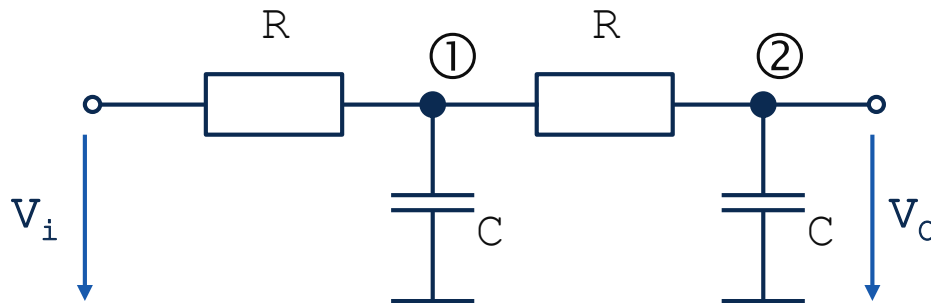


## Continuous Time Example

- 2-stage RC low-pass, equal R and C



Node equations (sum of outgoing currents):

$$\textcircled{1} \quad \mathbf{0} = -\frac{1}{R}V_i + \frac{2}{R}V_1 + C\dot{V}_1 - \frac{1}{R}V_2$$

$$\textcircled{2} \quad \mathbf{0} = -\frac{1}{R}V_1 + \frac{1}{R}V_2 + C\dot{V}_2$$

## Euler forward:

Solve ① for  $\dot{V}_1$  and ② for  $\dot{V}_2$

Introduce constant time steps  $\Delta t$ , sufficiently short.  
At every time step calculate

$$V_{1,2}(t + \Delta t) = V_{1,2}(t) + \Delta t \cdot \dot{V}_{1,2}(t)$$

with  $\dot{V}_{1,2}(t)$  as above.

Source code: `rc_lowpass/source/behavioral/euler_forward.vhd`

Euler **backward**:

Solve ① for  $\dot{V}_1$  and ② for  $\dot{V}_2$  and introduce constant time steps  $\Delta t$

Use this to solve the equation system of Euler backward formulae

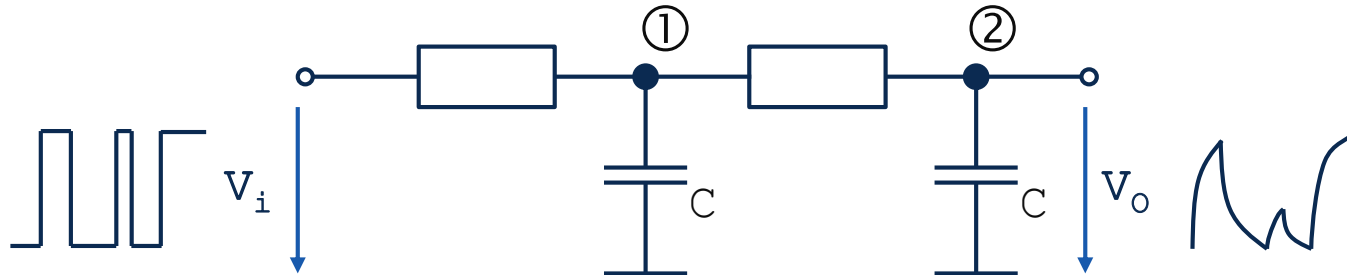
$$V_1(t + \Delta t) = V_1(t) + \Delta t \cdot \dot{V}_1(t + \Delta t)$$

$$V_2(t + \Delta t) = V_2(t) + \Delta t \cdot \dot{V}_2(t + \Delta t)$$

for  $V_1(t + \Delta t)$  and  $V_2(t + \Delta t)$ .

Source code: `rc_lowpass/source/behavioral/euler_backward.vhd`

Explicit: Known Waveform for piecewise constant input  $V_i(t)$



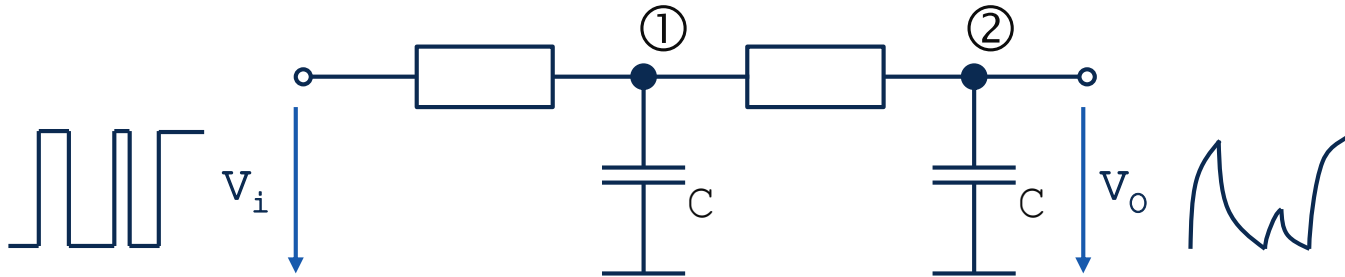
Start with node equations (sum of outgoing currents):

$$\textcircled{1} \quad 0 = -\frac{1}{R}V_i + \frac{2}{R}V_1 + C\dot{V}_1 - \frac{1}{R}V_2$$

$$\textcircled{2} \quad 0 = -\frac{1}{R}V_1 + \frac{1}{R}V_2 + C\dot{V}_2$$

Solve the differential equation system for  $V_1(t)$  and  $V_2(t)$  assuming  $V_i = \text{const}$

Explicit: Known Waveform for piecewise constant input  $V_i(t)$



Use  $\tau = \frac{1}{\omega} = RC$  to write the solution:

$$V_2(t) = V_i + V_A \cdot e^{z_1 t} + V_B \cdot e^{z_2 t}$$

$$V_1(t) = V_i + V_A \cdot e^{z_1 t} + V_B \cdot e^{z_2 t} + V_A \cdot \tau z_1 e^{z_1 t} + V_B \cdot \tau z_2 e^{z_2 t}$$

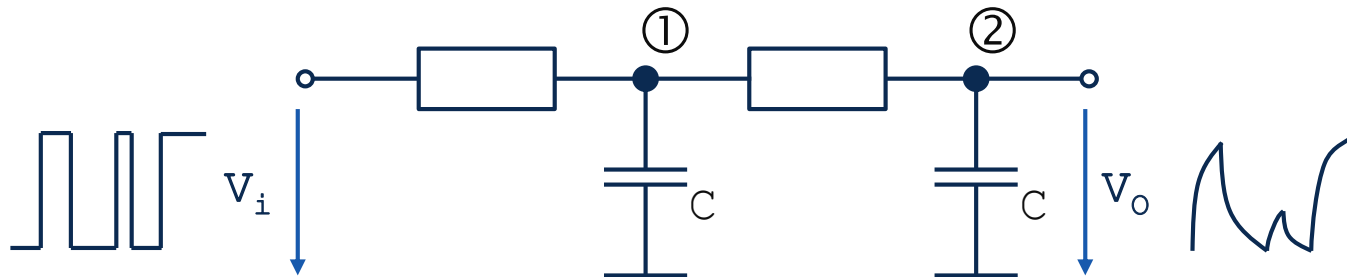
with

$$z_{1,2} = \frac{\omega}{2} \cdot (3 \pm \sqrt{5})$$

$V_A$  and  $V_B$  are constant voltage values

determined by the initial conditions for  $V_1$  and  $V_2$ .

Explicit: Known Waveform for piecewise constant input  $V_i(t)$

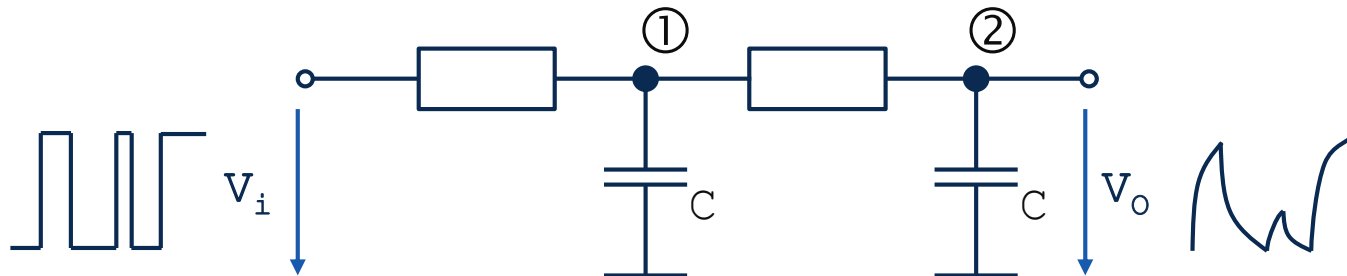


The input voltage  $V_i$  changes only at points in time  $t_1, t_2, t_3$ , etc.

Use known values of  $V_1(t_1)$  and  $V_2(t_1)$  to determine values of  $V_A$  and  $V_B$  for  $t \in [t_1, t_2]$ . Calculate  $V_1(t_2)$  and  $V_2(t_2)$ . Final values at  $t_2$  are the initial values for the next interval  $t \in [t_2, t_3]$ . Proceed to  $t_3, t_4$ , etc.

Source code: **`rc_lowpass/source/behavioral/explicit.vhd`**

Test Bench: PWM input  $V_i(t)$



Test Bench: `rc_lowpass/source/behavioral/rc_lowpass.vhd`

- creates a PWM signal with a single tone sinusoid as the payload signal
- applies the PWM signal as input to the RC lowpass.
- There are 3 instances of the lowpass: one for each modeling approach

Run for 400  $\mu$ s ("run 400 us" in the ncsim console)