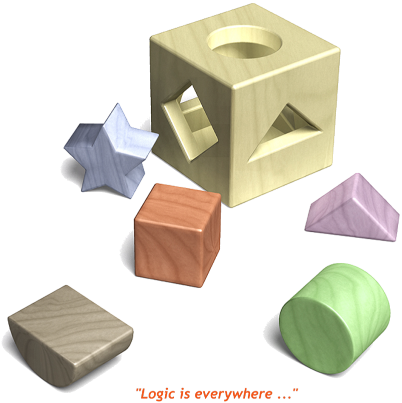
Logic Programming and Human Reasoning

Steffen Hölldobler
International Center for Computational Logic
Technische Universität Dresden
Germany



- **Introduction**
- **The Suppression Task**

Human Reasoning – Two Examples

- **Instructions on boarding card distributed at Amsterdam Schiphol Airport**
  - **If it's thirty minutes before your flight departure, make your way to the gate
    As soon as the gate number is confirmed, make your way to the gate**
- **Notice in London Underground**
  - **If there is an emergency then you press the alarm signal bottom
    The driver will stop if any part of the train is in a station**
- **Observations**
  - **Intended meaning differs from literal meaning**
  - **Rigid adherence to classical logic is no help in modeling the examples**
  - **There seems to be a reasoning process towards more plausible meanings**
    - **The driver will stop the train in a station
      if the driver is alerted to an emergency
      and any part of the train is in the station**
      Kowalski: Computational Logic and Human Life:
      How to be Artificially Intelligent. Cambridge University Press 2011

The Suppression Task

- Byrne: Suppressing Valid Inferences with Conditionals. Cognition 31, 61-83: 1989

C_1	**If she has an essay to write, then she will go to the library**
C_2	**if she has a textbook to read, then she will go to the library**
C_3	**If the library is open, then she will go to the library**
E	**She has an essay to write**
L	**She will go to the library**

	E	$\neg E$	L	$\neg L$
C_1	96%	46%	71%	92%
$C_1 \& C_2$	96%	4%	13%	96%
$C_1 \& C_3$	38%	63%	54%	33%
	(L)	$(\neg L)$	(E)	$(\neg E)$

The Suppression Task – Modus Ponens

- Stenning, van Lambalgen: Human Reasoning and Cognitive Science
  MIT Press: 2008

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false

- **Weakly completed programs & least models**

$$e \leftrightarrow \top$$
$$\ell \leftrightarrow e \wedge \neg ab_1$$
$$ab_1 \leftrightarrow \bot$$

true	*false*	
e	ab_1	$\Phi \uparrow 1$
ℓ		$\Phi \uparrow 2$

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle\{e, \ell\}, \{ab_1\}\rangle \models_{wcs} \ell$$

- Łukasiewicz: O logice trójwartościowej. Ruch Filozoficzny 5, 169-171: 1920

- H., Kencana Ramli: Logic Programs under Three-Valued Łukasiewicz's Semantics
  LNCS 5649, 464-478: 2009

The Suppression Task – Alternative Argument

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false
$\ell \leftarrow t \wedge \neg ab_2$	rule	definition of ℓ
$ab_2 \leftarrow \bot$	assumption	ab_2 is assumed to be false

- **Weakly completed programs & least models**

$$
\begin{array}{rcl}
e & \leftrightarrow & \top \\
\ell & \leftrightarrow & (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2) \\
ab_1 & \leftrightarrow & \bot \\
ab_2 & \leftrightarrow & \bot
\end{array}
$$

true	false	
e	ab_1	
	ab_2	$\Phi \uparrow 1$
ℓ		$\Phi \uparrow 2$

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e, \ell\}, \{ab_1, ab_2\} \rangle \models_{wcs} \ell$$

The Suppression Task – Additional Argument

- **Programs**

$e \leftarrow \top$	fact	definition of e
$\ell \leftarrow e \wedge \neg ab_1$	rule	definition of ℓ
$ab_1 \leftarrow \bot$	assumption	ab_1 is assumed to be false
$\ell \leftarrow o \wedge \neg ab_3$	rule	definition of ℓ
$ab_3 \leftarrow \bot$	assumption	ab_3 is assumed to be false
$ab_1 \leftarrow \neg o$	rule	definition of ab_1
$ab_3 \leftarrow \neg e$	rule	definition of ab_3

- **Weakly completed programs & least models**

$$
\begin{aligned}
e &\leftrightarrow \top \\
\ell &\leftrightarrow (e \wedge \neg ab_1) \vee (o \wedge \neg ab_3) \\
ab_1 &\leftrightarrow \bot \vee \neg o \\
ab_3 &\leftrightarrow \bot \vee \neg e
\end{aligned}
$$

true	false	
e		$\Phi \uparrow 1$
	ab_3	$\Phi \uparrow 2$

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e\}, \{ab_3\} \rangle \not\models_{wcs} \ell$$

The Suppression Task – Denial of the Antecedent

- **Programs**

$$\begin{array}{rcl} e & \leftarrow & \bot \\ \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array}$$

- **Weakly completed programs & least models**

$$\begin{array}{rcl} e & \leftrightarrow & \bot \\ \ell & \leftrightarrow & e \wedge \neg ab_1 \\ ab_1 & \leftrightarrow & \bot \end{array}$$

true	*false*
	e
	ab_1
	ℓ

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \emptyset, \{e, ab_1, \ell\} \rangle \models_{wcs} \neg \ell$$

The Suppression Task – Affirmation of the Consequent

- **Programs**

$$\begin{array}{rcl} \ell & \leftarrow & \top \\ \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \end{array}$$

- **Weakly completed programs & least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & \top \vee (e \wedge \neg ab_1) \\ ab_1 & \leftrightarrow & \bot \end{array}$$

true	*false*
ℓ	ab_1

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{\ell\}, \{ab_1\} \rangle \not\models_{wcs} e$$

  - Byrne 1989 **most humans conclude *e*!**

Abduction

- Hartshorn et. al.: Collected Papers of C. Sanders Peirce. Harvard Univ. Press: 1931
- **Programs & observations**

$$\ell \leftarrow e \wedge \neg ab_1 \qquad\qquad \ell$$
$$ab_1 \leftarrow \bot$$

- **Abducibles**

$$e \leftarrow \top \qquad\qquad e \leftarrow \bot$$

- **Weakly completed programs plus explanations & least models**

$$\ell \leftrightarrow e \wedge \neg ab_1$$
$$ab_1 \leftrightarrow \bot$$
$$e \leftrightarrow \top$$

true	false
e	ab_1
ℓ	

- **Computing logical consequences with respect to least models**

$$\mathcal{M} = \langle \{e, \ell\}, \{ab_1\} \rangle \models_{wcs} e$$

- H., Philipp, Wernhard: An Abductive Model for Human Reasoning. In: Proc. Tenth Int. Symposium on Logical Formalizations of Commonsense Reasoning: 2011

Alternative Arguments and Affirmation of the Consequent

- **Programs & observations**

$$\begin{array}{rcl} \ell & \leftarrow & e \wedge \neg ab_1 \\ ab_1 & \leftarrow & \bot \\ \ell & \leftarrow & t \wedge \neg ab_2 \\ ab_2 & \leftarrow & \bot \end{array} \qquad \qquad \ell$$

- **Abducibles**

$$e \leftarrow \top \qquad t \leftarrow \top \qquad e \leftarrow \bot \qquad t \leftarrow \bot$$

- **Weakly completed programs plus explanations & least models**

$$\begin{array}{rcl} \ell & \leftrightarrow & (e \wedge \neg ab_1) \vee (t \wedge \neg ab_2) \\ ab_1 & \leftrightarrow & \bot \\ ab_2 & \leftrightarrow & \bot \\ e & \leftrightarrow & \top \quad \text{or} \quad t \leftrightarrow \top \end{array}$$

true	false
e	ab_1
	ab_2
ℓ	

true	false
t	ab_1
	ab_2
ℓ	

- **Computing skeptical consequences with respect to both models**
  - e **does not follow**

- Dietz, H., Ragni: A Computational Logic Approach to the Suppression Task Proc. COGSCI, 1500-1505: 2012