Multi-layer beam with variable stiffness based on electroactive polymers

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ABSTRACT

The contribution describes a new kind of multi-layer beam with a variable stiffness based on electroactive polymers (EAP). These structures are supposed to be components of new smart, self-sensing and -controlling composite materials for lightweight constructions. Dielectric Elastomer foils from Danfoss PolyPower are used to control the beam’s stiffness.

The basic idea is to change the area moment of inertia of bending beams. These beams are built up as multi-layer stacks of thin metal or PMMA plates. Its internal structure can be changed by the use of the electroactive polymers for controlling the area moment of inertia. So it is possible to strongly change the stiffness of bending beams up to two orders of magnitude. Thereby, the magnitude of varying the stiffness can be scaled by the number of layers and the number and type of electroactive polymer elements used within the bending beam.

The mechanisms for controlling the area moment of inertia are described in detail. Modeling of the mechanical structure including the EAP uses a pseudo rigid-body model, a strain energy model as well as a finite element analysis. The theoretical calculations are verified by experiments. The prototype described here consists of two structural layers. First results show the feasibility of the proposed structure for mechanical components with stiffness control.

Keywords: Smart Composite, Variable Stiffness, Multilayer Beam, Electroactive Polymers

INTRODUCTION

In today’s markets there are great efforts in developing effective structures for lightweight constructions for nearly every engineering discipline. Especially in aeronautics and the space sector, but also in the automotive industry and for energy harvesting applications there is a need of new lightweight and reliable materials and structures. Such structures are often based on composite materials. Classic composite materials have the disadvantage that there is only little capability to adapt to varying environmental conditions. This is due to the fact that most materials, such as carbon or glass-fiber composites, show anisotropic material properties. To introduce new constructive solutions able to react to suddenly changing boundary conditions it is necessary to develop a new kind of material, like the co-called smart composites. These composites will be self-sensing and -controlling and will adapt to environmental conditions by their own. One main problem to be solved is how composites can change their stiffness to withstand or react elastically to an external load.[1]

In the recent years, there have been published a lot of studies trying to solve the problem of variable stiffness composites. In principle, there are two basic ways so vary the stiffness of a constructive component. First, by directly changing the material properties, especially the Young’s modulus, second, by influencing the geometry of the structures. McKnight et al. developed a new kind of morphing structure for aerospace application by using rigid reinforcement structures and a shape memory polymer (SMP) matrix [2][3][4]. By changing the temperature of the composite material it is possible to strongly change the Young’s modulus of the SMP matrix thus controlling the stiffness of the composite. In the heated state strains up to 20 % can be achieved. Gandhi et al. have taken a similar approach to develop structures with...
controllable stiffness [5][6]. They designed morphing multi-layered beams with changable stiffness. Since the aim of these studies was to develop new materials for aerospace applications the experimental set-ups were built up of stiff aluminum beams connected by SMP layers. If these layers are heated above the glass transition temperature then the stiffness of the SMP layers decrease and the aluminum layers can bend freely. In this case the bending stiffness is less than in the cold state, when the SMP layers transfer the sheer forces between the stiff aluminum layers.

Although smart composites based on SMPs can generate high reversal strains up to about 30 \% and in the cold state a high Young’s modulus is given, SMPs show also some serious disadvantages. Especially if these materials will be used in large structures such as airfoils, a lot of energy is needed to heat the SMPs above the glass transition temperature. Besides, it also needs proper heat isolation to avoid heat exchange with the cold air around the airfoils. Furthermore, heating and cooling large structures will always need long time and much power. Therefore, the application of smart SMP composites is limited to non-time-critical requirements.

An alternative way to create structures with controllable stiffness is to directly influence the structure itself. Most of such approaches have significant influence on the bending stiffness of beam structures or joints by changing the area moment of inertia or the active lengths. A comprehensive review of such approaches is given by van Ham et al. in [7]. Most of these solutions require complex mechanical structures. Therefore, most of them are not well-suited for being used in composite materials. The only exceptions seem to be multi layer stacks. This is shown in studies of Kawamura et al. [8][9] and Bergamini et al. [10]. The basic idea is to control the area moment of inertia of a multilayer stack of thin plates by influencing the friction between several layers (Figure 1). This could be achieved by controlling the normal force between the layers. In [8] the set-up is surrounded by a balloon. Applying a negative pressure by a vacuum pump, the atmospheric pressure compresses the balloon and with it the stack. This leads to an increase of friction between the layers and thereby to an increasing flexural stiffness. In [9] and [10] the friction force is applied by an electric field between the layers generating a Coulomb force which increases the friction and, hence, the bending stiffness. Both approaches show disadvantages for composite applications. The use of vacuum pumps requires a lot of energy to generate proper negative pressure and is limited to the difference between atmospheric and negative pressure. A sudden change in stiffness is quiet difficult especially for large structures. This problem is avoided by the use of Coulomb force. High electrical voltages can be generated fast, but it is difficult to realize proper insulation between the layers.

![Figure 1: Schematic comparison between the (a) soft and the (b) stiff state of a multilayer stack](image)

In multi-layer stacks each individual layer can be addressed separately. This enables the stiffness control over wide range with a large number of steps. This is the reason that multi-layer stacks have been chosen for this work. The contribution will describe the basic ideas and a prototype of a new kind of a multi-layer flexural beam with variable stiffness based on electroactive polymers.

**THEORETICAL BACKGROUND**

The stiffness $k$ of a beam-like structure is defined as the ratio between an external load $F$ and the resulting deformation $u$. It depends on the material properties, Young’s modulus $E$, the area moment of inertia $J$ and the beam length $l$ [11]:

$$k = \frac{F}{u} = \frac{3 \cdot E \cdot J}{l^3}$$

(1)
The area moment of inertia $J$ itself depends on the geometrical dimensions (height $h$ and width $b$) with respect to the geometrical shape of the beams’ cross-section area. For a simple rectangular cross-section $A = b \cdot h$ it yields to [11]:

$$J = \int_A z^2 \, dA = \frac{b \cdot h^3}{12}.$$  \hfill (2)

The multilayer stack consists of a finite number of layers with rectangular cross-sections. Figure 2 shows schematically the cross-section for the both cases with separate layers movable against each other (Figure 2a) and firmly connected layers acting as single bulk (Figure 2b).

![Figure 2: Multilayer stack with a) free separate layers movable against each other and b) firmly connected layers acting as single bulk](image)

For both cases the area $A$ is the same, but the moment of inertia is either the sum of the moments of inertia of $N$ flexure layers (Figure 2a) or the moment of inertia of a $N$-fold thick layer with a $N^2$-times higher value:

$$J_a = 4 \cdot \frac{b \cdot h_0^3}{12} = \frac{b \cdot h_0^3}{3}$$  \hfill (3)

$$J_b = \frac{b \cdot (4 \cdot h_0)^3}{12} = 16 \cdot \frac{b \cdot h_0^3}{3}$$  \hfill (4)

As result the moment of inertia scales with $N^2$:

$$\frac{J_b}{J_a} = 16 = N^2$$  \hfill (5)

Thus, already a stack of ten layers changes the stiffness by the factor of 100.

### DESIGN CONCEPTS

#### 3.1 Form closure connections of layers in a layer stack

As can be seen in equation (5) flexure beams with controlled stiffness can easily be realized by a stack of an appropriate number of layers if it is possible to switch between a first state where the layers can easily slide against each other and a second state where the layers are interlocked against each other.

To ensure a save and detachable interconnection between adjacent layers and to admit sliding among them, form closure is used. Therefore, the layers possess form closure structures which are ordered at their contact areas as shown in Figure 3. These structures consist of teeth and gaps. In the initial state all teeth of the layers 1 and 2 are ordered congruently, gap against gap. If one end of the double layer is fixed and the other end is bended, the teeth will slide over the opposite teeth. Because the layers are able to slide over each other, the structure shows large elastic compliance. To switch to higher stiffness a fitting structure is put into the gaps between the teeth, preventing the sliding against each other. This fitting structure locks the sliding between the layers and increases the area moment of inertia.
To move a fitting structure into the gaps between the teeth a new kind of flexible actuator is used. In order to achieve a wide ratio between soft and stiff state it is necessary to build up a multilayer stack of as much layers as possible, with layers and interconnection structures as flat as possible. The fitting structure and its actuator have to be positioned between the layers in the actuator gap. Hence, the actuator and its driving unit have to fit into the form closure structure. Electroactive polymer actuators are a good choice for this purpose, because EAPs are flat, flexible and lightweight.

![Figure 3: Double layer stack with form closure structure](image)

Furthermore, form closure connections can ensure a safe connection only limited by the material’s maximum load. Other advantages are that there is no limit for the maximum applicable friction forces and that form closure actuators based on EAPs can connect adjacent layers very fast.

### 3.2 Concepts of EAP actuators

The EAP actuators will be placed in the actuator gaps. Besides its flatness there are basic design requirements to the interlock actuator. Due to the fact, that the actuator itself should not influence the total stiffness too much, it is necessary that its Young’s modulus is in the order of that one of the layers. This is achieved by building up the actuators of a similar material as the layers. To avoid slipping and damage of the teeth and to prevent sliding of the layers, the interlock structure should move in the gaps as deep as possible. This requires large actuator deformations.

Electroactive polymers are principally able to perform large deformations [12] but the best performance can be achieved when a pre-straining force is applied [13][14]. In our experimental setup we used PolyPower foils from Danfoss [15], due to its commercial availability. These foils consist of a thin silicon-based elastomer layer with smart compliant electrodes. The electrodes made of a thin silver layer with a thickness of about 80 nm are deposited on a corrugated surface. Therefore, the electrodes are compliant only in one direction. The behavior of these foils in an interlock actuator can be calculated by means of a hyper-elastic material model. Iskandarani et al. [15] used a Mooney-Rivlin model [17] to calculate the behavior of the PolyPower foils. This material model is based on the strain energy $W$ calculated from the invariants $I_1$ and $I_2$ of the left Cauchy-Green deformation tensor [18]:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2,$$

$$I_2 = \lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2},$$

$$W = C_{10} (I_1 - 3) + C_{01} (I_2 - 3).$$

Elastomers are incompressible materials. For this, the principle Cauchy stresses $\sigma_i$ can be calculated by deriving the strain energy function to the principle stretch ratios $\lambda_i$, which are the ratio between actual and initial length in direction $x_i$. Considering, that the electrodes are stiff in one direction and the foil is loaded in the other direction, the calculations yield to the three Cauchy stresses:
\[ \sigma_i = \frac{\partial W}{\partial \lambda_i} - p = 2 \cdot (C_{10} + C_{01}) \cdot (\frac{\lambda_i^2}{\lambda_i} - \frac{1}{\lambda_i}) - p \]  

(9)

The hydrostatic pressure \( p \) can be calculated from the Cauchy stress in direction \( x_3 \). This stress depends upon the supplied driving voltage (Figure 4).

\[ \sigma_1 = K \cdot 2 \cdot (C_{10} + C_{01}) \cdot (\frac{\lambda_i^2}{\lambda_i}) - \frac{\varepsilon_0 \varepsilon_r V_i^2 \lambda_i^2}{d_0^2} = \frac{F \lambda_i}{w_0 d_0} \]  

(10)

\( K \) is the number of PolyPower layers, \( \varepsilon_0 \) and \( \varepsilon_r \) are the absolute and relative permittivity, and \( d_0 \) and \( w_0 \) the initial thickness and width of the film, respectively. Equation (10) describes the possibilities to design appropriate actuators with respect to the requirements described above.

### 3.3 Actuator design

An actuator made of an elastic material will always react on an external force by deformation. This deformation typically depends on the force value and is described by the actuator stiffness \( k \). As mentioned it is important to apply also a pre-straining force to the polymer. Simultaneously, it is necessary that the actuator is compliant to realize large deformations. For this, it is advantageous that the actuator has a strong non-linear force-deformation behavior. Thus, actuator stiffness \( k \) has to be a function of force:

\[ k = \frac{F}{u} = f(F) \]  

(11)
Equations (10) and (11) allow the calculation of the principle actuator behavior with respect to the applied external force mode. Figure 5a shows the stiffness functions for both a linear and a strongly non-linear actuator. Figure 5b depicts the actuator elongation versus supplied voltage. It shows that non-linear actuators possess a higher potential for large deformations of the electroactive polymer foils.

The actuator with linear stiffness (Figure 6a) comprises a planar network of flexural hinges. These flexures transform large deformations in one direction into small ones in the orthogonal direction. Figure 7 shows the geometric relationships of the actuator from Figure 6a which are the basis for calculating the actuator stiffness. The small angle $\alpha$ between the flexure and the columns leads to a high transformation factor $\beta$. The large deformation $u_1$ is transformed into the small one, $u_2$. This is absorbed by long leaf-spring structures at the sides. The transformation factor and the stiffness can be calculated by the combination of a pseudo rigid-body and a strain-energy model.

The transformation factor $\beta$ is given by the tangent of the angle $\alpha$ in Figure 7:

$$\beta = \tan(\alpha) = \frac{u_1}{u_2} = \frac{F_2}{F_1}. \quad (12)$$

The strain energy model sums up all strain energies resulting by the loads $F_1$ and $F_2$. Corresponding to Castiglianos theorem the deformation yields from the derivation of the strain energy to its cause [11]. The strain energy function
consists of three energy contributions, the normal, bending and shear energy. Those have to be summed up for every beams and flexure:

\[ U_{\text{ges}} = \sum_i (U_{N_i} + U_{M_i} + U_{Q_i}) \]  

(13)

Deformation \( u \) and stiffness \( k \) are defined by

\[ u_i = \frac{\partial U_i}{\partial F_i}, \]

(14)

\[ k_i = \frac{\partial^2 U_i}{\partial F_i^2} = \frac{\partial^2 U_i}{\partial F_i^2}. \]

(15)

The stiffness strongly depends on the shape of the flexure hinges and the material properties, especially by the Young's modulus. For ideal elastic material behavior the stiffness is constant and does not depend on the deflection. Figure 8 depicts a first actuator prototype, manufactured by stereolithography (section 4.1).

![Figure 8: Prototype of a linear interconnection actuator](image)

The second actuator type has a strongly non-linear stiffness, given by Euler-like buckling beams. Ideal Euler-like buckling beams deform only if the load exceeds a critical value (dotted line in Figure 5a). The deformation behavior in the buckled state can be described by a third-order theory [11]. In the case of double-clamped beams the Euler case IV has to be considered (Figure 9).

![Figure 9: Deformed buckling beam, Euler case IV](image)

The overcritical force \( F > F_{\text{crit}} \) causes a displacement \( u \) and thereby a buckling \( w(s) \). The arc length coordinate of the beam is \( s \). The angle \( \alpha_0 \) is the angle at the point of inflection of the beam. The critical force \( F_{\text{crit}} \) can be calculated by

\[ F_{\text{crit}} = \frac{\pi^2 \cdot E \cdot J}{(0.5 \cdot l)^2}. \]

(16)
An analytic solution for the post-buckling behavior can be found by the use of elliptic integrals. The corresponding differential equation is given by [19]

$$E \cdot J \cdot \frac{d\alpha}{ds} = -F \cdot w(s)$$

(17)

With respect to the geometrical boundary condition this equation yields

$$\frac{1}{2} \left( \frac{d\alpha}{ds} \right)^2 = \frac{F}{E \cdot J} (\cos \alpha - \cos \alpha_0)$$

(18)

According to [19] this leads to the displacement function

$$x(s) = \frac{l}{4K(p_0)} (2E(\varphi, p_0) - F(\varphi, p_0))$$

(19)

In equation (19) $K(p_0)$, $E(\varphi, p_0)$ and $F(\varphi, p_0)$ are elliptical integrals of the first and second kind. The values $\varphi$ and $p_0$ are transformed geometrical boundary conditions to solve these integrals:

$$p_0 = \sin \frac{\alpha_0}{2}$$

(20)

$$\sin \varphi = \frac{1}{p_0} \sin \frac{\alpha}{2}$$

(21)

Calculations have shown, that the deformation $x(l)$ of the end point can be linearized for small displacements:

$$F(u) = F_{crit} + k \cdot u$$

(22)

**PROTOTYPES AND EXPERIMENTAL SETUP**

4.1 Materials

The layers and interconnection structures were made of poly(methyl methacrylate) (PMMA). PMMA shows good material properties with respect to nearly ideal elastic stress-strain behavior and transparency. The single layers and their form closure structures were fabricated as two parts, the outer sheet and the form-closure structure. Both parts were manufactured individually by laser cutting and assembled by gluing. Figure 10 shows the construction of a double-layer set-up consisting of the outer sheets, form-closure structures and two interconnection actuators. Figure 11 depicts the assembled prototype structure consisting of a double-layer and two actuators.

The actuators were manufactured by rapid prototyping technologies, in particular by selective laser sintering and stereo lithography, respectively. As material Somos NeXt from DMS Somos (stereo lithography) and PA 3200 GF from EOS e-Manufacturing Solutions were chosen.
These materials have a Young’s modulus similar to PMMA. This is important, because a stiffer material would overlay the stiffness control effect of the PMMA layers by its own. Table 1 compares the Young’s modulus of the considered prototype materials.

### Table 1: Comparison of Young's moduli of prototype materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Purposed structure</th>
<th>Manufacturing technology</th>
<th>Young’s modulus in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA</td>
<td>Layers and form closure structures</td>
<td>Laser cutting</td>
<td>~3,000 [20]</td>
</tr>
<tr>
<td>PA 2200 Balance 1.0 (EOS)</td>
<td>Actuator</td>
<td>Selective laser sintering</td>
<td>1,500 [21]</td>
</tr>
<tr>
<td>PA 3200 GF (EOS)</td>
<td>Actuator</td>
<td>Selective laser sintering</td>
<td>2,900 [21]</td>
</tr>
<tr>
<td>Somos NeXt</td>
<td>Actuator</td>
<td>Stereo lithography</td>
<td>2,415 - 2,525 [22]</td>
</tr>
<tr>
<td>Accura 60</td>
<td>Actuator</td>
<td>Stereo lithography</td>
<td>2,700 - 3,000 [23]</td>
</tr>
<tr>
<td>Sheet steel</td>
<td>Buckling beams</td>
<td>-</td>
<td>210·10³ [11]</td>
</tr>
</tbody>
</table>
4.2 Simulation results

Simulation of deformation in consequence of applied load was carried out using equations (10) and (15) or (22). The following examples show the elongation ability of one linear and on nonlinear interconnection actuator made of somos NeXt. Analytical calculations were performed using Maple 14 whereas numerical simulations used ANSYS classic with a nonlinear post-buckling solution. The following figures show the force-elongation behavior.

The comparison between analytical and numerical results shows good agreement. The stiffness for both actuators taken from Figure 12 can be used to calculate the relationship between the elongation of the interconnection teeth at the actuators and the applied voltage. This was done using equation (10). Material properties were taken from [16]. The area of the EAP foils amounted to about 80x80 mm². The maximal driving voltage is given as 2500 V according to the Danfoss data sheets. Figure 13 compares the calculated elongation of the linear and the non-linear actuator types in the voltage range from 0 to 3000 V. As expected the non-linear actuator shows a larger possible elongation, because of its nearly constant pre-straining force. For the particular example the gain for the maximum actuation at 2500 V for the non-linear actuator accounts to 2.5.

![Figure 12: Deformation of (a) actuator I (Figure 5a) and of (b) actuator II (Figure 5b)](image)

![Figure 13: Comparison of elongations between a linear and non-linear actuator](image)
4.3 Experimental set up

For experiments double-layer structures have been used as described in section 4.1. The experimental set-up is shown schematically in Figure 14.

For safety reason, due to the high driving voltages of the PolyPower EAPs, all modules and devices except to the controlling computer is placed in a containment. The double-layer bending beam is clamped in a restraint screwed to the ground plate. The load is applied by a stepping motor (L4118 by Nanotec). Its shaft is connected to a force sensor (KAP-S/10N/0,1 by AST group). All devices are operated over USB by a computer. The measurement and control is performed by LabView, which allows the simultaneous and automated control, display and storage of data.

RESULTS

First measurements were performed at the double-layer prototype from Figure 11 without the EAP foils to proof the targeted variable stiffness of the device. The deformation was applied by a stepping motor. The resulting force was plotted versus the resulting deformation at the free end. The curves in Figure 15 were measured in three different actuator configurations, (a) when both, (b) only one or (c) none of the actuators were in the interconnection position, describing the stiff, intermediate and the compliant case. The results are summarized in Table 2. For the compliant state, where the two layers can slide freely over against each other, friction was to large leading to stick-and-slip effects and preventing the return of the sample in the initial state. Further work has to focus on improvements of the surface roughness if the sliding layers to avoid friction and hysteresis.

Table 2: Comparison of the three stiffness cases of the double-layer prototype structure

<table>
<thead>
<tr>
<th>Case</th>
<th>Actuator configuration</th>
<th>Stiffness k in N/mm</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>stiff</td>
<td>Both connected</td>
<td>0.45</td>
<td>Figure 15a</td>
</tr>
<tr>
<td>intermediate</td>
<td>One connected</td>
<td>0.11</td>
<td>Figure 15b</td>
</tr>
<tr>
<td>compliant</td>
<td>Both not connected</td>
<td>0.08</td>
<td>Figure 15c</td>
</tr>
</tbody>
</table>
According to, the stiffness-ratio between the stiff and the compliant state amounts to:

\[
\frac{k_{\text{stiff}}}{k_{\text{comp}}} = \frac{0.45 \text{ N/mm}}{0.08 \text{ N/mm}} \approx 5.6
\]  

(23)

This value is slightly higher than the predicted one calculated by using (5). This is caused by the approximations used for equation (5). This preliminary results show that the approach for EAP-based controlled change of the stiffness exhibits promising properties and will open up new opportunities for smart composites in lightweight construction. Further work will improve the construction, will enhance surface properties of the sliding faces and will expand the control range far beyond the factor of 5.6 achieved in our very first experiments.
ACKNOWLEDGEMENT

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