Aspects of Electromagnetic Coupling to Linear and Nonlinear Elements within a Rectangular Cavity

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Coupling path of a common EMC problem
Problem formulation

Ingredients:

- Maxwell’s equations
- Boundary conditions
- Radiation condition
- Edge conditions (finite energy)
- Material properties

But why an analytical approach?

- Yields understanding of underlying physical mechanisms
- Yields reference solutions for benchmark tests
Problems that are difficult to solve

Unsolved problems involve:

- cavities, containing metallic or dielectric scatterers
- apertures
- edges
- different media
- electrically large systems
Outline

We focus on:
Calculation of electric currents on one-dimensional structures inside of rectangular cavities

I. Analytical analysis of linear elements
   • Method of Regularization

II. Numerical analysis of linear elements
    • Method of Moments and effective Green’s functions

III. Experimental and analytical analysis of nonlinear elements
    • Strong cavity excitation by demodulation effect
I. The method of analytical regularization

Suppose we have an equation of the first kind,

\[ Lx = b \]

with \( L = L_0 + L_1 \) and \( L_0 \) contains a singular kernel. Suppose further that \( L_0 \) is analytically invertible. Then:

\[
L_0^{-1}(L_0 + L_1)x = L_0^{-1}b
\]

is of the form

\[ x + Hx = b' \]

where \( H \) is compact. This is an equation of the second kind.

Advantage: better convergent and more stable!
I. General coupling to one-dimensional structures

Electric Field Integral Equation (EFIE):

\[
\left( \int_{\text{1D-structure}} G(l, l') I(l') \, dl' \right)_t = -E_{t}^{\text{inc}}(l)
\]

or shorthand

\[
\hat{G} I = -E^{\text{inc}}
\]

Integral Equation of the first kind for the unknown current \( I \).

In free space:

\[
\hat{G} = \hat{G}_{\text{free}}
\]

Within cavity:

\[
\hat{G} = \hat{G}_{\text{cav}}
\]
I. Solution by analytical regularization

Within cavity: Split integral operator in two parts,

\[ \hat{G}_{\text{cav}} = \hat{G}_{\text{free}} + \hat{G}_{\text{mode}} \]

and know the inverse operator \( \hat{G}_{\text{free}}^{-1} \). Then:

\[ \hat{G}_{\text{free}} I_{\text{cav}} + \hat{G}_{\text{mode}} I_{\text{cav}} = -E^{\text{inc}} \]

\[ \implies I_{\text{cav}} = -\hat{G}_{\text{free}}^{-1} E^{\text{inc}} - \hat{G}_{\text{free}}^{-1} \hat{G}_{\text{mode}} I_{\text{cav}} \]

Second order integral equation for the unknown current \( I \). Solution by iteration:

\[ I_{\text{cav}} = \frac{I_{\text{free}}}{1 + \hat{G}_{\text{free}}^{-1} \hat{G}_{\text{mode}}} \]
I. Example: Electrically small antenna

We consider first a linear wire antenna (length $L$, radius $a$, along $z$-axis) in free space, which is excited by an incoming wave

$$E_z^{\text{inc}} = E_0 \sin \theta_i \exp(jkz \cos \theta_i)$$

Approximate solution for the induced current:

$$I_{\text{free}}(z, \omega) = I_0 \left[ \left( \cos(kz) - \exp(jkz \cos \theta_i) \right) + \left( \exp(jkL \cos \theta_i) - \cos(kL) \right) \frac{\sin(kz)}{\sin(kL)} \right]$$

where

$$I_0 := \frac{j4\pi E_0}{\eta_0 \Omega_0 k \sin \theta_i}, \quad \Omega_0 := 2 \ln(L/a).$$
I. Example: Electrically small antenna

Low frequency approximation \((kz, kL \ll 1)\):

\[
I_{\text{free}}(z, \omega) = K_{\text{free}}^E(\omega) f(z) E_z^{\text{inc}}
\]

where

\[
K_{\text{free}}(\omega) = j\omega \frac{\pi\varepsilon_0 L}{2 \ln(L/a)} \frac{L/2}{1 - (kL)^2/6}
\]

\[
f(z) = 1 - \frac{4z^2}{L^2}
\]

This approximate solution for the antenna current in free space will be used to obtain a solution for the antenna current within a rectangular cavity:

\[
I_{\text{cav}} = \frac{I_{\text{free}}}{1 + \hat{G}_{\text{free}}^{-1} \hat{G}_{\text{mode}}}
\]
I. Example: Electrically small antenna within a rectangular cavity

Antenna of length $L$, $\lambda \gg L$, radius $a$, directed in $z$-direction:

$$I_{\text{cav}}(z, \omega) = \frac{K_{\text{free}}(\omega)f(z)}{1 + K_{\text{free}}(\omega) \int G_{\text{mode}}(z, z')f(z')dz'} E_{\text{inc}}^{\text{inc}}$$

where

$$K_{\text{free}}(\omega) = j\omega \frac{\pi \varepsilon_0 L}{2 \ln(L/a)} \frac{L/2}{1 - (kL)^2/6}$$

$$f(z) = 1 - \frac{4z^2}{L^2}$$

Therefore we know: $J \rightarrow E$ and $E \rightarrow J$.

This results allows to calculate the coupling between two electrically small antennas within a rectangular cavity.
I. Example: Coupling between two electrically small antennas within a rectangular cavity

Consider an active and a passive antenna, both directed in $z$–direction:

- current on active antenna due to
  - free space part
  - selfinteraction with cavity
  - interaction with passive antenna within cavity
- current on passive antenna due to
  - interaction with active antenna within cavity
I. Example: Coupling between two electrically small antennas within a rectangular cavity

![Current transfer ratio graph](image-url)
Outline

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   - Strong cavity excitation by demodulation effect
II. Method of Moments and interior problems

General statement:
For interior problems and close to resonance the Method of Moments can become inaccurate.

Note:
Commercial Method of Moments codes take advantage of the Green’s function of free space.

Our strategy:

- Use cavities’ Green’s function.
- Use a representation which quickly converges both in the source region and close to resonance.
II. Modes and Rays within a cavity

Modes:

- Describe the electromagnetic field by eigenfunctions of the cavity, i.e., by its resonant properties.

Rays:

- Describe the electromagnetic field by direct, scattered, or diffracted wave fronts, i.e., by the electromagnetic propagator.

Note:

Modes and Rays have complementary properties!
## II. Modes versus Rays

<table>
<thead>
<tr>
<th>Modes</th>
<th>Rays</th>
</tr>
</thead>
<tbody>
<tr>
<td>yield <em>global</em> information</td>
<td>yield <em>local</em> information</td>
</tr>
<tr>
<td>characterize <em>late</em> response</td>
<td>characterize <em>early</em> response</td>
</tr>
<tr>
<td>advantageous for <em>low-frequency</em> components</td>
<td>advantageous for <em>high-frequency</em> components</td>
</tr>
</tbody>
</table>

**Hybrid-formulation:**
Combines advantages of both modes and rays!
II. Green’s function of a lossy rectangular cavity I

Well-known: Mode expansion

\[
\tilde{G}_{cav}^{zz}(r, r', k) = \frac{4\mu_0}{abc} \sum_{n_x=1}^{\infty} \sum_{n_y=1}^{\infty} \sum_{n_z=0}^{\infty} \frac{\varphi_{\nu}(r)\varphi_{\nu}(r')}{k_{\nu}^2 - k^2} \varepsilon_{n_z,0}
\]

\[
\varphi_{\nu}(r) := \sin(k_{x}x)\sin(k_{y}y)\cos(k_{z}z)
\]

\[
\nu \equiv (n_x, n_y, n_z)
\]

\[
\varepsilon_{n_z,0} = \begin{cases} 
1 & \text{for } n_z = 0, \\
2 & \text{for } n_z > 0.
\end{cases}
\]

- Advantage: Satisfying convergence close to resonance
- Disadvantage: Poor convergence close to the source region
II. Green’s function of a lossy rectangular cavity II

Also well-known: Ray expansion

\[
\tilde{G}_{\text{cav}}^{A}(r, r', k) = \frac{\mu_0}{4\pi} \sum_{n_1, n_2, n_3 = -\infty}^{\infty} (-1)^{n_1+n_2} e^{-jk\rho(r, r', n_1, n_2, n_3)} \frac{e^{-jk\rho(r, r', n_1, n_2, n_3)}}{\rho(r, r', n_1, n_2, n_3)}
\]

- Advantage: Satisfying convergence close to the source region
- Disadvantage: Poor convergence close to resonance
II. Green’s function of a lossy rectangular cavity

Use “Ewald-transformation” to construct hybrid representation

\[ G_{zz}^A = G_{zz1}^A + G_{zz2}^A \]

mode part \hspace{1cm} ray part

with excellent convergence properties!

\[ G_{zz1}^A = \frac{\mu_0}{8abc} \sum_{m,n,p=-\infty}^{\infty} \sum_{i=0}^{7} A_{i}^{zz} \exp\left(-\frac{k_0^2-k_i^2}{4E^2}\right) \exp\left(j(k_0xX_i + k_0yY_i + k_0zZ_i)\right) \]

\[ G_{zz2}^A = \frac{\mu_0}{8\pi} \sum_{m,n,p=-\infty}^{\infty} \sum_{i=0}^{7} A_{i}^{zz} \left[ \exp(jkR_{i,mnp}) \text{erfc}\left(\frac{R_{i,mnp}E + jk/2E}{R_{i,mnp}}\right) \right. \]
\[ + \left. \exp(-jkR_{i,mnp}) \text{erfc}\left(\frac{R_{i,mnp}E - jk/2E}{R_{i,mnp}}\right) \right] \]
II. Efficient Calculation of Complex Complementary Error Function

\[
erfc(z) := 1 - \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) \, dt
\]

Rational Expansion method, implemented in MATLAB (vector code):

```matlab
function w = cef(z,N)

z=-z/i;
M = 2*N; M2 = 2*M; k=[-M+1:1:M-1]';
L = sqrt(N/sqrt(2));
theta = k*pi/M; t=L*tan(theta/2);
f = exp(-t.^2).*(L^2+t.^2); f=[0;f];
a = real(fft(fftshift(f)))/M2;
a = flipud(a(2:N+1));
Z = (L+i*z)./(L-i*z); p=polyval(a,Z);
w = 2*p./(L-i*z).^2+(1/sqrt(pi))/(L-i*z);
w = 1-w./exp(-z*z);
```

– p.21
II. Example: Solution of Hallen’s equation inside a rectangular cavity
II. Input Impedance of a Dipole Antenna inside a cavity (imaginary part), (Q=500, Q=1000)
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III. Demodulation at nonlinear elements

- Known problem: back-door coupling
  modulated high frequency signal couples into a system
  → demodulation at nonlinear elements yields low
  frequency components
  → low frequency components disturb the system

- System: rectangular cavity (MSC)
  Signal: generated by a transmitting antenna
  Victim: receiving antenna

- First: no nonlinear element present
  Second: introduce a nonlinear scatterer within the cavity
III. The MSC at the University of Magdeburg
III. Experimental Setup

\[ U(t) = U_0 F(t), \quad F(t) = \sum_{n=0}^{N} f(t - nT_m) \]

\[ f(t) = \sin(\omega_c t)\theta(t)\theta(T_P - t) \]

\[ N \gg 1, \quad \theta(t): \text{unit step function}, \quad \omega_c \gg \omega_1 \]

\[ \tilde{U}(\omega) = U_0 e^{-j\omega \frac{T_P}{2}} \frac{2j\omega_c}{\omega_c^2 - \omega^2} \frac{1 - e^{-j\omega T_m}N}{1 - e^{-j\omega T_m}} \sin \frac{\omega T_P}{2} \]
III. Variation of Pulse Repetition Rate

harmonic: 7; carrier: 900 MHz, Po=-26 dBm

collected data
PRF=4.364143 MHz
PRF=4.376143 MHz
PRF=4.392143 MHz
III. Results – empty resonator

carrier: 900 MHz, $P_c=-26$ dBm

Receive Antenna Power / dBm vs. Frequency / Hz

- $n=2$
- $n=3$
- $n=4$
- $n=5$
- $n=6$
- $n=7$
- $n=8$
- $n=9$
- $n=10$
III. Theoretical description

Current induced in the receiving antenna (→ received power) is given by

$$I_R(\omega) = K_I(\omega)Y_{\text{in}}(\omega)\tilde{U}(\omega)$$

where

- $K_I(\omega)$: current transfer function (analytically known)
- $Y_{\text{in}}(\omega)$: input admittance of transmitting antenna (analytically and numerically known)
- $\tilde{U}(\omega)$: Fourier transform of feeding signal at input of transmitting antenna (analytically known)
III. Comparison Theory – Experiment

fc = 900 MHz, n = 3

- Theory (shifted by .65 MHz)
- Experiment
III. The nonlinear scatterer

Nonlinear Scatterer: 8 Schottky diodes

→ demodulation
→ current in loop contains low frequency components
→ low frequency components couple to cavity modes
III. Experimental Results
III. Amplification of low-frequency content

<table>
<thead>
<tr>
<th>$n$</th>
<th>with scatterer</th>
<th>without scatterer</th>
<th>$\Delta / \text{dB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-31.96 dBm</td>
<td>-85.39 dBm</td>
<td>53.43</td>
</tr>
<tr>
<td>3</td>
<td>-48.08 dBm</td>
<td>-94.23 dBm</td>
<td>46.15</td>
</tr>
<tr>
<td>4</td>
<td>-34.61 dBm</td>
<td>-89.98 dBm</td>
<td>55.37</td>
</tr>
<tr>
<td>5</td>
<td>-50.20 dBm</td>
<td>-98.95 dBm</td>
<td>48.75</td>
</tr>
<tr>
<td>6</td>
<td>-40.47 dBm</td>
<td>-93.66 dBm</td>
<td>53.19</td>
</tr>
<tr>
<td>7</td>
<td>-55.50 dBm</td>
<td>-105.01 dBm</td>
<td>49.51</td>
</tr>
<tr>
<td>8</td>
<td>-40.63 dBm</td>
<td>-95.95 dBm</td>
<td>55.32</td>
</tr>
<tr>
<td>9</td>
<td>-49.23 dBm</td>
<td>-104.05 dBm</td>
<td>54.82</td>
</tr>
<tr>
<td>10</td>
<td>-47.64 dBm</td>
<td>-98.17 dBm</td>
<td>50.53</td>
</tr>
</tbody>
</table>

Power level at 900 MHz: $-14$ dBm $\rightarrow$ $-18$ dBc
III. Theory: A simple model

Represent the electromagnetic field inside the resonator as independent, forced oscillators:

\[ E_z(r, t) = \sum_{\nu} E_{\nu, z}(r, t) \]

\[ \left( \frac{\partial^2}{\partial t^2} + 2\gamma_{\nu} \frac{\partial}{\partial t} + \omega_{\nu}^2 \right) E_{\nu, z}(r, t) = E_{\nu, 0}(r) \omega_{\nu, \rho}^2 F(t) \]

Suppose

\[ \omega_1 \approx n\omega_m \]

Then:

\[ E_z(r, t) \approx E_{1, 0}(r) \omega_1^2 \int_0^t F(t') K(t - t') dt' \]

with

\[ K(t) = \frac{\exp(-\gamma_{\nu} t) \sin(\sqrt{\omega_1^2 - \omega_1^2 t}) \theta(t)}{\sqrt{\omega_1^2 - \omega_1^2}} \]
III. Summary of results

- Without nonlinear scatterer the low frequency components are enhanced by about 30 dB if the modulation frequency corresponds to a resonance of the cavity.
- This can be explained by a theoretical model.
- The presence of a nonlinear scatterer further enhances the low frequency components by 55 dB.
- This can qualitatively be explained by a simple model (consider modes in the cavity as independent, forced harmonic oscillators).
Conclusion

- Electromagnetic coupling within cavities can greatly differ from that in free space.
- For analytical considerations it is advantageous to separate the coupling in a “free space” and a “mode part”.
- Numerical procedures must be able to deal both with source singularities and resonances.
- Within cavities, demodulation at nonlinearities can yield surprisingly large resonance effects.

Thank you for your attention!