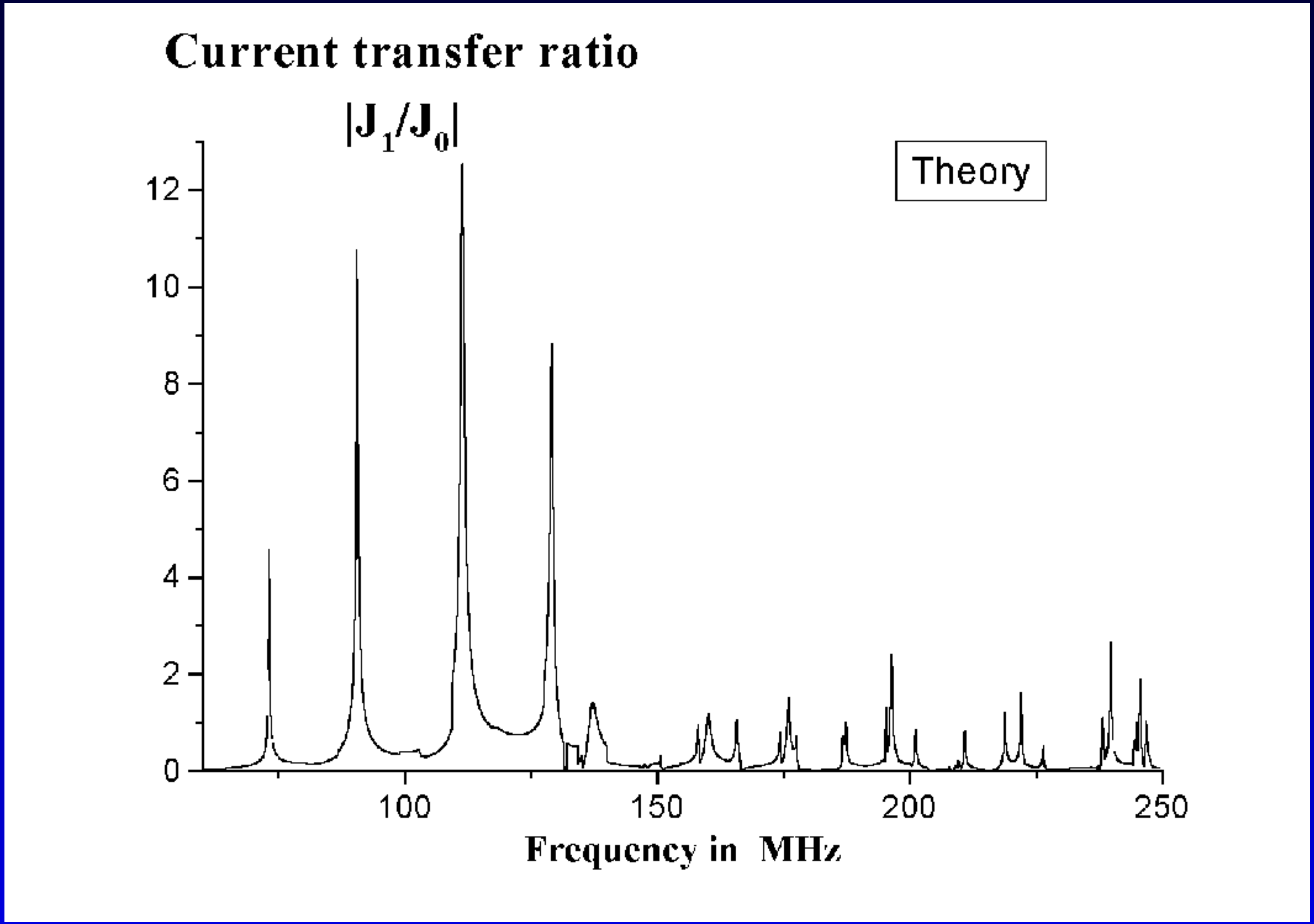
This results allows to calculate the coupling between two electrically small antennas within a rectangular cavity.

I. Example: Coupling between two electrically small antennas within a rectangular cavity

Consider an active and a passive antenna, both directed in $z-$direction:

- current on active antenna due to
  - free space part
  - selfinteraction with cavity
  - interaction with passive antenna within cavity
- current on passive antenna due to
  - interaction with active antenna within cavity

I. Example: Coupling between two electrically small antennas within a rectangular cavity



Outline

I. Analytical analysis of linear elements

- Method of Regularization

II. Numerical analysis of linear elements

- **Method of Moments and effective Green's functions**

III. Experimental and analytical analysis of nonlinear elements

- Strong cavity excitation by demodulation effect

II. Method of Moments and interior problems

General statement:

For interior problems and close to resonance the Method of Moments can become inaccurate.

Note:

Commercial Method of Moments codes take advantage of the Green's function of free space.

Our strategy:

- Use cavities' Green's function.
- Use a representation which quickly converges both in the source region and close to resonance.

II. Modes and Rays within a cavity

Modes:

- Describe the electromagnetic field by eigenfunctions of the cavity, i.e., by its resonant properties.

Rays:

- Describe the electromagnetic field by direct, scattered, or diffracted wave fronts, i.e., by the electromagnetic propagator.

Note:

Modes and Rays have complementary properties!

II. Modes versus Rays

Modes	Rays
yield *global* information	yield *local* information
characterize *late* response	characterize *early* response
advantageous for *low-frequency* components	advantageous for *high-frequency* components

<u>Hybrid-formulation:</u>

Combines advantages of both modes and rays!

II. Green's function of a lossy rectangular cavity I

Well-known: Mode expansion

$$\bar{\bar{G}}^{A}_{\mathrm{cav}\ zz}(\boldsymbol{r},\boldsymbol{r}',k) = \frac{4\mu_0}{abc}\sum_{n_x=1}^{\infty}\sum_{n_y=1}^{\infty}\sum_{n_z=0}^{\infty}\frac{\varphi_\nu(\boldsymbol{r})\varphi_\nu(\boldsymbol{r}')}{k_\nu^2-k^2}\varepsilon_{n_z,0}$$

$$\varphi_\nu(\boldsymbol{r}) := \sin(k_x x)\sin(k_y y)\cos(k_z z)$$

$$\nu \equiv (n_x, n_y, n_z)$$

$$\varepsilon_{n_z,0} = \begin{cases} 1 & \text{for } n_z = 0, \\ 2 & \text{for } n_z > 0. \end{cases}$$

- Advantage: Satisfying convergence close to resonance
- Disadvantage: Poor convergence close to the source region

II. Green's function of a lossy rectangular cavity II

Also well-known: Ray expansion

$$\bar{\bar{G}}^{A}_{\text{cav }zz}(\boldsymbol{r},\boldsymbol{r}',k) = \frac{\mu_0}{4\pi} \sum_{n_1,n_2,n_3=-\infty}^{\infty} (-1)^{n_1+n_2} \frac{e^{-jk\rho(\boldsymbol{r},\boldsymbol{r}',n_1,n_2,n_3)}}{\rho(\boldsymbol{r},\boldsymbol{r}',n_1,n_2,n_3)}$$

- Advantage: Satisfying convergence close to the source region
- Disadvantage: Poor convergence close to resonance

II. Green's function of a lossy rectangular cavity

Use "Ewald-transformation" to construct hybrid representation

$$\boxed{G^A_{zz} = \underbrace{G^A_{zz1}}_{\text{mode part}} + \underbrace{G^A_{zz2}}_{\text{ray part}}}$$

with excellent convergence properties!

$$G^A_{zz1} = \frac{\mu_0}{8abc} \sum_{m,n,p=-\infty}^{\infty} \sum_{i=0}^{7} A_i^{zz} \frac{\exp\left(-\frac{k_0^2-k^2}{4E^2}\right)}{k_0^2-k^2} \exp\big(j(k_{0x}X_i + k_{0y}Y_i + k_{0z}Z_i)\big)$$

$$G^A_{zz2} = \frac{\mu_0}{8\pi} \sum_{m,n,p=-\infty}^{\infty} \sum_{i=0}^{7} A_i^{zz} \Big[\frac{\exp(jkR_{i,mnp})\mathrm{erfc}(R_{i,mnp}E + jk/2E)}{R_{i,mnp}}$$

$$+ \frac{\exp(-jkR_{i,mnp})\mathrm{erfc}(R_{i,mnp}E - jk/2E)}{R_{i,mnp}}\Big]$$

II. Efficient Calculation of Complex Complementary Error Function

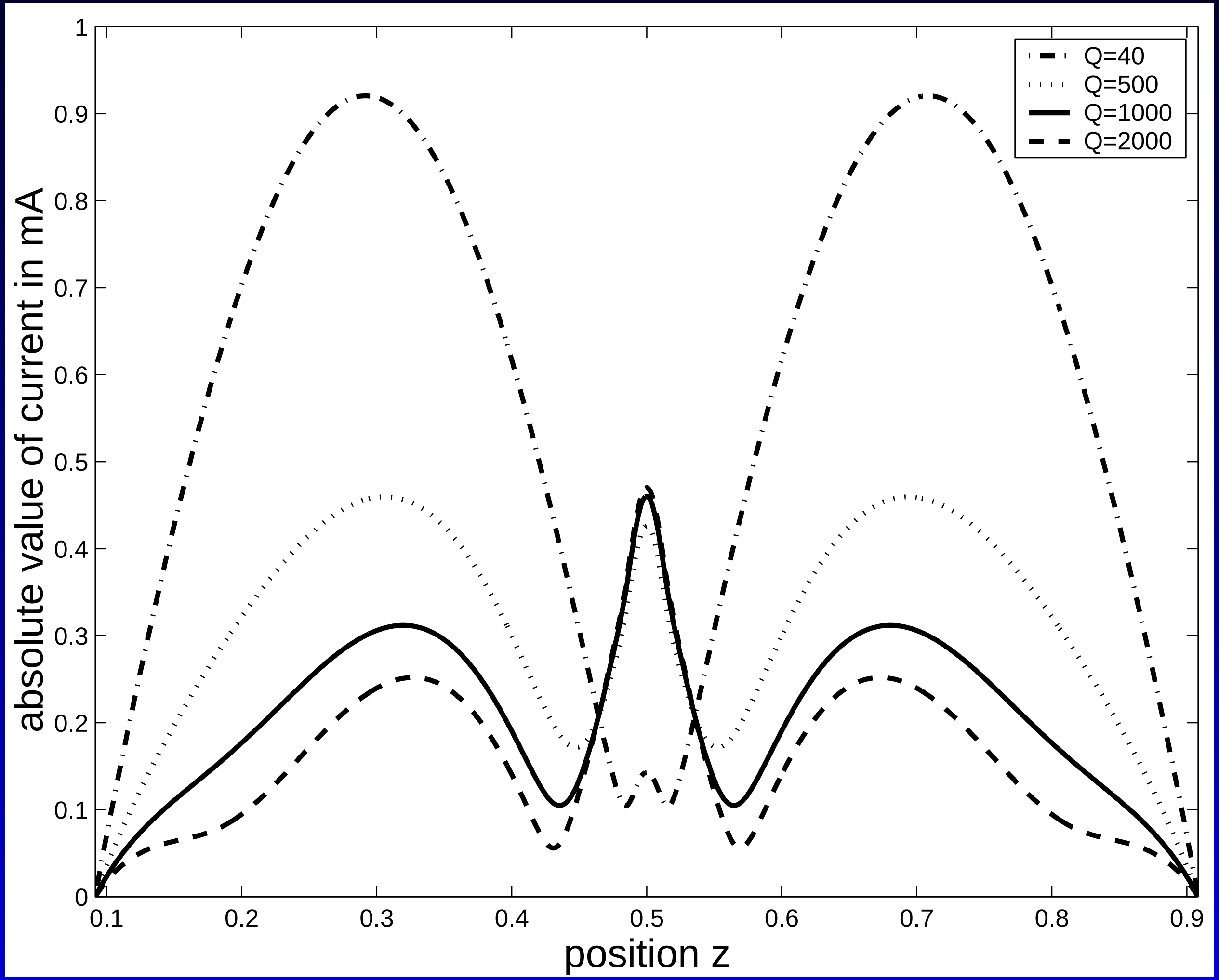
$$\boxed{erfc(z) := 1 - \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)\, dt}$$

Rational Expansion method, implemented in MATLAB (vector code):

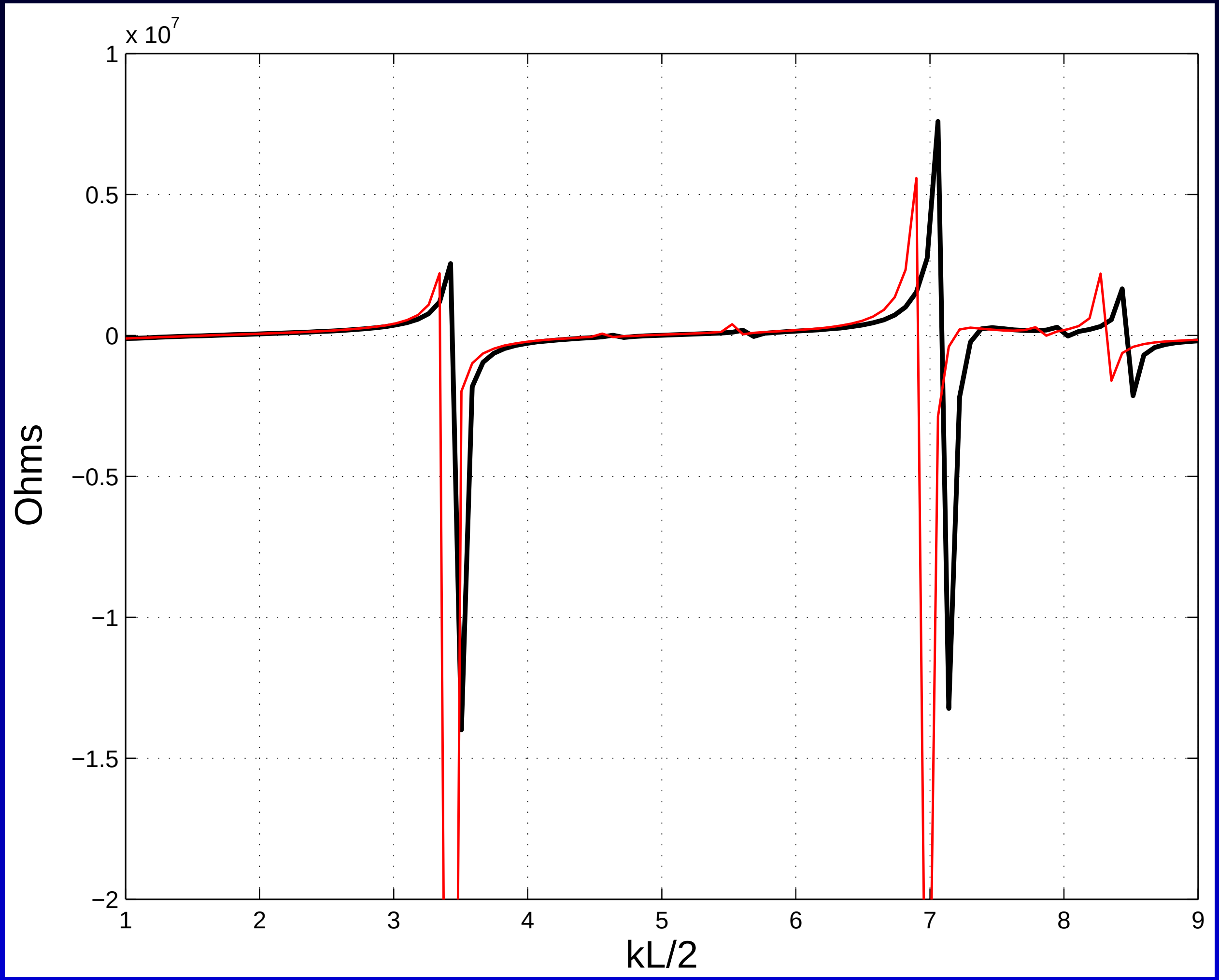
```
function w = cef(z,N)

z=-z/i;
M = 2*N; M2 = 2*M; k=[-M+1:1:M-1]';
L = sqrt(N/sqrt(2));
theta = k*pi/M; t=L*tan(theta/2);
f = exp(-t.^2).*(L^2+t.^2); f=[0;f];
a = real(fft(fftshift(f)))/M2;
a = flipud(a(2:N+1));
Z = (L+i*z)./(L-i*z); p=polyval(a,Z);
w = 2*p./(L-i*z).^2+(1/sqrt(pi))./(L-i*z);
w = 1-w./exp(-z*z);
```

II. Example: Solution of Hallen's equation inside a rectangular cavity



II. Input Impedance of a Dipole Antenna inside a cavity (imaginary part), (Q=500, Q=1000)



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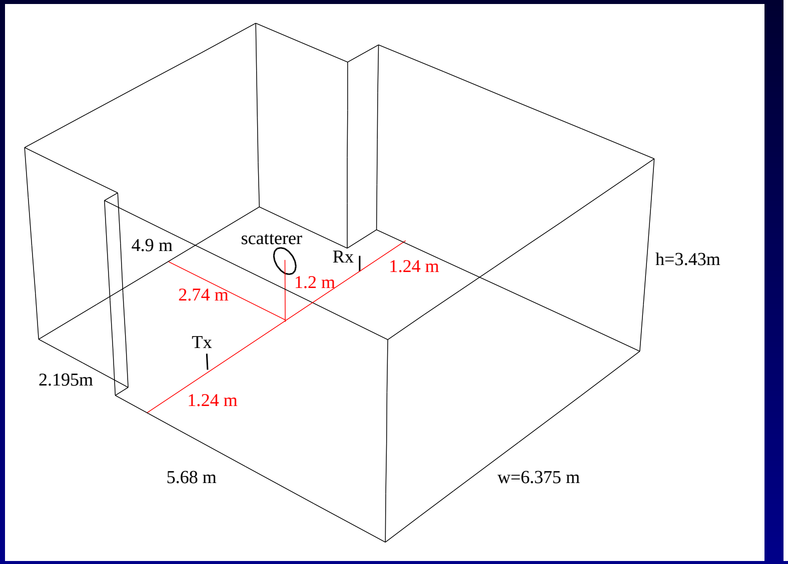
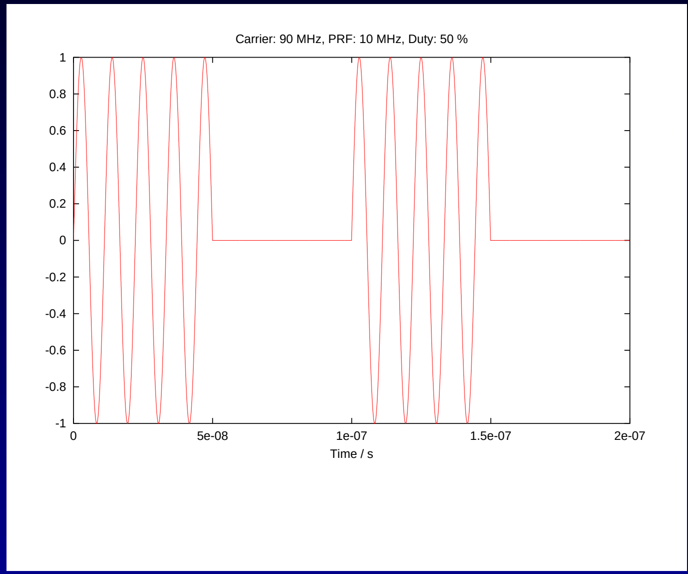
III. Demodulation at nonlinear elements

- Known problem: back-door coupling
  modulated high frequency signal couples into a system
  $\rightarrow$ demodulation at nonlinear elements yields low frequency components
  $\rightarrow$ low frequency components disturb the system
- System: rectangular cavity (MSC)
  Signal: generated by a transmitting antenna
  Victim: receiving antenna
- First: no nonlinear element present
  Second: introduce a nonlinear scatterer within the cavity

III. The MSC at the University of Magdeburg

III. Experimental Setup





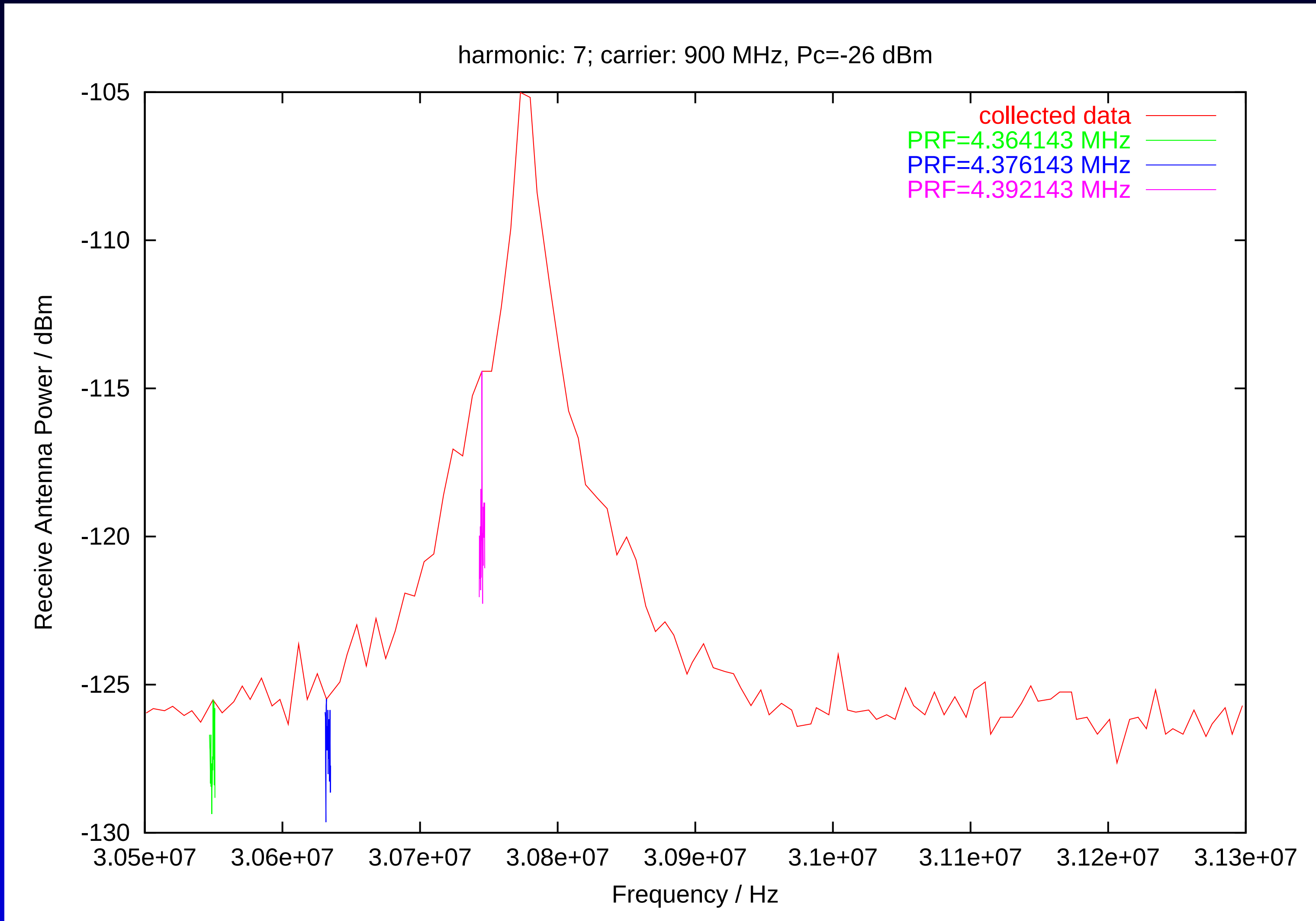
$$U(t) = U_0 F(t), \qquad F(t) = \sum_{n=0}^{N} f(t - nT_m)$$

$$f(t) = \sin(\omega_c t)\theta(t)\theta(T_p - t)$$

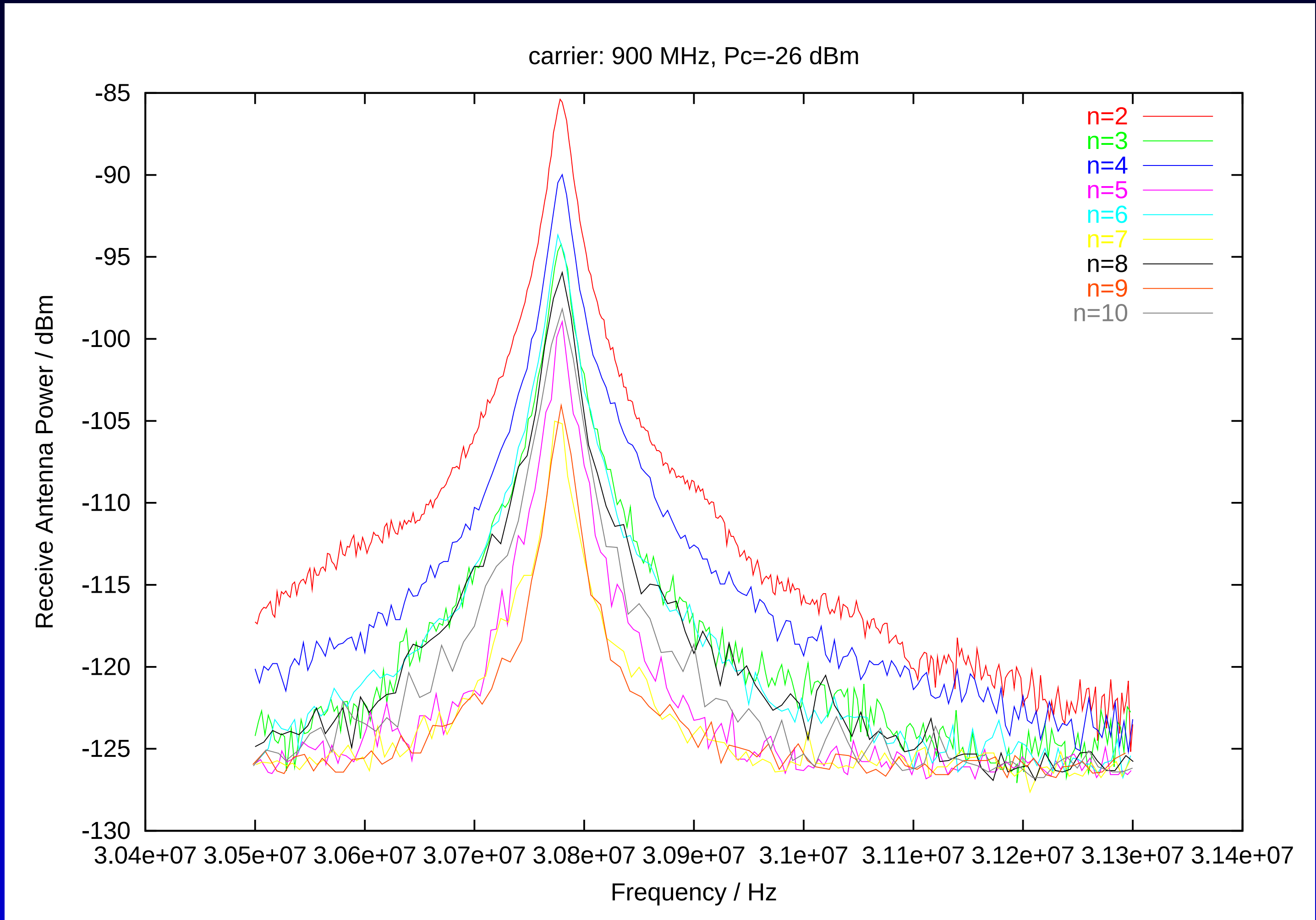
$N \gg 1, \quad \theta(t)$: unit step function, $\quad \omega_c \gg \omega_1$

$$\tilde{U}(\omega) = U_0 e^{-j\omega \frac{T_p}{2}} \frac{2j\omega_c}{\omega_c^2 - \omega^2} \frac{1 - e^{-j\omega T_m N}}{1 - e^{-j\omega T_m}} \sin \frac{\omega T_p}{2}$$

III. Variation of Pulse Repetition Rate



III. Results – empty resonator



III. Theoretical description

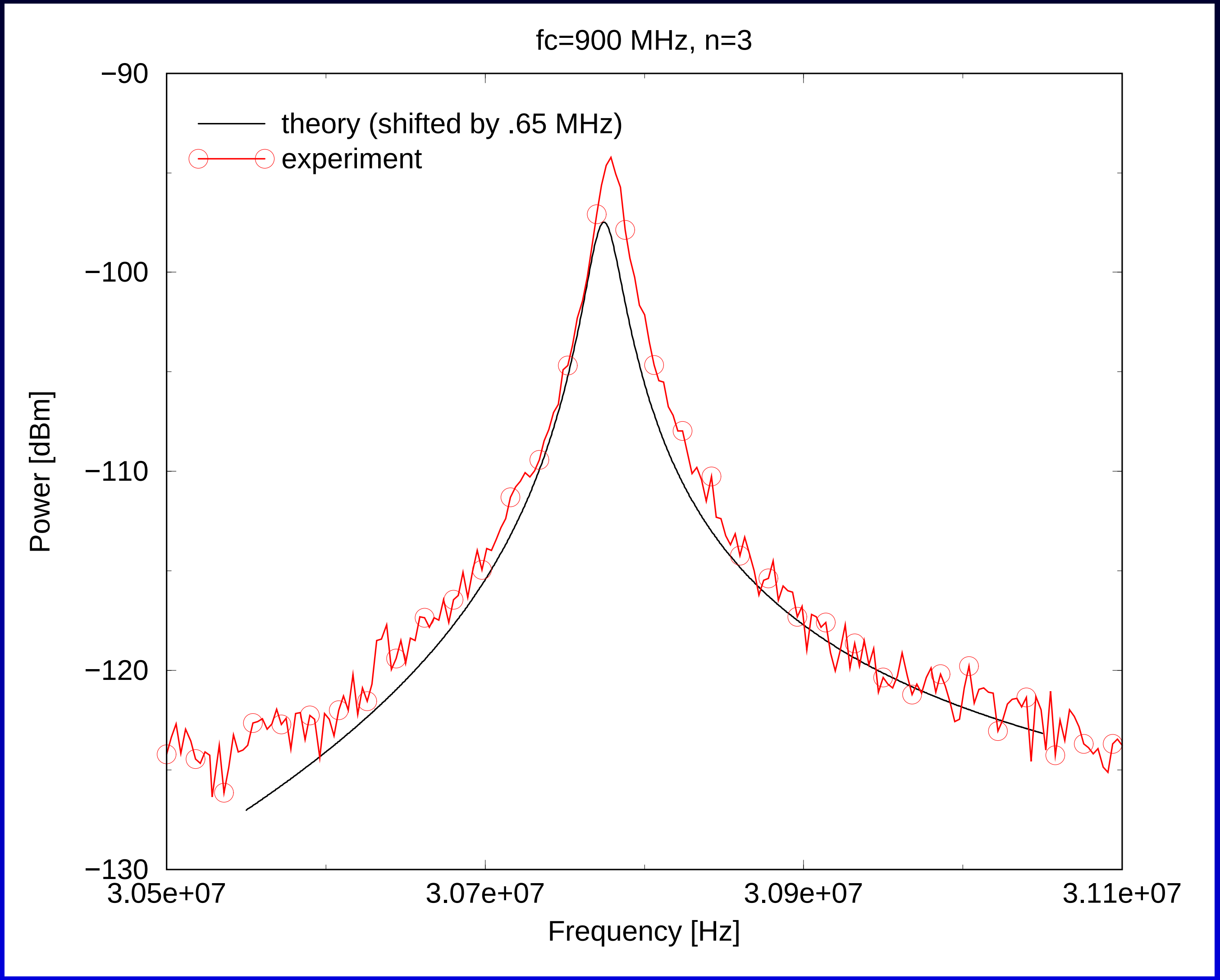
Current induced in the receiving antenna ($\longrightarrow$ received power) is given by

$$\boxed{I_R(\omega) = K_I(\omega) Y_{\text{in}}(\omega) \tilde{U}(\omega)}$$

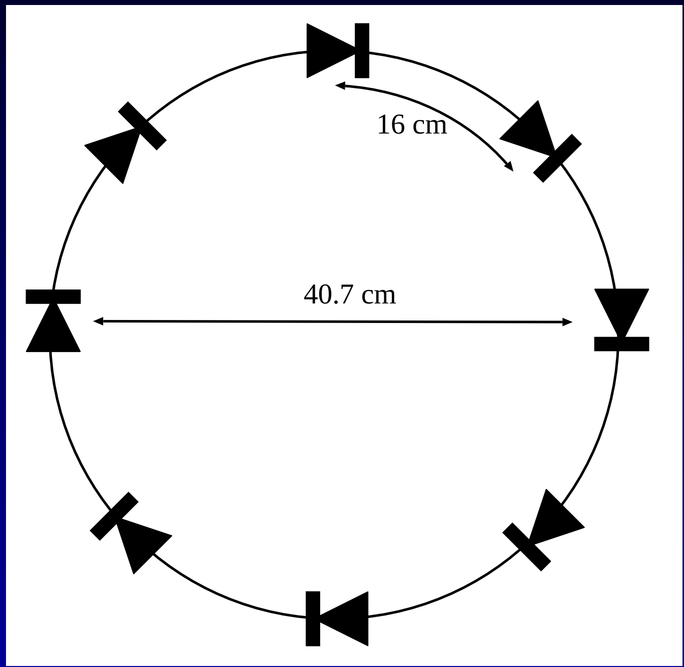
where

- $K_I(\omega)$: current transfer function (analytically known)
- $Y_{\text{in}}(\omega)$: input admittance of transmitting antenna (analytically and numerically known)
- $\tilde{U}(\omega)$: Fourier transform of feeding signal at input of transmitting antenna (analytically known)

III. Comparison Theory – Experiment



III. The nonlinear scatterer



Nonlinear Scatterer:

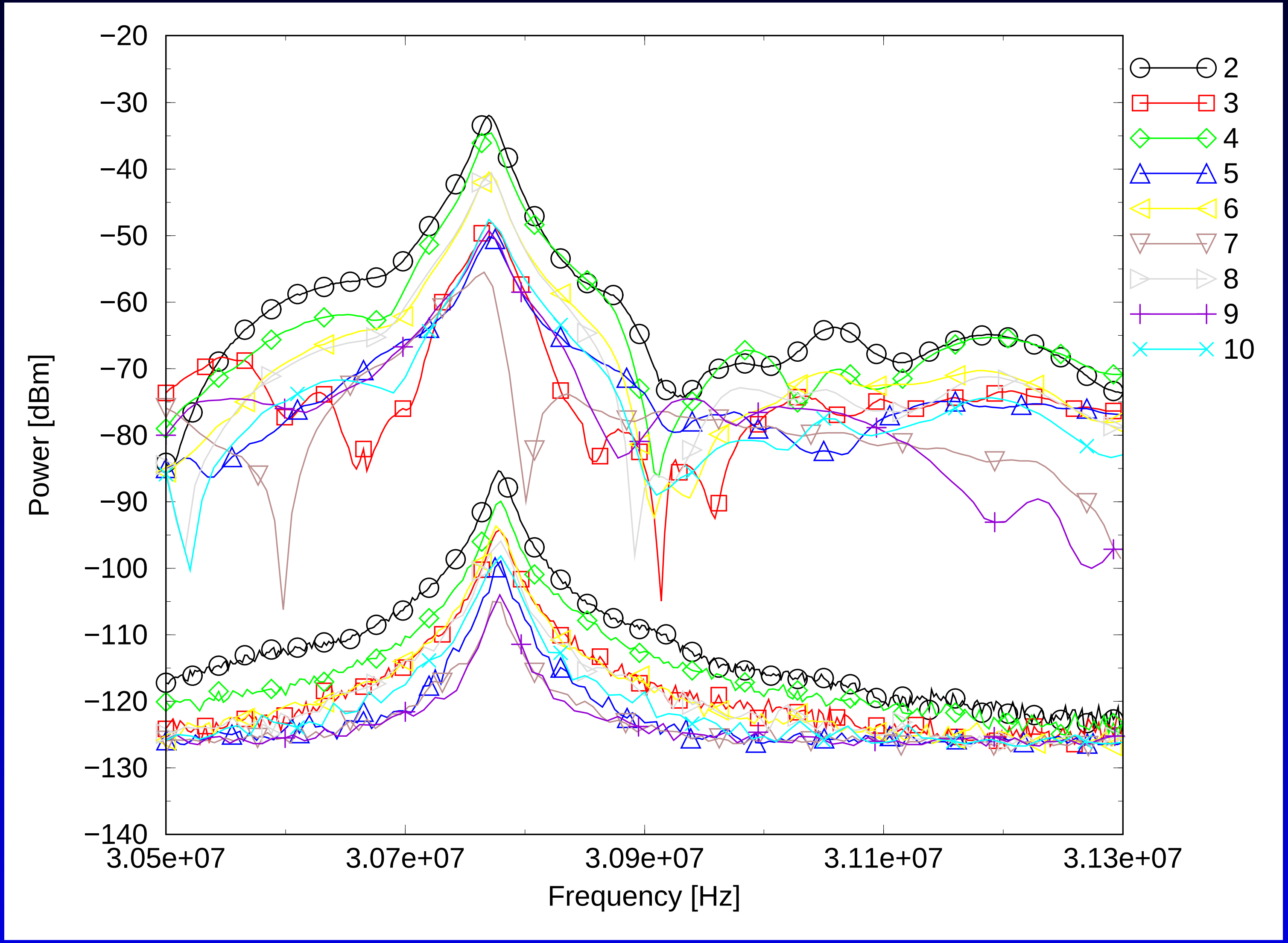
8 Schottky diodes

$\longrightarrow$ demodulation

$\longrightarrow$ current in loop contains low frequency components

$\longrightarrow$ low frequency components couple to cavity modes

III. Experimental Results



III. Amplification of low-frequency content

n	with scatterer	without scatterer	Δ / dB
2	-31.96 dBm	-85.39 dBm	53.43
3	-48.08 dBm	-94.23 dBm	46.15
4	-34.61 dBm	-89.98 dBm	55.37
5	-50.20 dBm	-98.95 dBm	48.75
6	-40.47 dBm	-93.66 dBm	53.19
7	-55.50 dBm	-105.01 dBm	49.51
8	-40.63 dBm	-95.95 dBm	55.32
9	-49.23 dBm	-104.05 dBm	54.82
10	-47.64 dBm	-98.17 dBm	50.53

power level at 900 MHz: -14 dBm $\longrightarrow$ -18 dBc

III. Theory: A simple model

Represent the electromagnetic field inside the resonator as independent, forced oscillators:

$$E_z(\boldsymbol{r},t) = \sum_\nu E_{\nu,z}(\boldsymbol{r},t)$$

$$\left(\frac{\partial^2}{\partial t^2} + 2\gamma_\nu \frac{\partial}{\partial t} + \omega_\nu^2\right) E_{\nu,z}(\boldsymbol{r},t) = E_{\nu,0}(\boldsymbol{r})\,\omega_{\nu,\rho}^2 F(t)$$

Suppose

$$\omega_1 \approx n\omega_m$$

Then:

$$\boxed{E_z(\boldsymbol{r},t) \approx E_{1,0}(\boldsymbol{r})\,\omega_1^2 \int_0^t F(t')K(t-t')\,dt'}$$

with

$$K(t) = \frac{\exp(-\gamma_\nu t)\sin(\sqrt{\omega_1^2-\gamma_1^2}\,t)\theta(t)}{\sqrt{\omega_1^2-\gamma_1^2}}$$

III. Summary of results

- Without nonlinear scatterer the low frequency components are enhanced by about 30 dB if the modulation frequency corresponds to a resonance of the cavity.
- This can be explained by a theoretical model.
- The presence of a nonlinear scatterer further enhances the low frequency components by 55 db.
- This can *qualitatively* be explained by a simple model (consider modes in the cavity as independent, forced harmonic oscillators).

Conclusion

- Electromagnetic coupling within cavities can greatly differ from that in free space.
- For analytical considerations it is advantageous to separate the coupling in a “free space” and a “mode part”.
- Numerical procedures must be able to deal both with source singularities and resonances.
- Within cavities, demodulation at nonlinearities can yield surprisingly large resonance effects.

Thank you for your attention!