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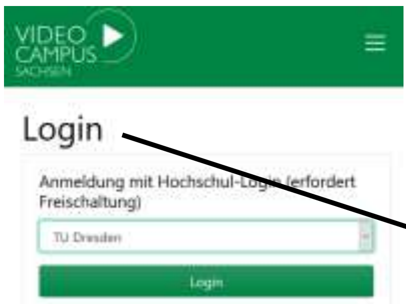

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Vacuum Technology WS 20/21 Virtually presented Lecture 4, Nov. 17, 2020

Prof. Dr. Johann W. Bartha

Inst. f. Halbleiter und Mikrosystemtechnik
Technische Universität Dresden

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“VT L04 a 19:43

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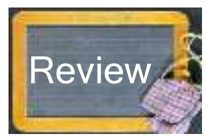

B04a

Review Kinetic Gas Theory

$p = \frac{\Delta p}{A \cdot \Delta t}$ ($2mv$!) → ($f(v)dv$) → $P = \frac{1}{3} n m \overline{v^2}$

Phenomenological: $\frac{P \cdot V}{T} = const$

STP $V_{Mol} = 22,4\ l$

$P \cdot V = \nu R T$

↑ # of Mols; ↖ ideal Gas constant

Units: Pa (N/m^2); Bar (mBar); Torr mm Hg

$P = n k T$

$k = \frac{R}{N_A}$ $\overline{E}_{kin} = \frac{m \overline{v^2}}{2} = \frac{3}{2} kT$

Independent of the kind of gas !!

Chapters

0. Introduction
Air pressure as a force to the walls of an empty container

1. Gas kinetic
Pressure as momentum transfer, Mol & Molvolume, Pressure units

2. Pressure Ranges

3. Vacuum technical terms

4. Vacuum generation

5. Pressure measurement

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Remember: For T=const. the Pressure depends only on the particle density!

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	(std) atm	Bar	mBar	Pa	Torr	psi
1(std) atm=	1	1,0132	$1{,}01\ 10^{-3}$	$101{,}32\ 10^{3}$	760	14,7
1 Bar =	0.987	1	10^{3}	10^{5}	750	14,5
1 mBar =	$0{,}987\ 10^{-3}$	10^{-3}	1	0,1	0,75	0,0145
1 Pa =	$9{,}87\ 10^{-6}$	10^{-5}	10^{-2}	1	$7{,}5\ 10^{-3}$	$145\ 10^{-6}$
1 Torr =	$1{,}31\ 10^{-3}$	$1{,}33\ 10^{-3}$	1,33	133	1	$19{,}3\ 10^{-3}$
1 psi =	$68\ 10^{-3}$	$69\ 10^{-3}$	69	$6{,}9\ 10^{3}$	51,7	1

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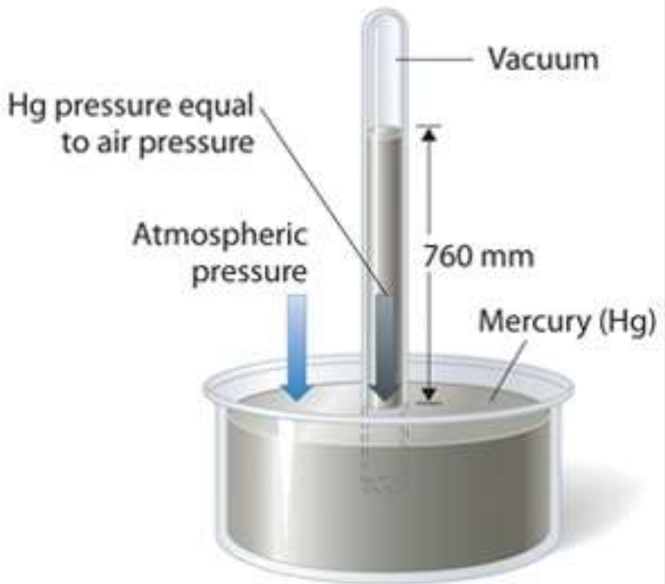

terms

n

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Torr is mm(Hg)

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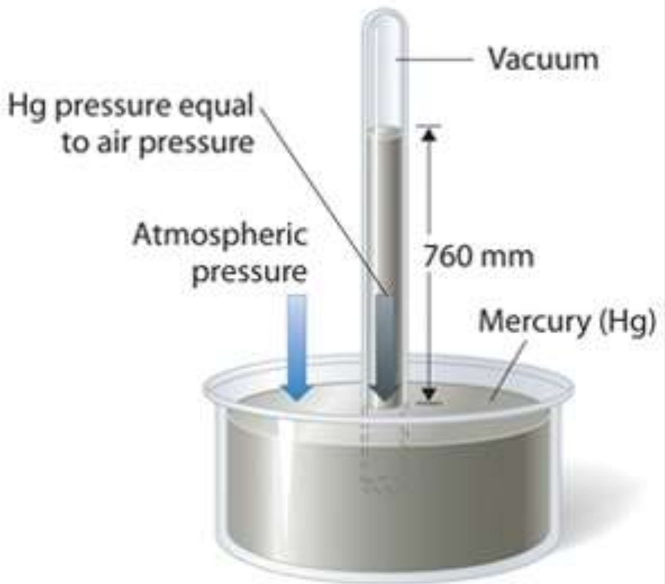

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Question: How many g SF_6 are inside a volume, when the pressure corresponds to that of 1g O_2 ?

$$P_{O_2} = n_{O_2} kT = Tk\, n_{SF_6} = P_{SF_6}$$

$$(O_2 \mathrel{\hat{=}} 32\ \text{amu})$$

$$\frac{N_A \cdot 1g_{O_2}}{Mol_{O_2} \cdot V} = \frac{N_A\, Xg_{SF_6}}{Mol_{SF_6}\, V}$$

$$Xg_{SF_6} = \frac{Mol_{SF_6} \cdot 1g}{Mol_{O_2}} = \frac{146 \cdot 1g}{32} = \underline{\underline{4{,}56\ g}}$$

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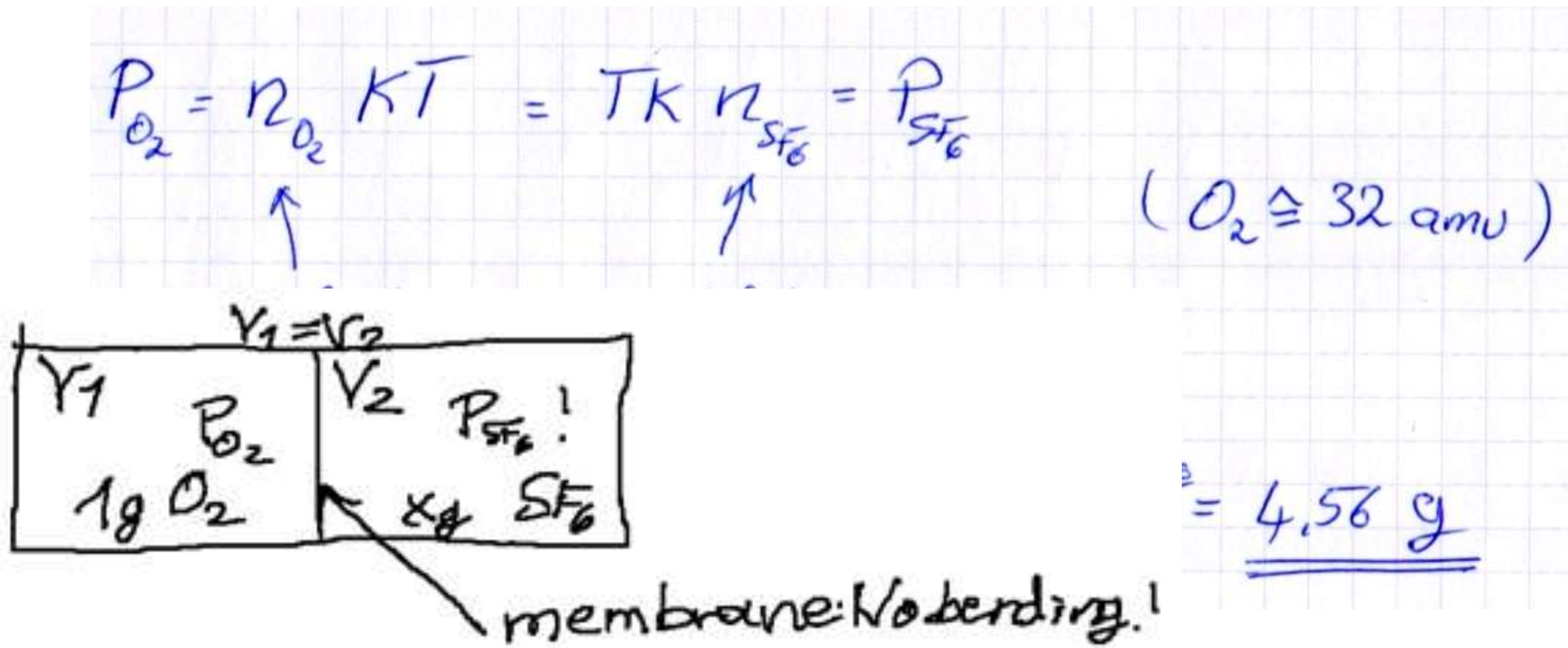

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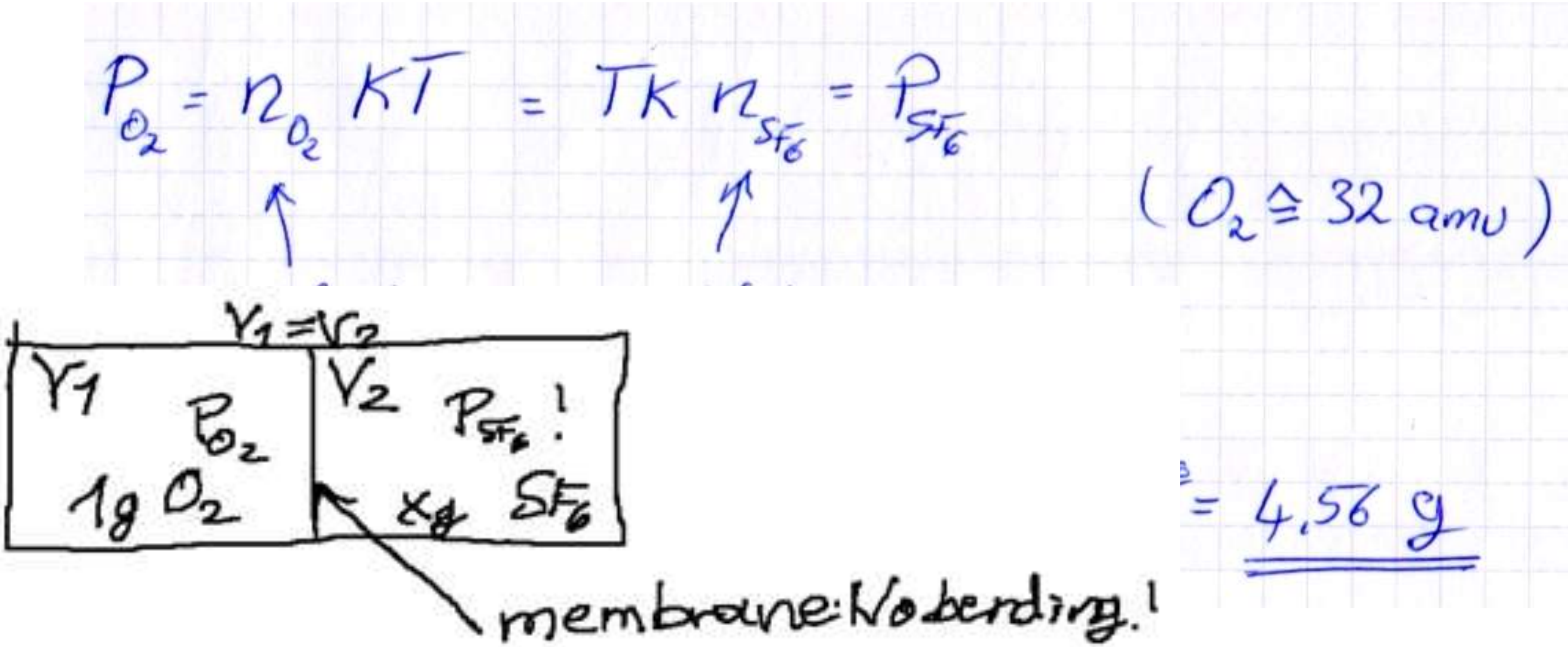

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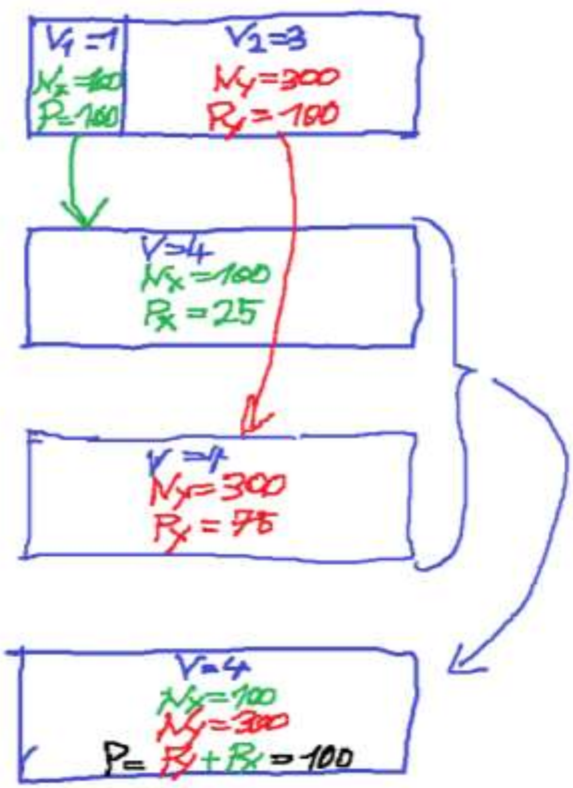
B04b

Now generalized particles x and y:



For T=const.

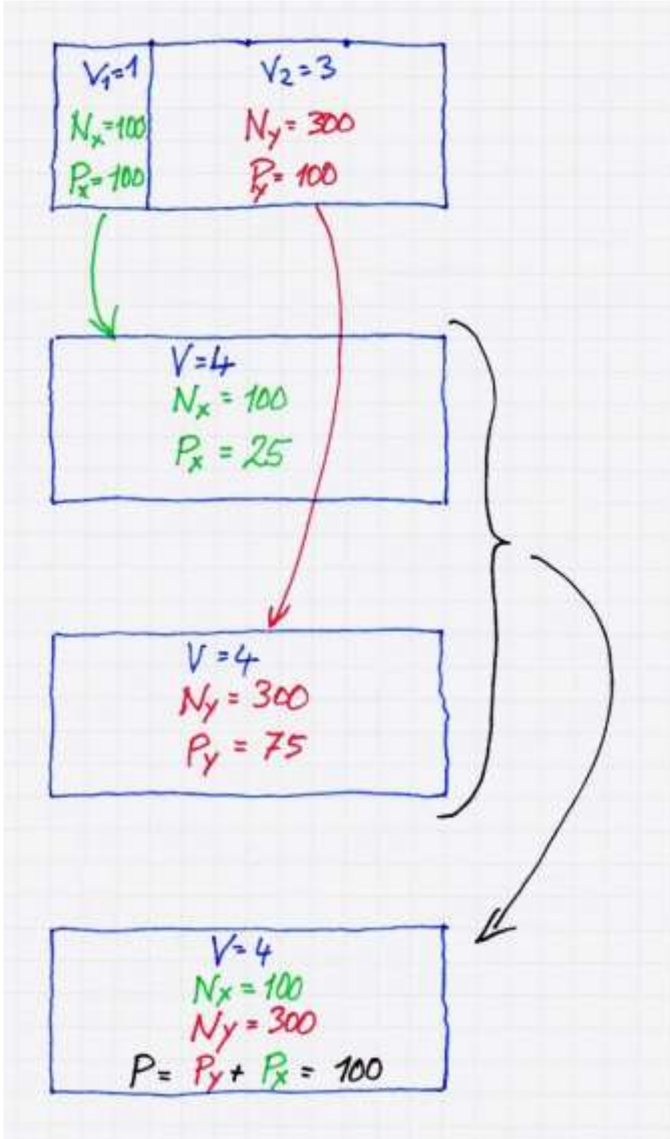
$P = n \cdot \text{const.}$ (kT), where const. $= 1$

Conclusion:
Each kind of particles may be handeled as an independent single gas. The total pressure is the sum of the pressures of the single gases.

Now generalized particles x and y:

For T=const. it holds
P = n · const.

In this case const.=1



Conclusion:

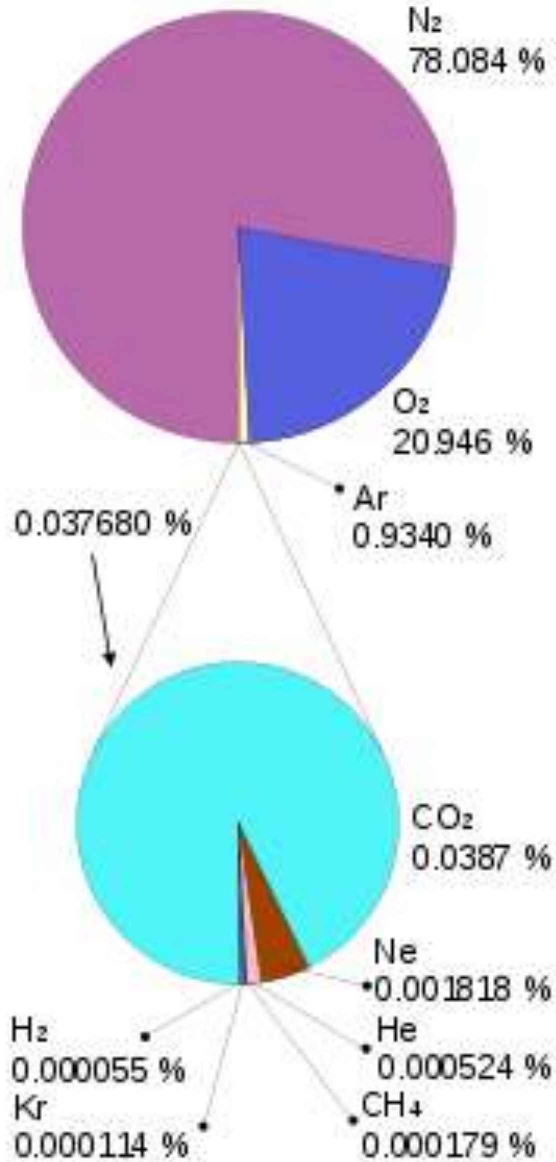
Each kind of particles may be handeled as an independent single gas. The total pressure is the sum of the presuures of the single gases.

If a gas consist of different kinds of molecules/atoms (i), the total pressure P of the gas mixture is the sum of the individual pressure's of the different kinds of particles P_i

$$P = \Sigma P_i$$

The pressure P_i of a specific kind of molecules/atoms is called partial pressure.

Example: Air


N_2
78.084 %
O_2
20.946 %
Ar
0.9340 %
0.037680 %
CO_2
0.0387 %
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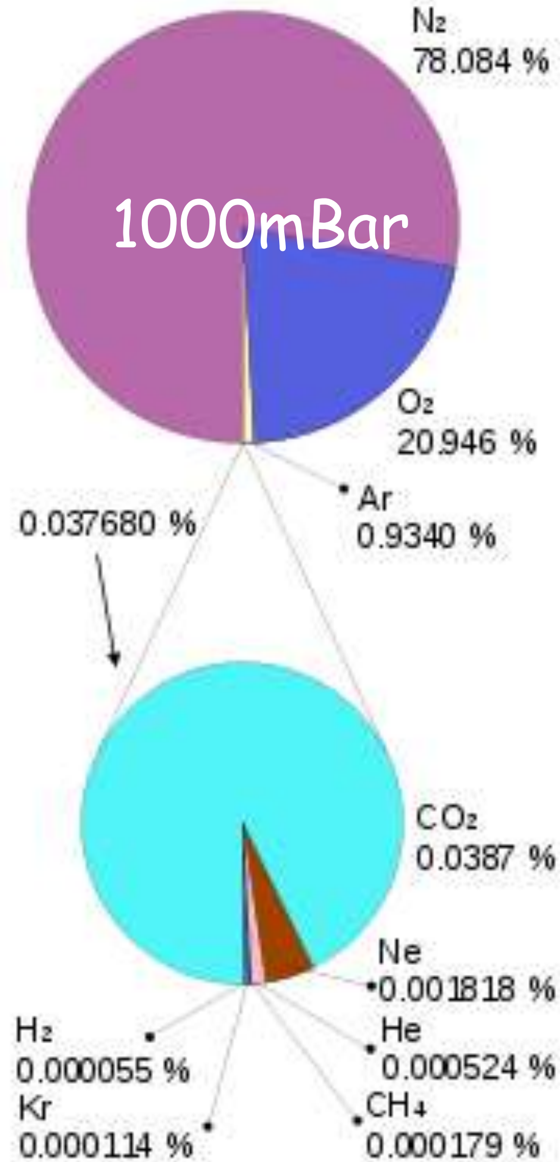
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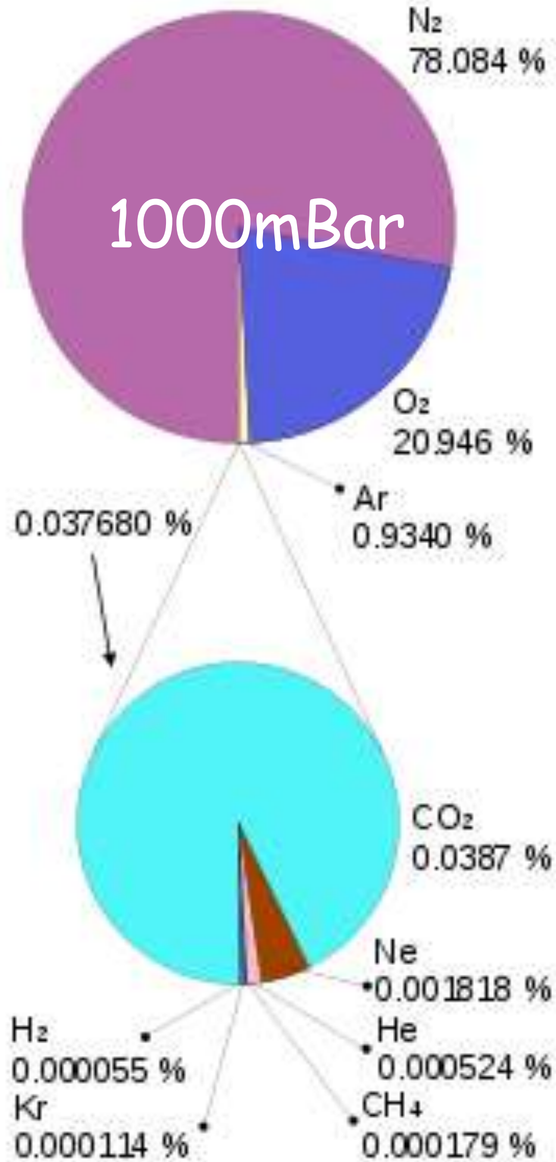
Example: Air

781mBar



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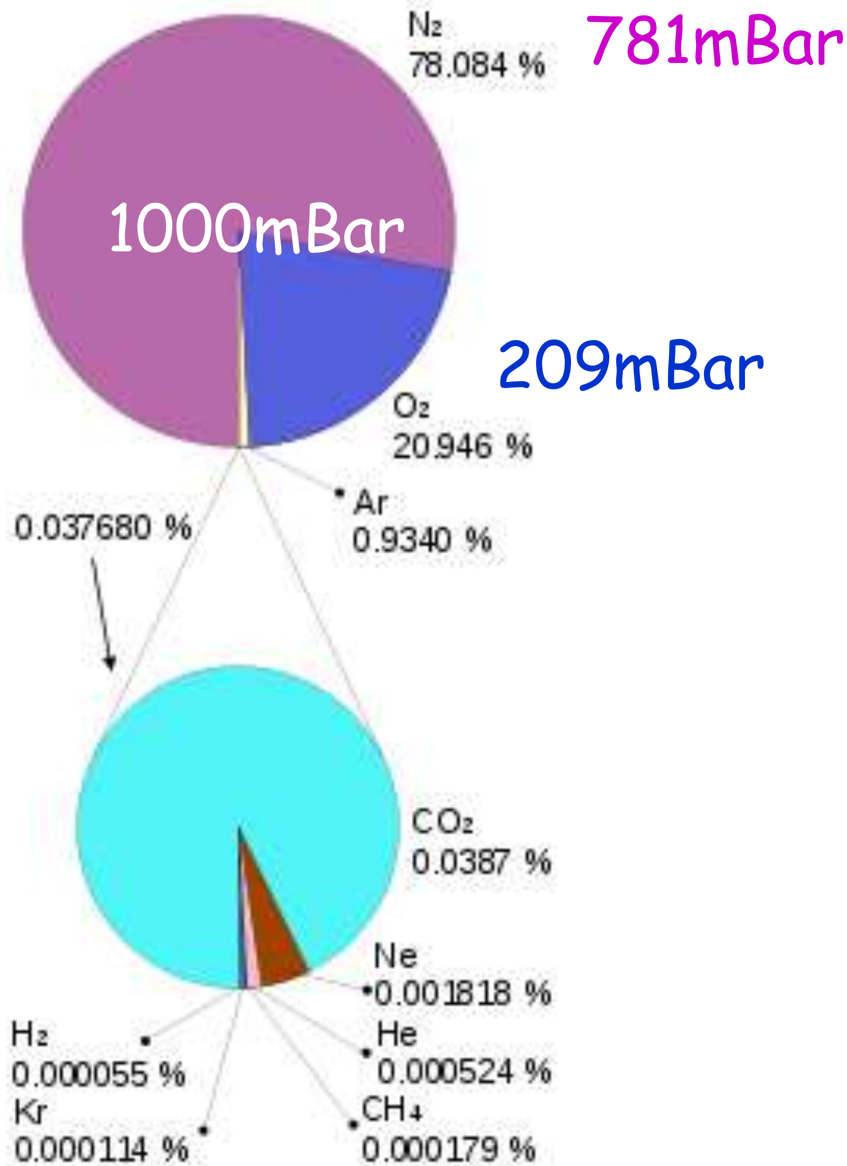
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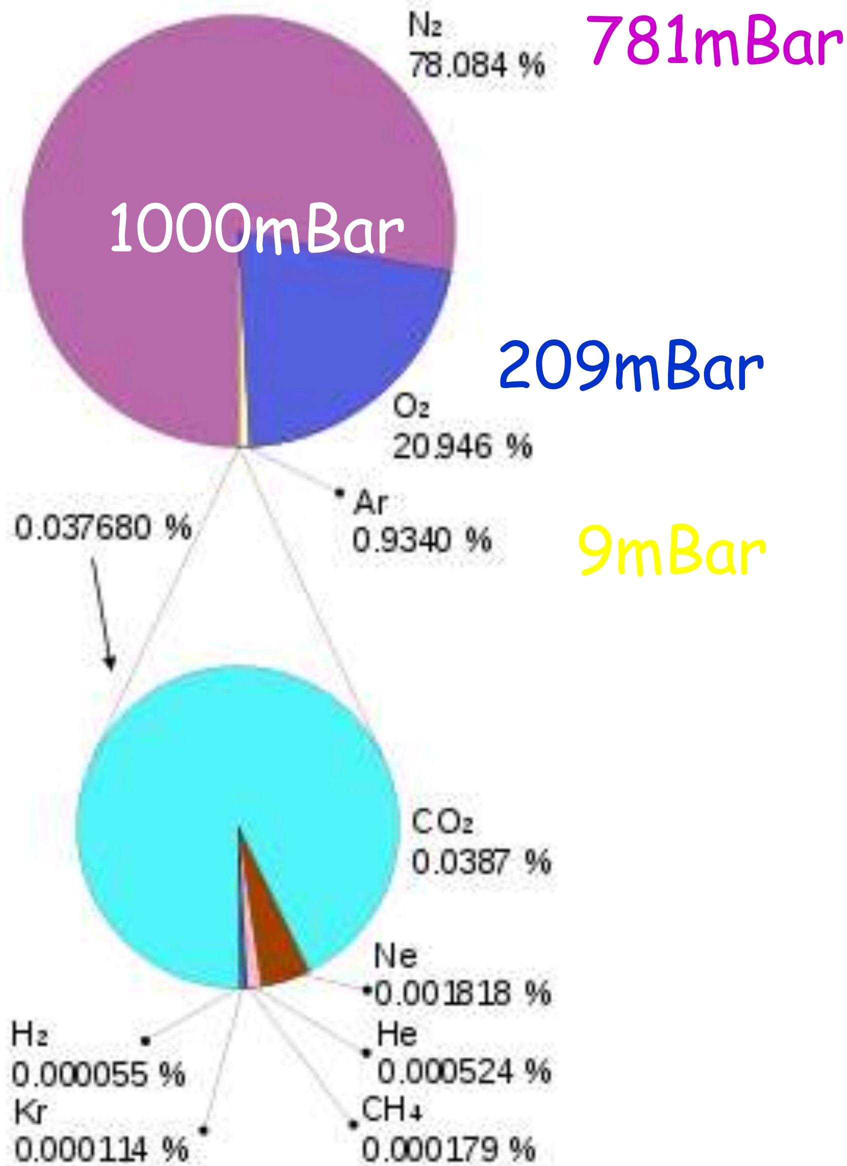

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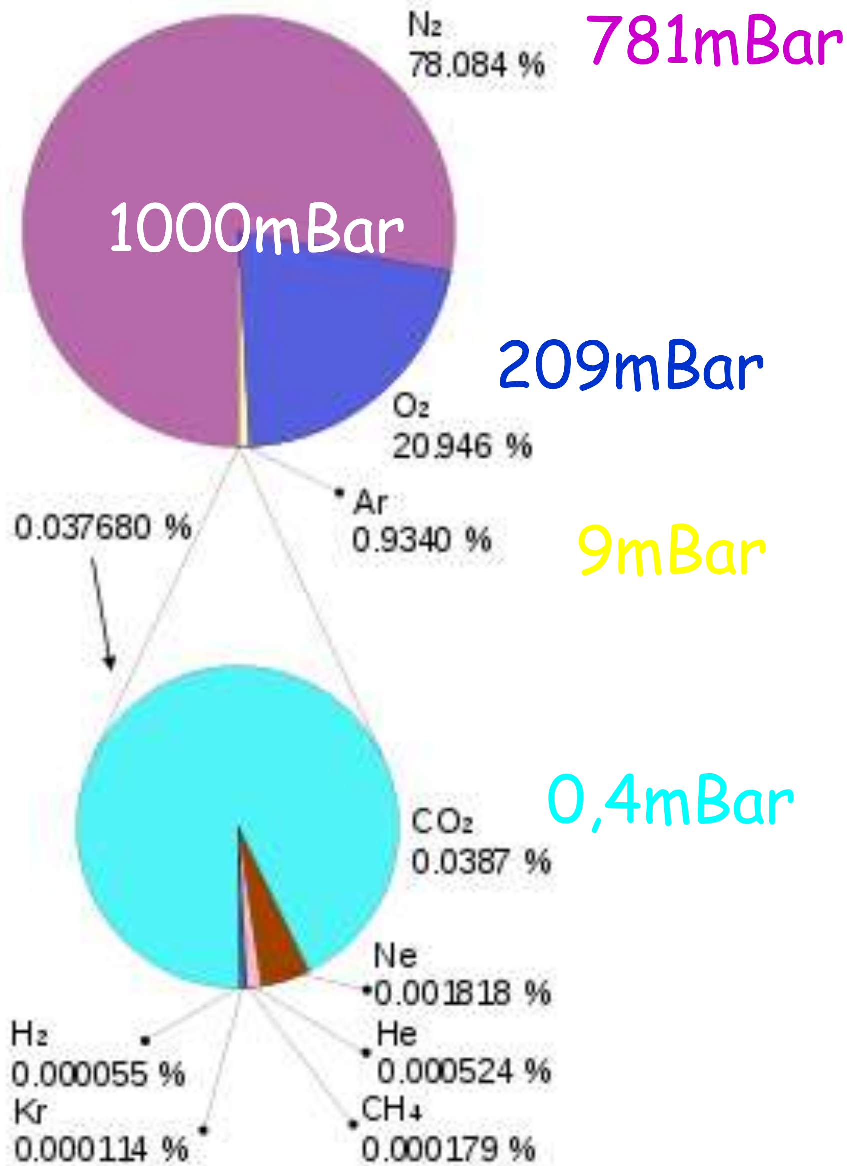

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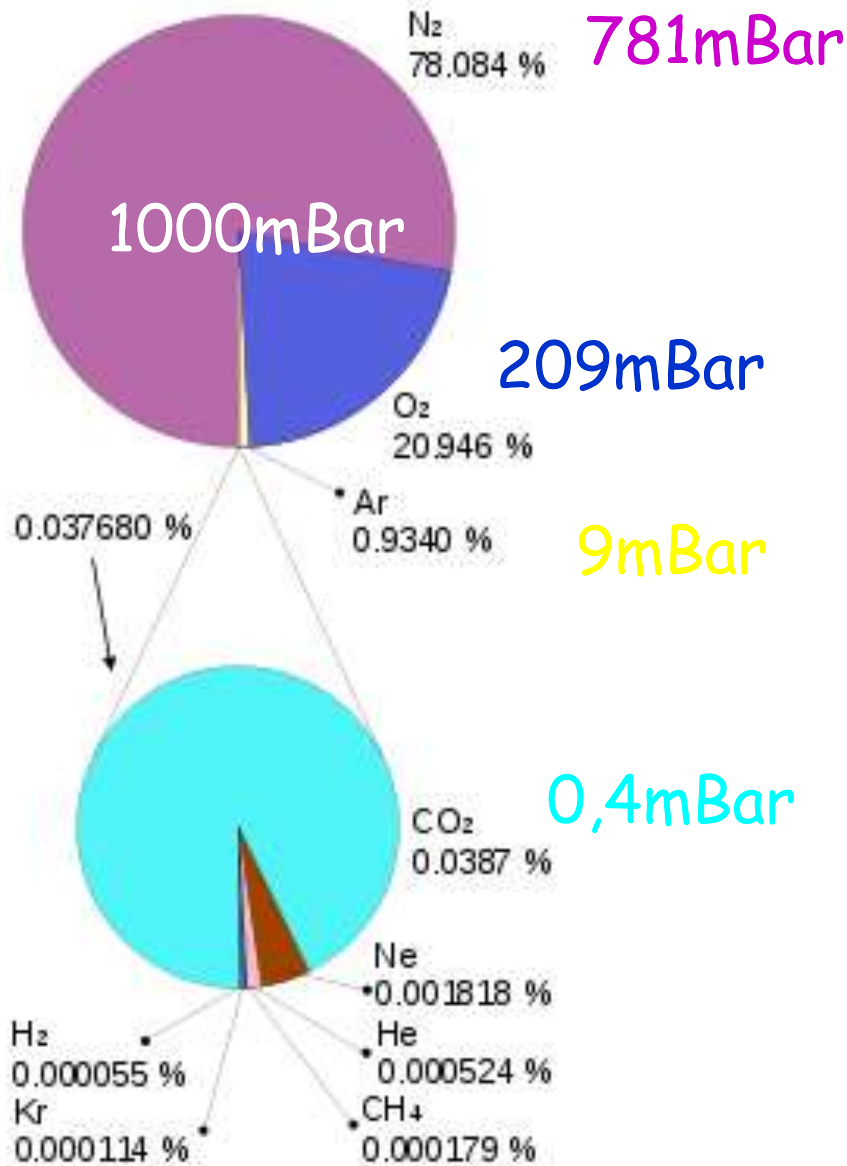

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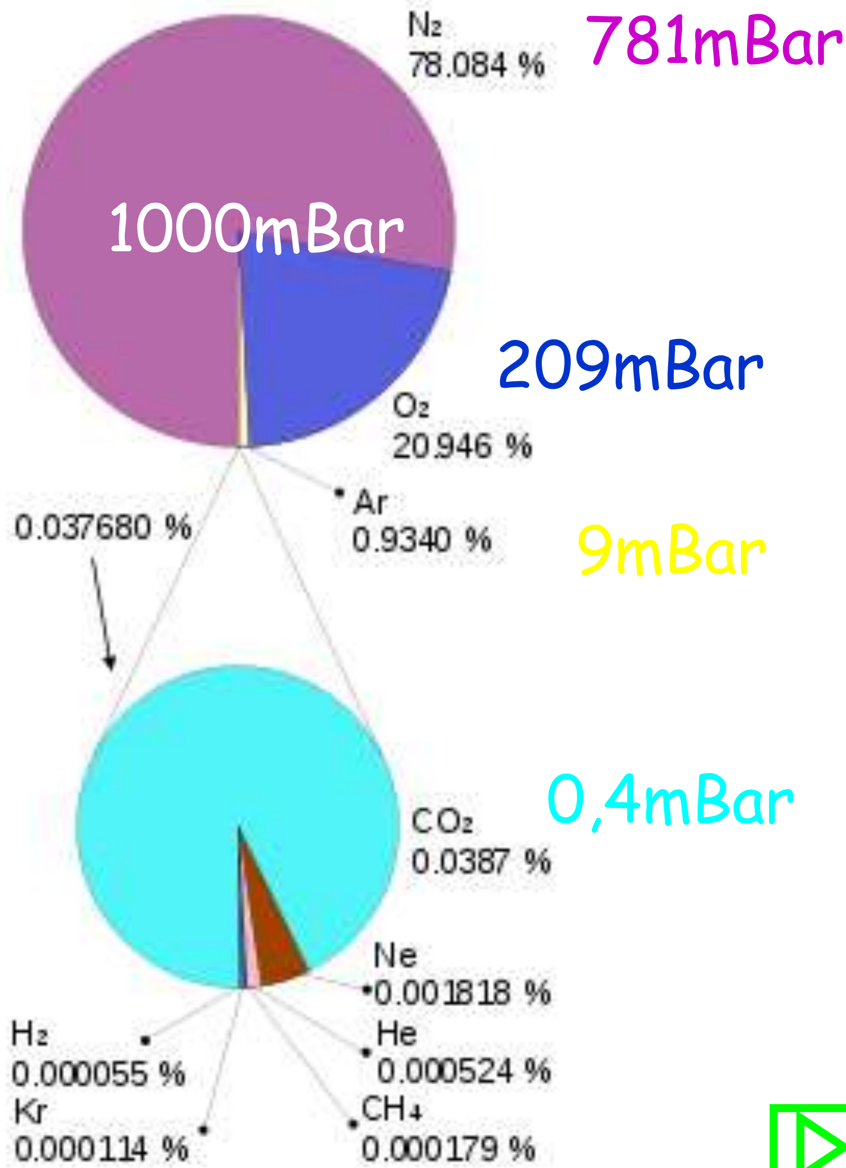

Back to blackboard one

"VT L04 c 54:33

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Independent of the kind of gas !!

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- Heavy gas compared to light gas of same density n ?
- How about momentum change?

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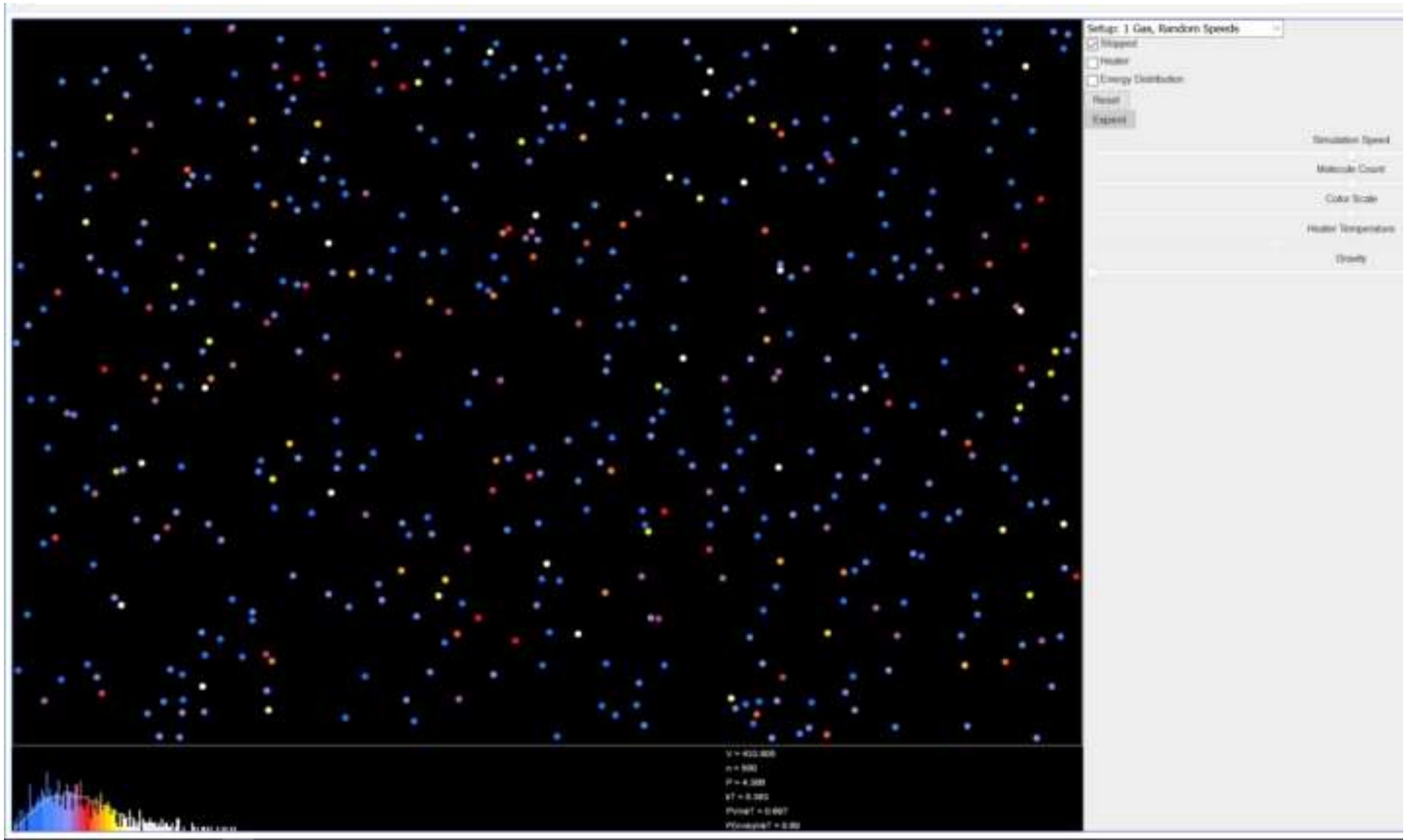
New Topic – Distribution of velocities:

http://www.falstad.com/gas/fullscreen.html



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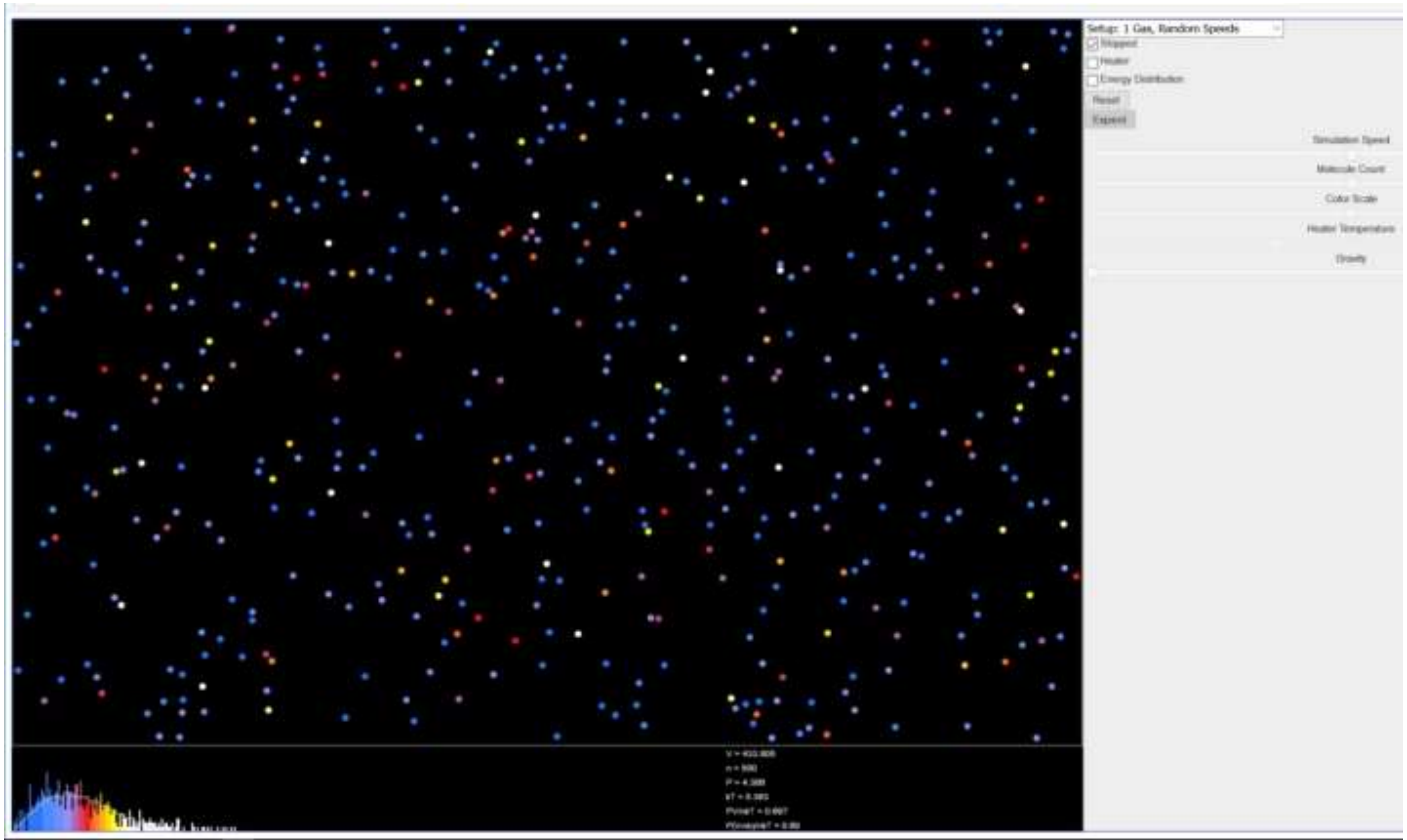
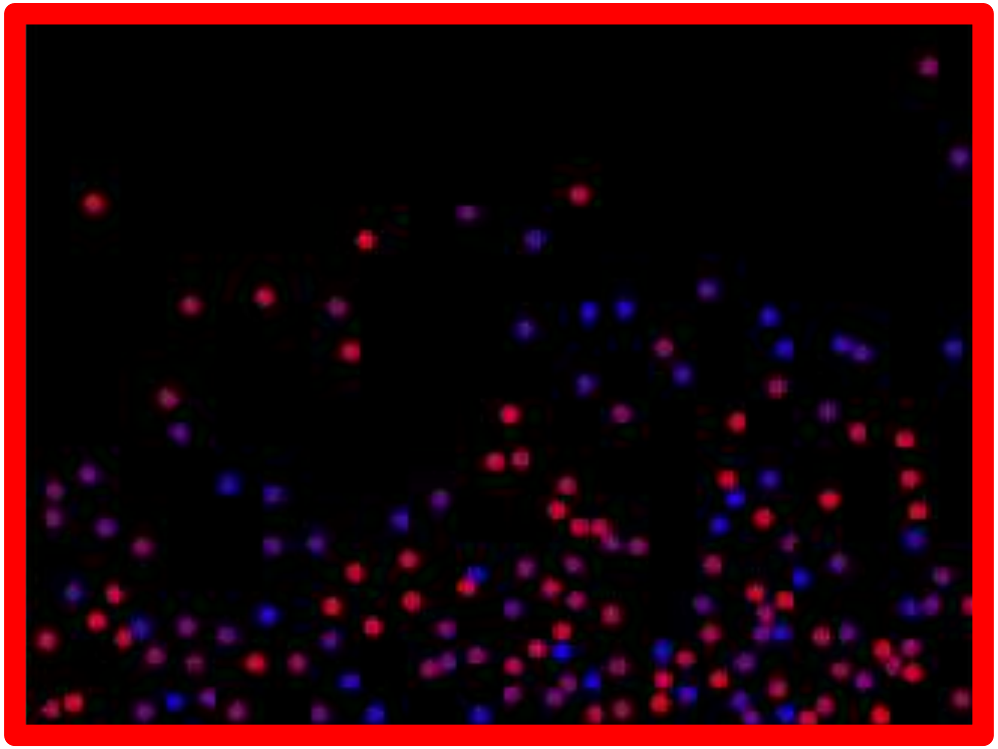


Particle velocity distribution approaches towards a stationary respectively equilibrium state!

The Boltzmann Theorem



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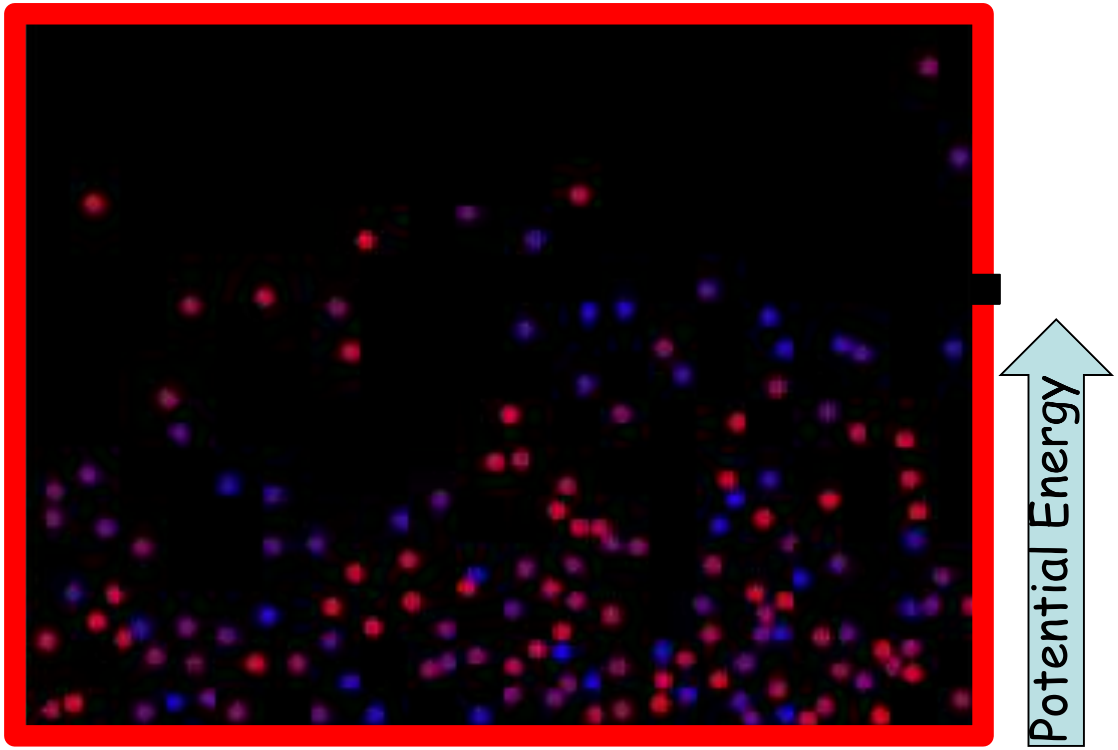

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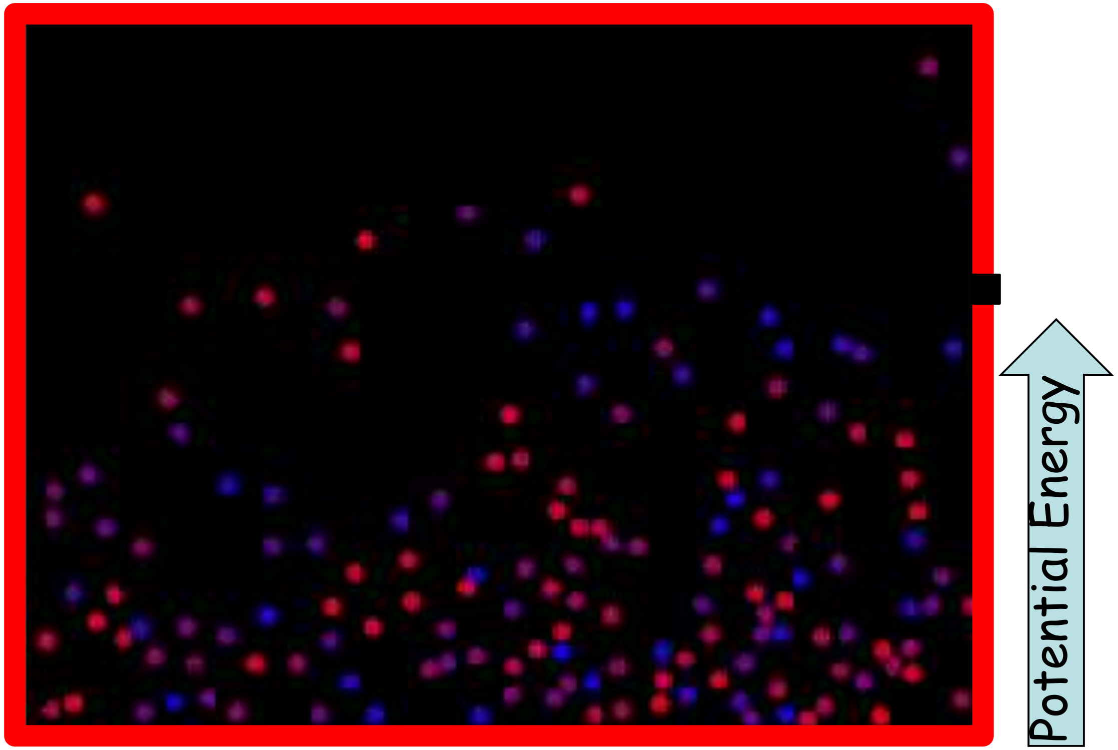
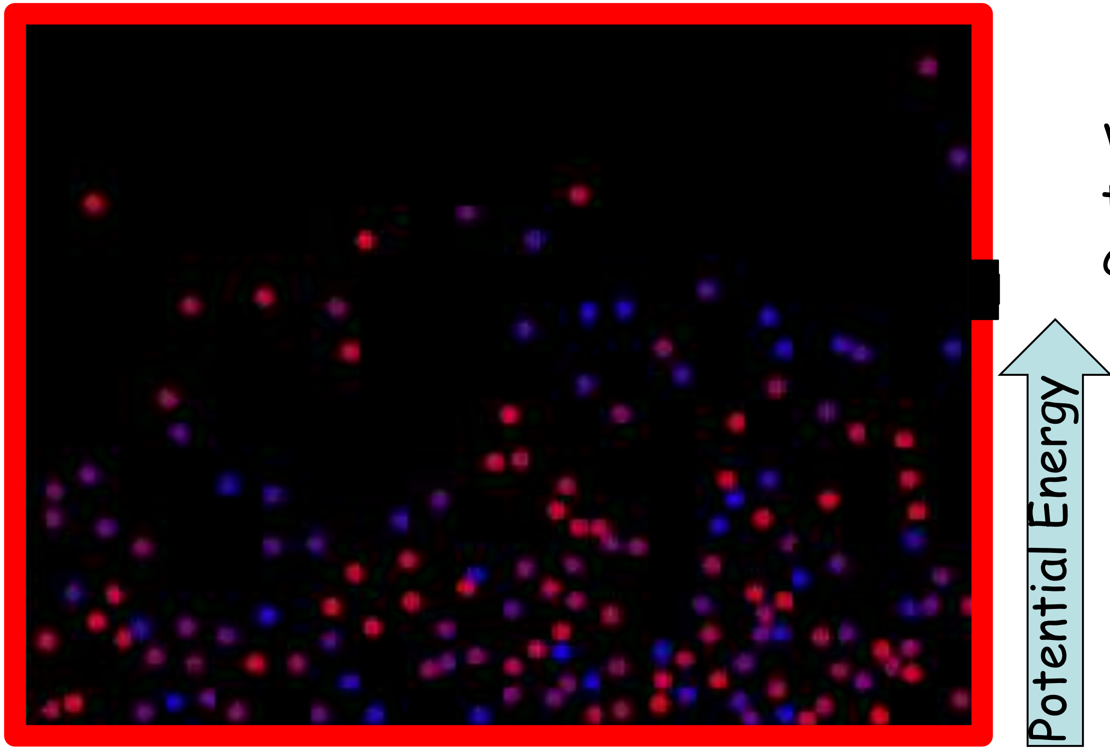


The Boltzmann Distribution

The Boltzmann distribution states that the probability of finding a physical system in a particular state is related to the energy of that state, by the formula

Probability ! ➡ $P(state) \propto e^{-E/kT}.$

The Boltzmann Theorem



Weight factor to be considered!

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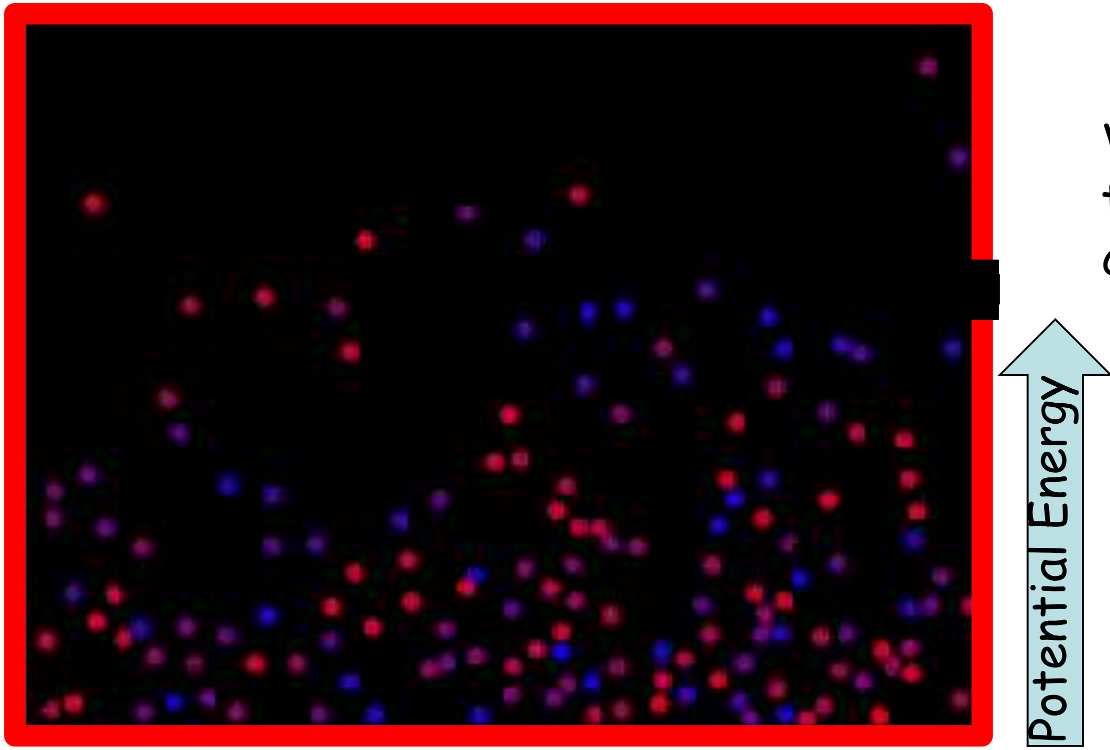

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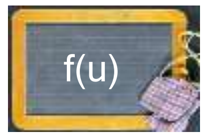

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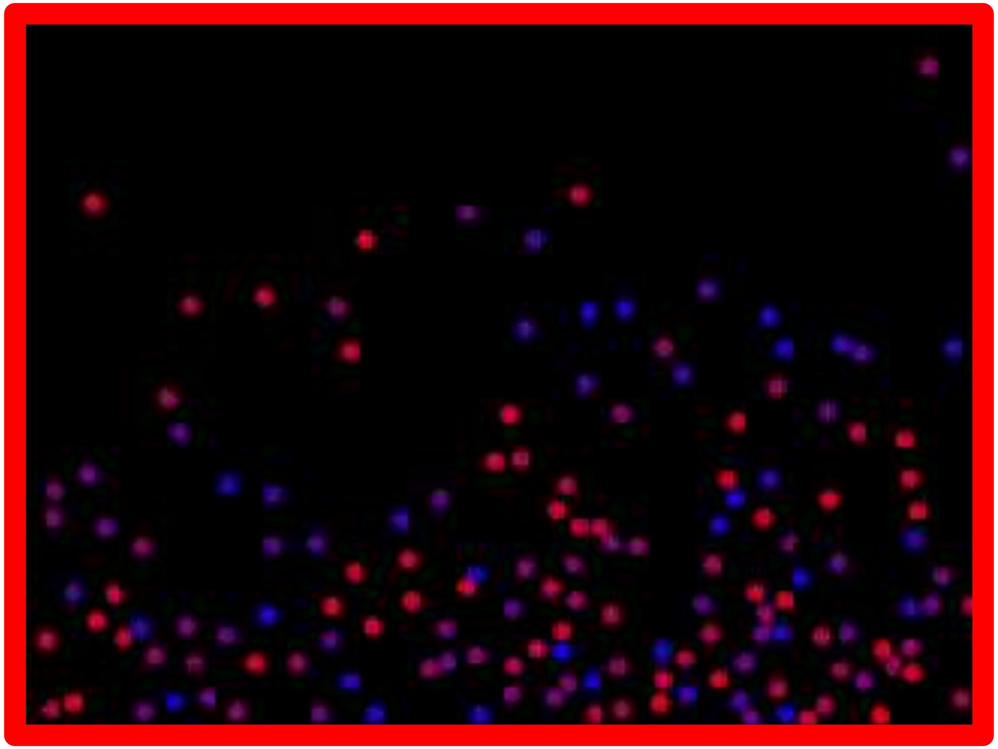
Boltzmann Distribution "Theorem"
The probability of finding a physical system in a particular state is related to the energy of that state by the relation $P_i \sim e^{-\frac{E_i}{kT}}$ (probability)

This must be multiplied with the number of possible states g_i at the corresponding energy E_i

$$\Rightarrow P_i = g_i \, e^{\frac{-E_i}{kT}}$$ (g_i: statistical weight)

The Boltzmann Theorem



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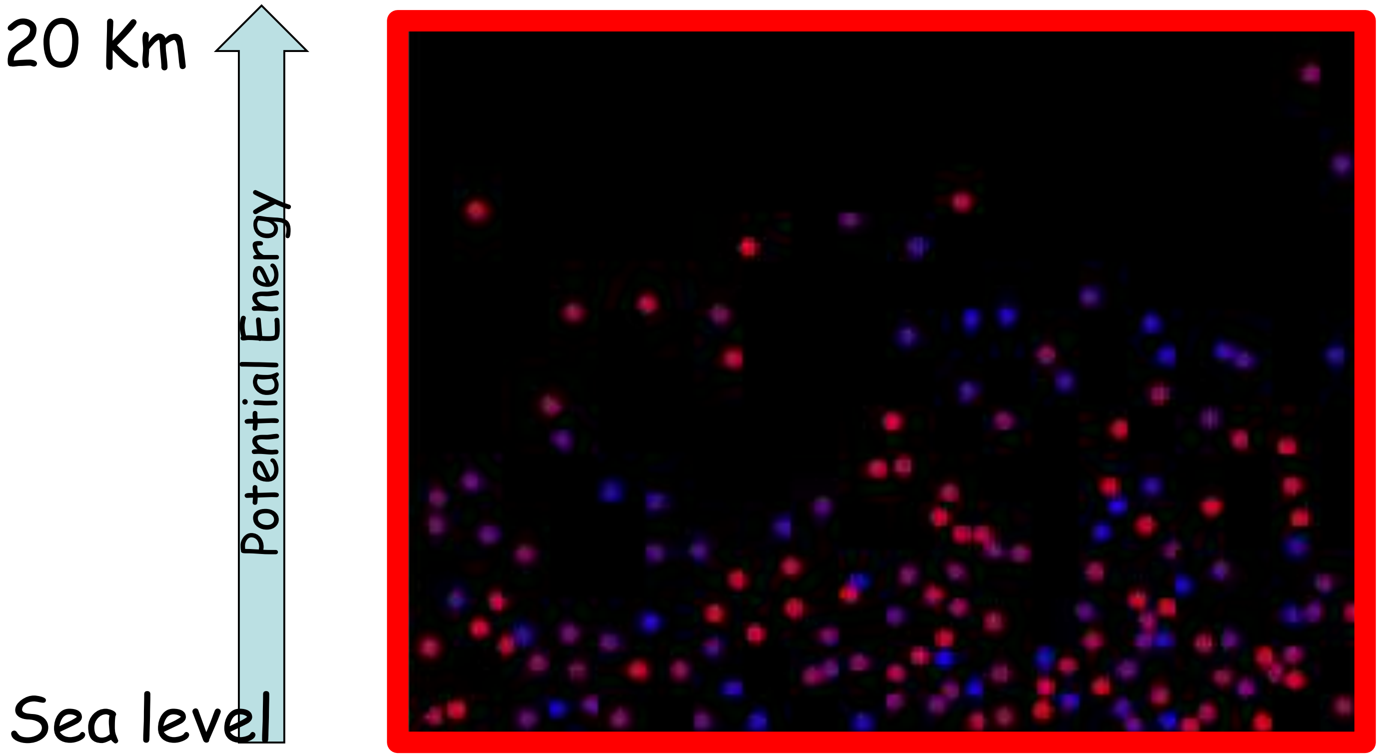

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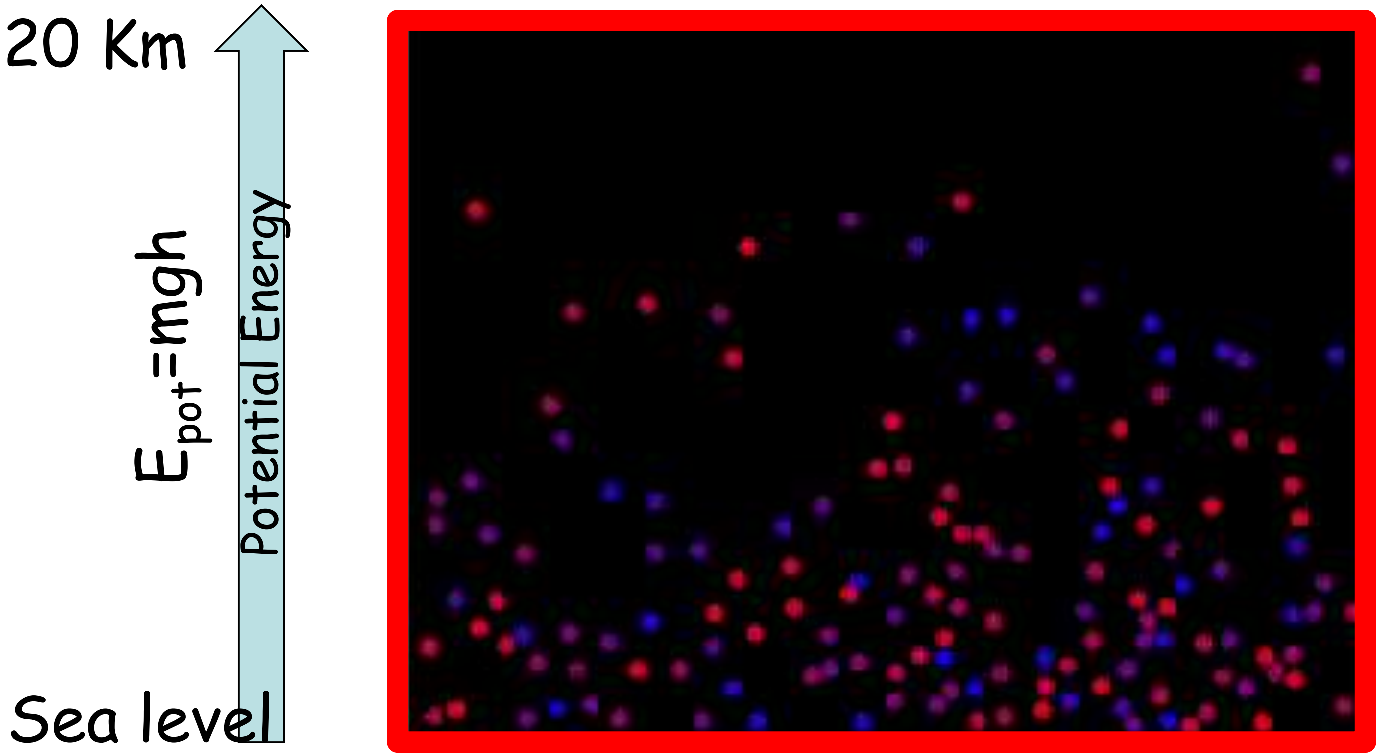

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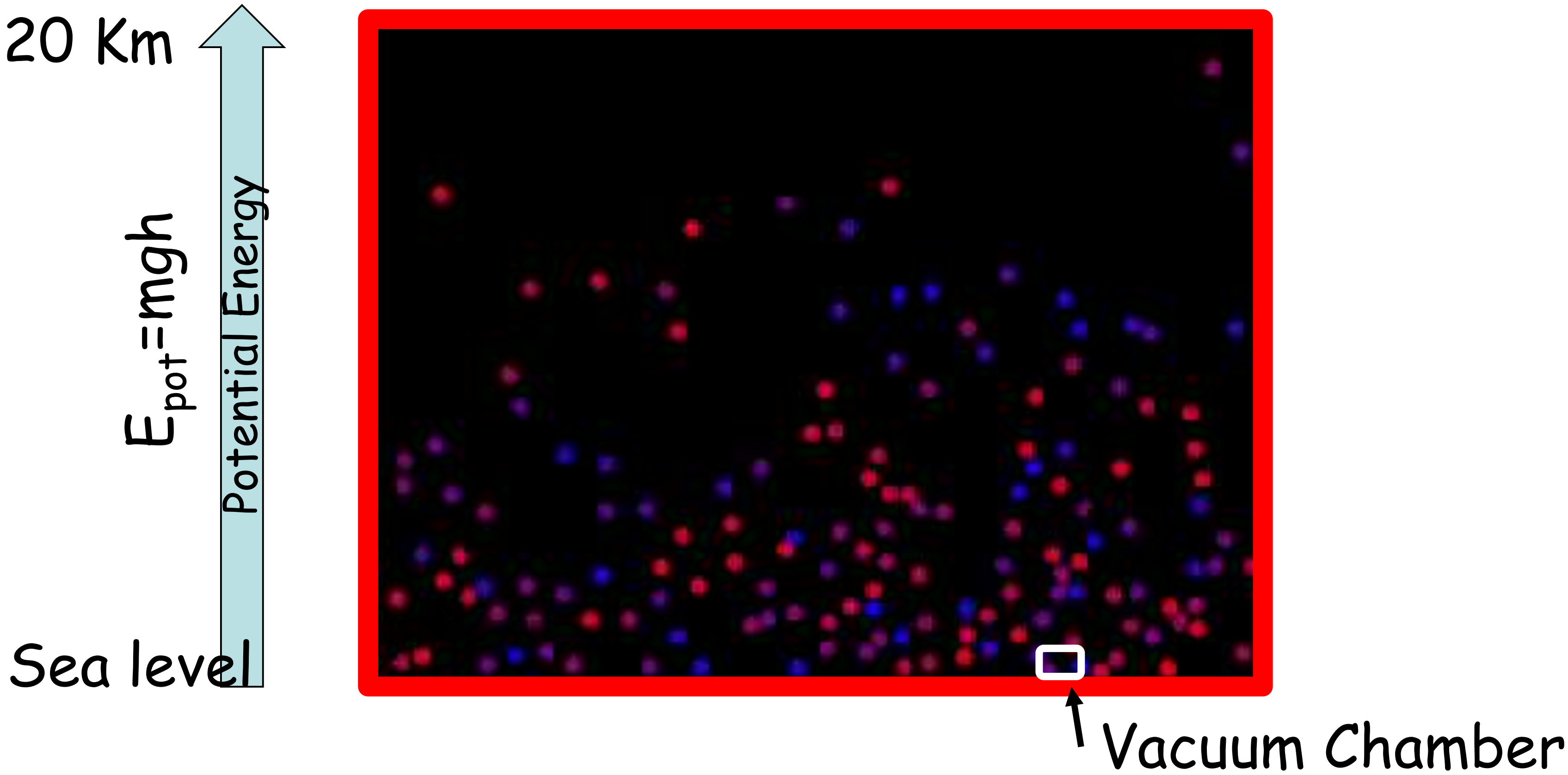

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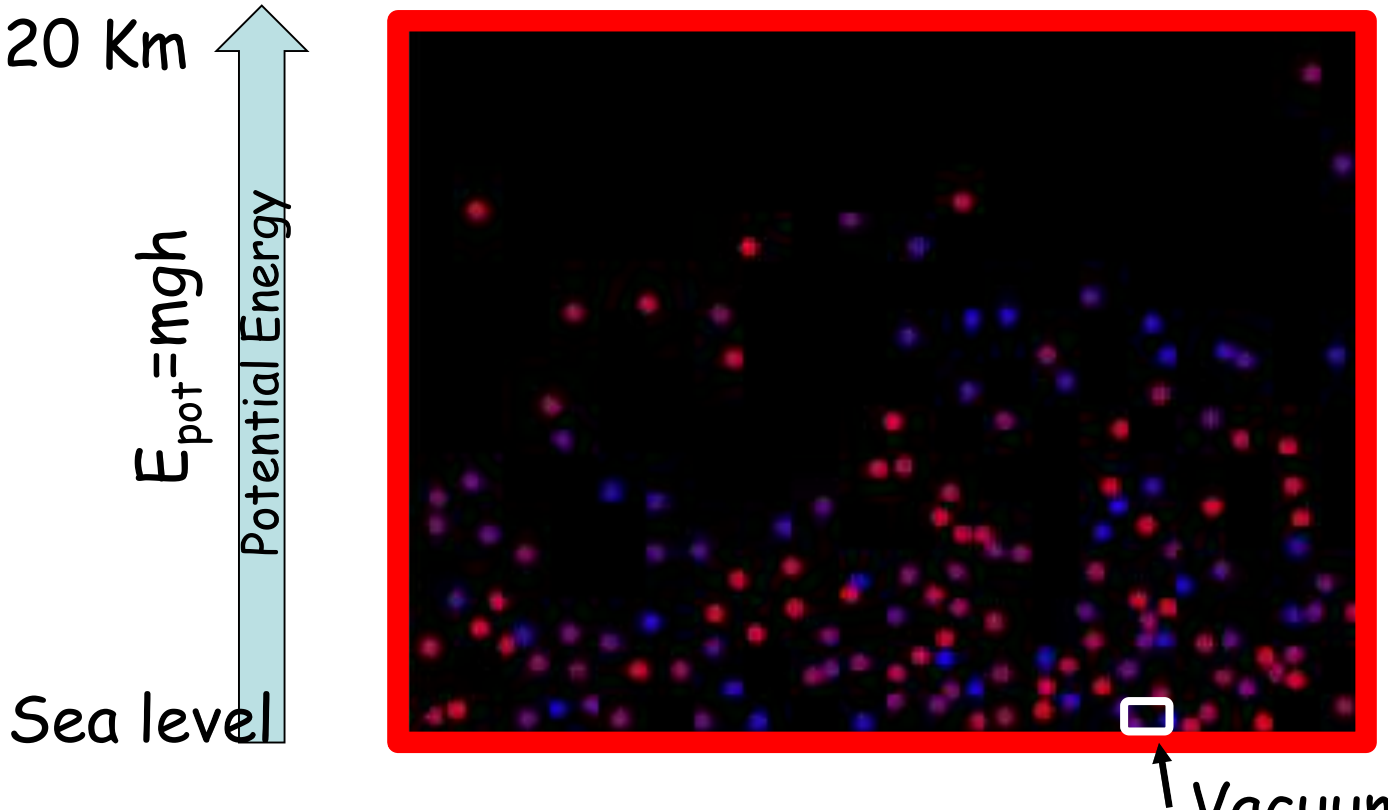

Difference in E_{pot} inside the chamber vanishes compared to E_{kin} !

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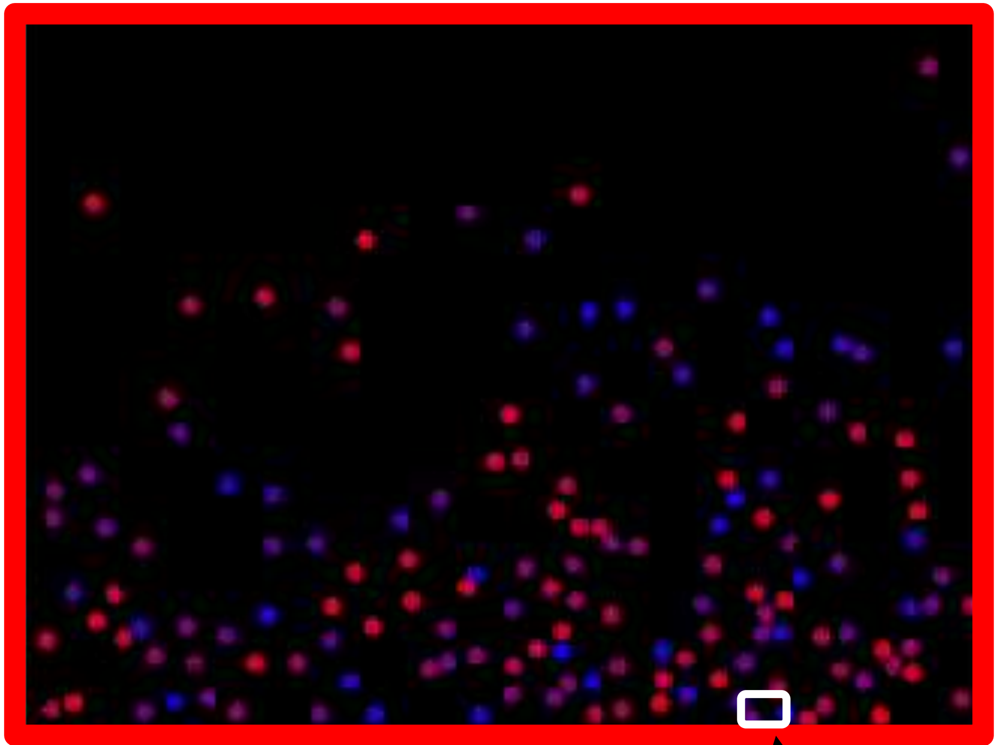
The Boltzmann Theorem

20 Km

$E_{pot}=mgh$

Potential Energy

Sea level



Difference in E_{pot} inside the chamber vanishes compared to E_{kin} !

Vacuum Chamber

$E_{kin}=\frac{1}{2}mu^2$

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B04c 2

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$$\Rightarrow p_i = g_i\, e^{\frac{-E_i}{kT}} \quad (g_i: \text{statistical weight})$$

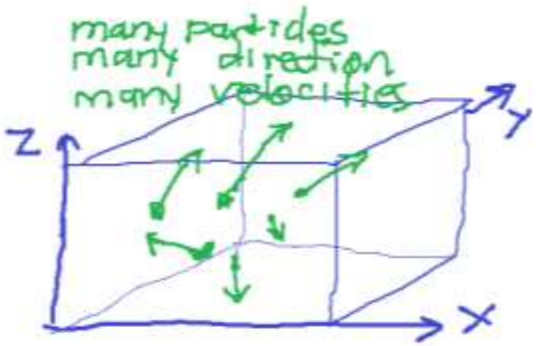
Application to ideal gas (neglecting gravity) $E_u = \frac{1}{2} m u^2$

u is continously changing however in average there is a constant fraction of particles: $f(u)\,du$ with the amount of u between u and $u+du$!
$f(u)\,du$ is equivalent to the probability p_u $\Rightarrow$

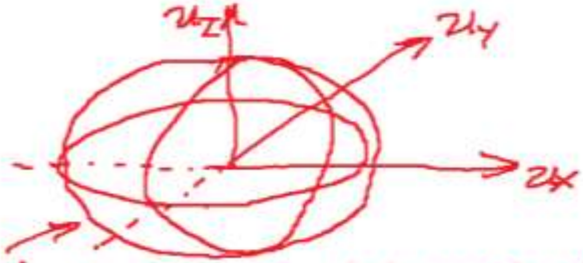
$$f(u)\,du = g(u)\, e^{-\frac{mu^2}{2kT}}$$

$\uparrow$?

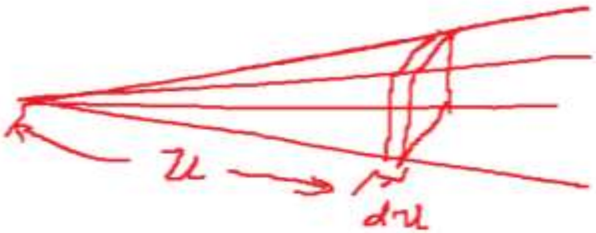
B04d



We sort the particles in the velocity space:



Location of particles with identical u → sphere $r = u$



Location of particles between u and $u + du$ within a shell of area $4\pi u^2$ and thickness du

$\Rightarrow g(u) \sim 4\pi u^2 du \Rightarrow f(u)du = C\,4\pi u^2 e^{-\frac{mu^2}{2kT}} du$

$\int_0^{+\infty} f(u)du = 1$! ↷ C can be determined as $C = \left(\frac{m}{2\pi kT}\right)^{3/2}$

B04e

$$\int_0^\infty f(u)\,du = C\int_0^\infty 4\pi u^2 e^{-\frac{mu^2}{2kT}}du = 1 \;\curvearrowright\; C = \frac{1}{\int_0^\infty 4\pi u^2 e^{-\frac{mu^2}{2kT}}du}$$

Analytic solution:
Integral table (Bronstein) $\int_0^\infty x^2 e^{-a^2x^2}dx = \frac{\sqrt{\pi}}{4a^3}$

$\curvearrowright$ substitute $x = u$, $a = \sqrt{\frac{m}{2kT}} = \left(\frac{m}{2kT}\right)^{\frac{1}{2}}$

$$4\pi\int_0^\infty u^2 e^{-\frac{mu^2}{2kT}}du = \frac{4\pi\sqrt{\pi}}{4\left(\frac{m}{2kT}\right)^{3/2}} = \frac{\pi^{3/2}}{\left(\frac{m}{2kT}\right)^{3/2}} = \frac{1}{C}$$

$$\Rightarrow C = \left(\frac{m}{2\pi kT}\right)^{3/2}$$

Finally $f(u)\,du = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi u^2 e^{-\frac{mu^2}{2kT}}du$

$$= \frac{4\pi}{(2\pi)^{3/2}}\cdot\left(\frac{m}{kT}\right)^{3/2} u^2 e^{-\frac{mu^2}{2kT}}du$$

$$\frac{4\pi}{2\pi\sqrt{2\pi}} = \frac{2}{\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}}$$

Both writings in literature

or $f(u)\,du = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT}\right)^{3/2} u^2 e^{-\frac{mu^2}{2kT}}du$

Boltzmann velocity distribution

Boltzmann theorem

Probability for state i $\quad P_i = g_i \, e^{-\frac{E_i}{kT}}$

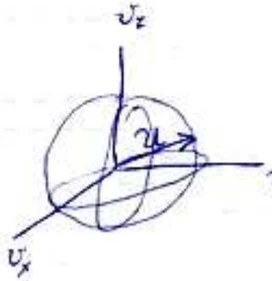
↑ statisticle weight

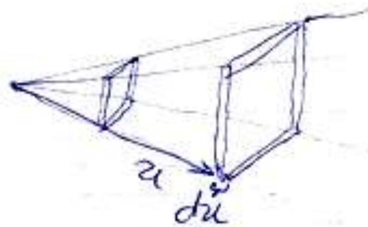
Ideal gas: $E = \frac{1}{2} m u^2$

Probability for velocity u (Fraction of u between u and $u+du$ is $f(u)du$)

$$f(u)du \; g(u) \, e^{-\frac{m u^2}{2kT}}$$

↳ will be proportional to a spere skin in the velocity space





Volume of the skin $4\pi u^2 du$

$$f(u)du = C 4\pi u^2 e^{-\frac{\frac{1}{2} m u^2}{kT}} du$$

↑ the unknown constant can be derived because $\int_0^\infty f(u)du = 1$!

$$\Rightarrow C = \left(\frac{m}{2\pi kT}\right)^{3/2}$$

So $f(u)du = 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} u^2 e^{-\frac{m u^2}{2kT}} du$

or $f(u) = \sqrt{\frac{2}{\pi}} \left(\frac{m}{kT}\right)^{3/2} u^2 e^{-\frac{m u^2}{2kT}}$

Derivation of the velocity distribution

$$f(u)\,du = C\,4\pi u^2 e^{-\frac{mu^2}{2kT}}\,du \quad \text{and} \quad C = \frac{1}{\int_0^\infty 4\pi u^2 e^{-\frac{mu^2}{2kT}}\,du}$$

Analytic solution:

Integral Table (Bronstein)

$$\int_0^\infty x^2 e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{4a^3}$$

→ substitute $x \equiv u$, $a = \sqrt{\frac{m}{2kT}} = \left(\frac{m}{2kT}\right)^{\frac{1}{2}}$

$$4\pi\int_0^\infty u^2 e^{-\frac{mu^2}{2kT}}\,du = \frac{4\pi\cdot\sqrt{\pi}}{4\left(\frac{m}{2kT}\right)^{3/2}} = \frac{\pi^{3/2}}{\left(\frac{m}{2kT}\right)^{3/2}} = \frac{1}{C}$$

$$\Rightarrow \quad C = \left(\frac{m}{2\pi kT}\right)^{3/2}$$

Finally

$$f(u)\,du = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi u^2 e^{-\frac{mu^2}{2kT}}\,du$$

$$= \frac{4\pi}{(2\pi)^{3/2}}\left(\frac{m}{kT}\right)^{3/2} u^2 e^{-\frac{mu^2}{2kT}}\,du$$

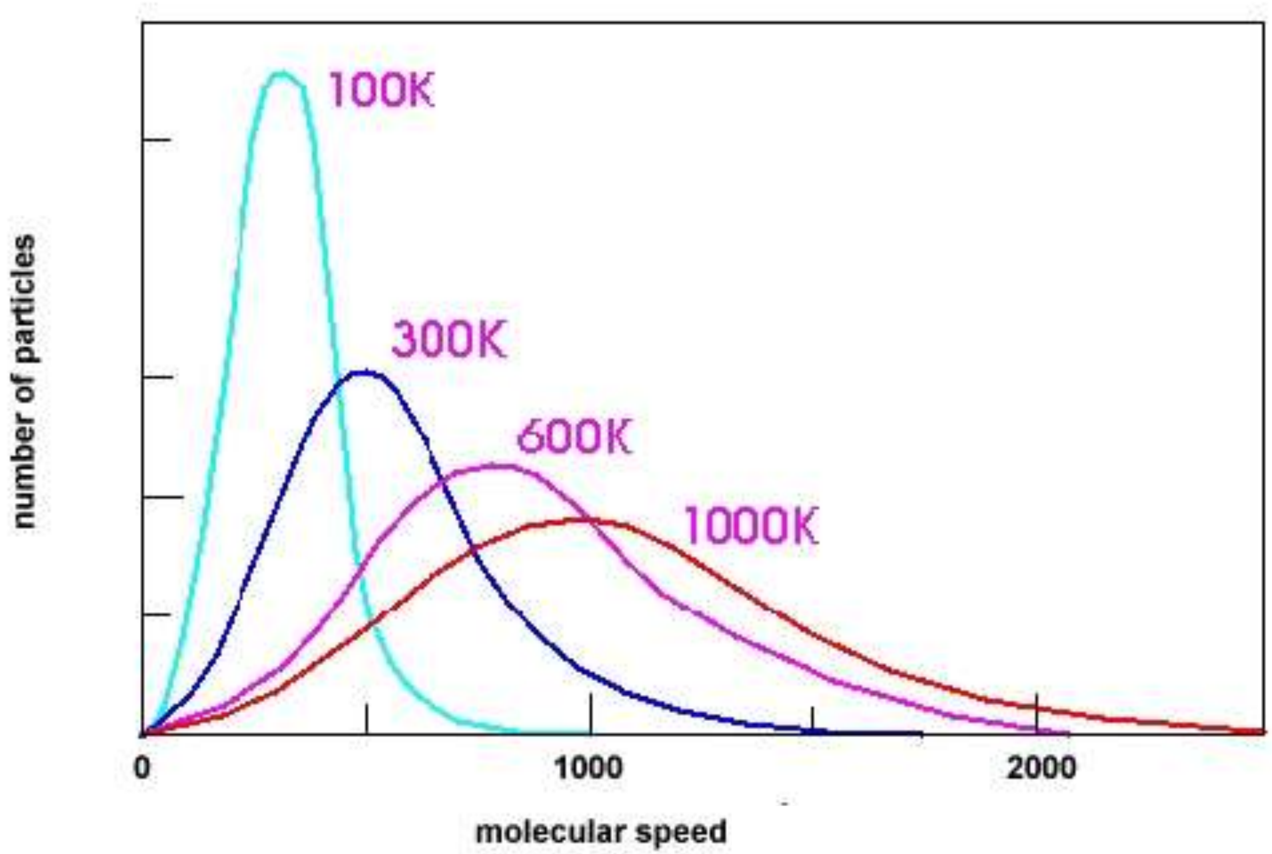
$$\frac{4\pi}{2\pi\cdot\sqrt{2\pi}} = \frac{2}{\sqrt{2\pi}} = \sqrt{\frac{2}{\pi}}$$

Or

$$f(u)\,du = \sqrt{\frac{2}{\pi}}\left(\frac{m}{kT}\right)^{3/2} u^2 e^{-\frac{mu^2}{2kT}}\,du$$

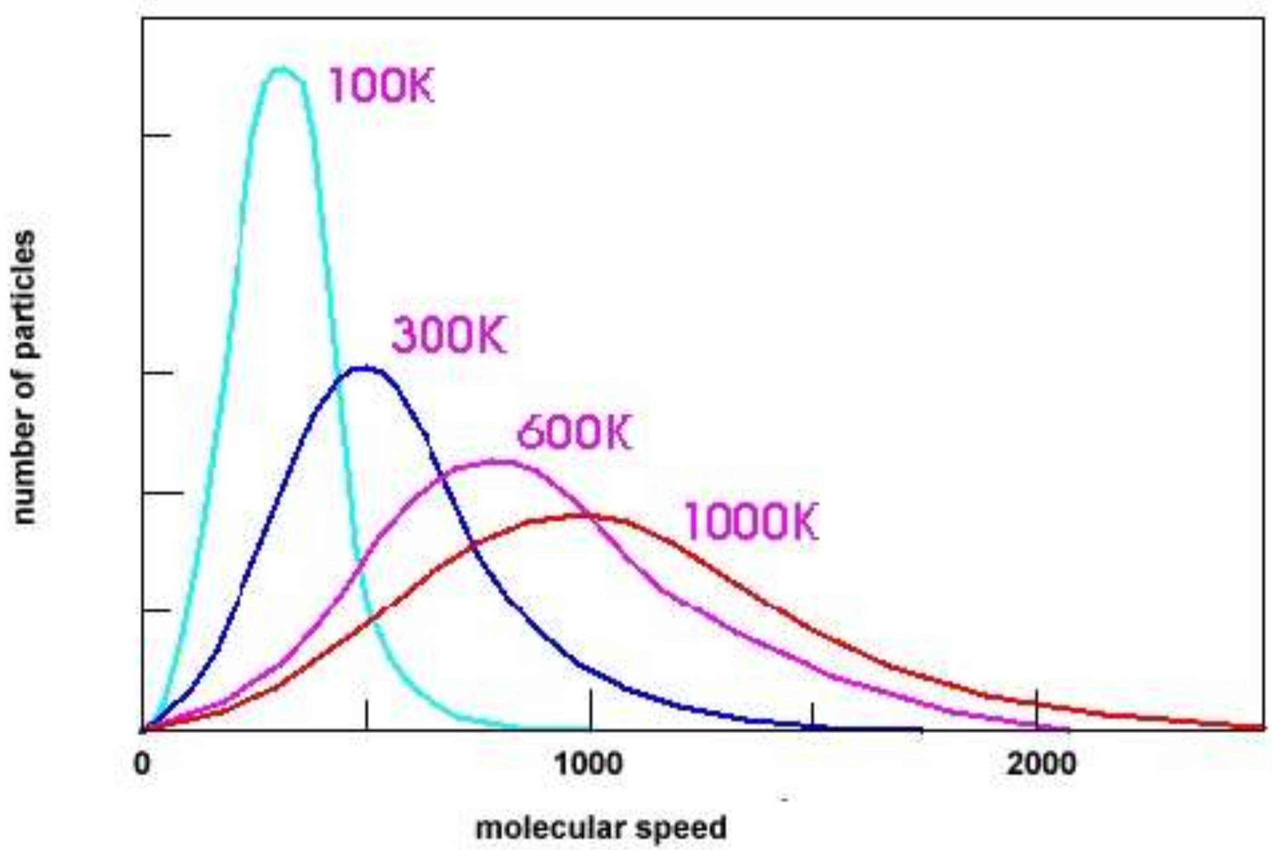
Both writings found in the literature

$$f(v) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$



$$f(v) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$

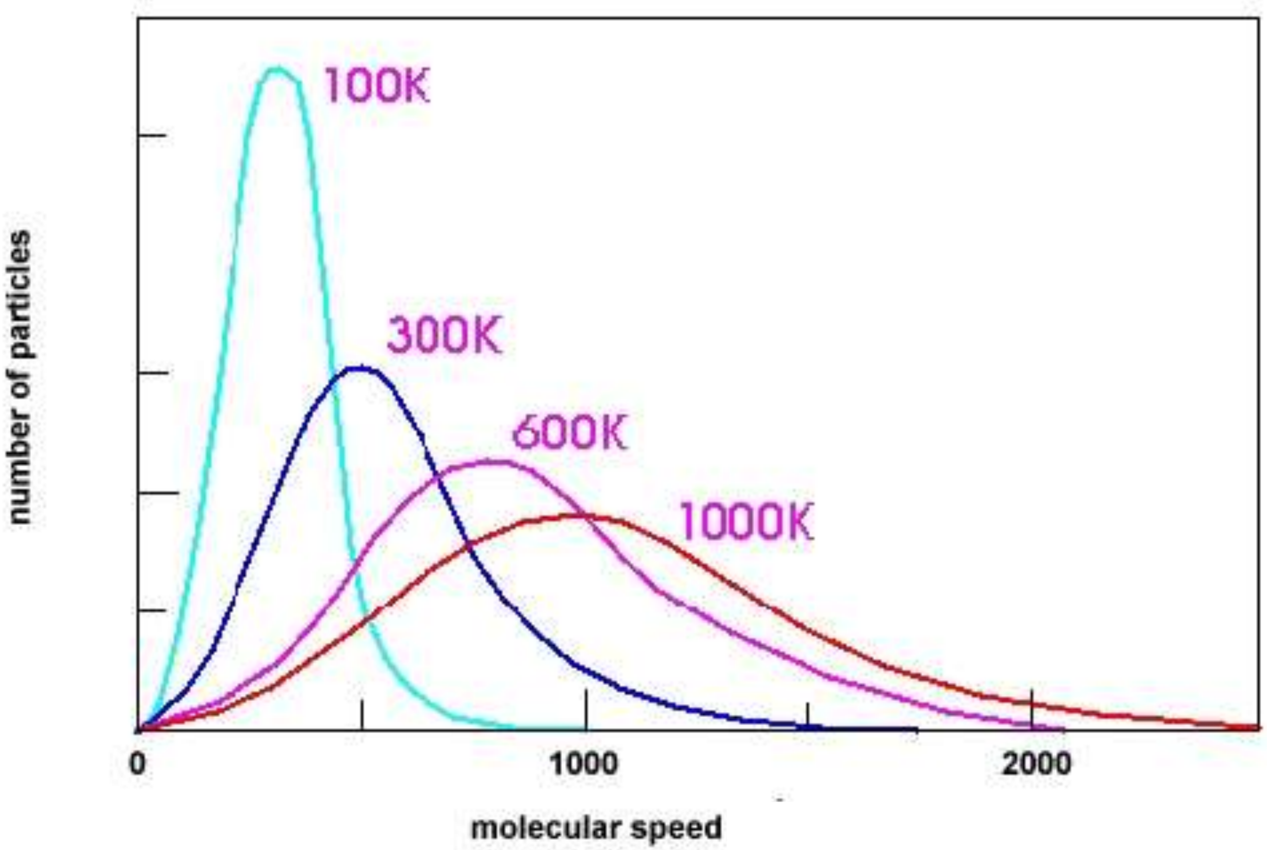
$$\int_0^\infty f(v)dv = C\int_0^\infty v^2 \exp\left(\frac{-mv^2}{2kT}\right)dv = 1$$



$$f(v) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$

$$\int_0^\infty f(v)dv = C\int_0^\infty v^2 \exp\left(\frac{-mv^2}{2kT}\right)dv = 1$$

The higher the temperature, the higher the speed at the most probable value, however the lower is the highest probability!



$$f(v) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} 4\pi v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$

»Wissen schafft Brücken.«