TECHNISCHE UNIVERSITÄT DRESDEN

Fakultät für Elektrotechnik und Informationstechnik • Institut für Automatisierungstechnik

Modeling and Control of a Fully-actuated Hexarotor with Fixed Tilted Rotors

Introduction

In recent times, research and development of unmanned aerial vehicles (UAVs) has become very popular. The conventional multicopters with collinear rotors can not exert any force parallel to the plane perpendicular to the vertical axis of the body. However, high precision force and motion control for physical interaction commonly requires fully actuated vehicle to apply full wrench on the target. This can be achieved by tilting the rotors of a hexarotor w.r.t. the vertical axis of the body frame. This work proposes and evaluates integral sliding mode control for a fully-actuated hexarotor.

System Modeling

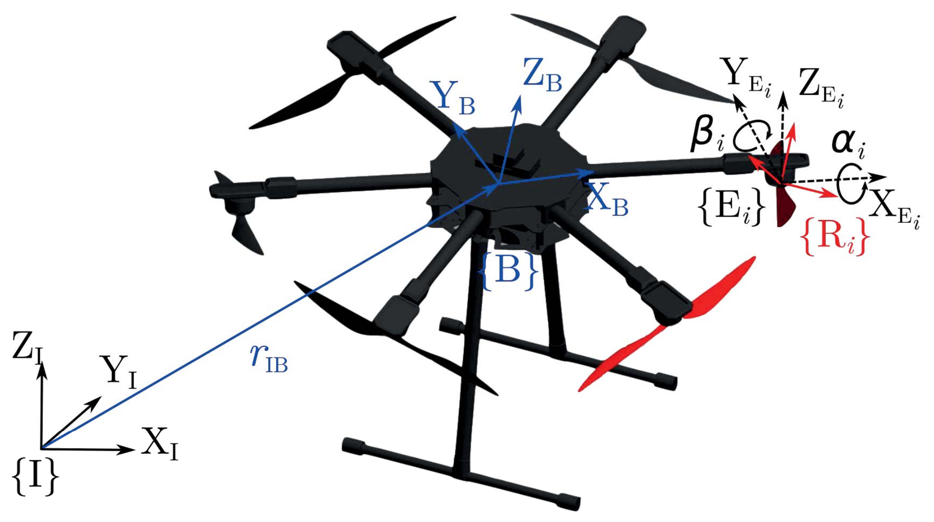

Figure1: Definition of coordinate frames

Translational motion: $m\ddot{\boldsymbol{p}} = \boldsymbol{f}_G + \boldsymbol{f}_T + \boldsymbol{f}_{ext}$

Rotational motion: $\boldsymbol{\tau}_T + \boldsymbol{\tau}_D + \boldsymbol{\tau}_{ext} = \boldsymbol{I}_B\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{I}_B\boldsymbol{\omega}$

with

$$\boldsymbol{f}_T = \sum_{i=1}^{6} {}^{I}_{B}\boldsymbol{R} \cdot {}^{B}_{E_i}\boldsymbol{R} \cdot {}^{E_i}_{R_i}\boldsymbol{R} \cdot \mathrm{sgn}(\omega_i) k\omega_i^2$$

$$\boldsymbol{f}_G = [0 \quad 0 \quad -mg]^{\mathrm{T}}$$

$$\boldsymbol{\tau}_T = \sum_{i=1}^{6} {}^{I}\boldsymbol{r}_{R_i} \times {}^{B}\boldsymbol{f}_{T,i}$$

$$\boldsymbol{\tau}_D = \sum_{i=1}^{6} {}^{B}_{E_i}\boldsymbol{R} \cdot {}^{E_i}_{R_i}\boldsymbol{R} \cdot (-1)^i \cdot \mathrm{sgn}(\omega_i) b\omega_i^2 \ ,$$

where $\boldsymbol{f}_T$ is the total thrust in inertial frame, $\boldsymbol{\tau}_T$ is the torque caused by thrust and $\boldsymbol{\tau}_D$ indicates the drag torque.

Control Design

Translational part:

- Sliding surface: $\boldsymbol{s} = \boldsymbol{z}_2 + \lambda_T \boldsymbol{z}_1$
- Stabilize $\boldsymbol{s}$ to $\boldsymbol{s} = \boldsymbol{0}$ using Lyapunov function $V = \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{s}$:

$$\dot{V} = \boldsymbol{s}^{\mathrm{T}}\dot{\boldsymbol{s}} = \boldsymbol{s}^{\mathrm{T}}(-\ddot{\boldsymbol{p}}_d - g\boldsymbol{e}_z + \frac{1}{m}\boldsymbol{u}_T + \lambda_T \boldsymbol{z}_2)$$

$$\rightarrow \boldsymbol{u}_T = m\left[\ddot{\boldsymbol{p}}_d + g\boldsymbol{e}_z - \lambda_T \boldsymbol{z}_2 - \boldsymbol{K} \cdot \frac{2}{\pi}\arctan(\boldsymbol{s})\right]$$

- Sliding manifold: $\boldsymbol{s}^* = \boldsymbol{s} - \boldsymbol{s}(0) - \int_0^t -\boldsymbol{K} \cdot \mathrm{sign}(\boldsymbol{s}) d\tau$

$$\rightarrow \boldsymbol{u}_T^I = -\boldsymbol{K}_I \cdot \frac{2}{\pi}\arctan(\boldsymbol{s}^*)$$

- Control law: $\boldsymbol{u}_T^* = \boldsymbol{u}_T^I + \boldsymbol{u}_T$

Rotational part

- Error represented with $\boldsymbol{z}_1 := [1 - |\tilde{\eta}| \quad \tilde{\varepsilon}]^{\mathrm{T}}$, $\boldsymbol{z}_2 := \dot{\boldsymbol{z}}_1$ and $\widetilde{\boldsymbol{\omega}} := \boldsymbol{\omega}$- $\boldsymbol{\omega}_d$
- Sliding surface $\boldsymbol{s}$ and Lyapunov function defined same as translational motion

$$\rightarrow \boldsymbol{u}_R = -\boldsymbol{I}_B \boldsymbol{G}(\tilde{\boldsymbol{q}}) \left[\lambda_R \boldsymbol{G}^{\mathrm{T}}(\tilde{\boldsymbol{q}}) \cdot \widetilde{\boldsymbol{\omega}} + \dot{\boldsymbol{G}}^{\mathrm{T}}(\tilde{\boldsymbol{q}}) \cdot \widetilde{\boldsymbol{\omega}} + 2\boldsymbol{K} \cdot\right.$$

- Integral part is designed analogously to translational motion:

$$\boldsymbol{u}_R^I = -\boldsymbol{K}_I \cdot \frac{2}{\pi}\arctan(\frac{1}{2}\boldsymbol{G}^{\mathrm{T}}(\tilde{\boldsymbol{q}}) \cdot \boldsymbol{I}_B^{-1} \cdot \boldsymbol{s}^*)$$

- Control law: $\boldsymbol{u}_T^* = \boldsymbol{u}_T^I + \boldsymbol{u}_T$

Simulation

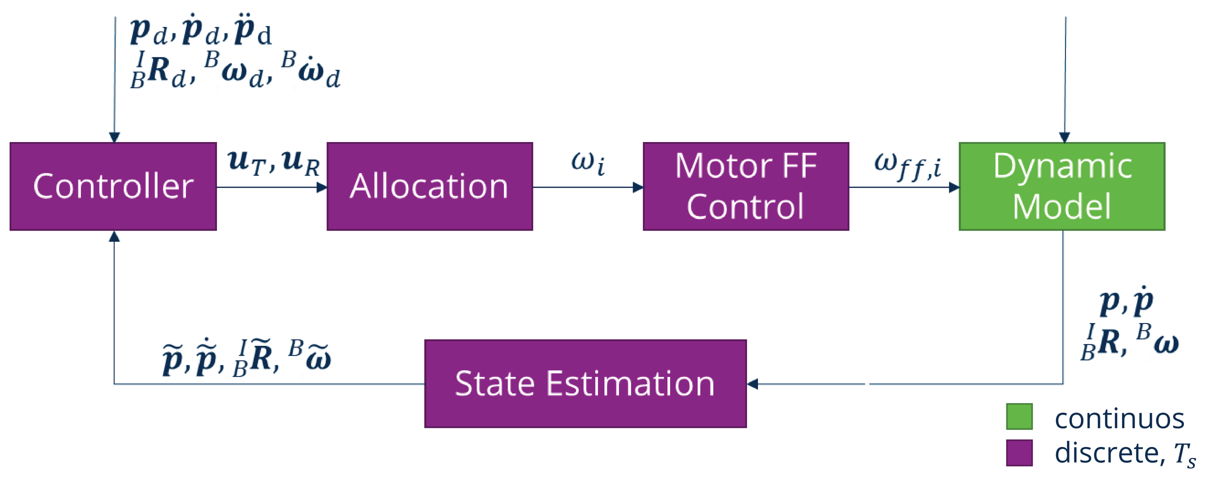

Figure 2: Control loop structure

The simulation environment simulates the state estimation by a VIVE® Lighthouse tracking system. The uncertainty parameters are listed in Table 1. Furthermore, the model parameter are randomly varied between an interval of ±15 %. The proposed controller is tested together with a PID controller with feedback linearization.

Table 1: Simulation parameters of state estimation

Parameter	$3\sigma_p$	$3\sigma_v$	$3\sigma_\varphi$	$3\sigma_\omega$	T_{delay}	T_s
Value	0.3 mm	0.11 m/s	0.02 °	7.1 °/s	4 ms	4 ms

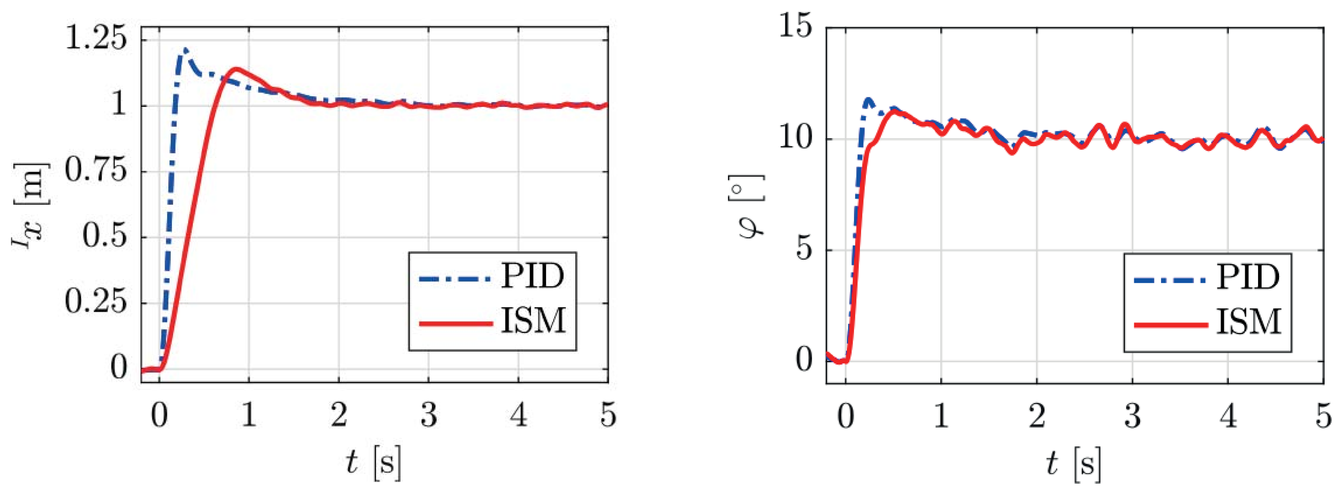

Figure 3: Step response for steps to $x = 1\mathrm{m}$ and $\varphi = 10°$

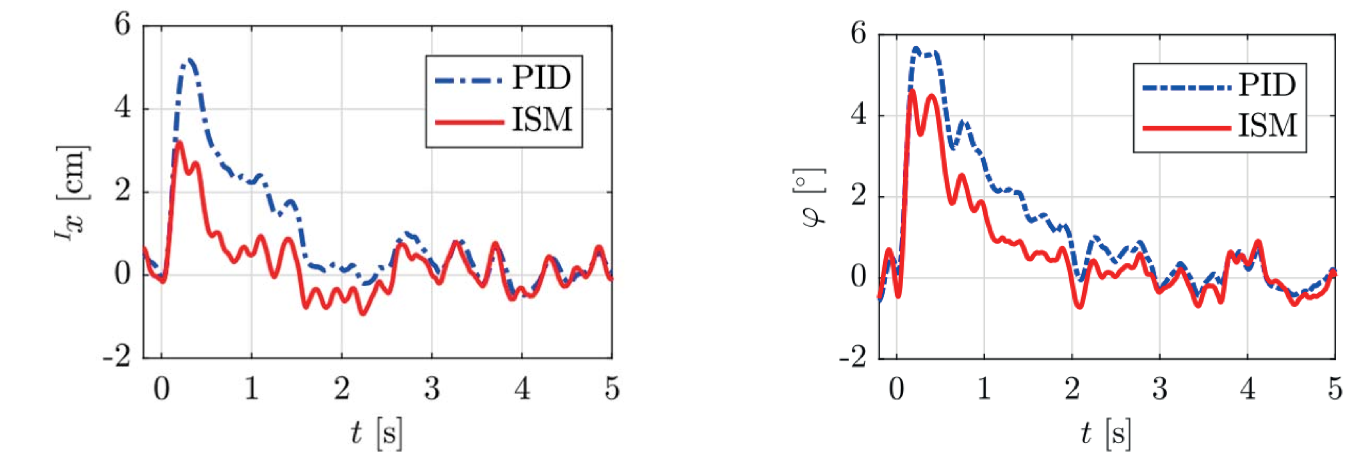

Figure 4: Disturbance step response to $f_{ext,x} = 10\mathrm{N}$ and $\tau_{ext,x} = 5\mathrm{Nm}$

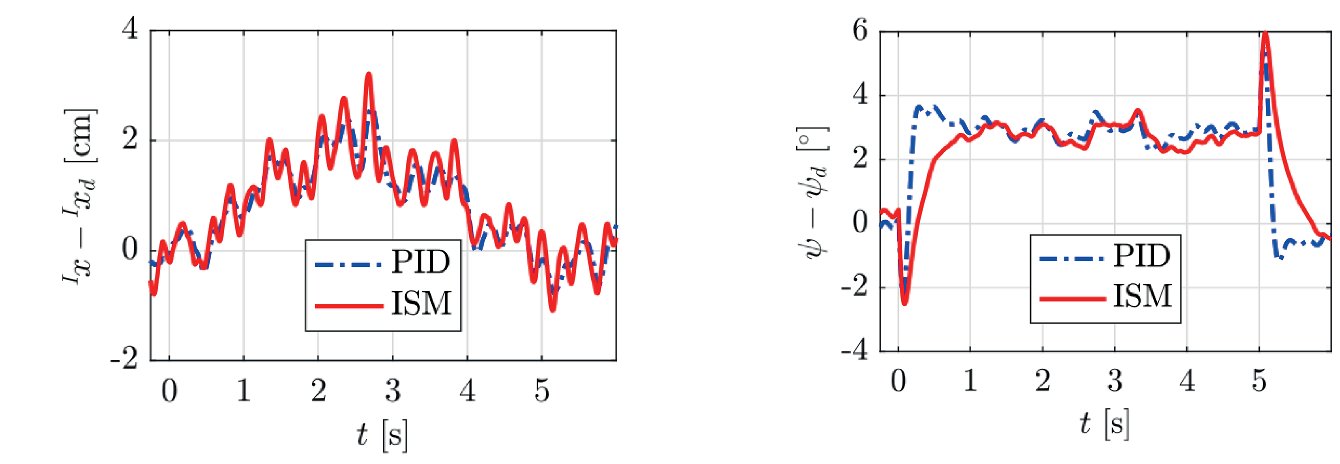

Figure 5: Reference tracking error

Summary

A robust integral sliding mode control is presented. The results indicate that our controller is able to control the hexarotor with a sufficient robustness by taking consideration of noisy state estimation and parameter uncertainty.

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