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Laser Metrology Exercise – List of Tasks

Introductory example: The planar parallel plate from different points of view: Calculate the offset of the beam path (geometrical optics) and the phase delay (wave optics) when a laser beam passes through a planar parallel plate.

Exercise 1: Show that a) the plane wave and b) the spherical wave are solutions of the wave equation.

Exercise 2: Jones matrix formalism I: Basics
a) Please derive the Jones vector for i) an oscillation of the electric field only in the x-direction, ii) an oscillation of the electric field in the diagonal direction, iii) an oscillation of the electric field with a phase shift of 90° between the x- and y-components.
b) Please derive the Jones matrix for an optical element that rotates the polarization angle α of linearly polarized light by a certain angle β.
c) Please derive the Jones matrix for an optical element that induces a phase shift of $\pi/2$ between the two polarization directions. How can this be achieved with a birefringent crystal, and what should be its thickness?

Exercise 3: Jones matrix formalism II: Wave Plate
a) How should a quarter-wave plate and linearly polarized light be oriented with respect to each other to generate circularly polarized light behind the wave plate?
b) Right- handed circular polarized light falls onto a half-wave plate. What is the polarization state of the transmitted light?
c) Please use the Jones matrix formalism, demonstrate how to realize an optical circulator with a polarizing beam splitter and a quarter-wave plate ($\lambda/4$ plate).

Exercise 4: Discussion of the Gaussian beam. Derive expressions for the intensity profile in radial and axial directions, as well as for the optical power.

Exercise 5: Please show: In a Keplerian telescope, the waist of a Gaussian beam transforms according to the laws of geometric optics.

Exercise 6: From the datasheet of a broad-area laser diode (wavelength λ = 830 nm), one extracts an emitter area of 1.5 µm x 100 µm as well as the far-field divergence angles of 40° and 10°, respectively, for directions perpendicular and parallel to the pn junction. Calculate the beam quality factors and the number of oscillating modes in both directions!

Exercise 7: What maximum beam quality factor M^2 is allowed for a laser beam with a wavelength of 633 nm, so that it can be focused to a diameter of $2w_0$ = 20 µm using a lens with a focal length of f = 100 mm and a usable free aperture (diameter) of D = 2 cm? What is the beam waist radius for an ideal Gaussian beam?

Exercise 8: Calculate the diffraction pattern a) of a double slit and b) of a diffraction grating (=multiple slit) using the phase-correct summation of the electric fields.

Exercise 9: Calculate the diffraction pattern of a single slit using the Fraunhofer diffraction theory.

Exercise 10: Derive the resolution limit of an optical system (Abbe limit) via the imaging of a periodic structure and compare the result with the beam parameter product of a Gaussian beam.

Exercise 11: Show using the rate equations: In a pure two-level system, neither thermal nor optical pumping can achieve population inversion.

Exercise 12: Mode coupling

a) How many longitudinal resonator modes can oscillate above the oscillation threshold for a gain bandwidth Δf_{gain} of the gain profile?

b) Derive an expression for the temporal intensity dependency of a mode-locked laser. What is the relationship between pulse power and the number of coupled modes?

c) Calculate the pulse peak intensity and pulse duration of a mode-locked Nd:YAG laser (wavelength λ=1064 nm, refractive index n=1.5, gain bandwidth Δf_{gain}= 3 THz, resonator length L= 1 dm).

Exercise 13: Coherence. Please discuss: What requirements must be placed on the spectrum of a light source and on an interferometer to still observe interference? How does a low degree of coherence manifest experimentally?

Exercise 14: Calculate the change in optical path length of a measurement path of L_{geo} = 1 m length, induced by the influence of environmental parameters using the Edlén formula. Standard values: Temperature variation ΔT = 2 °C, air pressure variation Δp = 120 hPa, and the variation of the relative humidity Δf_r= 20 %. How constant must these environmental conditions be kept to achieve a measurement uncertainty ΔL < 0.1 µm on the measurement path?

Exercise 15: Derive the free spectral range of a Fabry-Perot resonator (mirror spacing L, mirror reflectivity R) based on the interference of all (infinitely many) partial beams. Derive expressions for the maximum and minimum intensity as well as for the half-width.

Exercise 16: Acousto-Optic Modulator (AOM)
a) Derive the diffraction angle from the condition of constructive interference in Bragg reflection (classical derivation) for laser optics.
b) Derive the frequency shift between neutral and first order of diffraction using the Doppler effect (classical derivation).
c) Quantum mechanical consideration: Derive the diffraction angle and frequency shift using the conservation laws of energy and momentum in photon-phonon scattering.
d) Calculate the deflection angle of a HeNe laser beam (λ= 632.8 nm) when passing through an AOM made of TeO2 (sound velocity vs= 4200 m/s, sound frequency (80±2.5) MHz). How does the deflection angle of the light beam change when modulating the sound frequency over the entire available range?

Exercise 17: Faraday Effect
a) What is the relationship between the transmission of a Faraday modulator and the electric current I_s flowing through it?
b) Construct an optical isolator based on the Faraday effect, i.e., an element that allows light to pass only in one direction.

Exercise 18: Derive the phase shift induced by rotation at a frequency of Ω= 1 rad/s for a Sagnac interferometer (with radius R= 0.1 mm and wavelength λ= 0.5 µm). How can the sensitivity be increased by using fibers?

Exercise 19: Optical fibers
a) Derive calculate the acceptance angle of a light-guiding fiber from the refractive indices n_K and n_M of the core and cladding, respectively, as well as the critical wavelength λ_c , below which light propagation becomes multimode.
b) How many modes can be supported in a step-index fiber with a numerical aperture A_N =0.16 and a core diameter D=200 µm at a wavelength of λ=660 nm? What is the beam quality factor (diffraction factor) of the radiation emerging from the fiber?

Exercise 20: Doppler Effect / Laser Doppler Anemometry
Please determine the relationship between Doppler frequency and velocity in the process of laser Doppler anemometry (LDA) for laser optics:

a) through the optical Doppler effect in a linear approximation
b) by superimposing the electric fields of two intersecting plane waves
c) about the optical path length difference with a double slit (see exercise 8a)

(Proof of the equivalence of the Doppler effect and the interference fringe model).

Exercise 21: Show: If a diffraction grating is used instead of a prism for beam splitting in a laser Doppler anemometer, the fringe spacing in the middle of the measurement volume no longer depends on the wavelength (achromatic LDA).

Exercise 22: Show: The measurement of Doppler broadening (line broadening due to Doppler shift and Maxwell velocity distribution) can be utilized in gases for temperature determination.

Exercise 23: Discuss Brillouin Scattering in optical fibers