Determination of the Number of Statistically Independent Boundary Conditions of Mode-Stirred Chambers

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• **Mode-Stirred Chambers:**
  - Cavity resonator
  - Changing boundaries by the tuner(s)
  - Sufficient high mode density
  \[ \Rightarrow \text{get a statistical homogeneous field distribution} \]

• **IEC 61000-4-21**

[background: tuner (MSC Magdeburg)]
E-field measurements for a sample of tuner positions
\[ \{ E_1, E_2, \ldots E_{N-1}, E_N \} \]

Determination of statistical independency by means of autocorrelation
\[
r = \sum_{\nu=0}^{N-1} \frac{(E_\nu - \langle E \rangle)(E_\nu^* - \langle E \rangle)}{\sum_{\nu=0}^{N-1} (E_\nu - \langle E \rangle)^2}, \quad -1 \leq r \leq 1
\]

| \[ r \] | 1 perfect correlated, \( r = 0 \) perfect uncorrelated
Starting point: Measurement is a sample of size $N$

$\Rightarrow$ Correlation coefficient is affected by statistical variations

true autocorrelation coefficient only if sample size $N \to \infty$
\[
\psi(r) = \frac{N - 2}{\sqrt{2\pi}} \cdot \frac{\Gamma(N - 1)}{\Gamma\left(N - \frac{1}{2}\right)} \cdot \left(1 - \rho^2\right)^{\frac{(N-1)}{2}} \left(1 - r^2\right)^{\frac{N-4}{2}} \frac{1}{(1 - \rho r)^{N-\frac{3}{2}}} \left[1 + \frac{1 + \rho r}{4(2N - 1)} + \ldots\right]
\]
Distribution known from literature

\[ \Rightarrow \psi(r) \]

- \( \rho \) – expected value

- \( \text{true value for } N \to \infty \)

- \( N \) – sample size

- \( \rho_0 \) – limit

\[ \alpha = \frac{\rho_0}{\int_{-1}^{1} \psi(r) \, dr} \]

- \( \alpha \) – probability for values less than \( \rho_0 \) if the expected value is \( \rho \)
Comparison of two statistical approaches based on a publication of Lundén und Bäckström (IEEE EMC, 2000):

- $\rho = 0$, $\rightarrow$ perfect uncorrelated data

- **MD/BS:**
  - $\rho > \rho_{\text{critical}} > 0$, $\rightarrow$ correlation stronger than $\rho_{\text{critical}}$

![Graph showing comparison of statistical approaches](graph.png)
Results

original data set: \( N = 360 \) (1 degree)
reduced data sets: \( N = 120 \) (3 degrees) and \( N = 20 \) (18 degrees)

for LB approach: number of independent tuner positions increase with decreasing \( N \)

for MD/BS approach: number of independent tuner positions decrease with decreasing \( N \)
MSC MD: $f = 200$ MHz, #POS for different $N$, $\alpha=0.05$
Conclusions

- **Insufficiency in the standard IEC 61000-4-21:**
  - Limit for the autocorrelation function
    
    only given for a fixed $N$

- **Approach of Lundén und Bäckström:**
  - Pointed out that $N$ has to be taken into account
  - Calculations are correct, but improper starting point ($H_0$)

- **Addendum of the standard:**
  - Explanation of the critical limit in
    
    its statistical context
  - normative specification of the expected value for $\Psi(r)$
Test of Hypotheses

<table>
<thead>
<tr>
<th>$H_0$ correct</th>
<th>accept $H_0$</th>
<th>reject $H_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$ incorrect</td>
<td>no error</td>
<td>alpha error</td>
</tr>
<tr>
<td></td>
<td>beta error</td>
<td>no error</td>
</tr>
</tbody>
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$H_0$: $r > r_0$
$H_1$: $r \leq r_0$

Probability to obtain a measured $r \leq r_0 = 0.19$ from a set of $N = 36$ if the expected (true) value is $r = 0.43$ is less than $\alpha = 5\%$ (even smaller for larger $r$)

$\Psi(r)$ for $r = 0.43$, $N = 36$