THE WEAK COMPLETION SEMANTICS AND COUNTEREXAMPLES

MEGHNA BHADRA*
TU Dresden, 01062 Dresden, Germany
meghna.bhadra@tu-dresden.de

STEFFEN HÖLLDOBLER*
TU Dresden, 01062 Dresden, Germany and
North Caucasus Federal University, Stavropol, Russian Federation
sh43@posteo.de

Abstract

An experiment has revealed that if the antecedent of a conditional sentence is denied, then most participants conclude that the negation of the consequent holds. However, a significant number of participants answered *nothing follows* if the antecedent of the conditional sentence was non-necessary. The Weak Completion Semantics correctly models the answers of the majority, but cannot explain the number of *nothing follows* answers. In this paper we extend the Weak Completion Semantics by counterexamples. The extension allows to explain the experimental findings.

1 Introduction

Conditional sentences are propositions of the form $\mathit{if}\, A\ \mathit{then}\, C$ where A and C are atomic sentences called antecedent and consequent, respectively. Four kinds of conditional inference tasks have been a common area of research by psychologists till date:

1. Affirmation of the antecedent (AA): $\mathit{if}\, A\ \mathit{then}\, C$ and A, therefore C.
2. Denial of the antecedent (DA): $\mathit{if}\, A\ \mathit{then}\, C$ and $\neg A$, therefore $\neg C$.
3. Affirmation of the consequent (AC): $\mathit{if}\, A\ \mathit{then}\, C$ and C, therefore A.
4. Denial of the consequent (DC): $\mathit{if}\, A\ \mathit{then}\, C$ and $\neg C$, therefore $\neg A$.

*Authors are mentioned in alphabetical order.

In classical, two-valued propositional logic, conditional sentences are taken to mean material implications and bi-conditionals to mean (material) equivalence. The conclusion for the DA and the AC are hence considered to be logical fallacies (invalid) for a conditional sentence whereas they are considered valid for a bi-conditional. However, from our human experiences we know that it is not always the case in real life. When replacing the above abstract conditional sentences which have no everyday context with conditionals which do, the inferences largely depend on the semantics and pragmatics of human communication, culture, and context. In this paper, we therefore discuss how everyday conditional sentences can be categorized into four proposed semantic categories. We also share the results of an experiment reported in [6, 5] and (with particular regard to the DA) demonstrate how such classifications can help model an average human (DA) reasoner.

The Weak Completion Semantics (WCS) is a three-valued, non-monotonic cognitive theory, which can not only adequately model the suppression task by [2] as shown by [7], human syllogistic reasoning as shown by [20], and DC inferences as shown by [6] but also the AA, AC, and the majority $\neg C$ answers of the DA as shown by [5]. While the existing framework of the WCS adequately models the general consensus of the $\neg C$ responses generated in case of the DA inference task, it did not however, seem adequate to model the number of *nothing follows* responses, which is especially significant in case of conditional sentences with *non-necessary antecedents*. Here, *nothing follows* denotes no new inference or specific conclusion can be drawn with regard to the consequent of the conditional sentence.

In order to elaborate on what it really means for a conditional sentence to have a *non-necessary antecedent* and to propose a solution to the aforementioned problem, we begin by considering the following DA inference tasks:

1. *If Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age* and *Maria is not drinking alcoholic beverages in a pub.*

2. *If the plants get water, then they will grow* and *the plants get no water.*

Both of these examples appeared in the aforementioned experiment, and as was the case for every conditional sentence that was included in it, accompanied by a small background story. The curious reader may find the background stories for the above conditionals in the appendix. In the first example, 28 out of 56 participants answered *Maria must not be over 19 years of age*, whereas 25 answered *nothing follows*. In this example the antecedent is non-necessary; it is not considered necessary for a person to drink alcohol in order for her to be older than 19. In fact, there are many people who *do not drink alcoholic beverages* although *they are over 19 years of age*. In the second example, 47 out of 56 participants answered *the plants will not grow* whereas only 8 answered *nothing follows*. In this case, the antecedent is necessary. *Plants do not grow without water.* Table 4 gives a complete account of this experimental data.

Based on this observation, we propose an extension which allows WCS to account for the *nothing follows* answers. In Example 1, the existing framework of the WCS creates a model where given that *Maria is not drinking alcoholic beverages*, it can be concluded that *Maria is not older than 19 years of age*. With the proposed extension, however, a counterexample that *Maria is not drinking alcoholic beverages* and yet *Maria is older than 19 years of age* can be constructed. This leads to an alternative model, which when compared to the former model and reasoned skeptically, leads to the conclusion that it is unknown whether *Maria is older than 19 years of age*. In Example 2, WCS creates a model where given that *the plants do not get water*, it can be concluded that *they will not grow*. But in this case, a counterexample does not readily exist.

The paper is organized as follows. In the next section we formally introduce the WCS. A classification of conditional sentences is given in Section 3 where we discuss how conditionals may be classified as obligational or factual, and their antecedents as necessary or non-necessary. We discuss how pragmatics, culture and other such factors may affect how different individuals comprehend the same conditional sentence, and the different possibilities that arise from these comprehensions (in a spirit similar to [15]). Furthermore, we present how the WCS can handle the aforementioned classifications of conditionals. The experiment is described in Section 4. We extend the WCS framework with the search for counterexamples in Section 5. How the WCS may model the DA inference task is discussed in elaboration in Section 6 where we also revisit how the WCS models the AC and the DC inference tasks. This is followed by Section 7 which contains a brief discussion about the predictions of the Mental Model Theory (MMT) and the WCS with regard to the DA, AC and DC. Finally, in Section 8 we conclude and outline possible future research.

2 The Weak Completion Semantics

We assume the reader to be familiar with logic and logic programming as presented in e.g. [8] and [18]. Let $\top$, $\bot$, and $\cup$ be truth constants denoting *true*, *false*, and *unknown*, respectively. A *(logic) program* is a finite set of clauses of the form $B \leftarrow body$, where B is an atom and *body* is $\top$, or $\bot$, or a finite, non-empty set of literals. Clauses of the form $B \leftarrow \top$, $B \leftarrow \bot$, and $B \leftarrow L_1, \ldots, L_n$ are called *facts*, *assumptions*, and *rules*, respectively, where L_i, $1 \leq i \leq n$, are literals. We restrict our attention to propositional programs although the WCS extends to first-order programs as well [11].

Throughout this paper, $\mathcal{P}$ will denote a program. An atom B is *defined* in $\mathcal{P}$ if and only if $\mathcal{P}$ contains a clause of the form $B \leftarrow body$. As an example consider the program

$$\mathcal{P}_0 = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\},$$

where A, C, and ab are atoms. C and ab are defined, whereas A is undefined. ab is an abnormality predicate which is assumed to be false. In the WCS, this program represents the conditional sentence *if A then C*. In their everyday lives humans are often required to reason in situations where the information of all factors affecting the situation might not be complete. They still reason, unless new information which needs consideration comes to light. The abnormality predicate in the program serves the purpose of this (default) assumption, as was suggested in [21].

Consider the following transformation: (1) For all defined atoms B occurring in $\mathcal{P}$, replace all clauses of the form $B \leftarrow body_1,\ B \leftarrow body_2,\ \ldots$ by $B \leftarrow body_1 \vee body_2 \vee \ldots$. (2) Replace all occurrences of $\leftarrow$ by $\leftrightarrow$. The resulting set of equivalences is called the *weak completion* of $\mathcal{P}$. It differs from the program completion defined in [4] in that undefined atoms in the weakly completed program are not mapped to false, but to unknown instead. Weak completion is necessary for the WCS framework to adequately model the suppression task (and other reasoning tasks) as demonstrated in [7].

As shown in [12], each weakly completed program admits a least model under the three-valued Łukasiewicz logic [19] (see Table 1). This model will be denoted by $\mathcal{M}_{wc\mathcal{P}}$. It can be computed as the least fixed point of a semantic operator introduced in [22]. Let $\mathcal{P}$ be a program and I be a three-valued interpretation represented by the pair $\langle I^\top, I^\bot \rangle$, where $I^\top$ and $I^\bot$ are the sets of atoms mapped to true and false by I, respectively, and atoms which are not listed are mapped to unknown. We define $\Phi_\mathcal{P}\, I = \langle J^\top, J^\bot \rangle$,[1] where

$$\begin{array}{rcl} J^\top & = & \{B \mid \text{there is } B \leftarrow body \in \mathcal{P} \text{ and } I\ body = \top\}, \\ J^\bot & = & \{B \mid \text{there is } B \leftarrow body \in \mathcal{P} \text{ and} \\ & & \quad \text{for all } B \leftarrow body \in \mathcal{P} \text{ we find } I\ body = \bot\}. \end{array}$$

Following [16] we consider an *abductive framework* $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$, where $\mathcal{P}$ is a program, $\mathcal{A}_\mathcal{P} = \{B \leftarrow \top \mid B \text{ is undefined in } \mathcal{P}\} \cup \{B \leftarrow \bot \mid B \text{ is undefined in } \mathcal{P}\}$ is the *set of abducibles*. $\mathcal{IC}$ is a finite set of *integrity constraints* which are expressions of the form $\mathsf{U} \leftarrow L_1, \ldots, L_n$ and $\bot \leftarrow L_1, \ldots, L_n$ where each L_i, $1 \le i \le n$, is a literal. And $\mathcal{M}_{wc\mathcal{P}} \models_{wcs} F$ if and only if $\mathcal{M}_{wc\mathcal{P}}$ maps the formula F to true. Let $\mathcal{O}$ be an *observation*, i.e., a finite set of literals each of which does not follow from $\mathcal{M}_{wc\mathcal{P}}$. We apply abduction to explain $\mathcal{O}$, where $\mathcal{O}$ is called *explainable* in the abductive framework $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$ if and only if there exists a non-empty $\mathcal{X} \subseteq \mathcal{A}_\mathcal{P}$ called an *explanation* such that $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} \models_{wcs} L$ for all $L \in \mathcal{O}$ and $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})}$ satisfies $\mathcal{IC}$. We have assumed that explanations are non-empty as otherwise the observation already follows from the weak completion of the program. Formula F *follows credulously* from $\mathcal{P}$ and $\mathcal{O}$ if and only if there exists an explanation $\mathcal{X}$ for $\mathcal{O}$ such that $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} \models_{wcs} F$. F *follows skeptically*

[1] Whenever we apply a unary operator like $\Phi_\mathcal{P}$ to an argument like I, then we omit parenthesis and write $\Phi_\mathcal{P}\, I$ instead. Likewise, we write $I\ body$ instead of $I(body)$.

from $\mathcal{P}$ and $\mathcal{O}$, if and only if $\mathcal{O}$ can be explained and for all explanations $\mathcal{X}$ for $\mathcal{O}$ we find $\mathcal{M}_{wc(\mathcal{P}\cup\mathcal{X})} \models_{wcs} F$. The latter is an application of the so-called *Gricean implicature* [9]: humans normally do not quantify over things which do not exist. Meaning, (unlike classical logic) *all* explanations for an observation $\mathcal{O}$ may only be taken into account to skeptically decide on a formula F, when $\mathcal{O}$ is explainable and these so-called explanations exist in the first place. If a formula F does not follow skeptically from $\mathcal{P}$ and $\mathcal{O}$, we conclude *nothing follows*. Furthermore, one should also observe that if an observation $\mathcal{O}$ cannot be explained, then *nothing follows* credulously as well as skeptically. In all examples discussed in this paper the set of integrity constraints is empty. Integrity constraints are not very relevant to the goal of this paper. However they are needed in other applications of the WCS like human disjunctive reasoning [10].

Given premises, general knowledge, and observations, *reasoning in the WCS* is currently modeled in five steps:

1. Reasoning towards a logic program $\mathcal{P}$ following [22].
2. Weakly completing the program, which leads to $wc\mathcal{P}$.
3. Computing the least model $\mathcal{M}_{wc\mathcal{P}}$ of the weak completion of $\mathcal{P}$, $wc\mathcal{P}$, under the three-valued Łukasiewicz logic.
4. Reasoning with respect to $\mathcal{M}_{wc\mathcal{P}}$.
5. If observations cannot be explained, then applying skeptical abduction using the specified set of abducibles.

F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

Table 1: The truth tables for the Łukasiewicz logic. One should observe that $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U} \leftrightarrow \mathsf{U} = \top$ as shown in the grey cells.

In the following sections we will explain how these five steps work in the case of the DA reasoning tasks considered in this paper. More examples can be found, for example, in [7] or [20] or [10].

3 A Classification of Conditional Sentences

3.1 Obligational versus Factual Conditionals

Following [3], we call a conditional sentence an *obligational conditional* if the truth of the consequent appears to be obligatory given that its antecedent is true. For each obligational conditional there are two initial possibilities humans think about. The first possibility is the conjunction of the antecedent and the consequent, which is permitted. The second possibility is the conjunction of the antecedent and the negation of the consequent, which is forbidden. Exceptions are possible but unlikely. This can be exemplified by Example 1. In many countries the law demands that a person may only drink alcohol publicly when they are above a certain age group (for example, 19 years). This implies that *Maria is drinking alcoholic beverages in a pub* and *she is older than 19 years* is a permitted possibility, whereas *Maria is drinking alcoholic beverages in a pub* and *she is not older than 19 years* is a forbidden one. Hence, *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age* is an obligational conditional. Concerning Example 2, *plants getting water* and *plants are growing* is a permitted possibility. But *plants getting water* and *plants are not growing* is also possible; there are many other factors like, for example, over-watering, lack of light, pest infestation, etc. which may hinder their growth. Hence, *if the plants get water, then they will grow*, is not an obligational conditional.

Obligational conditionals may have different sources. They may be based on legal laws like Example 1 and are often called deontic conditionals, in which case words like *must*, *should* or *ought* may be explicitly used in the conditional sentence. Their usage however, does not seem mandatory in everyday communication and is skipped on many occasions. Knowledge or awareness that the consequent is obligatory given the antecedent suffices in these cases, and yields the same responses as when explicitly denoting the obligation. Obligational conditionals may also express moral or social obligations like *if somebody's parents are elderly, then he/she should look after them* [3]. Other obligational conditionals are based on causal or physical laws which hold on our planet like, *if an object is not supported, then it will fall to the ground*. In each case the conjunction of the antecedent and the consequent is permitted, whereas the conjunction of the antecedent and the negation of the consequent is forbidden.

On the other end of the spectrum, if the consequent of a conditional sentence is not obligatory given the antecedent, then it is called a *factual conditional*. In particular, the truth of the antecedent is *inconsequential* to that of the consequent; that is (even) if the antecedent is true, the consequent may or may not be true. This has already been exemplified using Example 2. The conditional *if the plants get water, then they will grow* is a factual one. As another example consider the conditional sentence *if Maria is over 19 years, then she may drink alcoholic beverages in a pub*. This sentence is a factual one, because given the atomic proposition *Maria is over 19 years* is true, one can imagine two permitted possibilities, one where *Maria drinks alcohol beverages* and another where *Maria does not drink alcoholic beverages in a pub*.

3.2 Necessary versus Non-Necessary Antecedents

As discussed in the previous section, the obligational or factual nature of a conditional sentence indicates if the consequent is obligatory or simply possible, provided the antecedent is satisfied. The question that may naturally arise at this point is, what happens when the antecedent of a conditional sentence is not satisfied? To that end, we now discuss the classifications of antecedents of conditional sentences. The antecedent A of a conditional sentence *if* A *then* C is said to be *necessary* with respect to the consequent C, if and only if C cannot be true unless A is true. This implies that if A does not hold, C cannot either. In Example 2, *plants get water* is a necessary antecedent for *plants will grow*. If a plant is not watered at all, it will very likely die.

The above does not imply however, that the antecedent need always be a precondition for the consequent, per se. The antecedent A of a conditional sentence *if* A *then* C is said to be *non-necessary* with respect to the consequent C, if C can be true irrespective of the truth or falsity of A. In particular this implies, if A does not hold, C may or may not hold. In Example 1, the falsity of *drinking alcoholic beverages in a pub* is inconsequential to the truth of the consequent *older than 19 years*. There are plenty of adults (over 19 years) who do not drink alcohol. The antecedent of the conditional sentence *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age*, in Example 1 is therefore called non-necessary.

3.3 Pragmatics

Generally, humans may recognize conditional sentences as obligational or factual and antecedents as necessary or non-necessary. This leads to an informal and pragmatic classification of four kinds: obligational conditional with necessary antecedent (ON) or non-necessary antecedent (ONN) and factual conditional with necessary antecedent (FN) or non-necessary antecedent (FNN). For an abstract conditional *if* A *then* C, *without* an everyday context, the classification of the conditional into any of the aforementioned kinds would be straghtforward and as discussed in Subsections 3.1 and 3.2, since they are independent of context. The classification of everyday conditionals (those *with* an everyday context), however, often depend on pragmatics: the context, the background knowledge and experience of a person. For example, the conditional sentence *if it is cloudy, then it is raining* discussed in [17] may be classified as an obligational conditional with necessary antecedent by people living in Java, whereas it may be classified as a factual conditional by people living in Central Europe. In another example [15], the authors conducted an experiment, where they categorized the proposition *if it's heated, then this butter will melt* as a bi-conditional. In particular they considered *if butter is not heated, it will not melt*. This corresponds to a necessary antecedent in our setting. While some of their subjects also gave it the same classification, many considered it possible that even *if butter is not heated (explicitly), it may still melt*. This implies that they considered the antecedent to be non-necessary.

3.4 Possibilities Arising from the Classifications of Conditional Sentences

In their paper [15], dedicated to the MMT and the meanings of conditionals, the authors discuss the notion of *sets of possibilities* arising from conditional sentences. Simply put, given a conditional $if\,A\;then\,C$, how humans comprehend or understand it depends on the following questions: what are the possibilities of C when A is satisfied, and when A is not satisfied? This notion of possibilities was harnessed for a detailed comparison between the MMT and the WCS in [1] and is not in the scope of the present discussion. For our current purposes, we limit our attention to characterizing the discussions in Subsections 3.1 and 3.2 using Table 2. It illustrates the meanings of the classifications ON, ONN, FN and FNN in terms of possibilities using the literals A, $\neg A$, C, and $\neg C$ in lines with the MMT. For the moment, we leave out the cases where any of aforementioned literals may be unknown. The sets of possibilities that an individual may use to characterize a conditional sentence may differ from another individual, depending upon factors like pragmatics, culture, context etc. as was discussed in Subsection 3.3.

$if\,A\;then\,C$			
ON	ONN	FN	FNN
$A \quad C$	$A \quad C$	$A \quad C$	$A \quad C$
$\neg A\; \neg C$	$\neg A\; \neg C$	$A\; \neg C$	$A\; \neg C$
	$\neg A \quad C$	$\neg A\; \neg C$	$\neg A\; \neg C$
			$\neg A \quad C$

Table 2: All possibilities of antecedent A and consequent C for each classification of everyday conditional sentences.

3.5 Handling Classifications in the WCS

In the WCS framework, the classification of conditional sentences can be taken into account by extending the definition of the set of abducibles to

$$\mathcal{A}^e_\mathcal{P} = \mathcal{A}_\mathcal{P} \cup \mathcal{A}^{nn}_\mathcal{P} \cup \mathcal{A}^f_\mathcal{P},$$

where $\mathcal{A}_\mathcal{P}$ is as defined earlier in this Section and,

$$\begin{aligned}\mathcal{A}^{nn}_\mathcal{P} &= \{C \leftarrow \top \mid C \text{ is the head of a rule occurring in } \mathcal{P} \text{ representing a} \\ &\qquad \text{conditional sentence with non-necessary antecedent}\}, \\ \mathcal{A}^f_\mathcal{P} &= \{ab \leftarrow \top \mid ab \text{ occurs in the body of a rule occurring in } \mathcal{P} \\ &\qquad \text{representing a factual conditional}\}.\end{aligned}$$

The set $\mathcal{A}^{nn}_\mathcal{P}$ contains facts for the consequents of conditional sentences with non-necessary antecedents. As was mentioned earlier, if an antecedent of a conditional sentence is non-necessary then the truth of the consequent does not depend on the truth of the antecedent. The abducible $C \leftarrow \top$ therefore implies that there may be other unknown reasons for establishing the consequent of the conditional sentence.

The set $\mathcal{A}^f_\mathcal{P}$ contains facts for the abnormality predicates occurring in the bodies of the (logic program) representation of factual conditionals. Owing to the factual nature of a conditional sentence, the antecedent of the conditional may be true, however its consequent may not hold, due to various reasons which we might broadly call abnormalities. As mentioned earlier, considerations of other plausible factors at play might override our default assumption that these abnormalities are false. Once we weakly complete our program, the abducible $ab \leftarrow \top$ shall cause the abnormality predicate to become true and its negation to become false. Hence, the body of the clause containing its negation will be false, causing the consequent to be false in turn. This technique is used in [7] to represent an enabling relation and model, for example, the suppression effect during the AA inference in the suppression task [2]. The original task was as follows. Given, *if she has an essay to write then she will study late in the library*, *if the library is open then she will study late in the library* and *she has an essay to write*. Only 38% of the participants had responded *she will study late in the library*. Although the percentage of participants who responded otherwise was not revealed in the paper, it is plausible, that many considered that *a library not being open* prevents a person from *studying in it*. This can be modeled using the abnormality predicate. Table 3 illustrates how the set of abducibles can be extended for each classification.

4 An Experiment

In [6, 5] an experiment concerning conditional reasoning is described, where 56 logically naive participants were tested on an online website (Prolific, `prolific.co`). The par-

$C \leftarrow A \wedge \neg ab$	A non-necessary	A necessary
factual conditional	$ab \leftarrow \top$, $C \leftarrow \top$	$ab \leftarrow \top$
obligational conditional	$C \leftarrow \top$	

Table 3: The additional facts in the set of abducibles for a rule of the form $C \leftarrow A \wedge \neg ab$ representing a conditional sentence *if* A *then* C.

ticipants were restricted to Central Europe and Great Britain in order to have a similar background knowledge about weather etc. It was also assumed that the participants had not received any education in logic beyond high school training. The participants were first presented with a story followed by a first assertion (a conditional premise), and a second assertion (a possibly negated atomic premise). Finally for each problem they had to answer the question "What follows?". Both parts were presented simultaneously. The participants responded by clicking one of the answer options. They could take as much time as they needed. Participants acted as their own controls.

The participants carried out 48 problems consisting of the 12 conditionals listed in the appendix and solved all four inference types (AA, DA, AC, DC). They could select one of three responses: *nothing follows*, the fact that had not been presented in the second premise, and the negation of this fact. For example in the case of DA, the first assertion was of the form *if* A *then* C, the second assertion was $\neg A$, and they could answer C, $\neg C$, or *nothing follows*. It should also be mentioned that the classification of the conditional sentences into the four aforementioned kinds was done by the authors of the experiment.

We can exemplify all that has been said above with the following short scenario taken from the experiment: *Peter has a lawn in front of his house. He is keen to make sure that the grass on the lawn does not dry out, so whenever it has been dry for multiple days, he turns on the sprinkler to water the lawn.* Along with this context the conditional sentence *if it rains, then the lawn is wet* and the negated atomic proposition *it does not rain* were provided. The participants were given three choices of answers: *the lawn is wet*, *the lawn is not wet*, and *nothing follows*.

As mentioned earlier, the WCS could well explain the findings of the experiment in the cases AA, AC, and DC (see [6, 5]), but failed to explain the findings in the case of DA. The data is shown in Table 4, where the total number of selected responses as well as the median response time in milliseconds for $\neg C$ ($Mdn\, \neg C$) and *nothing follows* ($Mdn\, nf$) responses are listed.

Everyday contexts for the DA inference task elicited a high response rate of about 78% (525 out of 672) for $\neg C$, but in case of *nothing follows* the rate varied from 8% (14 out of 168) up to 33% (56 out of 168). The number of participants answering C seems irrelevant. Until the present, the WCS could predict the $\neg C$ answered by the majority of the participants, but it could not yet model the significant number of *nothing follows* responses. We now propose a solution to the latter. Before we elaborate further, one might first observe that as per the data *nothing follows* was answered much more often in case of conditional sentences with non-necessary antecedents than in the case of conditional sentences with necessary ones (30% vs. 8%, Wilcoxon signed rank, $W = 0$, $p < .001$). More importantly,

Conditional/Classification	C	pct.	$\neg C$	pct.	*nf*	pct.	Sum	$Mdn\ \neg C$	$Mdn\ nf$
(1)	0		45		11		56	2863	4901
(2)	2		54		0		56	3367	na
(3)	2		51		3		56	3647	10477
ON	4	2%	150	89%	14	8%	168	3356	5115
(4)	1		40		15		56	3722	7189
(5)	3		28		25		56	5735	7814
(6)	4		36		16		56	3602	6240
ONN	8	5%	104	62%	56	33%	168	4064	7471
(7)	2		51		3		56	3928	7273
(8)	1		47		8		56	3296	5728
(9)	1		52		3		56	3549	8735
FN	4	2%	150	89%	14	8%	168	3605	6582
(10)	1		39		16		56	3725	6874
(11)	0		41		15		56	3374	5887
(12)	1		41		14		56	3205	7002
FNN	2	1%	121	72%	45	27%	168	3374	6221
Obligational Conditional (O)	12	4%	254	76%	70	21%	336	3583	6613
Factual Conditional (F)	6	2%	271	81%	59	18%	336	3518	6221
Necessary Antecedent (N)	8	2%	300	89%	28	8%	336	3474	5808
Non-Necessary Antecedent (NN)	10	3%	225	67%	101	30%	336	3646	6700
Total	18	3%	525	78%	129	19%	672	3558	6450

Table 4: The results for DA inferences given a conditional sentence $if\, A\ then\, C$ and a negated atomic sentence $\neg A$. The grey lines show the numbers for the examples discussed in the introduction. If the antecedent is non-necessary, then *nothing follows (nf)* is answered significantly often (gray cells at the bottom). ON: obligational conditional with necessary antecedent, ONN: obligational conditional with non-necessary antecedent, FN: factual conditional with necessary antecedent, and FNN: factual conditional with non-necessary antecedent. All percentages (*pct.*) have been rounded off to the nearest natural number for the convenience of the reader.

the reader may observe that when the classification of the antecedents changed from *necessary* to *non-necessary* the number of $\neg C$ responses decreased to 225 and *nothing follows* increased to (a significant) 101. **The goal of this paper is to extend the WCS in order to model this observed phenomenon.**

5 Extending the WCS to Search for Counterexamples

As shown in Table 4 the majority of the participants always answered $\neg C$ when given the premises *if* A *then* C and $\neg A$. The classification of conditional sentences seems irrelevant at this point. However, this general consensus is sometimes only barely met. Indeed, some humans seem to be responding *nothing follows* during the DA task. Upon a closer look at Table 4, the reader may observe that the number of *nothing follows* responses increases when the classification of the antecedent of the conditional changes from necessary to non-necessary. This is because unlike a necessary antecedent, a non-necessary one makes room for counterexamples where even if the antecedent does not hold, the consequent might still hold (that is $\neg A$ and C is possible). This observation hints at two reasoning patterns. The first is the reasoner who responds $\neg C$ to any DA inference task and does not deliberate upon the task, and the second, the reasoner who does. The latter, whom we may also call the *careful reasoner* searches for counterexamples before drawing a definite conclusion such as $\neg C$, unlike the former who does not, that is the *general reasoner*. And counterexamples in the DA task are possible when an individual deems the antecedent to be non-necessary.

The aforementioned difference between the two kinds of reasoning patterns is unfortunately not very noticeable if the so-called careful reasoner has deliberated upon the problem but considered the antecedent to be necessary. In what follows, we attempt to clarify this statement for the reader while also illustrating an approach to model the general consensus of $\neg C$. For example, consider Example 2 ((8) in Table 4), for which 47 participants responded $\neg C$ while only 8 responded *nothing follows*. Assuming it is known that *the plants do not get water* we obtain the program

$$\mathcal{P}_1 = \{g \leftarrow w \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ w \leftarrow \bot\},$$

where g and w denote that *the plants will grow* and *the plants get water*, respectively, and ab_1 is an abnormality predicate which is implicitly assumed to be false. Weakly completing $\mathcal{P}_1$ we obtain:

$$\{g \leftrightarrow w \wedge \neg ab_1,\ ab_1 \leftrightarrow \bot,\ w \leftrightarrow \bot\},$$

whose least model is

$$\mathcal{M}_{wc\mathcal{P}_1} = \langle \emptyset, \{g, ab_1, w\} \rangle,$$

where nothing is true, and g, ab_1, and w are all false. Because the antecedent, *the plants get water* (w), is generally considered to be *necessary* for the consequent, *plants will grow* (g), the falsity of w allows us to falsify g. Hence, we conclude that *the plants will not grow*. This is the general consensus for this particular example. Please note that the authors of the experiment classified this conditional as FN, that is, the antecedent as necessary. Although it is difficult to ascertain how humans comprehend conditionals without inquiring of them, it is plausible that some careful reasoners who deliberate upon this DA task may not find a counterexample where the plants receive no water but they still grow, that is the possibility,

$$\neg \mathrm{A} \quad \mathrm{C}.$$

In other words, these reasoners comprehend the antecedent to be necessary for the consequent and so conclude $\neg C$, much like the general reasoner. In such a case, there seems to be no apparent way to distinguish between the general and the careful reasoner.

Now we discuss the case of the non-necessary antecedent in the DA task where the difference between the reasoning patterns is more pronounced, and how the WCS may model the two. Let us reconsider Example 1 ((5) in Table 4) where 28 out of 56 participants answered $\neg C$, whereas 25 participants answered *nothing follows*. Interestingly, the latter took more time in their response compared to the former. For these 25 participants the antecedent of the conditional may plausibly have been non-necessary for the consequent. That is upon deliberation, they may have been able to construct a counterexample to the putative conclusion $\neg C$, where *Maria is not drinking alcoholic beverages in a pub* but she may nevertheless be *over 19 years of age*. Because Maria may simply abstain from alcohol.

Overall, the data in Table 4 suggests that the difference between the reasoning patterns during a DA inference task becomes more apparent when the antecedent of the conditional is taken to be non-necessary as it leads to the possibility of counterexamples. In this paper we hence propose to model DA inferences by extending the WCS with the addition of a sixth step to the procedure presented in Section 2:

1. Reasoning towards a logic program $\mathcal{P}$ following [22].

2. Weakly completing the program, which leads to $wc\mathcal{P}$.

3. Computing the least model $\mathcal{M}_{wc\mathcal{P}}$ of the weak completion of $\mathcal{P}$, $wc\mathcal{P}$, under the three-valued Łukasiewicz logic.

4. Reasoning with respect to $\mathcal{M}_{wc\mathcal{P}}$.

5. If observations cannot be explained, then applying skeptical abduction using the specified set of abducibles.

6. **Search for counterexamples.**

The sixth step corresponds to the validation step in the Mental Model Theory [14] in that alternative models falsifying a putative conclusion are searched for. Particularly in the case of the DA task, $\neg C$ may be considered as the putative conclusion generated due to steps 1 to 5. In the sixth step using the extended set of abducibles $\mathcal{A}^e_\mathcal{P}$, the extended procedure searches for models where $\neg A$ is true, but $\neg C$ is not. If such models are found, then skeptical reasoning with respect to all constructed models is applied. This will be illustrated and discussed in more detail in the next section.

6 Modeling the DA Inference Task

In order to discuss how the WCS along with its extension can model the general consensus of $\neg C$ in the DA task and also explain the significant number of *nothing follows* answers in case of non-necessary antecedents, we return to Example 1 and assume that *Maria is not drinking alcoholic beverages in a pub*. In the WCS this is formalized by

$$\mathcal{P}_2 = \{o \leftarrow a \wedge \neg ab_2,\ ab_2 \leftarrow \bot,\ a \leftarrow \bot\},$$

where o and a denote that *Maria is over 19 years old* and *she is drinking alcoholic beverages*, respectively, and ab_2 is an abnormality predicate which is initially assumed to be false. As the weak completion of $\mathcal{P}_2$ we obtain

$$\{o \leftrightarrow a \wedge \neg ab_2,\ ab_2 \leftrightarrow \bot,\ a \leftrightarrow \bot\},$$

whose least model is

$$\mathcal{M}_{wc\mathcal{P}_2} = \langle \emptyset, \{a, ab_2, o\}\rangle.$$

Here, a, ab_2, and o are all false. A general reasoner following this approach will draw the conclusion *Maria is not over 19 years old* and stop reasoning at this point. This accounts for the 28 $\neg C$ responses for this particular conditional in our data. Overall it so appears that these participants treated the conditional sentence as a bi-conditional, hence considering only the possibilities corresponding to ON in Table 2,

a	o
¬a	¬o.

Classifying an antecedent as non-necessary however, would also allow the consequent to be true despite the falsity of the former. In other words, recognizing an antecedent as non-necessary, might allow humans to consider two possibilities: *Maria does not drink alcohol in a pub* and *she is younger than 19 years*, and *Maria does not drink alcohol in a pub* but *she is older than 19*. Meaning, these participants did not treat the conditional sentence as a bi-conditional, but instead regarded the possibilities,

a	o
¬a	¬o
¬a	o

corresponding to that of ONN in Table 2. That is, careful reasoners not only consider the aforementioned model $\mathcal{M}_{wc\mathcal{P}_2}$, where o is mapped to false, but also search for a counterexample to $\neg o$. Investigating the third possibility listed above may lead them to

$$\langle\{o\}, \{a, ab_2\}\rangle,$$

which is also a model for the program $\mathcal{P}_2$, but not a model for $wc\mathcal{P}_2$. **The question now stands, how can this (and similar) counterexamples in the DA inference task be modeled by the WCS?** Before proposing a plausible answer to this, we will first turn our attention to how the WCS models the AC and the DC inference tasks.

6.1 Modeling the AC Inference Task

To illustrate how the WCS models the AC inference task we reconsider the previously discussed conditional sentence,

> *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age*

and the atomic premise

> *Maria is over 19 years of age.*

Unlike the DA, in this case only the conditional premise is represented as a logic program, that is,

$$\mathcal{P}_3 = \{o \leftarrow a \wedge \neg ab_2,\ ab_2 \leftarrow \bot\},$$

where o and a denote that *Maria is over 19 years old* and *she is drinking alcoholic beverages*, respectively. ab_2 is the abnormality predicate. The atomic premise, o, is not considered as a fact because the program $\mathcal{P}_3$ already contains a definition of o and the addition of the fact $o \leftarrow \top$ would override this definition upon weak completion thus not giving us much information about a. Therefore, o is considered as an observation that needs to be explained. Meaning, we apply abduction. Because a is undefined in $\mathcal{P}_3$ and the conditional sentence is classified as obligational with a *non-necessary* antecedent, we obtain,

$$\mathcal{A}_{\mathcal{P}_3} = \{a \leftarrow \top,\ a \leftarrow \bot\} \quad \text{and} \quad \mathcal{A}^e_{\mathcal{P}_3} = \mathcal{A}_{\mathcal{P}_3} \cup \{o \leftarrow \top\}$$

respectively. The set of integrity constraints is empty. Considering $\mathcal{A}_{\mathcal{P}_3}$, the observation o is explained by the minimal explanation

$$\{a \leftarrow \top\}. \tag{1}$$

Adding this explanation to $\mathcal{P}_3$, weakly completing the extended program, and computing its least model, a general reasoner obtains

$$\langle\{o,\, a\},\ \{ab_2\}\rangle \tag{2}$$

and concludes that *Maria is drinking alcoholic beverages in a pub*. However, a careful reasoner additionally searching for counterexamples may discover a second explanation to the observation, viz.

$$\{o \leftarrow \top\} \tag{3}$$

by considering $\mathcal{A}^e_{\mathcal{P}_3}$. Here, o being true signifies the possibility that *Maria might still be over 19 irrespective of whether she is drinking alcohol in a pub or not*. Adding such an explanation to $\mathcal{P}_3$, weakly completing the extended program, and computing its least model the careful reasoner obtains

$$\langle\{o\},\ \{ab_2\}\rangle. \tag{4}$$

Comparing the least models (2) where a is true and (4) where a is unknown, and reasoning skeptically, a careful reasoner concludes *nothing follows*. We would like to point out to the reader that the explanations (1) and (3) are independent in that neither is a subset nor a superset of the other.

To summarize, it appears that the general reasoner considers $\mathcal{A}_{\mathcal{P}_3}$ and concludes that a and o hold. On the other hand, the careful reasoner considers $\mathcal{A}^e_{\mathcal{P}_3}$, investigates counterexamples, reasons that although o holds a need not necessarily hold, and finally concludes *nothing follows*. This is also supported by the time measured in the experiment according to [5]: the answer a was generated on average in 4704 ms, whereas the answer *nothing follows* was generated on average in 6044 ms. Overall as an investigation of Table 5 would suggest, like in the DA, it is the (necessary or non-necessary) type of the antecedent of the conditional which plausibly influences the search for counterexamples.

6.2 Modeling the DC Inference Task

Here we discuss how the WCS framework currently models the DC inference task. For this purpose, let us consider the conditional sentence *if Ron scores a goal, then he is happy*

and the atomic sentence

Ron is not happy.

The following program represents the conditional premise

$$\mathcal{P}_4 = \{h \leftarrow g \wedge \neg ab_3,\ ab_3 \leftarrow \bot\},$$

where g and h denote *Ron scores a goal* and *Ron is happy*, respectively, and ab_3 is an abnormality predicate. Here, $\neg h$ is considered as an observation that needs to be explained because the program $\mathcal{P}_4$ already contains a definition for h. Adding $h \leftarrow \bot$ to the program has no effect as it will be overridden once the program is weakly completed. As g is undefined and the conditional sentence is classified as a *factual* conditional with non-necessary antecedent, therefore,

$$\mathcal{A}_{\mathcal{P}_4} = \{g \leftarrow \top,\ g \leftarrow \bot\} \quad \text{and} \quad \mathcal{A}^e_{\mathcal{P}_4} = \mathcal{A}_{\mathcal{P}_4} \cup \{h \leftarrow \top,\ ab_3 \leftarrow \top\},$$

respectively. The set of integrity constraints is empty. Considering $\mathcal{A}_{\mathcal{P}_4}$ the observation $\neg h$ is explained by the minimal explanation

$$\{g \leftarrow \bot\}. \tag{5}$$

Adding this explanation to $\mathcal{P}_4$, weakly completing the extended program, and computing its least model a general reasoner will obtain

$$\langle \emptyset,\ \{h,\ g,\ ab_3\}\rangle \tag{6}$$

and conclude, that *Ron does not score a goal*. This is where most reasoners seem to halt their reasoning. However, there may be some individuals who recognize the conditional sentence as factual, meaning, they recognize that $\neg h$ need not just be caused or explained by $\neg g$. More precisely, a careful reasoner will recognize that h may be false, even if g is not. Analogously, such a reasoner will search for counterexamples to the putative conclusion $\neg g$, which can also explain $\neg h$. Using $\mathcal{A}^e_{\mathcal{P}_4}$,

$$\{ab_3 \leftarrow \top\} \tag{7}$$

can be used as an another minimal explanation for $\neg h$. Here ab_3 being true indicates that *Ron may have other reasons to be unhappy*. Adding this abducible to $\mathcal{P}_4$ and weakly completing the resulting extended program leads to the least model

$$\langle \{ab_3\},\ \{h\}\rangle. \tag{8}$$

As g is false in the first model (6) whereas unknown in the second, (8), skeptical abduction is applied which leads to the conclusion, *nothing follows*. The overall point of importance is that, aside from the falsity of g it is also possible to find other reasons which can cause h to be false, and this leads to the consideration of more than one model, which may lead humans to reason skeptically.

To summarize, similar to the AC, we stipulate that the general reasoner considers $\mathcal{A}_{\mathcal{P}_4}$ and concludes that $\neg g$ holds, whereas a careful reasoner considers $\mathcal{A}^e_{\mathcal{P}_4}$, finds a counterexample, where $\neg h$ holds and g is unknown, and concludes skeptically *nothing follows*. However, in case of the DC inference task it is the (obligational or factual) type of the conditional which plausibly influences this said search for counterexamples. Comparing the explanations (5) and (7) we find that they are independent. It must also be pointed out that the average time taken by participants to respond $\neg A$, for this particular task, was 3726 ms and that to respond *nothing follows* was 3813 ms, which are quite comparable.

6.3 Modeling the $\neg C$ and Nothing Follows Responses in the DA Task

In Subsections 6.1 and 6.2, when modeling the general consensus as well as the *nothing follows* responses in the AC and DC inference tasks, the premise C and $\neg C$ are considered as observations, respectively. The *nothing follows* responses can be accounted for by using the extended set of abducibles and applying skeptical abduction in order to explain the observation. This may lead to models which act as counterexamples to each other. On the other hand, when modeling the general consensus for the DA, the atomic premise was a part of the logic program representation of the premises, as illustrated in the beginning of this section.

We now propose that this negated premise be considered as an observation instead, in a fashion similar to the AC and the DC modeling techniques. To illustrate the proposal we reconsider the premises

> *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age*

and

> *Maria is not over 19 years of age.*

In this revised proposal, the conditional premise is represented by

$$\mathcal{P}_5 = \{o \leftarrow a \wedge \neg ab_2,\ ab_2 \leftarrow \bot\},$$

where the meanings of the atomic predicates are unchanged. Now $\neg a$ is considered as an observation. As was also mentioned earlier, it is the (necessary or non-necessary) type of the antecedent of the conditional which plausibly influences the search for counterexamples in a DA task. Since this particular conditional sentence is classified as obligational with non-necessary antecedent, we obtain

$$\mathcal{A}_{\mathcal{P}_5} = \{a \leftarrow \top,\ a \leftarrow \bot\} \quad \text{and} \quad \mathcal{A}^e_{\mathcal{P}_5} = \mathcal{A}_{\mathcal{P}_5} \cup \{o \leftarrow \top\}.$$

The set of integrity constraints is empty. Considering $\mathcal{A}_{\mathcal{P}_5}$ the observation is explained by the minimal explanation

$$\{a \leftarrow \bot\}. \tag{9}$$

The reader may note that adding this explanation to $\mathcal{P}_5$ leads to the program $\mathcal{P}_2$ introduced at the beginning of this section. Again, weakly completing $\mathcal{P}_2$ and computing its least model we obtain

$$\langle \emptyset,\ \{o,\ ab_2,\ a\}\rangle. \tag{10}$$

Hence the conclusion is, that *Maria is not older than 19 years of age*. The data in Table 4 suggests that many reasoners stop reasoning at this point, thereby treating the conditional premise as a bi-conditional. Following this revised technique allows us to model the general consensus or the general reasoner.

Now we turn our attention to the careful reasoner, the one who searches for counterexamples to the aforementioned conclusion and, in particular, considers $\mathcal{A}^e_{\mathcal{P}_5}$. Such a reasoner may discover a second, non-minimal explanation to $\neg a$, viz.

$$\{a \leftarrow \bot,\ o \leftarrow \top\}. \tag{11}$$

This translates to the possibility that *Maria is not drinking alcoholic beverages but she is over 19 years of age*. Adding this explanation to $\mathcal{P}_5$, weakly completing the extended program, now leads us to the second model earlier mentioned in the section, that is the least model

$$\langle \{o\},\ \{ab_2,\ a\}\rangle. \tag{12}$$

Comparing the least models (10) where o is false and (12) where o is true and reasoning skeptically, we conclude *nothing follows*. Comparing the explanations (9) and (11) we find

$$\{a \leftarrow \bot\} \subset \{a \leftarrow \bot,\ o \leftarrow \top\},$$

meaning, the second explanation is a superset of the first.

To summarize, in a manner similar to the AC and DC the second premise in a DA reasoning task is considered as an observation which needs to be explained. The general reasoner considers $\mathcal{A}_{\mathcal{P}_5}$ and concludes that $\neg o$ holds. On the other hand, a careful reasoner considers $\mathcal{A}^e_{\mathcal{P}_5}$, reasons that although $\neg a$ holds o may nevertheless hold, and concludes *nothing follows*. This is also supported by the time measured in the experiment as shown in Table 4. The answer $\neg o$ was generated on average in 5735 ms, whereas the answer *nothing follows* was generated on average in 7814 ms.

7 A Brief Discussion about the Predictions of the Mental Model Theory

The scope of the MMT is broad and has been applied to quite a few areas of human reasoning till date. At present we restrict ourselves to a brief discussion of some of the predictions of the MMT, as discussed by Philip Johnson-Laird and Ruth Byrne, in their paper [15] in case of the inference tasks, DA, AC and DC.

The MMT suggests that in a DA task, given a conditional sentence *if* A *then* C and $\neg A$, humans *intuitively* refrain from responding $\neg C$ and favor *nothing follows*. It thus predicts that when reasoners respond with $\neg C$, it is a result of deliberately comprehending the conditional sentence as a bi-conditional, meaning the possibilities,

$$\begin{array}{rr} \text{A} & \text{C} \\ \neg\text{A} & \neg\text{C}. \end{array}$$

However, in the case that a reasoner comprehends a conditional sentence as a conditional, meaning the possibilities,

$$\begin{array}{rr} \text{A} & \text{C} \\ \neg\text{A} & \neg\text{C} \\ \neg\text{A} & \text{C}, \end{array}$$

the reasoner refrains from the response of $\neg C$ upon deliberation. Now, if the *nothing follows* response in Table 4 is an intuitive one in comparison to the $\neg C$ responses, they should be quicker, that is take lesser time. But according to the experimental data $\neg C$ responses took 3558 ms on average while *nothing follows* responses took 6450 ms. While the discussion about intuition and deliberation may be reserved for a later occasion, the WCS predicts, that most reasoners respond with $\neg C$ for everyday conditional sentences during a DA inference task. In this case, most reasoners seem to (inherently) treat the antecedent of the conditional sentence as necessary, hence responding how they would in case of a bi-conditional sentence. Reasoners who upon deliberation have found a counterexample to the putative conclusion of $\neg A$ respond with *nothing follows*. This is the case, when these reasoners have comprehended the antecedent to be non-necessary. As mentioned in the beginning of Section 6, a non-necessary antecedent is one which allows the possibilities,

$$\begin{array}{rr} \neg\text{A} & \neg\text{C} \\ \neg\text{A} & \text{C}. \end{array}$$

A necessary antecedent on the other hand, disallows the latter. It must be pointed out that this also suggests that even if a reasoner deliberately searches for counterexamples to $\neg C$ they will be unable to find one, if they have comprehended the antecedent of the conditional as necessary.

The MMT predicts that in an AC task, given *if* A *then* C and C, most reasoners intuitively respond with A. When reasoners deliberate they respond with *nothing follows* in case they comprehend the conditional sentence as a conditional, meaning the possibilities,

A	C
¬A	¬C
¬A	C.

On the other hand, they stick to the putative response of A if they comprehend the conditional as a bi-conditional, meaning the possibilities,

A	C
¬A	¬C.

Like the MMT, the WCS also predicts that most reasoners will answer A in the AC inference task. They seem to (inherently) treat the antecedents of conditional sentences as necessary; thereby treating the sentence as a bi-conditional. Furthermore, it predicts that when individuals look for counterexamples to their putative conclusion of A, they will skeptically respond *nothing follows*. In the search for counterexamples, the necessary or non-necessary nature of the antecedent with respect to the consequent seems to gain relevance, like in the DA task.

The MMT predicts that in the DC inference task, given *if* A *then* C and $\neg C$, most reasoners respond *nothing follows* as a result of intuition, and reasoners respond $\neg A$ as a result of deliberation. This implies that the response *nothing follows* should be more rapid (take less time) than the response $\neg A$. However, the data from Table 6 in the appendix, suggests that the median response time for *nothing follows* responses is 5162 ms, whereas the median response time for $\neg A$ responses is 4311 ms. The WCS on the other hand predicts that given an everyday conditional sentence most individuals may conclude $\neg A$. The response may plausibly be a result of the application of the *modus tollens* rule of inference as discussed in [6]. Upon deliberation, the previously discussed *factual* nature of the conditional may motivate the search for counterexamples which in turn may lead individuals to respond nothing follows. This prediction is limited to everyday conditionals where the antecedent and the consequent are related to each other to an acceptable or believable degree. Conditionals such as *if the sky is blue, then horses can speak English*, which may be considered bizarre, unacceptable or unbelievable are beyond the scope of the present discussion.

8 Conclusion

In this paper, we have discussed how the classification of conditional sentences and their antecedents help gain an insight into how humans understand or comprehend conditional sentences. On this basis, we have presented how the WCS along with its proposed extension can adequately model the average and the careful human reasoner in case of the DA inference task while also revisiting how the WCS can model the AC and the DC inference tasks.

In case of the DA, although most reasoners seem to respond with $\neg C$, the (necessary or non-necessary) type of the antecedent seems to be a relevant feature of the conditional sentence when a reasoner deliberates upon the task. As suggested by Philip Johnson-Laird in [13], given a set of premises, if one is beginning to form a conclusion, one should believe or adopt the same only if they are able to find no counterexamples strong enough to refute it. The data in Table 4 in fact suggests that the reasoners responding *nothing follows* may actually be doing so, and such a response is due to the presence of counterexamples to their putative conclusion of $\neg C$. In case of the AC (like in the DA), reasoners who recognize the antecedent as non-necessary respond with *nothing follows*. In case of the DC, it is possibly the obligational or factual nature of the conditional sentence which is taken into consideration (see [6]) and reasoners with appropriate counterexamples respond *nothing follows*.

The case for the AA seems to be a ceiling effect, as an overwhelming majority of the responses were C (640 out of 672). The data has been omitted from the present discussion. At present, it suffices to state that the WCS can well model this majority which also indicates that the conditional sentences were inherently taken to be obligational by most reasoners. This means, when A was affirmed they simply concluded C, probably due to the application of the *modus ponens* rule of inference. Nonetheless, although not significantly reflected in the current data for the AA, we do recognize that in case of factual conditionals where even if A holds, C may or may not, a reasoner might choose to respond *nothing follows*. Consider for example the conditional sentence, *if it is Monday, then Rita goes to school*. Given the factual nature of the conditional it seems plausible that skeptical reasoners may respond with *nothing follows*. WCS can also account for these reasoners, but it is outside the present scope of discussion. We believe the data at hand motivates further research about the AA and why humans accord with the response C so easily.

Returning to the DA task on the other hand, if we were to deny the antecedent, that is, *it is not Monday*, then although many reasoners might respond *Rita does not go to school*, once again, WCS with the extension proposed in this paper can account for such reasoners as well as those who choose to respond with skepticism that *nothing follows*.

9 Appendix

9.1 Conditionals used in the Experiment with Classification[2]

Obligational Conditionals with Necessary Antecedent (ON)

(1) *If it rains, then the roofs must be wet.*
(2) *If water in the cooking pot is heated over* $99°C$*, then the water starts boiling.*
(3) *If the wind is strong enough, then the sand is blowing over the dunes.*

Obligational Conditionals with Non-Necessary Antecedent (ONN)

(4) *If Paul rides a motorbike, then Paul must wear a helmet.*
(5) *If Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age.*
(6) *If it rains, then the lawn must be wet.*

Factual Conditionals with Necessary Antecedent (FN)

(7) *If the library is open, then Sabrina is studying late in the library.*
(8) *If the plants get water, then they will grow.*
(9) *If my car's start button is pushed, then the engine will start running.*

Factual Conditionals with Non-Necessary Antecedent (FNN)

(10) *If Nancy rides her motorbike, then Nancy goes to the mountains.*
(11) *If Lisa plays on the beach, then Lisa will get sunburned.*
(12) *If Ron scores a goal, then Ron is happy.*

9.2 Short Background Story for Example 1

Maria and her friends are visiting a local pub to enjoy the evening with drinks and good food. Maria knows the local rules and regulations and obeys them.

[2]Note: The classification was done by the authors of [6, 5].

9.3 Short Background Story for Example 2

The Presleys have moved into their newly built house and have hired a gardener to lay out the garden. They are sitting on their terrace and are looking at the bushes, small trees, and shrubs which were planted by the gardener two months ago.

9.4 Experiment Results for the AC Inference Task

Conditional/Classification	A	pct.	$\neg A$	pct.	*nf*	pct.	Sum	$Mdn\ A$	$Mdn\,nf$
(1)	37		1		18		56	3952	7995
(2)	48		1		7		56	4003	4170
(3)	43		1		12		56	3458	9001
ON	128	76%	3	2%	37	22%	168	3797	8175
(4)	42		1		13		56	3659	8828
(5)	32		1		23		56	4704	6044
(6)	29		1		26		56	3593	4396
ONN	103	61%	3	2%	62	37%	168	3968	5939
(7)	51		1		4		56	3767	4397
(8)	42		1		13		56	3798	4565
(9)	45		1		10		56	3492	4598
FN	138	82%	3	2%	27	16%	168	3699	4565
(10)	34		2		20		56	5224	6289
(11)	29		2		25		56	3218	6205
(12)	33		1		22		56	3483	4992
FNN	96	57%	5	3%	67	40%	168	3885	6116
Obligational Conditional (O)	231	69%	6	2%	99	29%	336	3888	6044
Factual Conditional (F)	234	70%	8	2%	94	28%	336	3769	5650
Necessary Antecedent (N)	266	79%	6	2%	64	19%	336	3735	5450
Non-Necessary Antecedent (NN)	199	59%	8	2%	129	38%	336	3906	6039
Total	465	69%	14	2%	193	29%	672	3826	5802

Table 5: The results for AC inferences given a conditional sentence $if\,A\,then\,C$ and an atomic fact C. In case of factual conditionals, *nf* is answered significantly more often. ON: obligational conditional with necessary antecedent, ONN: obligational conditional with non-necessary antecedent, FN: factual conditional with necessary antecedent, and FNN: factual conditional with non-necessary antecedent. All percentages (*pct.*) have been rounded off to the nearest natural number for the convenience of the reader.

9.5 Experiment Results for the DC Inference Task

Conditional/Classification	A	pct.	$\neg A$	pct.	*nf*	pct.	Sum	$Mdn\ \neg A$	$Mdn\ nf$
(1)	1	2%	45	80%	10	18%	56	3449	4758
(2)	0	0%	50	89%	6	11%	56	4058	7922
(3)	2	4%	46	82%	8	14%	56	3796	4517
ON	3	2%	141	84%	24	14%	168	3767	5732
(4)	3	5%	46	82%	7	13%	56	3872	4154
(5)	1	2%	54	96%	1	2%	56	4946	8020
(6)	0	0%	36	64%	20	36%	56	4062	5235
ONN	4	2%	136	81%	28	17%	168	4293	5803
(7)	1	2%	37	66%	18	32%	56	5974	4744
(8)	3	5%	42	75%	11	20%	56	4367	5013
(9)	0	0%	47	84%	9	16%	56	4208	3966
FN	4	2%	126	75%	38	23%	168	4849	4574
(10)	2	4%	35	63%	19	34%	56	4879	4167
(11)	0	0%	39	70%	17	30%	56	4411	5647
(12)	0	0%	34	61%	22	39%	56	3726	3813
FNN	2	1%	108	64%	58	35%	168	4338	4542
Obligational Conditional (O)	7	2%	277	82%	52	15%	336	4053	4790
Factual Conditional (F)	6	2%	234	70%	96	29%	336	4459	4345
Necessary Antecedent (N)	7	2%	267	79%	62	18%	336	4096	4758
Non-Necessary Antecedent (NN)	6	2%	244	73%	86	26%	336	4325	4555
Total	13	2%	511	76%	148	22%	672	4311	5162

Table 6: The results for DC inferences given a conditional sentence $if\,A\ then\,C$ and a negated atomic fact $\neg C$. In case of factual conditionals, *nf* is answered significantly more often. ON: obligational conditional with necessary antecedent, ONN: obligational conditional with non-necessary antecedent, FN: factual conditional with necessary antecedent, and FNN: factual conditional with non-necessary antecedent. All percentages (*pct.*) have been rounded off to the nearest natural number for the convenience of the reader.

References

[1] Meghna Bhadra. Comparative studies of the weak completion semantics and the mental model theory. Master's thesis, Technische Universität Dresden, Fakultät für Informatik, 2021.

[2] Ruth M. J. Byrne. Suppressing valid inferences with conditionals. *Cognition*, 31(1):61–83, 1989.

[3] Ruth M. J. Byrne. *The Rational Imagination: How People Create Alternatives to Reality*. MIT Press, Cambridge, MA, USA, 2005.

[4] Keith L. Clark. Negation as failure. In H. Gallaire and J. Minker, editors, *Logic and Databases*, pages 293–322. Plenum, New York, 1978.

[5] Marcos Cramer, Steffen Hölldobler, and Marco Ragni. Modeling human reasoning about conditionals. `https://tu-dresden.de/ing/informatik/ki/krr/ressourcen/dateien/chr2021b.pdf/view`, 2021. Accepted at NMR2021.

[6] Marcos Cramer, Steffen Hölldobler, and Marco Ragni. When are humans reasoning with modus tollens? *Proceedings of the Annual Conference of the Cognitive Science Society,*, 43:2337–2343, 2021. Retrieved from `https://escholarship.org/uc/item/9x33q50g`.

[7] Emmanuelle-Anna Dietz, Steffen Hölldobler, and Marco Ragni. A computational logic approach to the suppression task. *Proceedings of the Annual Conference of the Cognitive Science Society,*, 34:1500–1505, 2012. Retrieved from `https://escholarship.org/uc/item/2sd6d61q`.

[8] Melvin Fitting. *First–Order Logic and Automated Theorem Proving*. Springer-Verlag, Berlin, 2nd edition, 1996.

[9] Herbert Paul Grice. Logic and conversation. In P. Cole and J. L. Morgan, editors, *Syntax and Semantics*, volume 3, pages 41–58. Academic Press, New York, 1975.

[10] Islam Hamada and Steffen Hölldobler. On disjunctions and the weak completion semantics. In *Proceedings of the Virtual MathPsych/ICCM*. via `mathpsych.org/presentation/571`, 2021.

[11] Steffen Hölldobler. Weak completion semantics and its applications in human reasoning. In Ulrich Furbach and Claudia Schon, editors, *Bridging 2015 – Bridging the Gap between Human and Automated Reasoning*, volume 1412 of *CEUR Workshop Proceedings*, pages 2–16. CEUR-WS.org, 2015. `http://ceur-ws.org/Vol-1412/`.

[12] Steffen Hölldobler and Carroline D. P. Kencana Ramli. Logic programs under three-valued Łukasiewicz's semantics. In P. M. Hill and D. S. Warren, editors, *Logic Programming*, volume 5649 of *Lecture Notes in Computer Science*, pages 464–478. Springer-Verlag Berlin Heidelberg, 2009.

[13] Philip N Johnson-Laird. Mental models in cognitive science. *Cognitive science*, 4(1):71–115, 1980.

[14] Philip N. Johnson-Laird and Ruth M. J. Byrne. *Deduction*. Lawrence Erlbaum Associates, Hove and London (UK), 1991.

[15] Philip N. Johnson-Laird and Ruth M. J. Byrne. Conditionals: A theory of meaning, pragmatics, and inference. *Psychological Review*, 109:646–678, 2002.

[16] Antonis C. Kakas, Robert A. Kowalski, and Francesca Toni. Abductive Logic Programming. *Journal of Logic and Computation*, 2(6):719–770, 1992.

[17] Sangeet S. Khemlani, Ruth M. J. Byrne, and Philip N. Johnson-Laird. Facts and possibilities: A model-based theory of sentenial reaoning. *Cognitive Science*, pages 1–38, 2018.

[18] John W. Lloyd. *Foundations of Logic Programming*. Springer-Verlag, 1984.

[19] Jan Łukasiewicz. O logice trójwartościowej. *Ruch Filozoficzny*, 5:169–171, 1920. English translation: On Three-Valued Logic. In: *Jan Łukasiewicz Selected Works*. (L. Borkowski, ed.), North Holland, 87-88, 1990.

[20] Ana Oliviera da Costa, Emmanuelle-Anna Dietz Saldanha, Steffen Hölldobler, and Marco Ragni. A computational logic approach to human syllogistic reasoning. *Proceedings of the Annual Conference of the Cognitive Science Society,*, 39:883–888, 2017.

[21] Keith Stenning and Michiel van Lambalgen. Semantic interpretation as computation in non-monotonic logic: The real meaning of the suppression task. *Cognitive Science*, 29:919–960, 2005.

[22] Keith Stenning and Michiel van Lambalgen. *Human Reasoning and Cognitive Science*. MIT Press, 2008.

Received \jreceived