The Weak Completion Semantics and Counter Examples

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Abstract. An experiment has revealed that if the antecedent of a conditional sentence is denied, then most participants conclude that the negation of the consequent holds. However, a significant number of participants answered *nothing follows* if the antecedent of the conditional sentence was non-necessary. The weak completion semantics correctly models the answers of the majority, but cannot explain the number of *nothing follows* answers. In this paper we extend the weak completion semantics by counter examples. The extension allows to explain the experimental findings.

Keywords: Conditional Reasoning · Denial of Antecedent · Weak Completion Semantics · Counter Examples.

1 Introduction

Conditional sentences are propositions of the form $if\, A\, then\, C$ where A and C are atomic sentences called antecedent and consequent, respectively. Four kinds of conditional inference tasks have been a common area of research by psychologists till date:

1. Affirmation of the antecedent (AA): $if\, A\, then\, C$ and A, therefore C.
2. Denial of the antecedent (DA): $if\, A\, then\, C$ and $\neg A$, therefore $\neg C$.
3. Affirmation of the consequent (AC): $if\, A\, then\, C$ and C, therefore A.
4. Denial of the consequent (DC): $if\, A\, then\, C$ and $\neg C$, therefore $\neg A$.

In classical, two-valued propositional logic, conditional sentences are taken to mean material implications and biconditionals to mean (material) equivalence. The conclusion for the DA and the AC are hence considered to be logical fallacies (invalid) for a conditional sentence whereas they are considered valid for a biconditional. When replacing the above abstract conditional sentences with everyday ones, however, the inferences largely depend on the semantics and pragmatics of human communication, culture, and context. In this paper, we therefore discuss how everyday conditional sentences can be categorized into four proposed semantic categories. We also share the results of an experiment reported in [5,4] and (with particular regard to the DA) demonstrate how such classifications can help model an average human (DA) reasoner.

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The Weak Completion Semantics (WCS) is a three-valued, non-monotonic cognitive theory, which can not only adequately model the suppression task by [1] as shown by [6], human syllogistic reasoning as shown by [18], and DC inferences as shown by [5] but also the AA, AC, and the majority $\neg C$ answers of the DA as shown by [4]. While the existing framework of the WCS adequately models the general consensus of the $\neg C$ responses generated in case of the DA inference task, it did not however, seem adequate to model the number of *nothing follows* (nf) responses, which is especially significant in case of conditional sentences with *non-necessary antecedents*. Here, *nothing follows* denotes no new inference or specific conclusion can be drawn with regard to the consequent of the conditional sentence.

In order to elaborate on what it really means for a conditional sentence to have a *non-necessary antecedent* and to propose a solution to the aforementioned problem, we begin by considering the following DA inference tasks:

1. *If Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age* and *Maria is not drinking alcoholic beverages in a pub*.
2. *If the plants get water, then they will grow* and *the plants get no water*.

Both of these examples appeared in the aforementioned experiment, and as was the case for every conditional sentence that was included in the experiment, accompanied with a small background story. A curious reader may find the background stories in the Appendix. In the first example, 28 out of 56 participants answered *Maria must not be over 19 years of age*, whereas 25 answered *nf*. In this example the antecedent is non-necessary; it is not considered necessary for a person to drink alcohol in order for her to be older than 19. There are many people who *do not drink alcoholic beverages* although *they are over 19 years of age*. In the second example, 47 out of 56 participants answered *the plants will not grow* whereas only 8 answered *nf*. In this case, the antecedent is necessary. *Plants do not grow without water.* Table 3 gives a complete account of this experimental data.

Based on this observation, we propose an extension which allows WCS to account for the nf answers. In Example 1, the existing framework of the WCS creates a model where given that *Maria is not drinking alcoholic beverages*, it can be concluded that *Maria is not older than 19 years of age*. With the proposed extension, however, a counter example can be constructed based on a possible observation that *Maria is not drinking alcoholic beverages* and yet *Maria is older than 19 years of age*. This leads to an alternative model, which when compared to the former model and reasoned skeptically, leads to the conclusion that it is unknown whether *Maria is older than 19 years of age*. In Example 2, WCS creates a model where given that *the plants do not get water*, it can be concluded that *they will not grow*. But in this case, a counter example does not readily exist.

The paper is organized as follows: In the next section we formally introduce the WCS. A classification of conditional sentences is given in Section 3. The experiment is described in Section 4. We demonstrate how the WCS models the general consensus in Section 5. The search for counter examples is presented in Section 6. Counter Examples are modeled in the WCS in Section 7. Finally, in Section 8 we conclude and outline further possible research.

Table 1: The truth tables for the Łukasiewicz logic. One should observe that $\mathsf{U} \leftarrow \mathsf{U} = \mathsf{U} \leftrightarrow \mathsf{U} = \top$ as shown in the grey cells.

F	$\neg F$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	$\top$	U
$\bot$	$\bot$	U	$\top$

2 The Weak Completion Semantics

We assume the reader to be familiar with logic and logic programming as presented in e.g. [7] and [16]. Let $\top$, $\bot$, and U be truth constants denoting *true*, *false*, and *unknown*, respectively. A *(logic) program* is a finite set of clauses of the form $B \leftarrow body$, where B is an atom and $body$ is $\top$, or $\bot$, or a finite, non-empty set of literals. Clauses of the form $B \leftarrow \top$, $B \leftarrow \bot$, and $B \leftarrow L_1, \ldots, L_n$ are called *facts*, *assumptions*, and *rules*, respectively, where L_i, $1 \leq i \leq n$, are literals. We restrict our attention to propositional programs although the WCS extends to first-order programs as well [10].

Throughout this paper, $\mathcal{P}$ will denote a program. An atom B is *defined* in $\mathcal{P}$ iff $\mathcal{P}$ contains a clause of the form $B \leftarrow body$. As an example consider the program

$$\mathcal{P}_c = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\},$$

where A, C, and ab are atoms. C and ab are defined, whereas A is undefined. ab is an abnormality predicate which is assumed to be false. In the WCS, this program represents the conditional sentence *if* A *then* C. In their everyday lives humans are often required to reason in situations where the information of all factors affecting the situation might not be complete. They still reason, unless new information which needs consideration comes to light. The abnormality predicate in the program serves the purpose of this (default) assumption, as was suggested in [19].

Consider the following transformation: (1) For all defined atoms B occurring in $\mathcal{P}$, replace all clauses of the form $B \leftarrow body_1,\ B \leftarrow body_2,\ \ldots$ by $B \leftarrow body_1 \vee body_2 \vee \ldots$. (2) Replace all occurrences of $\leftarrow$ by $\leftrightarrow$. The resulting set of equivalences is called the *weak completion* of $\mathcal{P}$. It differs from the program completion defined in [3] in that undefined atoms in the weakly completed program are not mapped to false, but to unknown instead. Weak completion is necessary for the WCS framework to adequately model the suppression task (and other reasoning tasks) as demonstrated in [6].

As shown in [11], each weakly completed program admits a least model under the three-valued Łukasiewicz logic [17] (see Table 1). This model will be denoted by $\mathcal{M}_{\mathcal{P}}$. It can be computed as the least fixed point of a semantic operator introduced in [20]. Let $\mathcal{P}$ be a program and I be a three-valued interpretation represented by the pair $\langle I^\top, I^\bot \rangle$, where $I^\top$ and $I^\bot$ are the sets of atoms mapped to true and false by I, respectively, and atoms which are not listed are mapped to unknown. We define

$\Phi_{\mathcal{P}}\, I = \langle J^\top, J^\bot \rangle$,[4] where

$$\begin{array}{rcl} J^\top & = & \{B \mid \text{there is } B \leftarrow \mathit{body} \in \mathcal{P} \text{ and } I\ \mathit{body} = \top\}, \\ J^\bot & = & \{B \mid \text{there is } B \leftarrow \mathit{body} \in \mathcal{P} \text{ and} \\ & & \quad \text{for all } B \leftarrow \mathit{body} \in \mathcal{P} \text{ we find } I\ \mathit{body} = \bot\}. \end{array}$$

Following [14] we consider an *abductive framework* $\langle \mathcal{P}, \mathcal{A}_{\mathcal{P}}, \mathcal{IC}, \models_{wcs} \rangle$, where $\mathcal{P}$ is a program, $\mathcal{A}_{\mathcal{P}} = \{B \leftarrow \top \mid B \text{ is undefined in } \mathcal{P}\} \cup \{B \leftarrow \bot \mid B \text{ is undefined in } \mathcal{P}\}$ is the *set of abducibles*, $\mathcal{IC}$ is a finite set of *integrity constraints*, and $\mathcal{M}_{\mathcal{P}} \models_{wcs} F$ iff $\mathcal{M}_{\mathcal{P}}$ maps the formula F to true. Let $\mathcal{O}$ be an *observation*, i.e., a finite set of literals each of which does not follow from $\mathcal{M}_{\mathcal{P}}$. We apply abduction to explain $\mathcal{O}$, where $\mathcal{O}$ is called *explainable* in the abductive framework $\langle \mathcal{P}, \mathcal{A}_{\mathcal{P}}, \mathcal{IC}, \models_{wcs} \rangle$ iff there exists a non-empty $\mathcal{X} \subseteq \mathcal{A}_{\mathcal{P}}$ called an *explanation* such that $\mathcal{M}_{\mathcal{P} \cup \mathcal{X}} \models_{wcs} L$ for all $L \in \mathcal{O}$ and $\mathcal{M}_{\mathcal{P} \cup \mathcal{X}}$ satisfies $\mathcal{IC}$. We have assumed that explanations are non-empty as otherwise the observation already follows from the weak completion of the program. Formula F *follows credulously* from $\mathcal{P}$ and $\mathcal{O}$ iff there exists an explanation $\mathcal{X}$ for $\mathcal{O}$ such that $\mathcal{M}_{\mathcal{P} \cup \mathcal{X}} \models_{wcs} F$. F *follows skeptically* from $\mathcal{P}$ and $\mathcal{O}$, iff $\mathcal{O}$ can be explained and for all explanations $\mathcal{X}$ for $\mathcal{O}$ we find $\mathcal{M}_{\mathcal{P} \cup \mathcal{X}} \models_{wcs} F$. The latter is an application of the so-called *Gricean implicature* [8]: humans normally do not quantify over things which do not exist. Meaning, (unlike classical logic) *all* explanations for an observation $\mathcal{O}$ may only be taken into account to skeptically decide on a formula F, when $\mathcal{O}$ is explainable and these so-called explanations exist in the first place. If a formula F does not follow skeptically from $\mathcal{P}$ and $\mathcal{O}$, we conclude *nothing follows*. Furthermore, one should also observe that if an observation $\mathcal{O}$ cannot be explained, then *nothing follows* credulously as well as skeptically. In all examples discussed in this paper the set of integrity constraints is empty. Integrity constraints are not relevant to the goal of this paper. However they are needed in other applications of the WCS like human disjunctive reasoning [9].

Given premises, general knowledge, and observations, *reasoning in the WCS* is currently modeled in five steps:

1. Reasoning towards a logic program $\mathcal{P}$ following [20].
2. Weakly completing the program.
3. Computing the least model $\mathcal{M}_{\mathcal{P}}$ of the weak completion of $\mathcal{P}$ under the three-valued Łukasiewicz logic.
4. Reasoning with respect to $\mathcal{M}_{\mathcal{P}}$.
5. If observations cannot be explained, then applying skeptical abduction using the specified set of abducibles.

In Section 5 we will explain how these five steps work in the case of the DA reasoning tasks considered in this paper. More examples can be found, for example, in [6] or [18] or [9].

[4] Whenever we apply a unary operator like $\Phi_{\mathcal{P}}$ to an argument like I, then we omit parenthesis and write $\Phi_{\mathcal{P}}\ I$ instead. Likewise, we write $I\ \mathit{body}$ instead of $I(\mathit{body})$.

3 A Classification of Conditional Sentences

3.1 Obligation versus Factual Conditionals

Following [2], we call a conditional sentence an *obligation conditional* if the truth of the consequent appears to be obligatory given that its antecedent is true. For each obligation conditional there are two initial possibilities humans think about. The first possibility is the conjunction of the antecedent and the consequent which is permitted. The second possibility is the conjunction of the antecedent and the negation of the consequent which is forbidden. Exceptions are possible but unlikely. This can be exemplified by Example 1. In many countries the law demands that a person may only drink alcohol publicly when they are above a certain age group (for example, 19 years). This implies that *Maria is drinking alcoholic beverages in a pub* and *she is older than 19 years* is a permitted possibility, whereas *Maria is drinking alcoholic beverages in a pub* and *she is not older than 19 years* is a forbidden one. Hence, *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age* is an obligation conditional. Concerning Example 2, *plants getting water* and *plants are growing* is a permitted possibility. But *plants getting water* and *plants are not growing* is also possible; it holds in particular if a plant is watered too much, but there are many other factors like, for example, lack of light, pest infestation, etc. which may hinder their growth. Hence, *if the plants get water, then they will grow*, is not an obligation conditional.

Obligation conditionals may have different sources. They may be based on legal laws like Example 1 and are often called deontic conditionals, in which case words like *must*, *should* or *ought* may be explicitly used in the conditional sentence. Their usage however, does not seem mandatory in everyday communication and is skipped on many occasions. Knowledge or awareness that the consequent is obligatory given the antecedent suffices in these cases, and yields the same responses as when explicitly denoting the obligation. Obligation conditionals may also express moral or social obligations like *if somebody's parents are elderly, then he/she should look after them* [2]. Other obligation conditionals are based on causal or physical laws which hold on our planet like *if an object is not supported, then it will fall to the ground*. In each case, the conjunction of the antecedent and the consequent is permitted, whereas the conjunction of the antecedent and the negation of the consequent is forbidden.

On the other end of the spectrum, if the consequent of a conditional sentence is not obligatory given the antecedent, then it is called a *factual conditional*. In particular, the truth of the antecedent is *inconsequential* to that of the consequent; that is (even) if the antecedent is true, the consequent may or may not be true. This has already been exemplified using Example 2. The conditional *if the plants get water, then they will grow* is a factual one. As another example consider the conditional sentence *if Maria is over 19 years, then she may drink alcoholic beverages in a pub*. This sentence is a factual one, because given the atomic proposition *Maria is over 19 years* is true, one can imagine two permitted possibilities, one where *Maria drinks alcohol beverages* and another where *Maria does not drink alcoholic beverages in a pub*.

3.2 Necessary versus Non-Necessary Antecedents

As discussed in the previous section, the obligation or factual nature of a conditional sentence indicates if the consequent is obligatory or simply possible provided the antecedent is satisfied. The question that may naturally arise at this point is, what happens when the antecedent of a conditional sentence is not satisfied? To that end, the antecedent A of a conditional sentence $\mathit{if}\, A\ \mathit{then}\, C$ is said to be *necessary* with respect to the consequent C, if and only if C cannot be true unless A is true. This implies that if A does not hold, C cannot either. For example, in Example 2 *plants get water* is a necessary antecedent for *plants will grow*. If a plant is not watered at all, it will very likely die.

The above does not imply however, that the antecedent need always be a precondition for the consequent, per se. The antecedent A of a conditional sentence $\mathit{if}\, A\ \mathit{then}\, C$ is said to be *non-necessary* with respect to the consequent C, if C can be true irrespective of the truth or falsity of A. In particular this implies, if A does not hold, C may or may not hold. In Example 1 the falsity of *drinking alcoholic beverages in a pub* is inconsequential to the truth of the consequent *older than 19 years*. There are plenty of adults (over 19 years) who do not drink alcohol. The antecedent of the conditional sentence *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age*, in Example 1 is therefore called non-necessary.

3.3 Pragmatics

Generally, humans may recognize conditional sentences as obligation or factual and antecedents as necessary or non-necessary. This leads to an informal and pragmatic classification of four kinds: obligation conditional with necessary antecedent (ON) or non-necessary antecedent (ONN) and factual conditional with necessary antecedent (FN) or non-necessary antecedent (FNN). For an abstract conditional $\mathit{if}\, A\ \mathit{then}\, C$, without an everyday context, the classification of the conditional into any of the aforementioned kinds would be as discussed in the above section. The classification of everyday conditionals, however, often depend on pragmatics: the context, the background knowledge and experience of a person. For example, the conditional sentence *if it is cloudy, then it is raining* discussed in [15] may be classified as an obligational conditional with necessary antecedent by people living in Java, whereas it may be classified as a factual conditional by people living in Central Europe. In another example [13], the authors conducted an experiment, where they categorized the proposition *if it's heated, then this butter will melt* as a biconditional. In particular they considered *if butter is not heated, it will not melt*. This corresponds to a necessary antecedent in our setting. While some of their subjects also gave it the same classification, many considered it possible that even *if butter is not heated (explicitly), it may still melt*. This implies that they considered the antecedent to be non-necessary.

3.4 Handling Classifications in WCS

The classification of conditional sentences can be taken into account by extending the definition of the set of abducibles:

$$\mathcal{A}^e_\mathcal{P} = \mathcal{A}_\mathcal{P} \cup \mathcal{A}^{nn}_\mathcal{P} \cup \mathcal{A}^f_\mathcal{P},$$

Table 2: The additional facts in the set of abducibles for a rule of the form $C \leftarrow A \wedge \neg ab$ representing a conditional sentence *if* A *then* C.

$C \leftarrow A \wedge \neg ab$	A non-necessary	A necessary
factual conditional	$ab \leftarrow \top,\ C \leftarrow \top$	$ab \leftarrow \top$
obligation conditional	$C \leftarrow \top$	

where $\mathcal{A}_\mathcal{P}$ is as defined above,

$$\begin{array}{rcll}\mathcal{A}_\mathcal{P}^{nn} & = & \{C \leftarrow \top \mid & C \text{ is the head of a rule occurring in } \mathcal{P} \text{ representing a} \\ & & & \text{conditional sentence with non-necessary antecedent}, \\ \mathcal{A}_\mathcal{P}^{f} & = & \{ab \leftarrow \top \mid & ab \text{ occurs in the body of a rule occurring in } \mathcal{P} \\ & & & \text{representing a factual conditional}\}.\end{array}$$

The set $\mathcal{A}_\mathcal{P}^{nn}$ contains facts for the consequents of conditional sentences with non-necessary antecedents. As mentioned earlier, if an antecedent of a conditional sentence is non-necessary, then the truth of the consequent does not depend on the truth of the antecedent. The abducible $C \leftarrow \top$ therefore implies that there may be other unknown reasons for establishing the consequent of the conditional sentence.

The set $\mathcal{A}_\mathcal{P}^{f}$ contains facts for the abnormality predicates occurring in the bodies of the (logic program) representation of factual conditionals. Owing to the factual nature of a conditional sentence, the antecedent of the conditional may be true, however its consequent may not hold, due to various reasons which we might broadly call abnormalities. As mentioned earlier, considerations of other plausible factors at play might override our default assumption that these abnormalities are false. Once we weakly complete our program, the abducible $ab \leftarrow \top$ shall cause the abnormality predicate to become true and its negation to become false. Hence, the body of the clause containing its negation will be false, causing the consequent to be false in turn.[5] Table 2 illustrates how the set of abducibles can be extended for each classification.

4 An Experiment

In [5,4] an experiment concerning conditional reasoning is described, where 56 logically naive participants were tested on an online website (Prolific, `prolific.co`). The participants were restricted to Central Europe and Great Britain to have a similar background knowledge about weather etc. It was also assumed that the participants had not received any education in logic beyond high school training. The participants were first presented with a story followed by a first assertion (a conditional premise), and a second assertion (a (possibly negated) atomic premise). Finally for each problem they had to answer the question “What follows?”. Both parts were presented simultaneously. The participants responded by clicking one of the answer options. They could take as much time as they needed. Participants acted as their own controls.

[5] This technique is used in [6] to represent an enabling relation and model the suppression effect. In particular, *a library not being open* prevents a person from *studying in it*.

The participants carried out 48 problems consisting of the 12 conditionals listed in the Appendix and solved all four inference types (AA, DA, AC, DC). They could select one of three responses: *nothing follows (nf)*, the fact that had not been presented in the second premise, and the negation of this fact. E.g., in the case of DA, the first assertion was of the form $if\,A\;then\,C$, the second assertion was $\neg A$, and they could answer C, $\neg C$, or nf. It should also be mentioned that the classification of the conditional sentences into the four aforementioned kinds was done by the authors of the experiment.

As an example consider the following short scenario from the experiment: *Peter has a lawn in front of his house. He is keen to make sure that the grass on the lawn does not dry out, so whenever it has been dry for multiple days, he turns on the sprinkler to water the lawn.* Along with this context the conditional sentence *if it rains, then the lawn is wet* and the negated atomic proposition *it does not rain* were provided. The participants were given three choices of answers: *the lawn is wet*, *the lawn is not wet*, and *nothing follows*.

As mentioned earlier, the WCS could well explain the findings of the experiment in the cases AA, AC, and DC (see [5,4]), but failed to explain the findings in the case of DA. The data is shown in Table 3, where the total number of selected responses as well as the median response time (in milliseconds) for $\neg C$ ($Mdn\,\neg C$) and *nf* ($Mdn\; nf$) responses are listed.

Everyday contexts for the DA inference task elicited a high response rate of about 78% (525 out of 672) for $\neg C$, but in case of nf the rate varied from 8% (14 out of 168) up to 33% (56 out of 168). The number of participants answering C seems irrelevant. Until the present, the WCS could predict the $\neg C$ answered by the majority of the participants, but it could not yet model the significant number of nf responses. We now propose a solution to the latter. Before we elaborate further, one might first observe that as per our data nf was answered much more often in case of conditional sentences with non-necessary antecedents than in the case of conditional sentences with necessary ones (30% vs. 8%, Wilcoxon signed rank, $W = 0$, $p < .001$). More importantly, the reader may observe that when the classification of the antecedents changed from *necessary* to *non-necessary* the number of $\neg C$ responses decreased to 225 and nf increased to (a significant) 101. **The goal of this paper is to extend the WCS in order to model this observed phenomenon.**

5 Modeling the General Consensus

As shown in Table 3 the majority of the participants always answered $\neg C$ when given the premises $if\,A\;then\,C$ and $\neg A$ no matter how the conditional sentence was classified. To illustrate how WCS models the majority consensus, let us consider Example 2 ((8) in Table 3). Assuming it is known that *the plants do not get water* we obtain the program

$$\mathcal{P}_1 = \{g \leftarrow w \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ w \leftarrow \bot\},$$

where g and w denote that *the plants will grow* and *the plants get water*, respectively, and ab_1 is an abnormality predicate. Weakly completing $\mathcal{P}_1$ we obtain:

$$\{g \leftrightarrow w \wedge \neg ab_1,\ ab_1 \leftrightarrow \bot,\ w \leftrightarrow \bot\},$$

Table 3: The results for DA inferences given a conditional sentence *if* A *then* C and a negated atomic sentence $\neg A$. The grey lines show the numbers for the examples discussed in the introduction. If the antecedent is non-necessary, then *nf* is answered significantly often (grey cells at the bottom). ON: obligation conditional with necessary antecedent, ONN: obligation conditional with non-necessary antecedent, FN: factual conditional with necessary antecedent, and FNN: factual conditional with non-necessary antecedent. All percentages (*pct.*) have been rounded off to the nearest natural number for the convenience of the reader.

Conditional/Classification	C	$pct.$	$\neg C$	$pct.$	nf	$pct.$	Sum	$Mdn\,\neg C$	$Mdn\,nf$
(1)	0		45		11		56	2863	4901
(2)	2		54		0		56	3367	na
(3)	2		51		3		56	3647	10477
ON	4	2%	150	89%	14	8%	168	3356	5115
(4)	1		40		15		56	3722	7189
(5)	3		28		25		56	5735	7814
(6)	4		36		16		56	3602	6240
ONN	8	5%	104	62%	56	33%	168	4064	7471
(7)	2		51		3		56	3928	7273
(8)	1		47		8		56	3296	5728
(9)	1		52		3		56	3549	8735
FN	4	2%	150	89%	14	8%	168	3605	6582
(10)	1		39		16		56	3725	6874
(11)	0		41		15		56	3374	5887
(12)	1		41		14		56	3205	7002
FNN	2	1%	121	72%	45	27%	168	3374	6221
Obligation Conditional (O)	12	4%	254	76%	70	21%	336	3583	6613
Factual Conditional (F)	6	2%	271	81%	59	18%	336	3518	6221
Necessary Antecedent (N)	8	2%	300	89%	28	8%	336	3474	5808
Non-Necessary Antecedent (NN)	10	3%	225	67%	101	30%	336	3646	6700
Total	18	3%	525	78%	129	19%	672	3558	6450

whose least model is

$$\mathcal{M}_{\mathcal{P}_1} = \langle \emptyset, \{g, ab_1, w\} \rangle,$$

where nothing is true, and g, ab_1, and w are all false. As mentioned earlier, because water (the antecedent) is generally considered to be *necessary* for the growth of a plant (the consequent), the falsity of w allows us to falsify g. Hence, we conclude that *the plants will not grow*.

6 Extending WCS to Seach for Counter Examples

But the general consensus to answer $\neg C$ when given the premises *if* A *then* C and $\neg A$ is sometimes only barely met. Reconsider again Example 1 ((5) in Table 3): 28 out of 56 participants answered $\neg C$, whereas 25 participants answered *nf*. In general, the increase in the number of nf responses occurs when the classification of the antecedent of the conditional sentence changes from necessary to non-necessary. This is because unlike a necessary antecedent, a non-necessary one makes room for counter examples where even if the antecedent does not hold, the consequent might still hold. For example, *if Maria is not drinking alcoholic beverages in a pub* she may nevertheless be *over 19 years of age*. Maria may simply abstain from alcohol. One should observe that this cannot be modeled within the WCS so far, even if the set of abducibles is extended from $\mathcal{A}_\mathcal{P}$ to $\mathcal{A}^e_\mathcal{P}$, as in the case of DA no abductive reasoning takes place.

In this paper we propose to extend WCS by adding a sixth step to the procedure presented in Section 2:

1. Reasoning towards a logic program $\mathcal{P}$ following [20].
2. Weakly completing the program.
3. Computing the least model $\mathcal{M}_\mathcal{P}$ of the weak completion of $\mathcal{P}$ under the three-valued Łukasiewicz logic.
4. Reasoning with respect to $\mathcal{M}_\mathcal{P}$.
5. If observations cannot be explained, then applying skeptical abduction using the specified set of abducibles.
6. **Search for counter examples.**

The sixth step corresponds to the validation step in the mental model theory [12] in that alternative models falsifying a putative conclusion are searched for. In the case of DA $\neg C$ may be considered as the putative conclusion generated due to steps 1 to 5. In the sixth step using the extended set of abducibles $\mathcal{A}^e_\mathcal{P}$, the extended procedure searches for models where $\neg A$ is true, but $\neg C$ is not. If such models are found, then skeptical reasoning with respect to all constructed models is applied. This will be illustrated in the next section.

7 Modeling Counter Examples

In order to illustrate how WCS along with its extension can explain the significant number of nf answers in case of the non-necessary antecedents, we return to Example 1 and assume that *Maria is not drinking alcoholic beverages in a pub*. In the WCS this is formalized by

$$\mathcal{P}_2 = \{o \leftarrow a \wedge \neg ab_2,\ ab_2 \leftarrow \bot,\ a \leftarrow \bot\},$$

where o and a denote that *Maria is over 19 years old* and *she is drinking alcoholic beverages*, respectively, and ab_2 is an abnormality predicate which is initially assumed to be false. As the weak completion of $\mathcal{P}_2$ we obtain

$$\{o \leftrightarrow a \wedge \neg ab_2,\ ab_2 \leftrightarrow \bot,\ a \leftrightarrow \bot\},$$

whose least model is

$$\mathcal{M}_{\mathcal{P}_2} = \langle \emptyset, \{a, ab_2, o\} \rangle.$$

Here, a, ab_2, and o are all false. An average reasoner following this approach will draw the conclusion *Maria is not over 19 years old* and stop reasoning at this point. This accounts for the 28 $\neg C$ responses for this conditional in our data.

Classifying an antecedent as non-necessary, however, would allow the consequent to be true or false despite the falsity of the former. In other words, recognizing an antecedent as non-necessary, might allow humans to consider two possibilities: *Maria does not drink alcohol in a pub* and *she is younger than 19 years*, and *Maria does not drink alcohol in a pub* but *she is older than 19*. Hence, for such a reasoner the extended WCS not only creates the previous model $\mathcal{M}_{\mathcal{P}_2}$, where o is mapped to false, but also searches for a counter example by considering o as a possible observation that needs to be explained. As mentioned earlier in Section 3.4, because the conditional sentence is classified as an obligation conditional with non-necessary antecedent, the extended set of abducibles for $\mathcal{P}_2$ is

$$\mathcal{A}^e_{\mathcal{P}_2} = \{a \leftarrow \top,\ a \leftarrow \bot,\ o \leftarrow \top\}.$$

The abducible $\{o \leftarrow \top\}$ can be used as a minimal explanation for the observation o. Hence, adding this abducible to $\mathcal{P}_2$ leads to

$$\mathcal{P}_3 = \{o \leftarrow a \wedge \neg ab_2,\ ab_2 \leftarrow \bot,\ a \leftarrow \bot,\ o \leftarrow \top\}.$$

Weakly completing $\mathcal{P}_3$ we obtain

$$\{o \leftrightarrow (a \wedge \neg ab_2) \vee \top,\ ab_2 \leftrightarrow \bot,\ a \leftrightarrow \bot\},$$

whose least model is

$$\mathcal{M}_{\mathcal{P}_3} = \langle \{o\}, \{a, ab_2\} \rangle.$$

Here o is true, whereas a and ab_2 are false. As o is false in the model $\mathcal{M}_{\mathcal{P}_2}$ but true in $\mathcal{M}_{\mathcal{P}_3}$, reasoning skeptically WCS concludes nf. This accounts for the 25 nf responses for this conditional sentence in our data. Similar counter examples can be constructed for the examples (4), (6), and (10)-(12) depicted in Table 3 which explain the *nf* answers given by a significant number of participants. But similar counter examples cannot be constructed for the remaining examples (1)-(3) and (7)-(9).

8 Conclusion

In this paper, we have presented how the WCS along with its proposed extension can adequately model the average human reasoner in case of the DA inference task. Whereas the majority consensus of $\neg C$ suggests reasoners who might not have considered the necessity or non-necessity of the antecedent, the significant number of nf answers suggests reasoners who might have. Accordingly, we have discussed how the classification of conditional sentences and their antecedents help gain an insight into how humans understand or recognize conditional sentences. This not only allows us to model the

DA reasoning task but also model the average human reasoner in case of the AA, AC, and DC. In case of the AC (like in the DA) reasoners might recognize the antecedent as non-necessary which influences their response. In case of the DC, it is possibly the obligation or factual nature of the conditional sentence which is taken into consideration (see [5]).

The case for the AA seems to be a ceiling effect however, as an overwhelming majority of our responses were C (640 out of 672). The WCS can well model this majority which indicates that the conditional sentences were taken to be obligatory by most reasoners, meaning, when A was affirmed, they simply concluded C. Nonetheless, we also realize that in case of factual conditionals where although A holds, C may or may not, a reasoner might respond *nothing follows*. This, however, does not reflect significantly on the current data. Consider a seemingly strange yet everyday conditional sentence uttered by humans, viz. *if I take an umbrella then it will not rain*. Affirming the antecedent, *I take an umbrella* the conclusion *it will not rain* seems arguable. Given the factual nature of the conditional it seems plausible that skeptical reasoners may respond with *nothing follows*. WCS can also account for these reasoners. Such a discussion motivates further research about the AA and why humans accord with the response C so easily. Coming back to the DA, if we were to deny the antecedent on the other hand, this is, *I do not take an umbrella*, then although most reasoners might respond $\neg C$ meaning, *it will rain*, once again, WCS with the extension proposed in this paper can account for the former as well as for reasoners who choose to respond with skepticism that *nothing follows*.

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Appendix: Conditionals of the Experiment

Obligation Conditionals with Necessary Antecedent (ON)

(1) *If it rains, then the roofs must be wet.*
(2) *If water in the cooking pot is heated over* $99°C$, *then the water starts boiling.*
(3) *If the wind is strong enough, then the sand is blowing over the dunes.*

Obligation Conditionals with Non-Necessary Antecedent (ONN)

(4) *If Paul rides a motorbike, then Paul must wear a helmet.*
(5) *If Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age.*
(6) *If it rains, then the lawn must be wet.*

Factual Conditionals with Necessary Antecedent (FN)

(7) *If the library is open, then Sabrina is studying late in the library.*
(8) *If the plants get water, then they will grow.*
(9) *If my car's start button is pushed, then the engine will start running.*

Factual Conditionals with Non-Necessary Antecedent (FNN)
(10) *If Nancy rides her motorbike, then Nancy goes to the mountains.*
(11) *If Lisa plays on the beach, then Lisa will get sunburned.*
(12) *If Ron scores a goal, then Ron is happy.*

The classification was done by the authors of [5,4]. One should observe that for each obligation conditional the conjunction of the antecedent and the negation of the consequent is usually considered to be a forbidden possibility, whereas this does not hold for each factual conditional. Likewise, in each case of a non-necessary antecedent one can easily come up with a different reason for the consequent to hold, whereas this is not the case for each of the necessary antecedents.

8.1 Short background story for Example 1.

Maria and her friends are visiting a local pub to enjoy the evening with drinks and good food. Maria knows the local rules and regulations and obeys them.

8.2 Short background story for Example 2.

The Presleys have moved into their newly built house and have hired a gardener to lay out the garden. They are sitting on their terrace and are looking at the bushes, small trees, and shrubs which were planted by the gardener two months ago.