

# Chair of Computer Graphics and Visualization

Institute for Software and Multimedia  
Technology

Prof. Dr. Stefan Gumhold



# Content

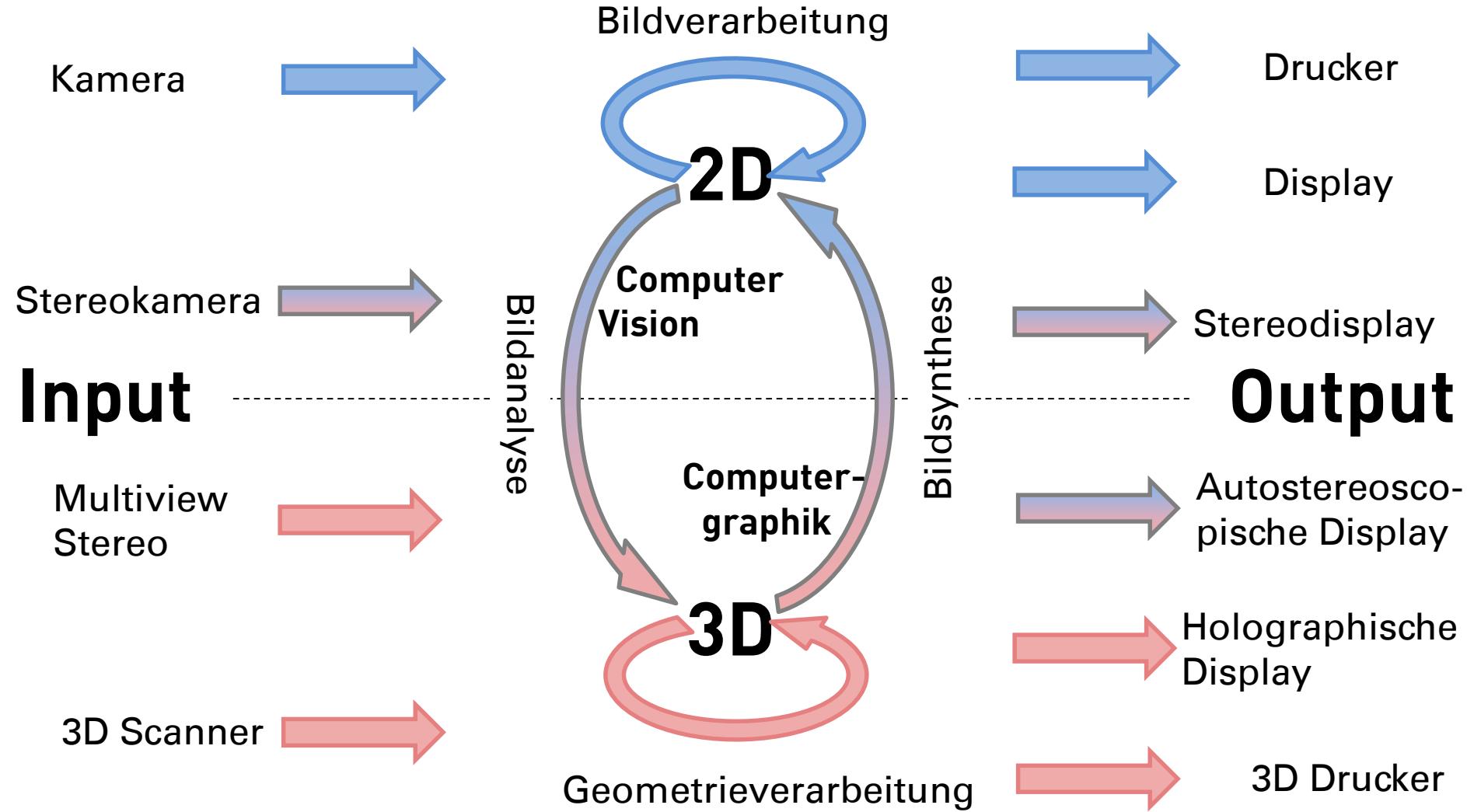
- ◆ Overview
  - ◆ Computer Graphics
  - ◆ Scientific Visualization
  - ◆ Research Areas at CGV
  - ◆ Courses at CGV
- ◆ Example Research Questions
  - ◆ Normal Estimation for 3D Scanning
  - ◆ Efficient Particle Rendering for Visualization
  - ◆ Object Pose Estimation
- ◆ Oral Exam



# OVERVIEW CG, SCIVIS, RESEARCH AND COURSES

# Einordnung und Begriffsbestimmung

## Übersicht der GDV





## Computer Graphics

Modelling

Rendering /  
Image Synthesis

Animation

## Application Domains

Visualization

Natural Sciences

Medicine

Geoinformatics

Information

Engineering

CAD/CAM

Simulation

Round Trip

3D Scanning

Interaction

Operating Systems

User Interaction

virtual Reality

augmented Reality

Entertainment

Games

Special Effects

Cartoons

CGV research areas



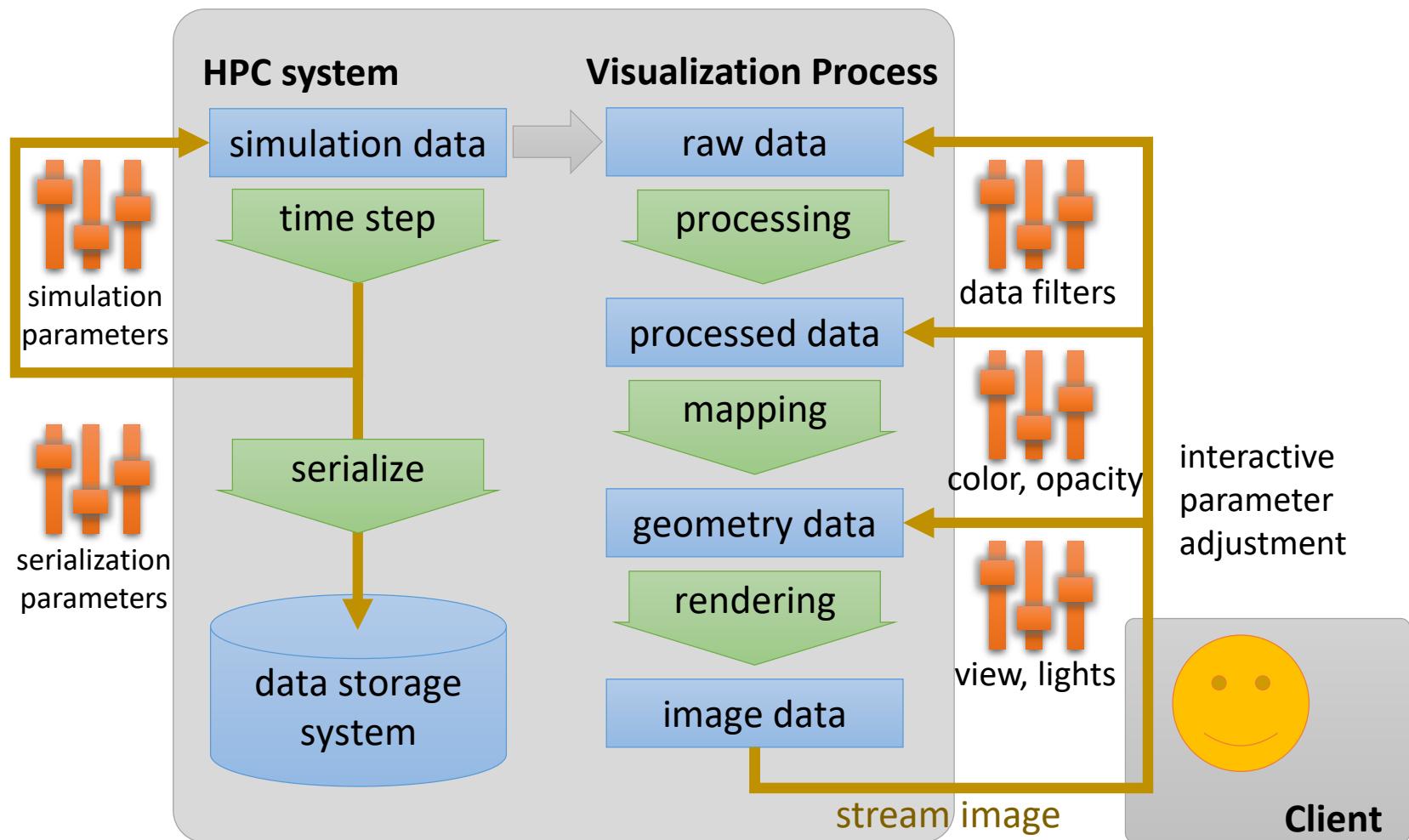
## A Reduced Model for Interactive Hairs

Menglei Chai<sup>1</sup> Changxi Zheng<sup>2</sup> Kun Zhou<sup>1</sup>

Zhejiang University<sup>1</sup> Columbia University<sup>2</sup>

© Chai et al., [A Reduced Model for Interactive Hairs](#), SIGGRAPH 2014

# Scientific Visualization Pipeline



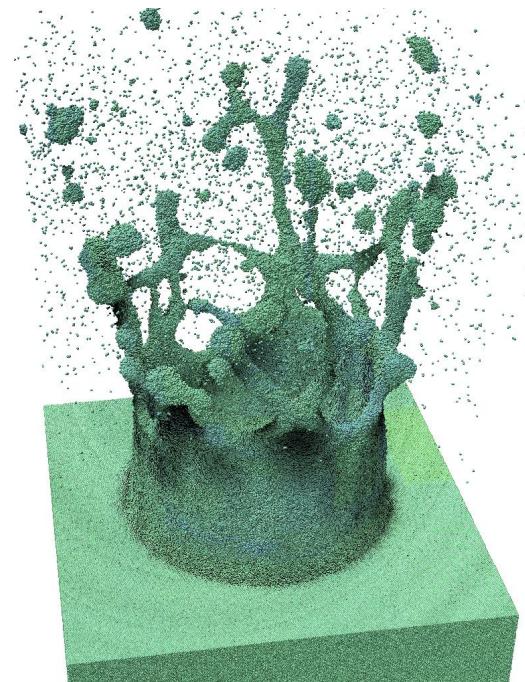
# Sample Visualizations



Volumes  
© Bruckner et al. 2005



Vector Fields  
© Günther et al. 2013



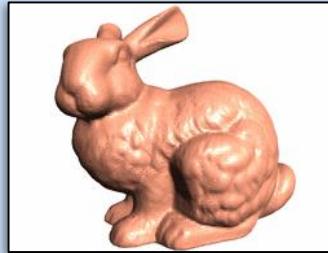
Particles  
© Grottel 2014

# Overview – Research Areas at CGV

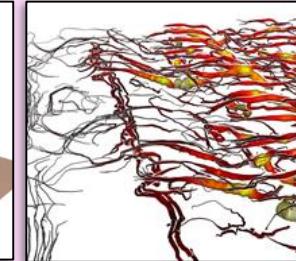
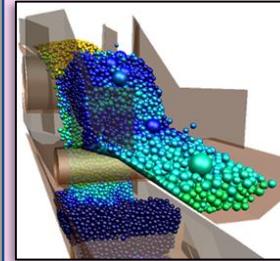


Computer Graphics  
and Visualization

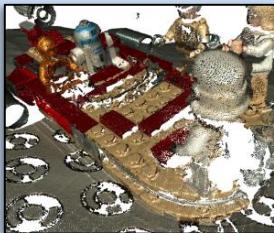
## Geometry Processing



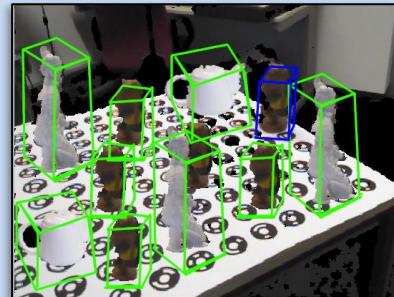
## Scientific Visualization



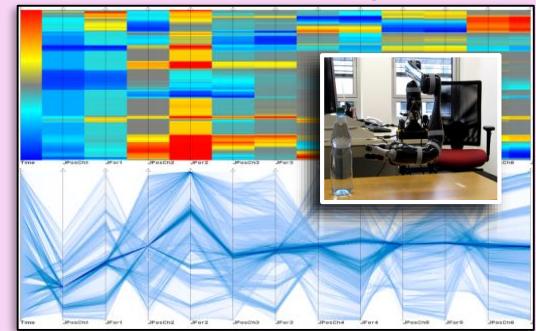
## Scene Acquisition



## Scene Understanding



## Visual Analysis



# Overview – Research Areas at CGV



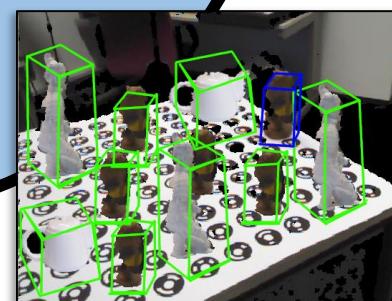
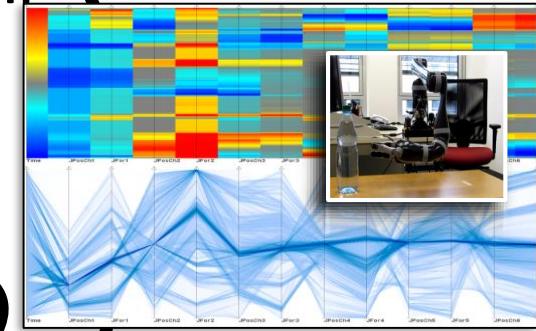
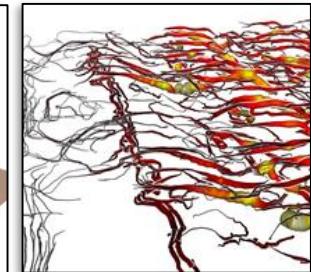
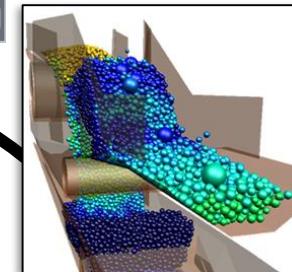
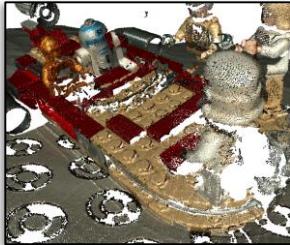
Computer Graphics  
and Visualization

## INTERACTIVE VISUAL COMPUTING

3D Scanning  
and Geometry  
Processing

Scientific  
Visualization  
and Visual  
Analysis

Scene  
Understanding



# Overview – Course at CGV



	WS	SS	BAS-7	VERT7	VMI-8	INF-PM-FOR/ANW
<b>ECG</b>		X				
<b>CG1</b>	X		X			
<b>CG2</b>		X	X	X	X	X
<b>CG3</b>	X			X	X	X
<b>WV</b>	X		X			
<b>CPP4CG</b>		X	X			

- ❖ ECG ... Einführung Computergraphik
- ❖ CG1 ... Computer Graphics 1 – enabling technologies
- ❖ CG2 ... Computer Graphics 2 – acquisition & geometry processing
- ❖ CG3 ... Computer Graphics 3 – physical based graphics
- ❖ WV ... Wissenschaftliche Visualisierung
- ❖ CPP4CG ... C++-Programmierung für Computergraphik

# Course Content

## ● ECG

- Modellierung
- Rasterisierung
- Transformationen
- Basic GPU Prog.
- Kurven
- Beleuchtung
- Raytracing

## ● CG4CPP

- usage of IDE
- polymorphism
- Templates, stl
- exceptions
- advanced features

## ● CG1

- Grand Tour
- Modern GPU Prog.
- Mesh Processing
- Acceleration-DS
- Optimization

## ● CG2 – Geometry

- Representations  
(implicit surfaces  
subdivision surf.)
- Inverse Kinematics
- Rigging & Skinning
- 3D Scanning

## ● SciVis

- Foundations
- Preprocessing
- Mapping
- Volumes
- Vector Fields
- Particles

## ● CG3 - Physical

- Physics for CG
- Global Illumination
- Rigid Bodies
- Fluids



# EXAMPLE RESEARCH QUESTIONS

Normal Estimation for 3D Scanning

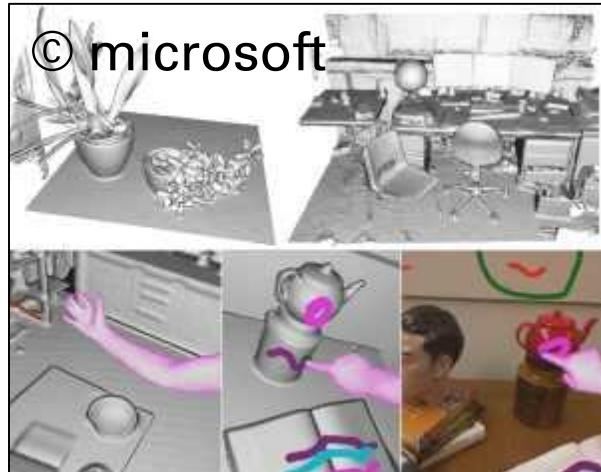
Efficient Particle Rendering for Visualization

Object Pose Estimation

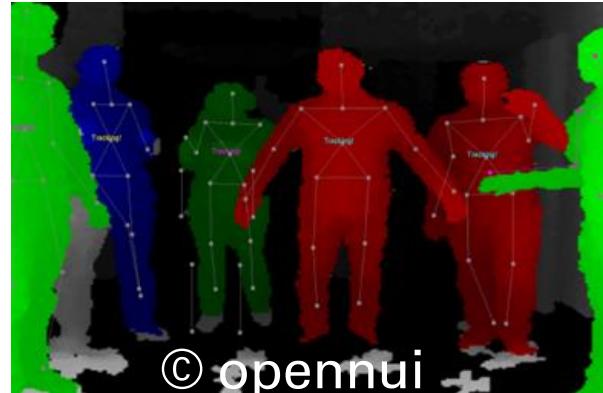
# 3D Acquisition



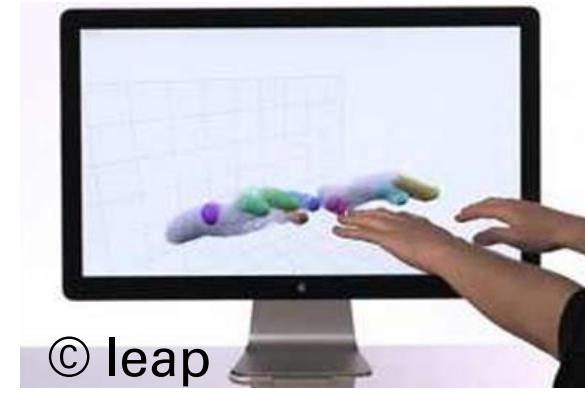
commercial realtime 3D scanning devices



kinect fusion



skeleton tracking



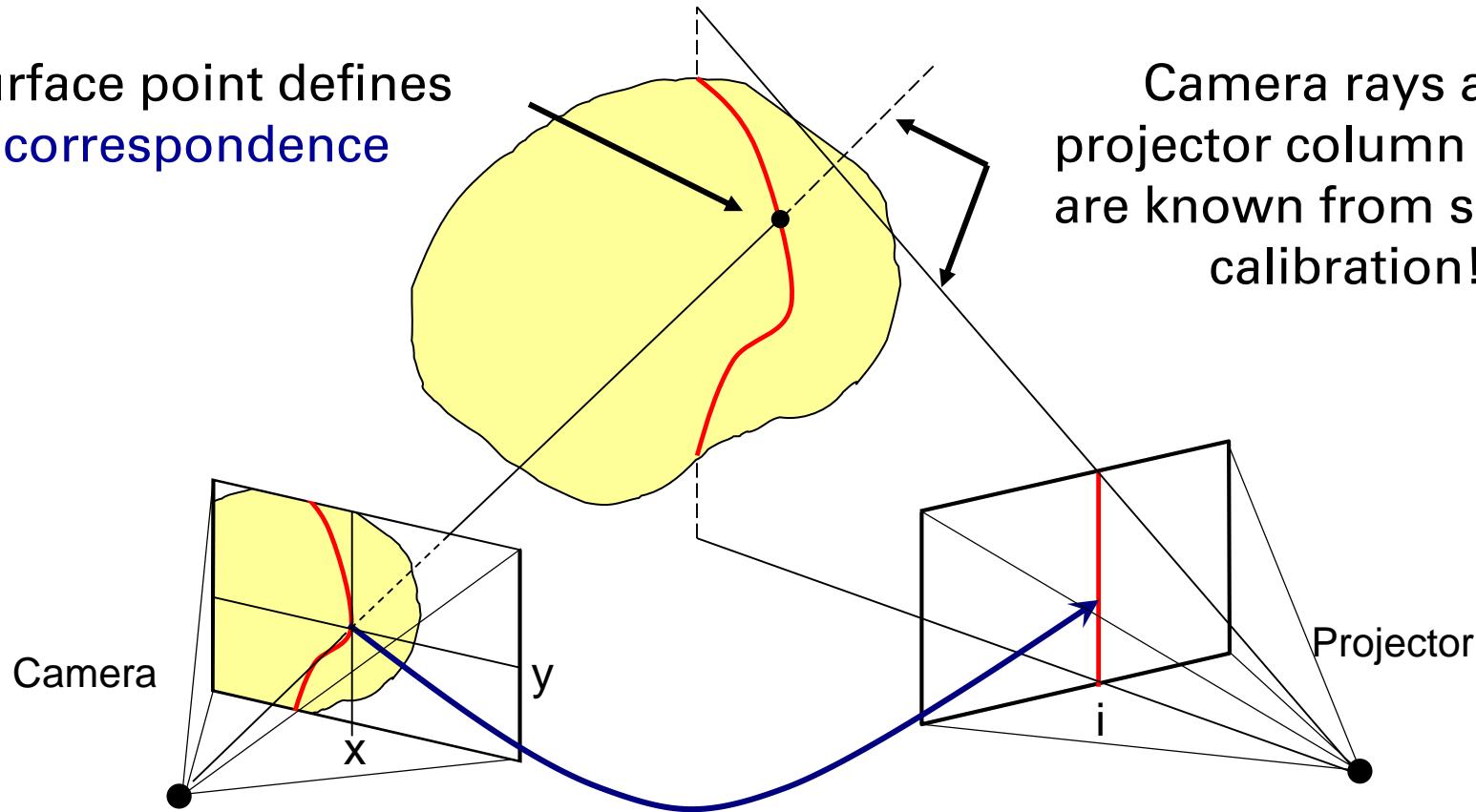
hand tracking



# Structured Light Setup

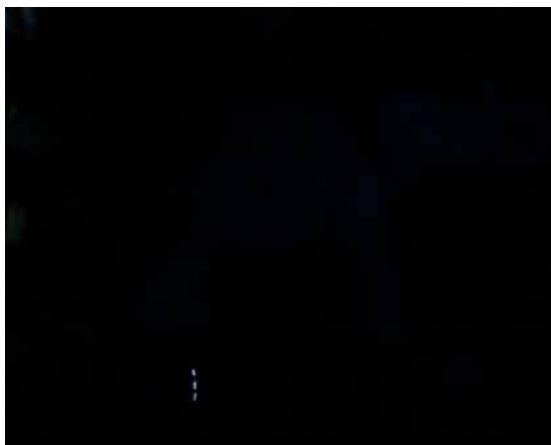
Surface point defines  
correspondence

Camera rays and  
projector column planes  
are known from scanner  
calibration!

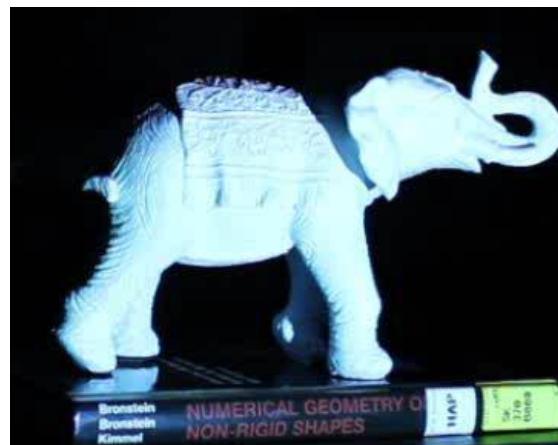


Reconstruction: Solve Correspondence problem  
Camera pixel  $\rightarrow$  projector column + Ray-Plane  
Intersection

# Column Coding Techniques



lineshift



binary code

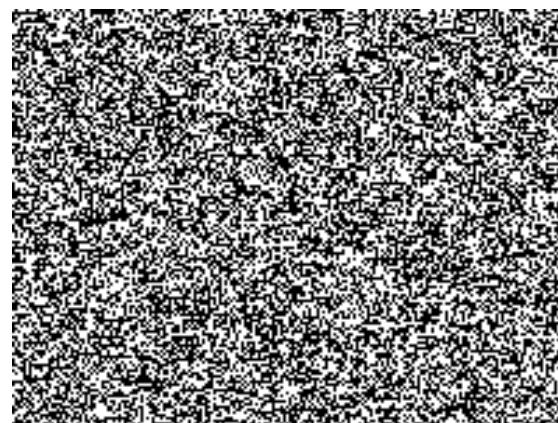


hierarchical phaseshift

Motion Compensation for Dynamic Case



no compensation



motion compensation pattern



with compensation

# Sample Dynamic 3D Scans



Computer Graphics  
and Visualization

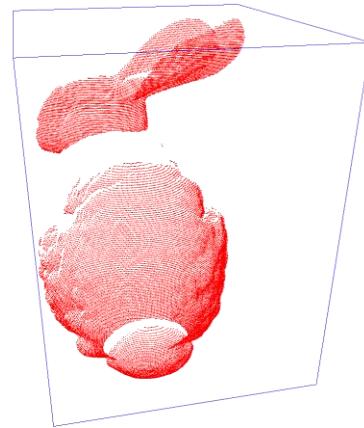


dynamic face scan

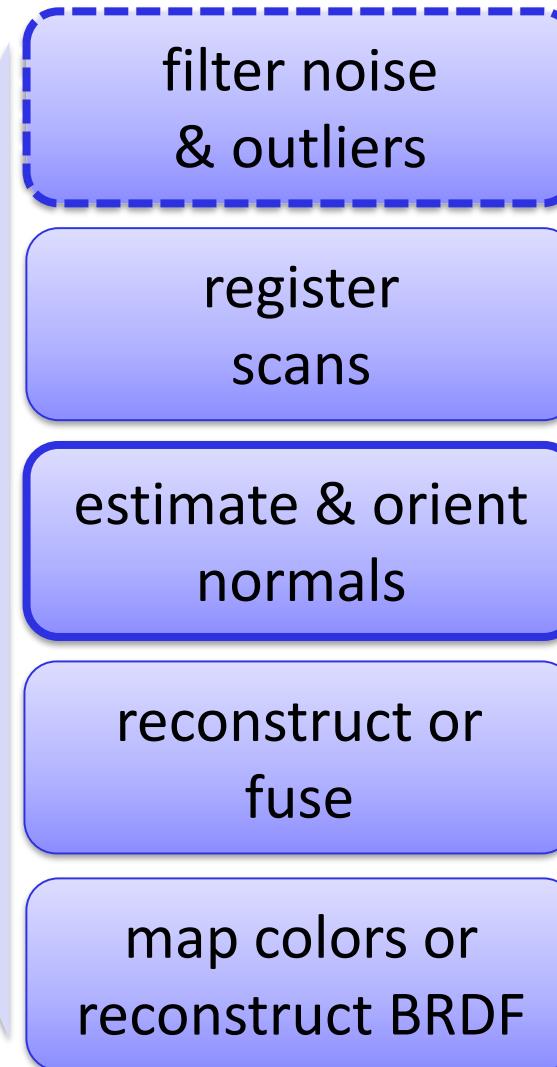
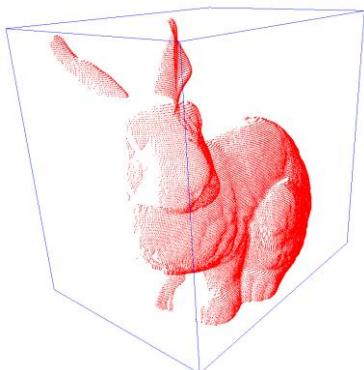


dynamic hand scan

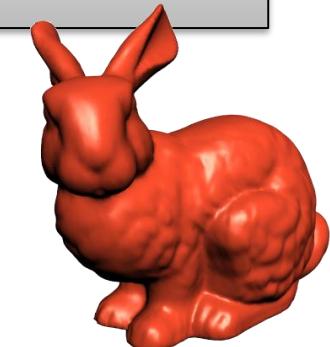
# Shape Reconstruction Pipeline



multiple  
3D scans



3D shape  
model

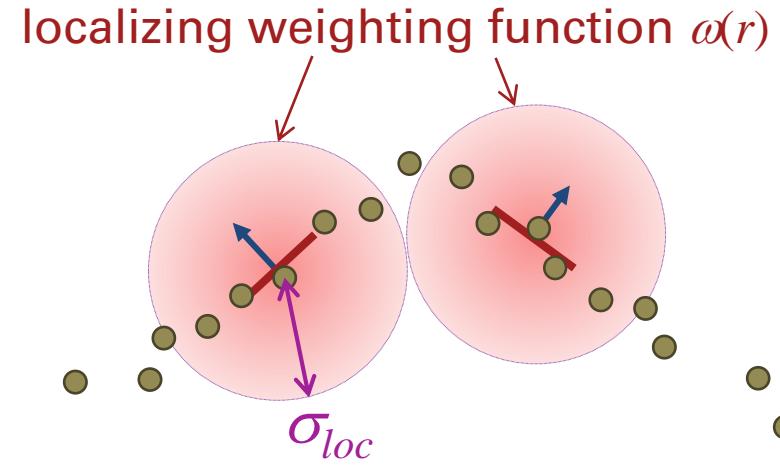




# Estimation of Tangent Plane

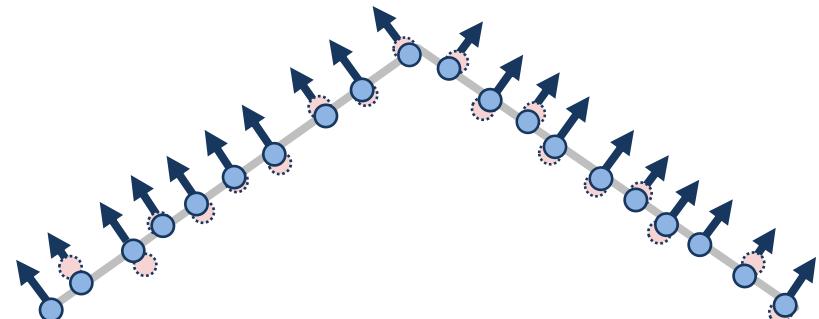
## Localized Fit

- For each point in 3D scan estimate a local tangent plane by some weighted fitting procedure
- Crutial is to choose the correct width  $\sigma_{loc}$  of the weighting function

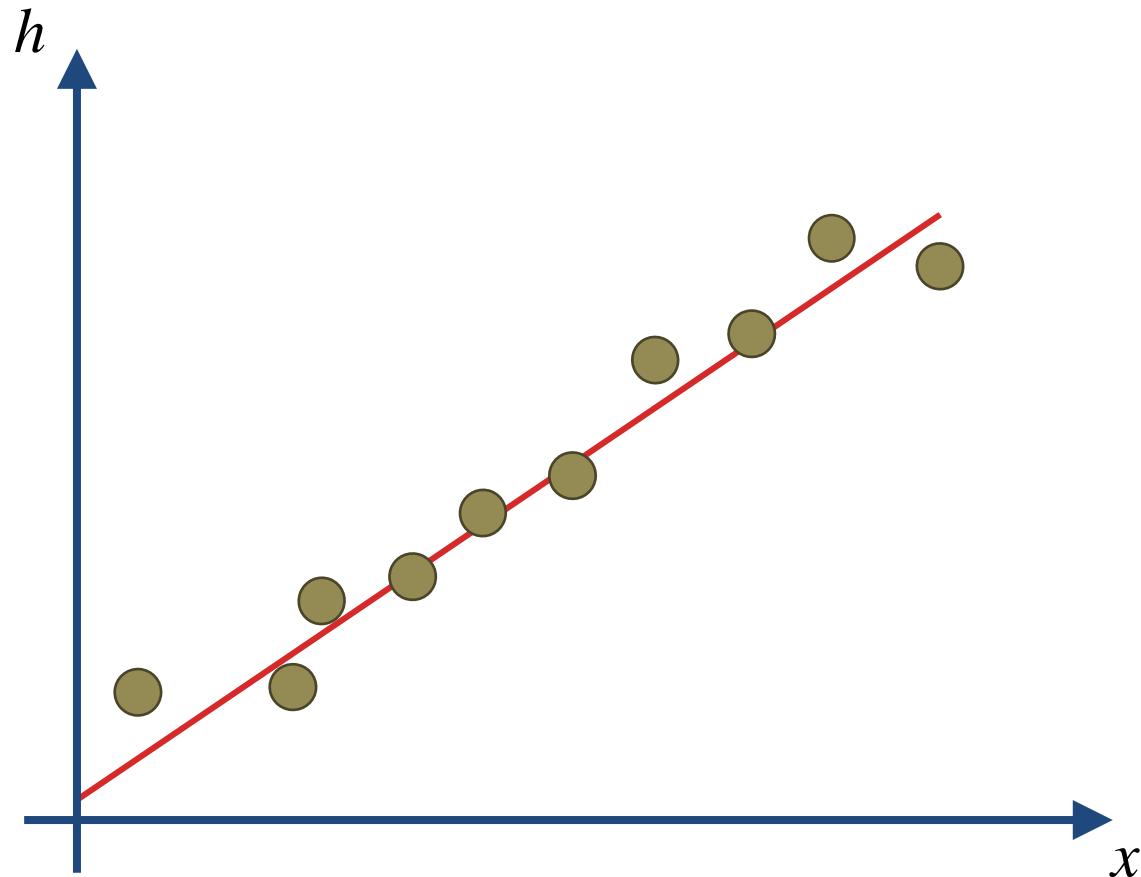


## Denoising

- Project each point onto its local tangent plane for denoising



# Fit Line to Heights



data are  $k$  pairs:

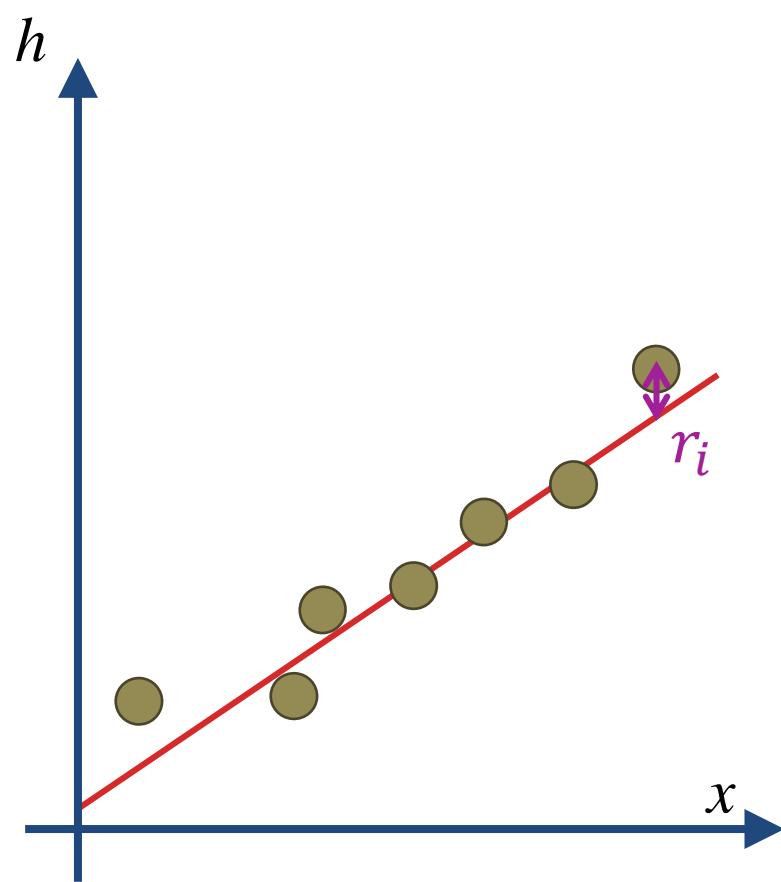
$$(x_i, h_i)$$

model is line:

$$h(x) = mx + c$$

- Goal: find model parameters  $\theta = (m, c)$  of line that best fits the data

# Least Squares of Height Differences



data are  $k$  pairs:  $(x_i, h_i)$

model is line:  $h(x) = mx + c$

**residuals** form heights

$$r_i = h(x_i) - h_i$$

fit energy from sum of squares

$$E(m, c) = \sum_i r_i^2$$

least squares approach

$$\theta^* = (m^*, c^*) = \min_{(m,c)} E(m, c)$$



# Normal Equations

data are  $k$  pairs:  $(x_i, h_i)$

residuals:  $r_i = mx_i + c - h_i$

model is line:  $h(x) = mx + c$

fit energy:  $E(m, c) = \sum_i r_i^2$

normal equations:

$$0 = \frac{\partial}{\partial m} E(m, c) = \sum_i 2r_i x_i = 2 \sum_i (mx_i^2 + cx_i - h_i x_i)$$

$$0 = \frac{\partial}{\partial c} E(m, c) = \sum_i 2r_i = 2 \sum_i (mx_i + c - h_i)$$

some shortcuts:

$$\begin{aligned} S_x &= \sum_i x_i & S_h &= \sum_i h_i & S_{xx} &= \sum_i x_i^2 \\ \bar{x} &= \frac{1}{k} S_x & \bar{h} &= \frac{1}{k} S_h & S_{hx} &= \sum_i h_i x_i \end{aligned}$$

0 =  $mS_{xx} + cS_x - S_{hx}$   
0 =  $S_x + kc - S_h$

normal equations



# Transform – Centering of Average

data are  $k$  pairs:  $(x_i, h_i)$

model is line:  $h(x) = mx + c$

residuals:  $r_i = mx_i + c - h_i$

fit energy:  $E(m, c) = \sum_i r_i^2$

normal equations:

$$0 = mS_{xx} + cS_x - S_{hx}$$

$$0 = S_x + kc - S_h$$

compute averages:

$$\bar{x} = \frac{1}{k}S_x \quad \bar{h} = \frac{1}{k}S_h$$

transform data:

$$(x'_i, h'_i) = (x_i - \bar{x}, h_i - \bar{h})$$

sums vanish

$$S_{x'} = 0 \quad S_{h'} = 0$$

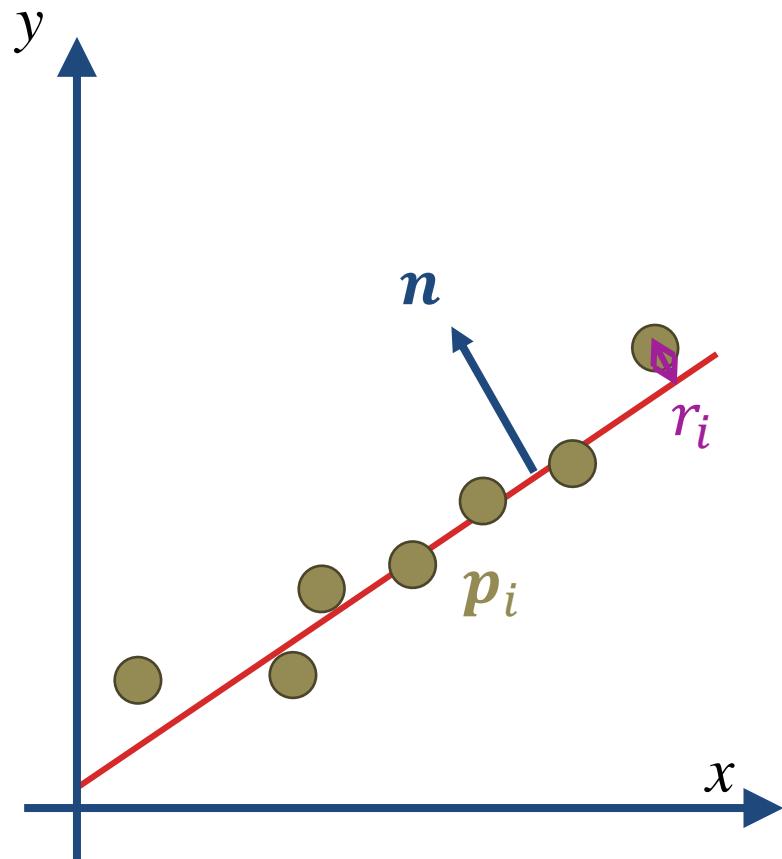
normal equations become:

$$c' = 0 \quad m' = \frac{S_{h'x'}}{S_{x'x'}}$$

→ by centering to averages constant term  $c'$  vanishes



# Orthogonal distances



data are  $k$  points:  $\mathbf{p}_i = (x_i, y_i)$

Hessian form                     $0 = \langle \mathbf{n}, \mathbf{p} \rangle + c$   
with normal  $\mathbf{n}$ :                 $\|\mathbf{n}\| = 1$

residuals form orthogonal dist.:

$$r_i = \langle \mathbf{n}, \mathbf{p}_i \rangle + c$$

transformation:  $\mathbf{p}'_i = \mathbf{p}_i - \bar{\mathbf{p}}$

without proof  $\rightarrow c' = 0$

least squares approach

$$\mathbf{n}'^* = \min_{\mathbf{n}' \mid \|\mathbf{n}'\|=1} \underbrace{E(\mathbf{n}')}_{\sum_i r'^2_i}$$



# Solve Orthogonal Least Squares Fit

$k$  data points:  $\mathbf{p}_i = (x_i, y_i)$

transformed:  $\mathbf{p}'_i = \mathbf{p}_i - \bar{\mathbf{p}}$

reduced model:  $0 = \langle \mathbf{n}', \mathbf{p}' \rangle, \|\mathbf{n}'\| = 1$

residuals:  $r_i = \langle \mathbf{n}', \mathbf{p}'_i \rangle$

$$S_{\mathbf{p}'} = \sum_i \mathbf{p}'_i$$

$$S_{\mathbf{p}'\mathbf{p}'} = \sum_i \mathbf{p}'_i \mathbf{p}'_i^T$$

fit energy  $E(\mathbf{n}') = \sum_i r_i^2 = \sum_i \langle \mathbf{n}', \mathbf{p}'_i \rangle^2 = \mathbf{n}'^T S_{\mathbf{p}'\mathbf{p}'} \mathbf{n}'$

$$\langle \mathbf{n}', \mathbf{p}'_i \rangle^2 = \mathbf{n}'^T \mathbf{p}'_i \mathbf{p}'_i^T \mathbf{n}'$$

$$\mathbf{p}'_i \mathbf{p}'_i^T = \begin{pmatrix} x'_i \\ y'_i \end{pmatrix} (x'_i & y'_i) = \begin{pmatrix} x'^2_i & x'_i y'_i \\ x'_i y'_i & y'^2_i \end{pmatrix}$$

least squares fit:  $\mathbf{n}'^* = \min_{\mathbf{n}' \mid \|\mathbf{n}'\|=1} \mathbf{n}'^T S_{\mathbf{p}'\mathbf{p}'} \mathbf{n}'$

→  $\mathbf{n}'^*$  is eigenvec. to smallest eigenval.

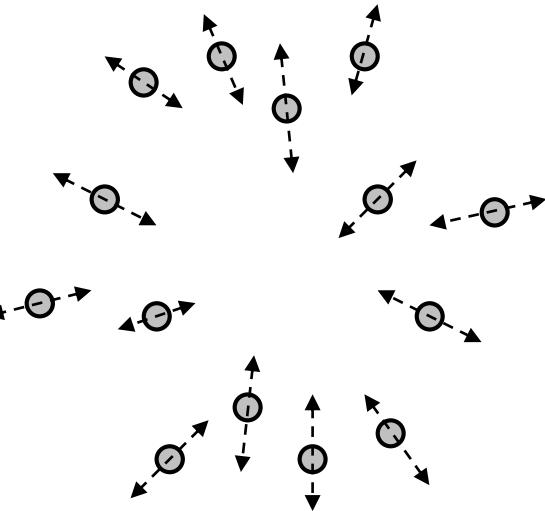
$$\mathbf{n}'^* = \pm EV(S_{\mathbf{p}'\mathbf{p}'})_0$$



# Discussion of Orthogonal Fit

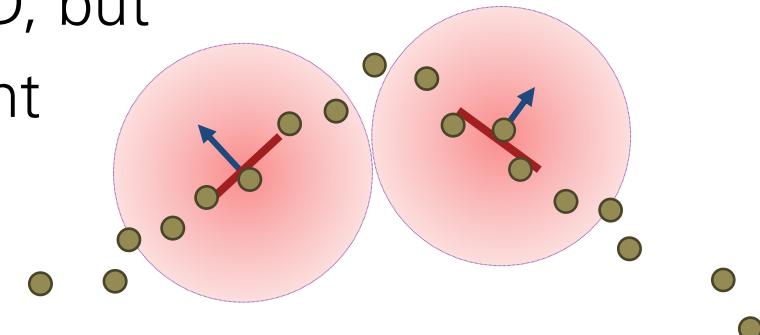
## Normal Orientation

- sign of normal is called orientation
- orientation defines inside of shape
- globally consistent orientation needed
- scanner location defines orientation



## 3D Case

- all generalizes nicely to 3D or nD, but
- single line / plane is not sufficient
- we need some technique to  
localize the fit



# Weighted Least Squares



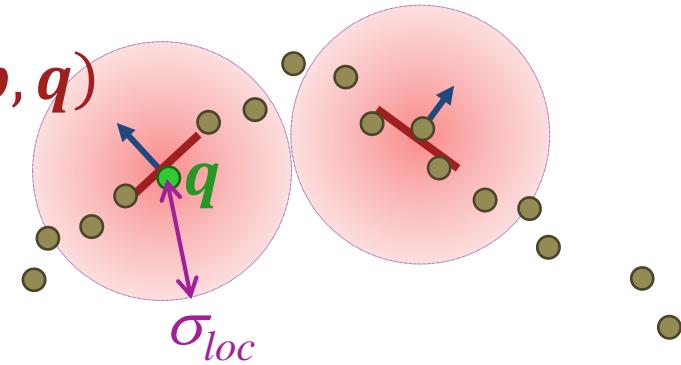
reference point:  $q$

weight function:  $\omega(p, q)$

influence scale:  $\sigma_{loc}$

often Gaussian weights:

$$\omega(p, q) = \theta(\|p - q\|), \quad \theta(d) = \exp(-d^2/\sigma_{loc}^2)$$



weighted fit energy:  $E(\mathbf{n}'(q)) = \sum_i \omega_i(q) r_i^2$

only few changes:

$$S_\omega = \sum_i \omega_i, \quad S_p = \sum_i \omega_i \mathbf{p}_i, \quad \bar{\mathbf{p}} = \frac{S_p}{S_\omega}, \quad S_{pp} = \sum_i \omega_i \mathbf{p}_i \mathbf{p}_i^T$$



# Discussion of Localized Fit

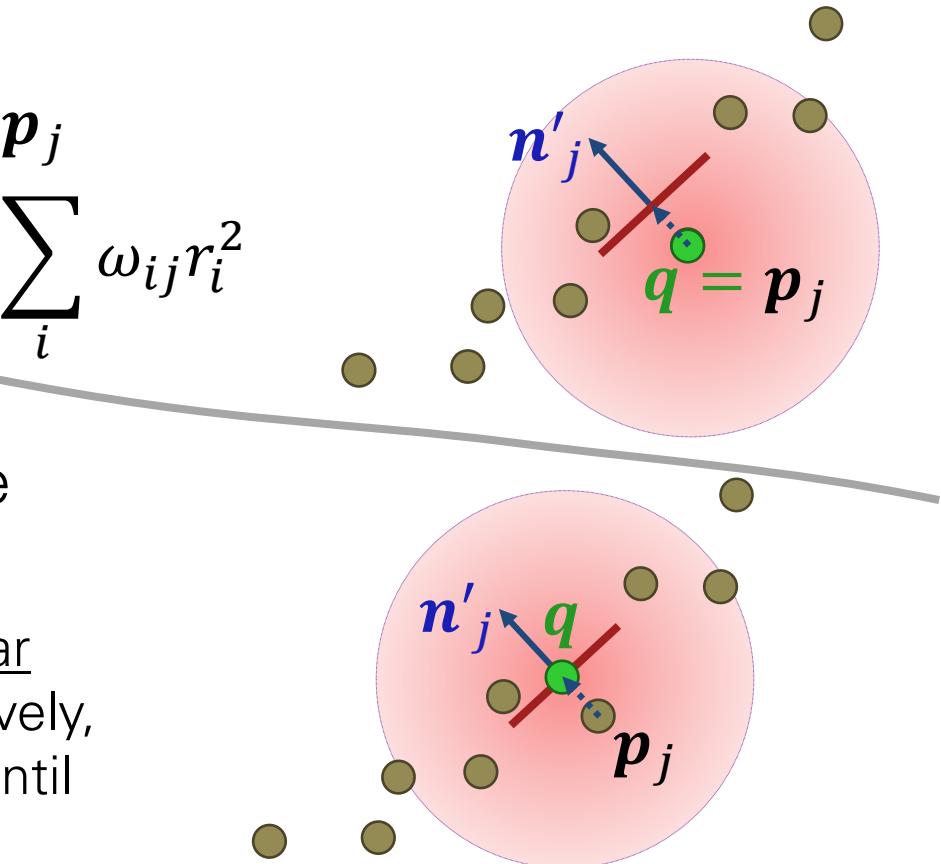
## choice of reference point

- choose each data point  $\mathbf{q} = \mathbf{p}_j$

$$\omega_{ij} = \omega(\mathbf{p}_i, \mathbf{p}_j), \quad E(\mathbf{n}'_j) = \sum_i \omega_{ij} r_i^2$$

- choose projection  $\mathbf{q}$  of data point  $\mathbf{p}_j$  onto resulting plane as reference point
  - fit problem becomes non-linear and needs to be solved iteratively, by recomputing the weights until convergence

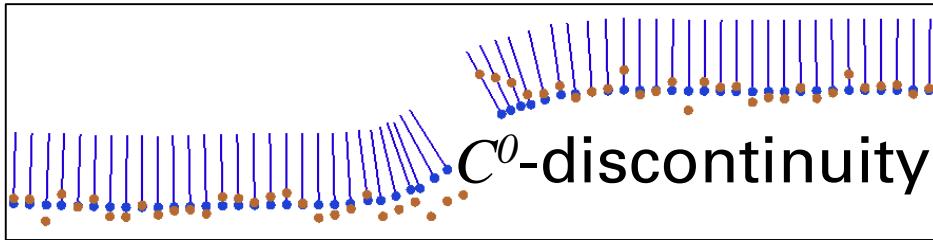
- besides  $\mathbf{p}_j$  one can choose any initial point  $\mathbf{p}$
- all points that project onto themselves form a  $C^\infty$ -continuous surface called the Moving Least Squares (MLS) surface



# Discussion of Localized Fit

## Problems

- outliers and  $C^0$ -discontinuities significantly influence fit

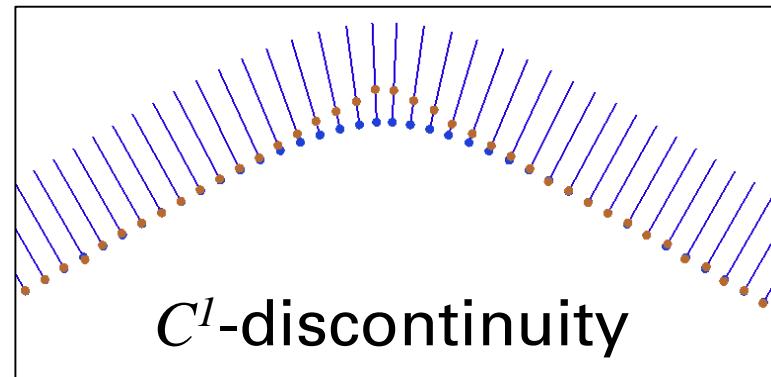
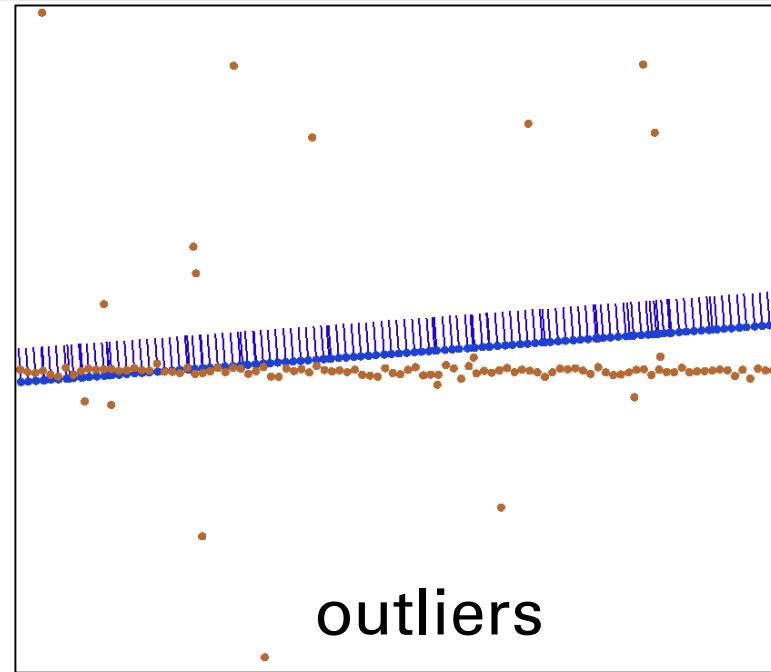


- $C^1$ -discontinuities are smoothed out

## Solution?

- use robust norm  $\rho(r)$

$$E(\mathbf{n}') = \sum_i \rho(r_i)$$

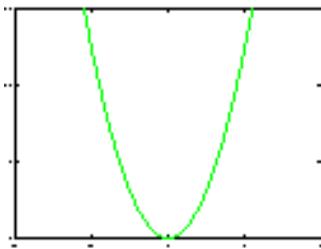




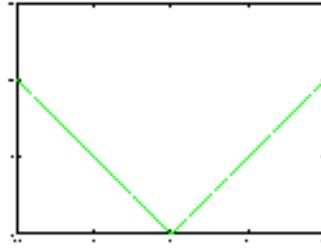
# Zoo of Robust Norms

- for Gaussian noise, the  $L_2$ -norm  $\rho(r) = r^2$  is optimal
- Otherwise a large number of norms can be chosen from, which need to be scaled by the noise scale  $\sigma_{noise}$ :

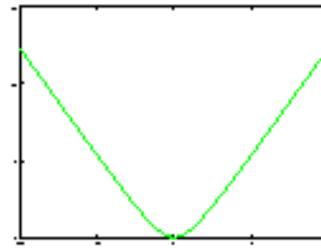
**Least-squares**



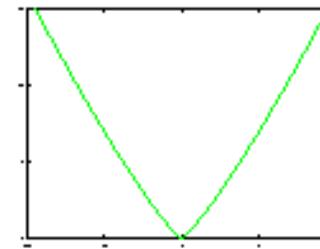
**Least-absolute**



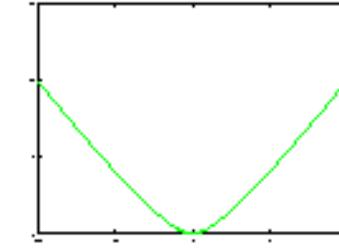
**$L_1 - L_2$**



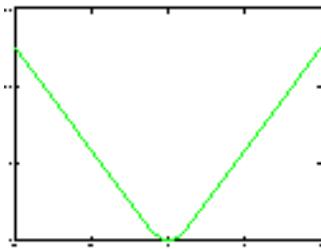
**Least-power**



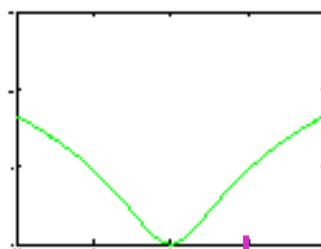
**Fair**



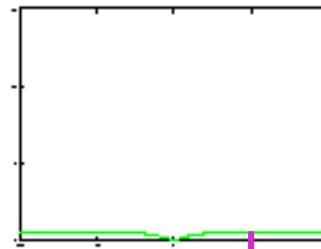
**Huber**



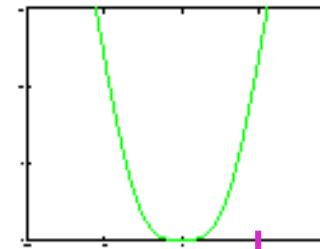
**Cauchy**



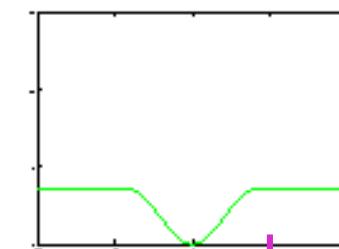
**Geman-McClure**



**Welsch**



**Tukey**



$\sigma_{noise}$

$\sigma_{noise}$

$\sigma_{noise}$

$\sigma_{noise}$

# M-estimation with Iterated Re-weighted Least Squares



- Idea: choose weights not for localization but to emulate robust norm with weighted least squares fit

robust norm:  $\rho(r)$

influence function:  $\psi(r) = \frac{\partial \rho}{\partial r}(r)$

## Weighted LS

$$E = \frac{1}{2} \sum_i \varpi_i r_i^2$$

$$\frac{\partial E}{\partial \theta} = \sum_i \varpi_i r_i \frac{\partial r_i}{\partial \theta}$$

IRWLS:

$$WLS(\varpi_i^0 \equiv 1) \Rightarrow r_i^0 \Rightarrow \varpi_i^1 = \psi(r_i^0)/r_i^0$$

$$WLS(\varpi_i^l) \Rightarrow r_i^l \Rightarrow \varpi_i^{l+1} = \psi(r_i^l)/r_i^l$$

$$l \leftarrow l + 1$$

## Robust LS

$$E = \sum_i \rho(r_i)$$

$$\frac{\partial E}{\partial \theta} = \sum_i \frac{\partial \rho}{\partial r}(r_i) \frac{\partial r_i}{\partial \theta}$$

weight function:  $\varpi_i = \frac{\psi(r_i)}{r_i}$

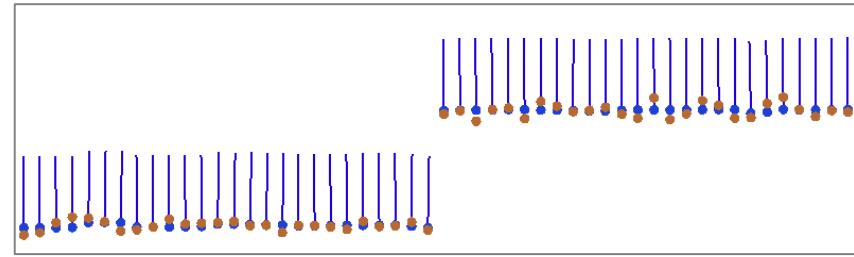
typically  $l = 3..5$   
iterations suffice



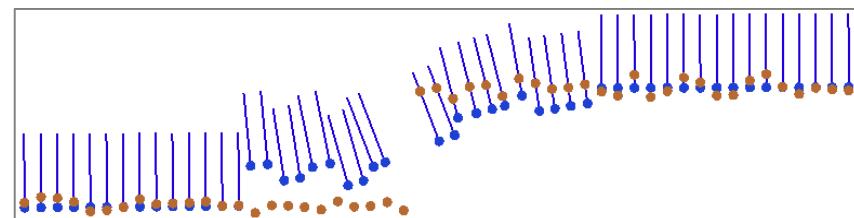
# Robust Least Squares – Discussion

- localization of robust fit similar to weighted LS
- most robust norms do not help much with outliers,  $C^0$  and  $C^1$ -discontinuities
- Example  $C^0$ -discontinuity with best robust norm being German-McClury:

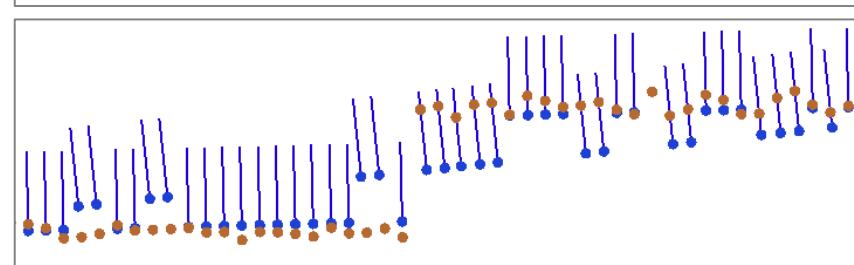
good choices  
for  $\sigma_{loc}$  and  $\sigma_{noise}$ :



for  $\sigma_{loc}$  too large:



for  $\sigma_{noise}$  too large:





- How to locally estimate  $\sigma_{loc}$  and  $\sigma_{noise}$  from the data?  
→ maximize local goodness of fit over  $\sigma_{loc}$  and  $\sigma_{noise}$
  
- How to support  $C^1$ -discontinuities?  
→ add weight that depends on difference in estimated normals
  
- How to get real-time performance?  
→ work on hierarchical redundant grid partition of data



# EXAMPLE RESEARCH QUESTIONS

Normal Estimation for 3D Scanning

Efficient Particle Rendering for Visualization

Object Pose Estimation



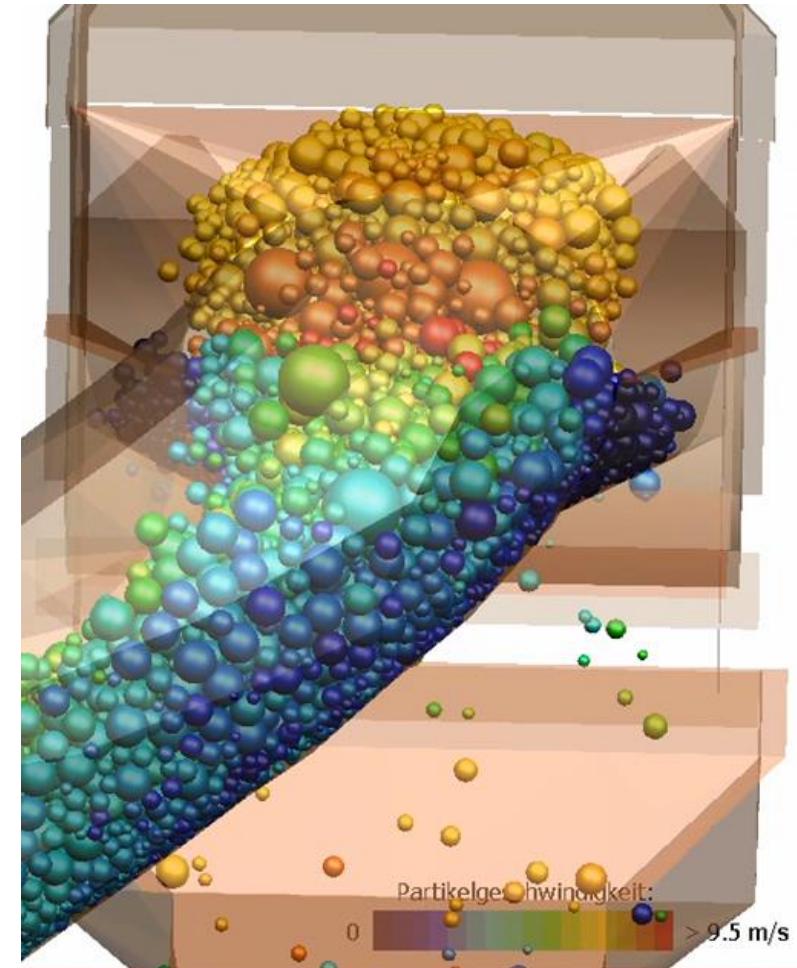
## Discrete Element Method

### ◆ Applications

- ◆ liquids and solutions, for instance of sugar or proteins;
- ◆ bulk materials in storage silos, like cereal;
- ◆ granular matter, like sand;
- ◆ powders, like toner.
- ◆ Blocky or jointed rock masses

### ◆ Simulation

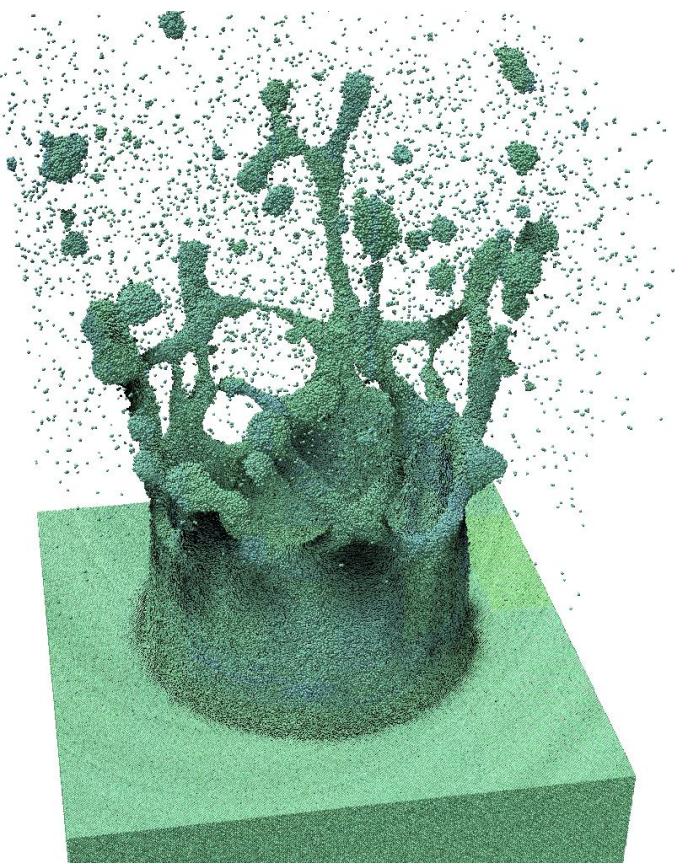
- ◆ Discretize matter on particles
- ◆ Model particle particle interaction forces



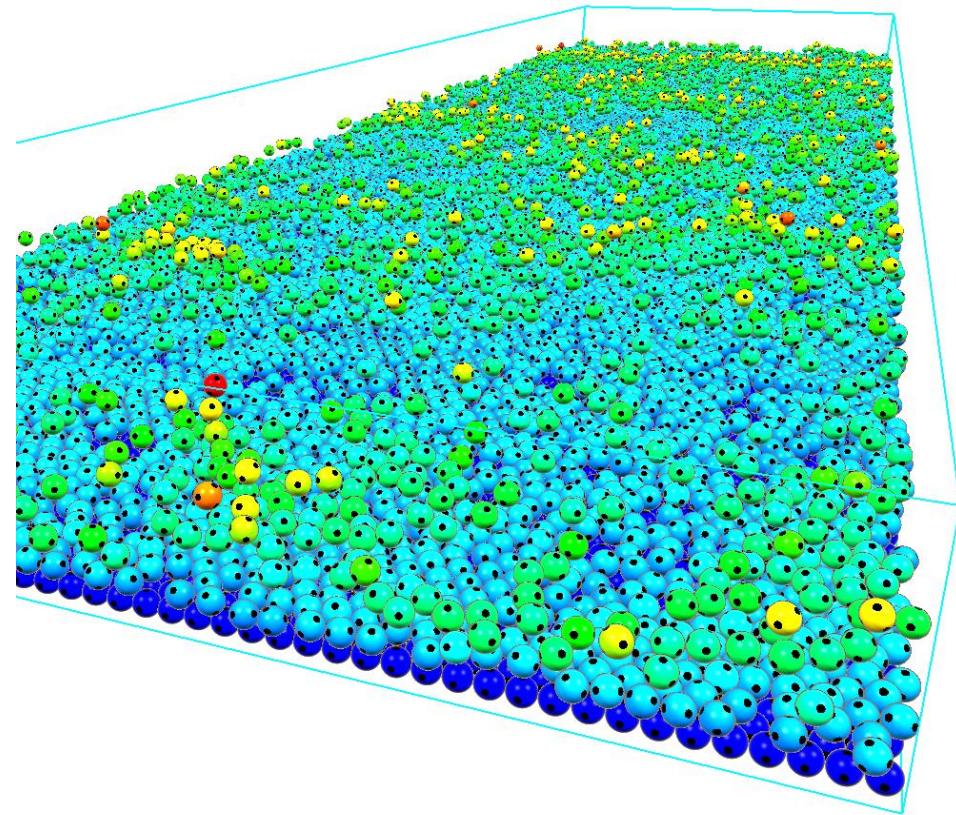
# Particle Based Visualization



- Molecular Dynamics  
Simulation of Laser Ablation



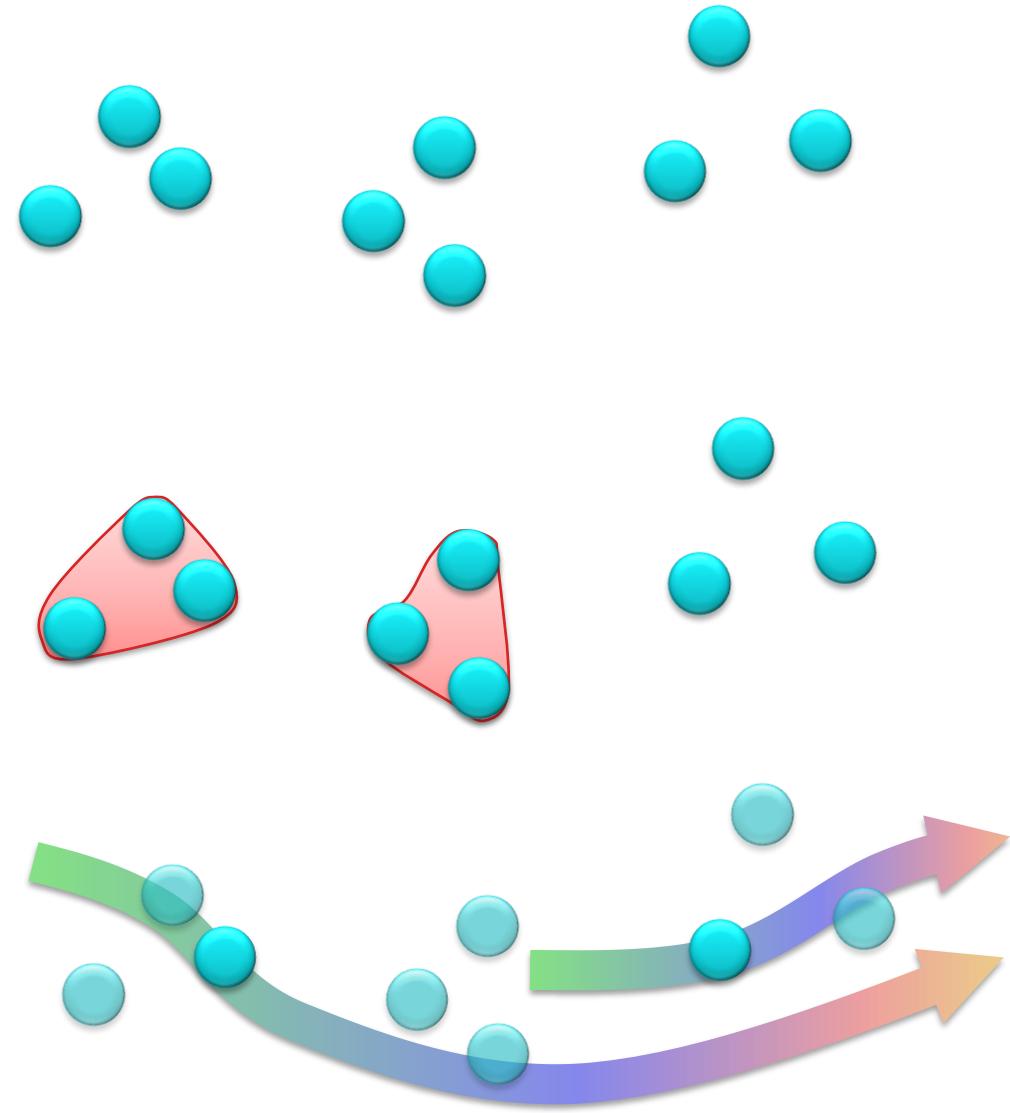
- Simulation of Sedimentation  
with rotating particles





# Visual Analysis of Particle Data

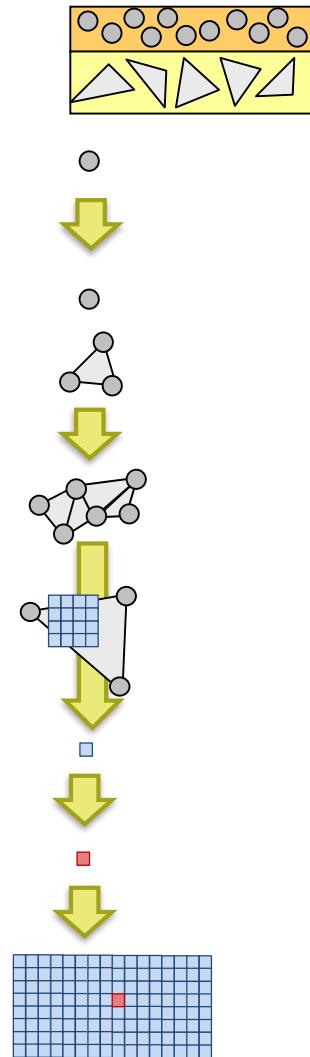
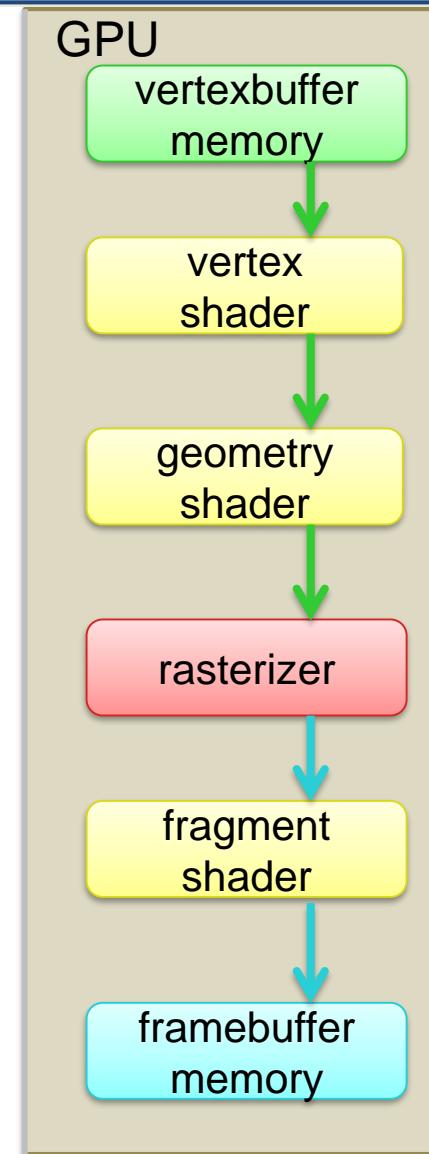
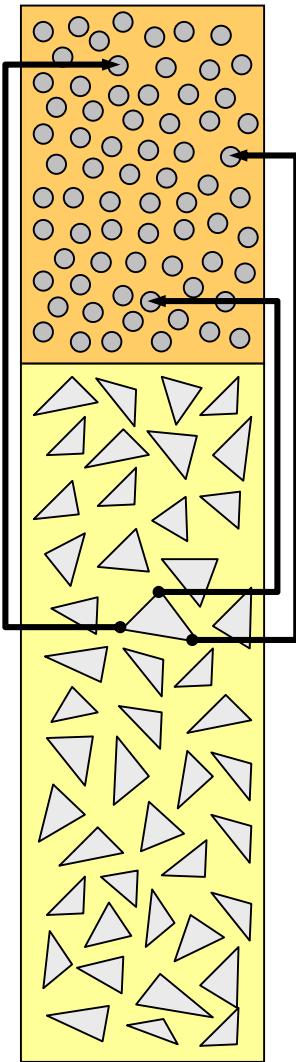
- ◆ show animation of particles
- ◆ cluster particle and trace clusters over time
- ◆ extract and filter particle traces



# Graphics Pipeline



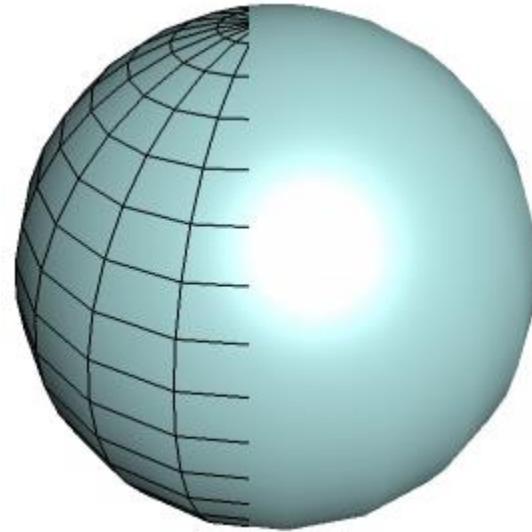
## Indexed mesh rendering



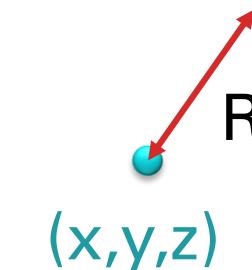


# Problem with Pipeline

- points, lines and triangles are only supported graphics primitives
- all other primitives must be tessellated



$20 \times 20 = 400$  vertices plus  
800 triangles

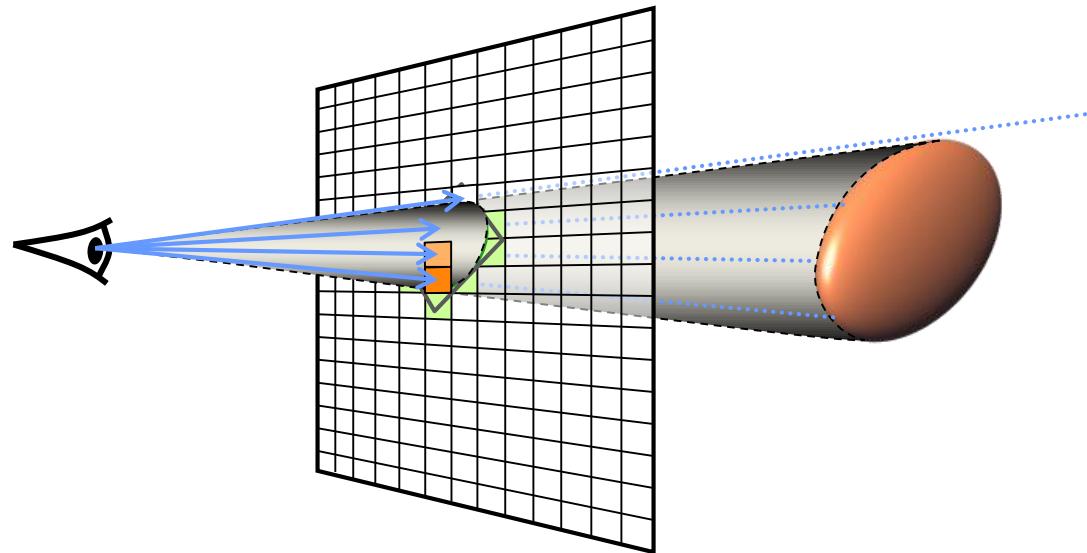


vs 4 floats



# Particle Raycasting

- ◆ compute projected silhouette
- ◆ cover silhouette with graphics primitive[s]

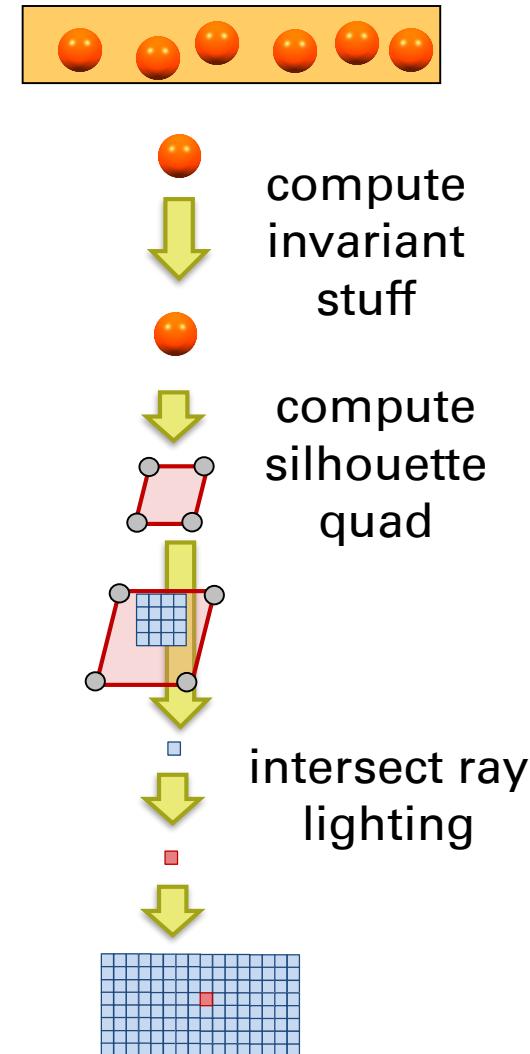
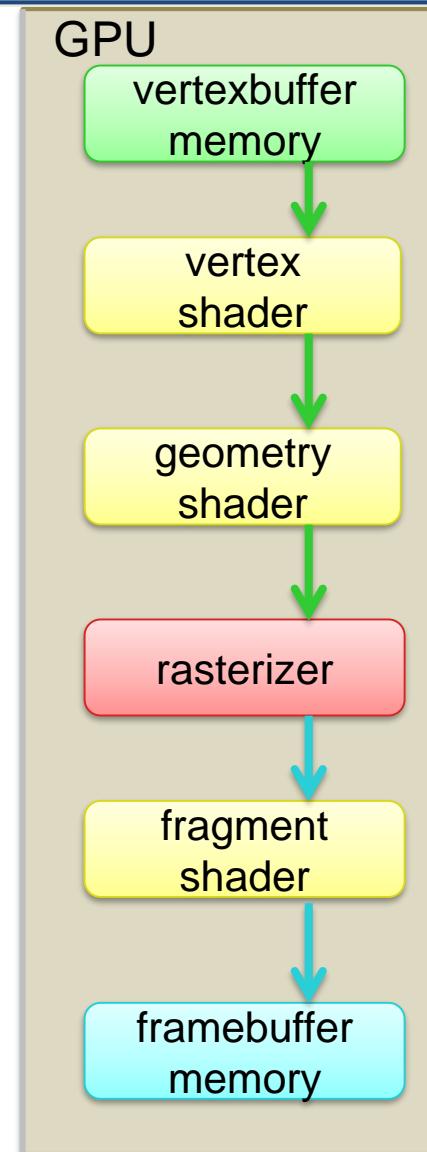
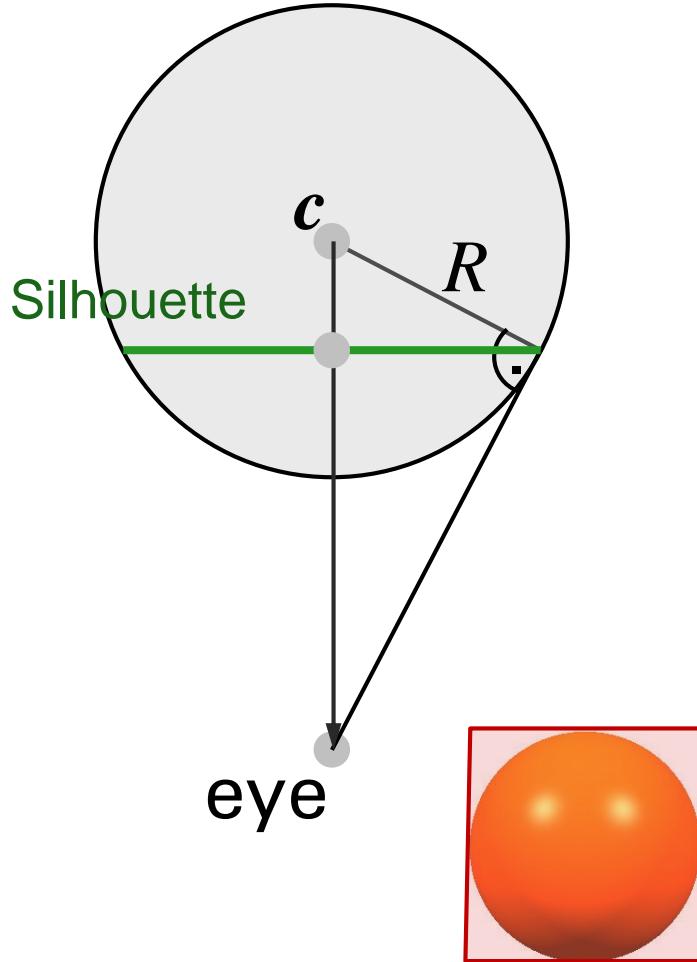


- ◆ per fragment ray intersect with particle
- ◆ compute illumination

# Graphics Pipeline



## Particle rendering Overview



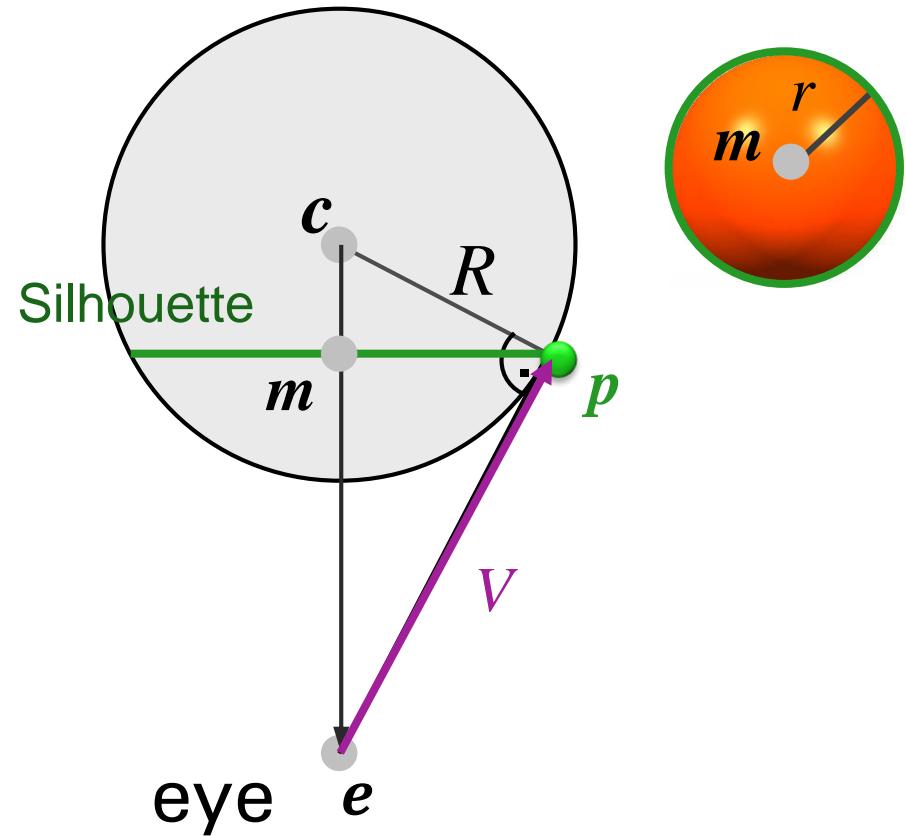


# Particle Silhouette

- condition for particle points  $\mathbf{p}$  on silhouette:

$$\mathbf{p} - \mathbf{c} \perp V$$

- silhouette points form a circle of some radius  $r < R$  around  $\mathbf{m}$

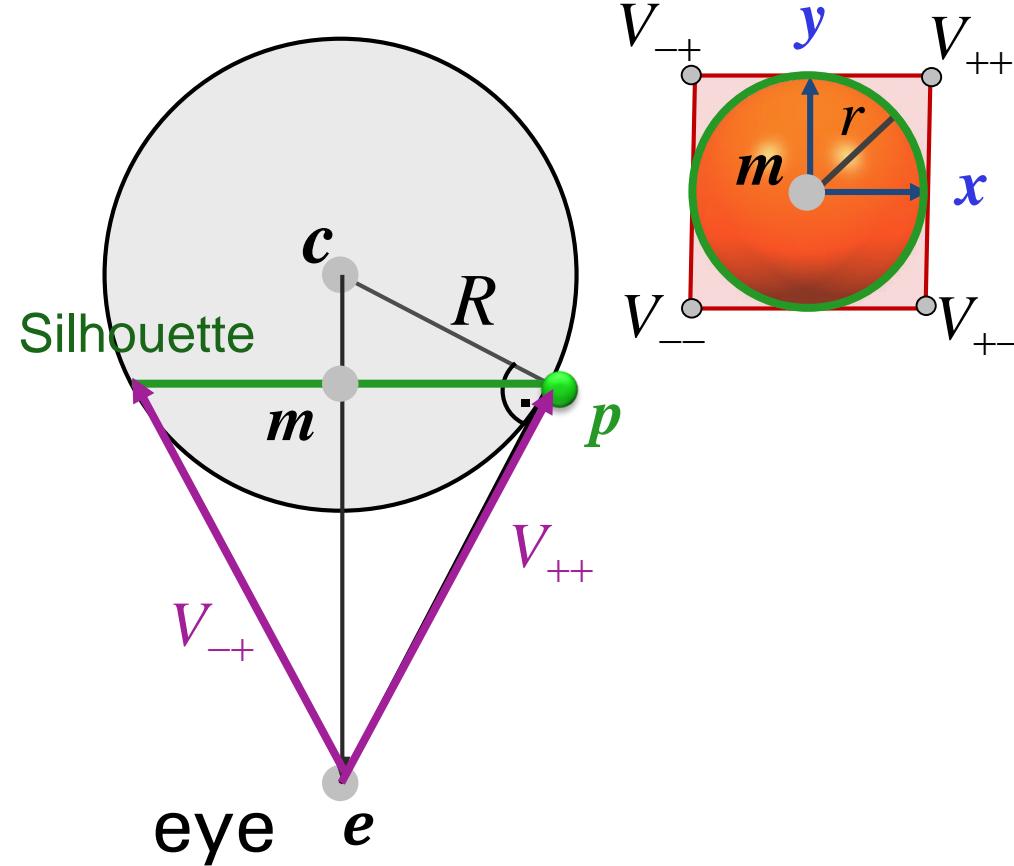


# Silhouette Quad (Geometry Shader)



- the particle can be completely covered, by covering the silhouette circle with a quad as graphics primitive
- for this  $\mathbf{m}$  and the directions  $\mathbf{x}$  and  $\mathbf{y}$  are computed
- The 4 quad corners are  $V_{\pm\pm} = \mathbf{m} \pm \mathbf{x} \pm \mathbf{y}$
- $\mathbf{x}$  and  $\mathbf{y}$  parametrize all quad location  $\mathbf{q}$ :

$$\mathbf{q} \in [-1, 1]^2$$



simple test for intersection:  
 $\|\mathbf{q}\|^2 \leq 1$

# Ray Intersection (Fragment Shader)

- rasterizer interpolates  $V_{\pm\pm}$  over quads to  $V$
- equations for ray-sphere intersection:

$$\mathbf{p} = \mathbf{e} + \lambda V$$

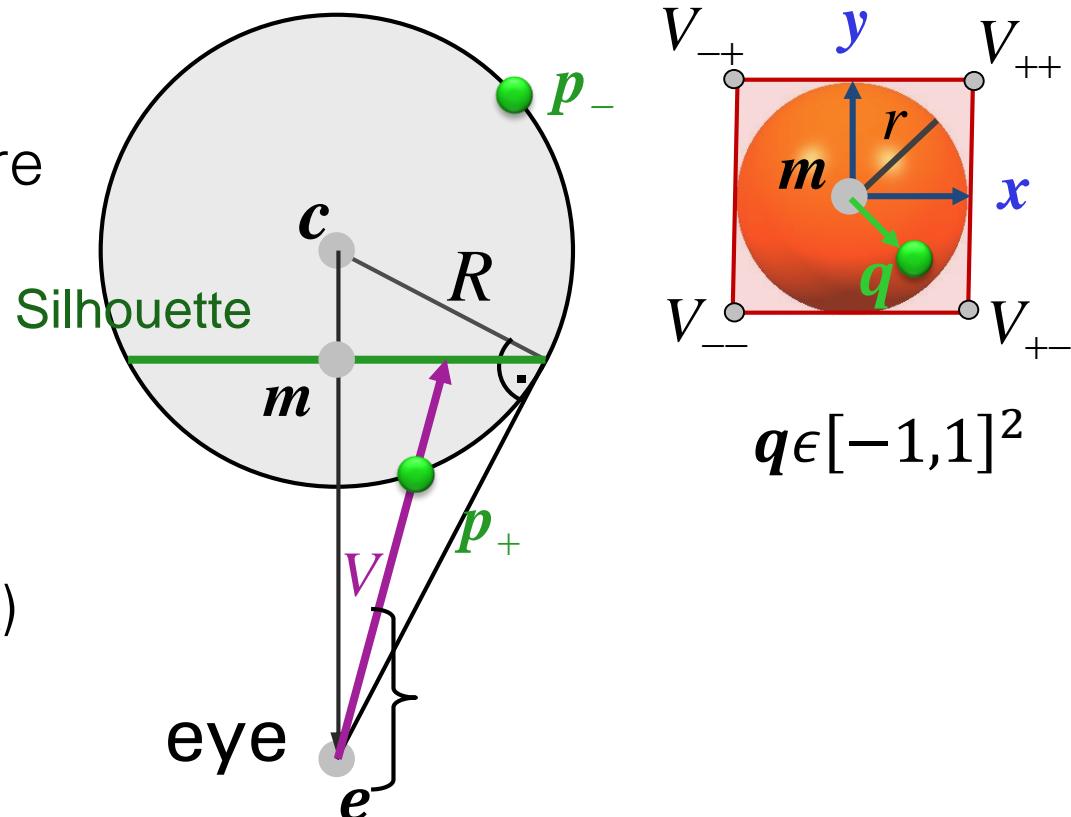
$$\|\mathbf{p} - \mathbf{c}\|^2 = R^2$$

- solution (as homework)

$$\mathbf{p}_{\pm} = \mathbf{e} + \lambda_{\pm} V$$

$$\lambda_{\pm} = \frac{1}{1 \pm \beta}$$

$$\beta = \frac{R}{\|\mathbf{e}-\mathbf{c}\|} \sqrt{1 - \|\mathbf{q}\|^2}$$



important consequence:  
 $\lambda_+$  is always first intersection

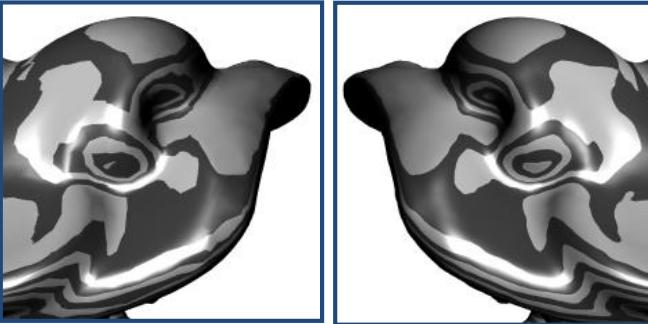


# Discussion

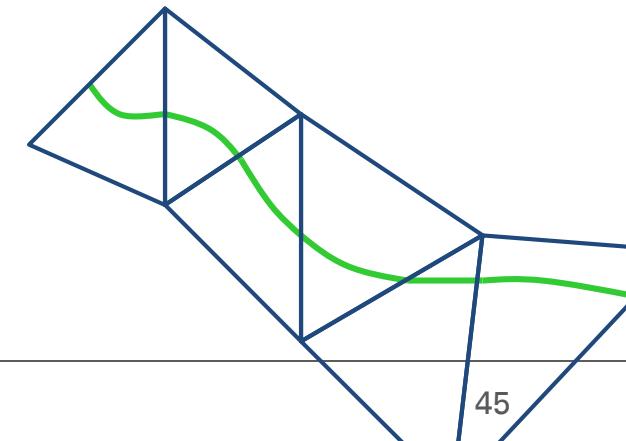
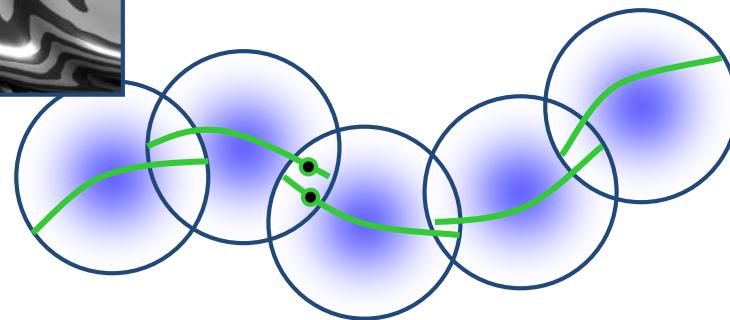
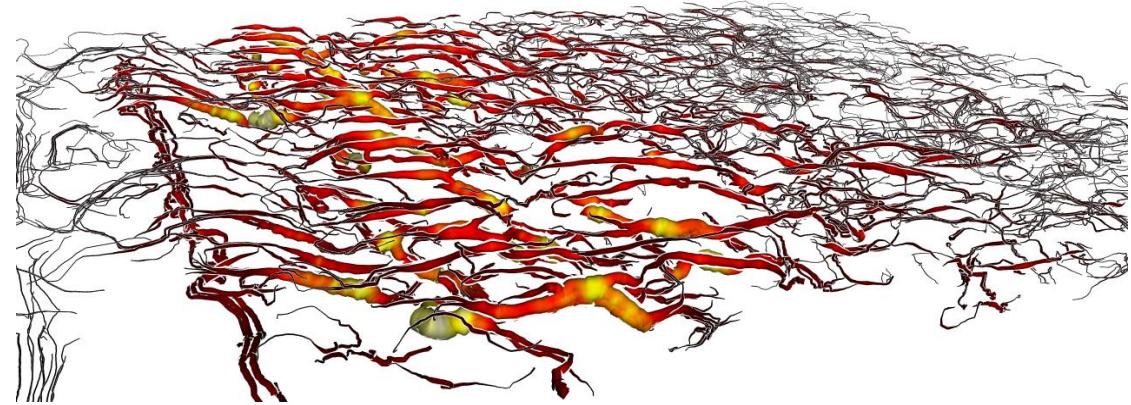
- Some more details needed to allow implementation of shaders (see CG1)
- Ellipsoids can be supported very easily by considering it as an affinely transformed sphere

## further primitives

- tubes



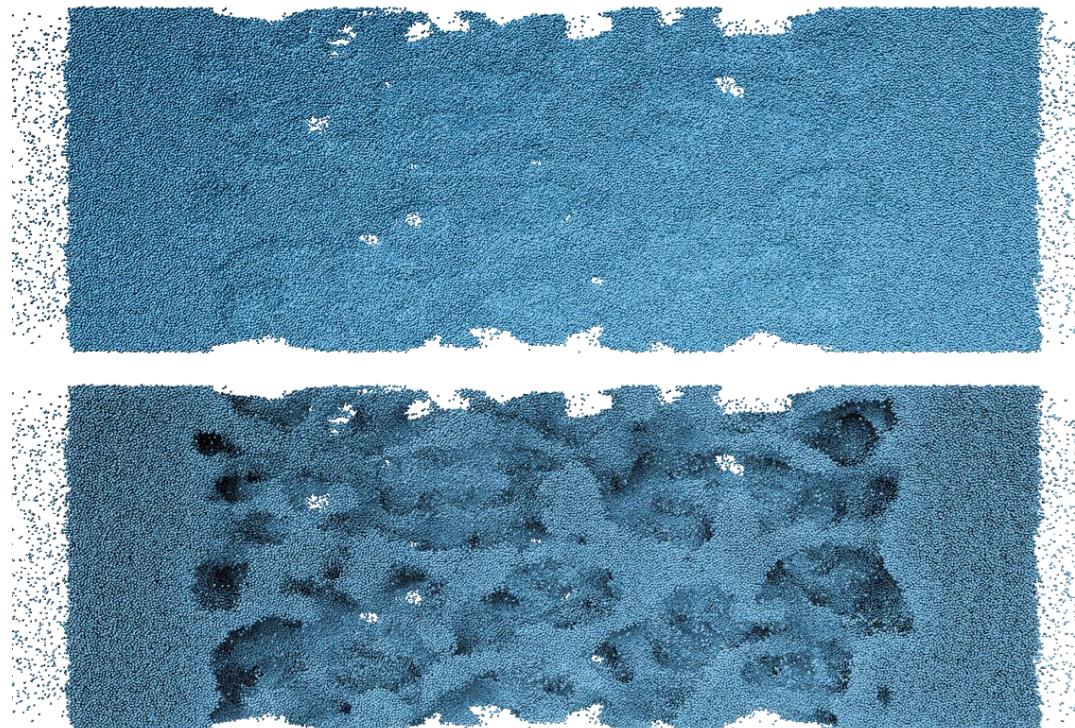
- locally smooth surfaces



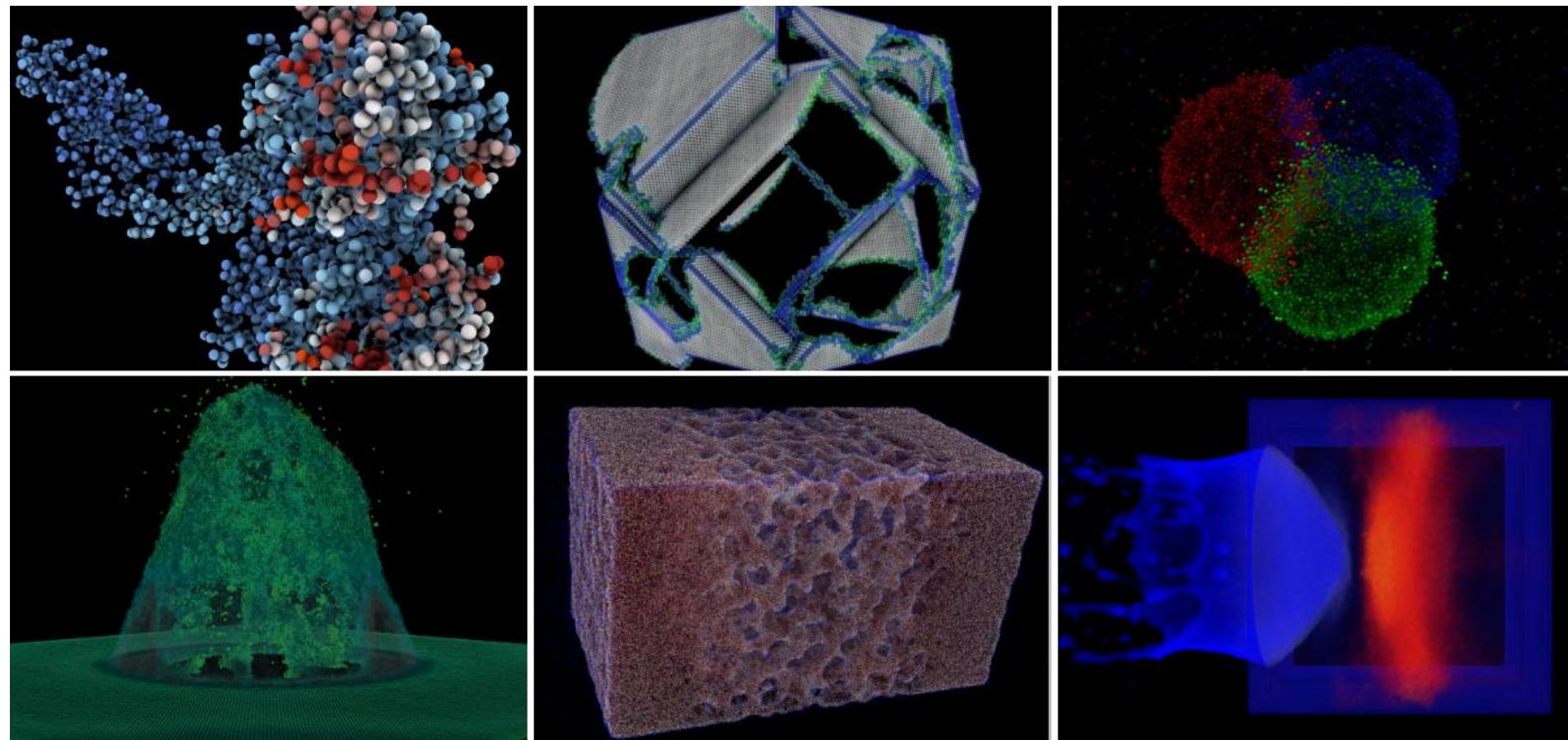


# Ambient Occlusion

- ◆ Ambient Occlusion approximates incoming light per point through the fraction of hemisphere that sees environment
- ◆ Fast Approximation by resampling particle data on volume



# Recent Research

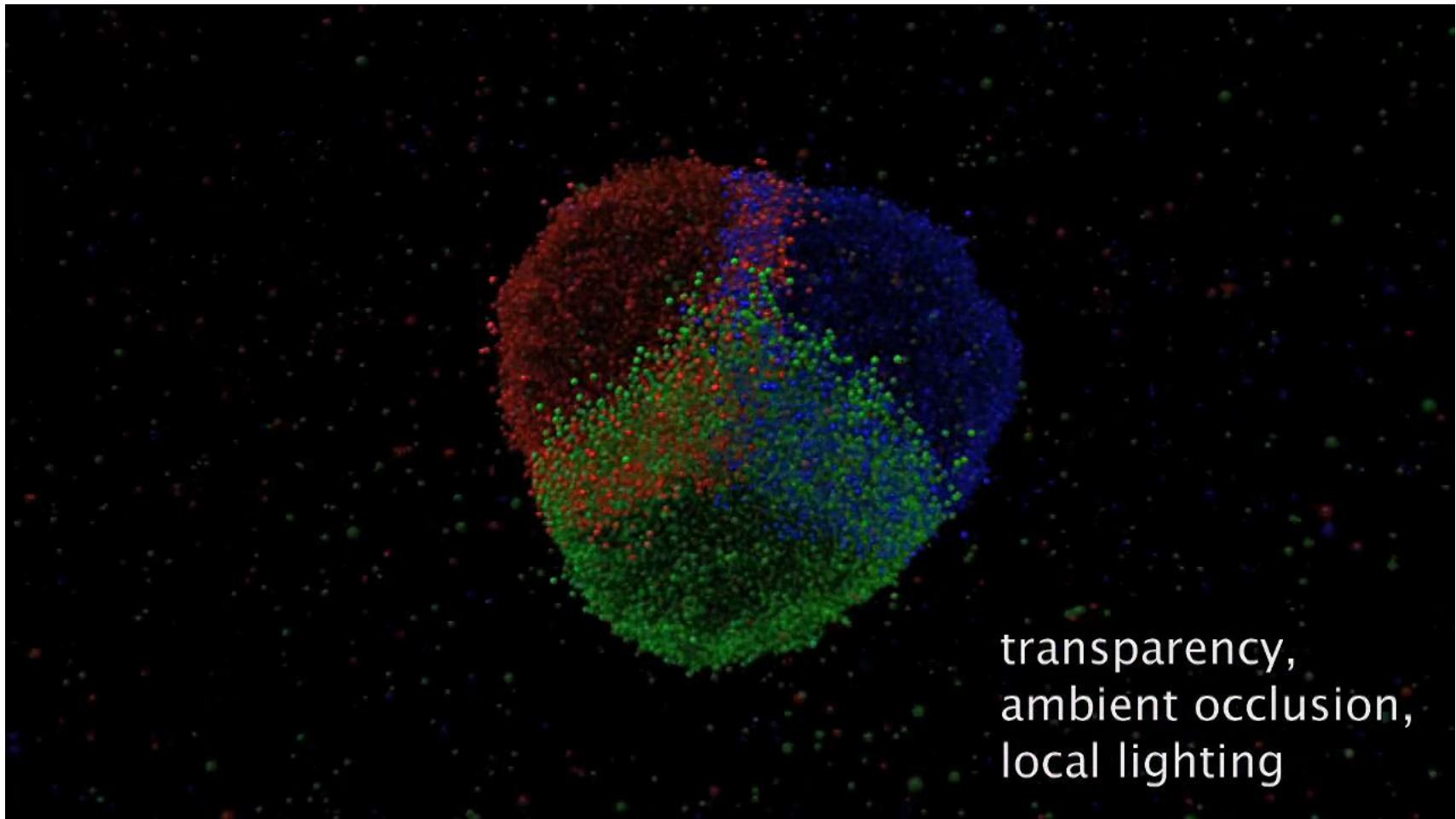


J. Staib, Sebastian Grottel, S. Gumhold, *Visualization of Particle-based Data with Transparency and Ambient Occlusion*, EuroVis 2015

# Recent Research



Computer Graphics  
and Visualization



transparency,  
ambient occlusion,  
local lighting

J. Staib, Sebastian Grottel, S. Gumhold, *Visualization of Particle-based Data with Transparency and Ambient Occlusion*, EuroVis 2015





# EXAMPLE RESEARCH QUESTIONS

Normal Estimation for 3D Scanning

Efficient Particle Rendering for Visualization

Object Pose Estimation

# Object Recognition & Pose Estimation

## Problem Statement



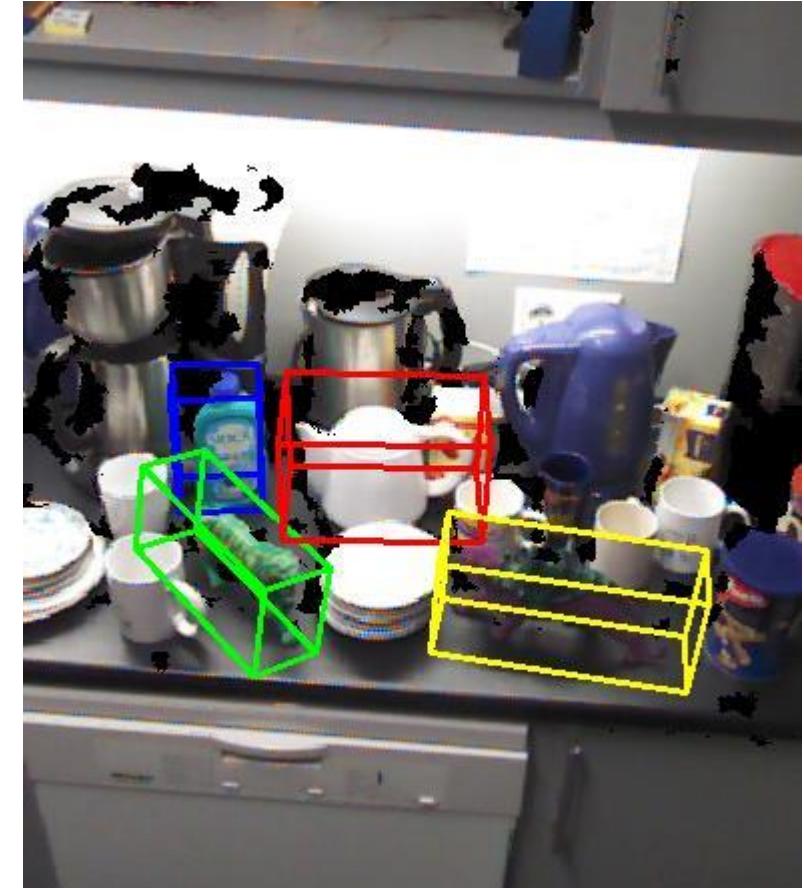
Computer Graphics  
and Visualization

### Given a single RGBD frame

- ➊ recognize objects present
- ➋ segment objects
- ➌ estimate full 6 DoF poses

### Challenges

- ➊ small objects with occlusion
- ➋ cluttered, unknown background
- ➌ changing lighting conditions



# Object Recognition & Pose Estimation

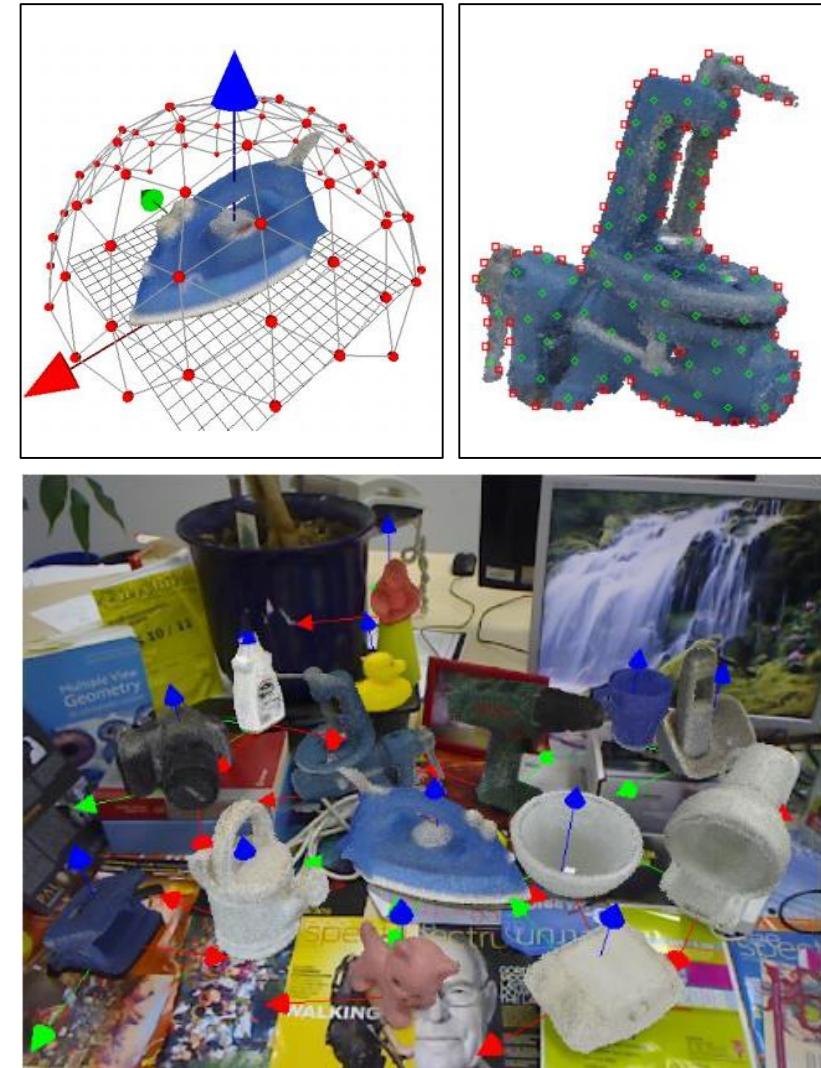
## Related Work – Template Based

### Learning

- acquire 3D-scan of to be recognized model
- synthesize RGB-D views from a dense sampling of hemisphere
- extract and store gradient & color features in templates

### Recognition

- sliding window based template matching
- ICP with 3D model



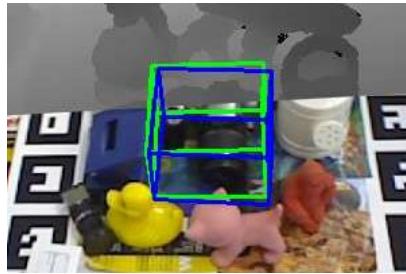
Hinterstoisser 2011, 2012; Rios-Cabrera 2013

# Object Recognition & Pose Estimation

## Overview of Our Approach



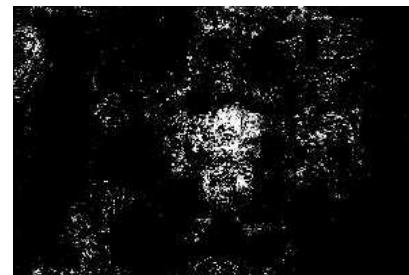
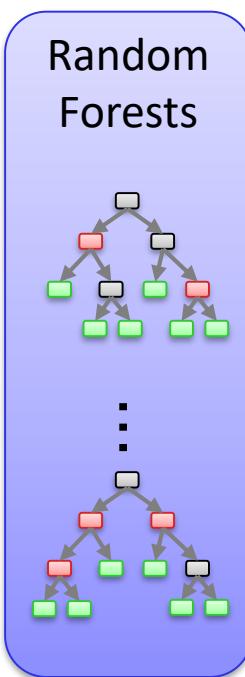
- define object coordinate system
- train random forest to predict object coordinates
- use forest output to detect objects and to initialize energy minimization



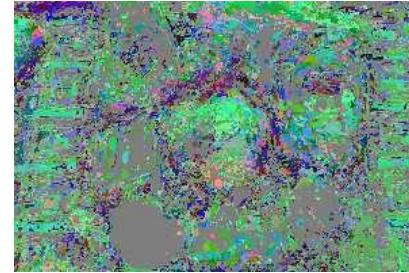
RGBD



object coordinates



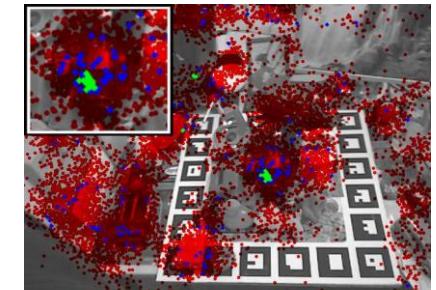
object probability



object coordinates

object  
detection

RANSAC  
based  
energy  
minimi-  
zation



RANSAC sampling



energy minimum

# Object Recognition & Pose Estimation

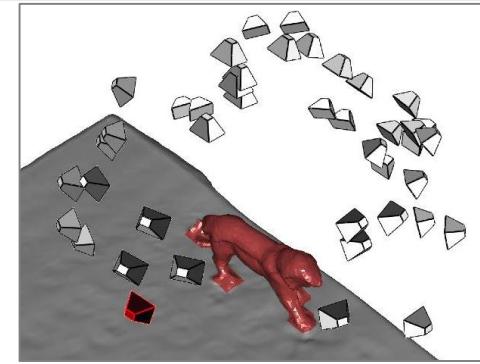
## Data Acquisition – 2 Approaches



Computer Graphics  
and Visualization

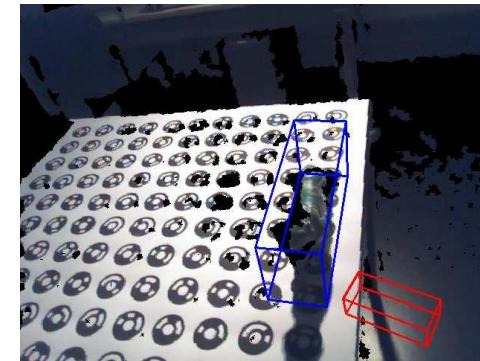
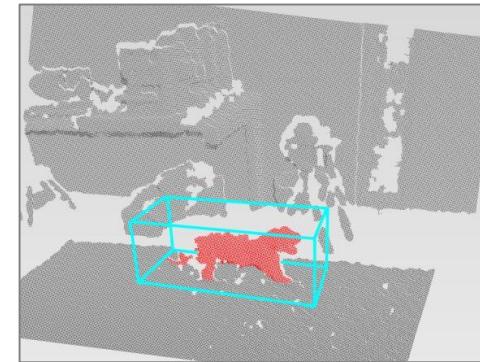
### 3D Scan Approach

- ➊ perform 3D Scan
- ➋ define object coordinates
- ➌ regularly sample views
- ➍ render ground truth with GPU



### Kinect Fusion Approach

- ➊ scan with Kinect Fusion
- ➋ ensure good sampling
- ➌ define object coordinates from fusion result
- ➍ repeat acquisition for several lighting conditions



# Object Recognition & Pose Estimation

## Dataset of 20 Models



Computer Graphics  
and Visualization

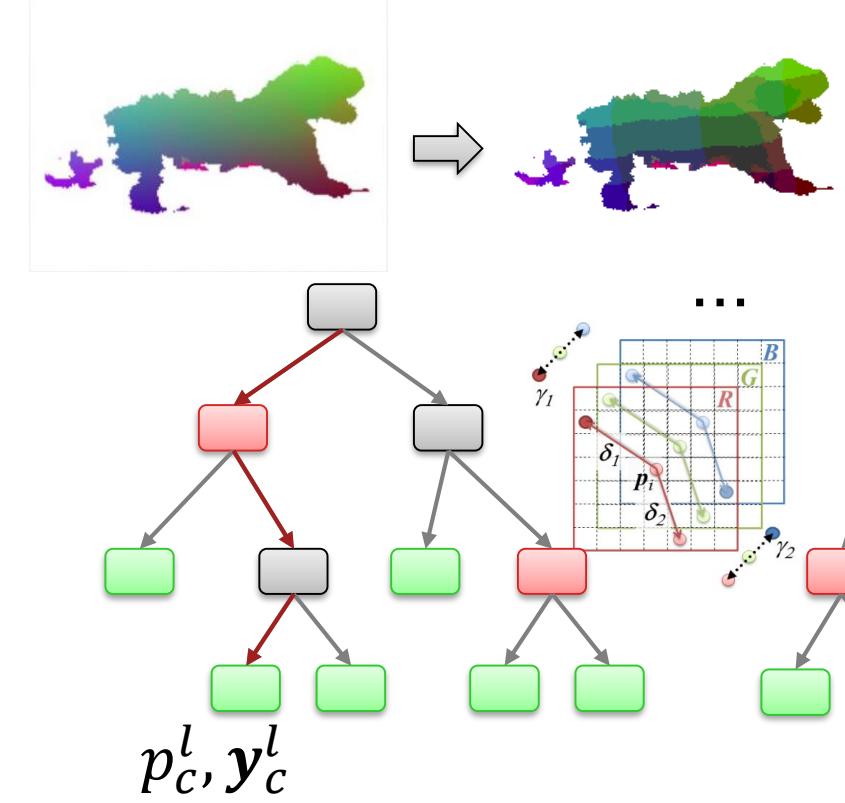


# Object Recognition & Pose Estimation

## Random Forest

### Building the Forest

- quantize object coordinates
- use simple depth and color features
- use information gain to select features
- in each leaf store probability of objects and per object one object coordinate



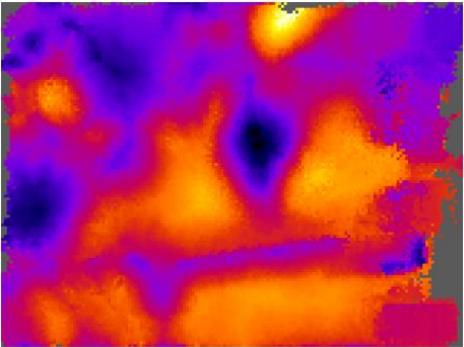
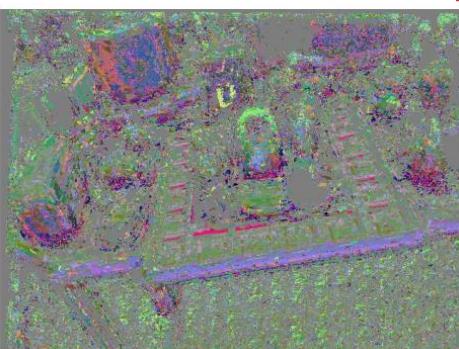
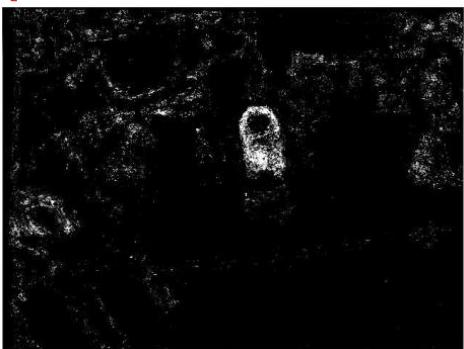
### Usage of Forest

- traverse per pixel all trees and average object probabilities and coordinates

$$p_{c,i}^l, y_{c,i}^l$$

# Object Recognition & Pose Estimation Energy Function

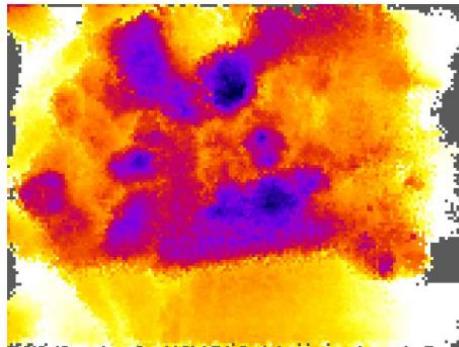
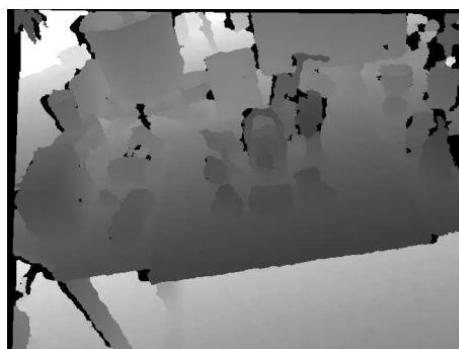
From Random Forest



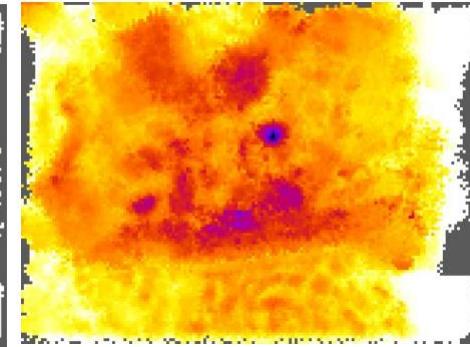
object probabilities

$$p_{c,i}^l$$

From Rendering



depth comparison



final energy

$$\mathbf{y}_{c,i}^l$$

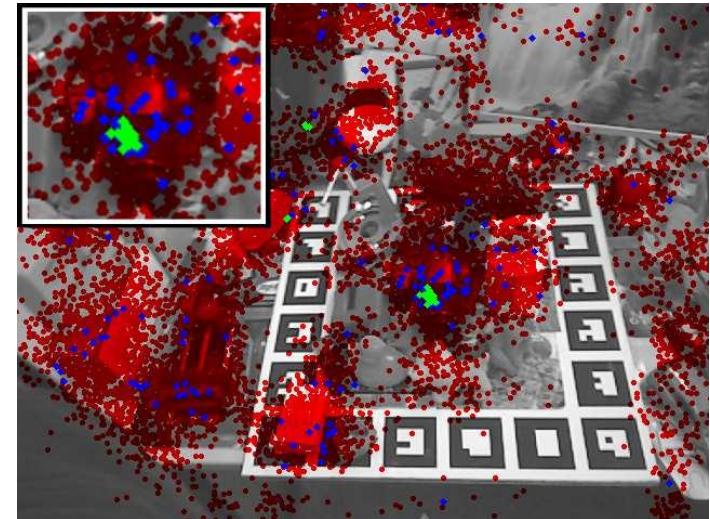
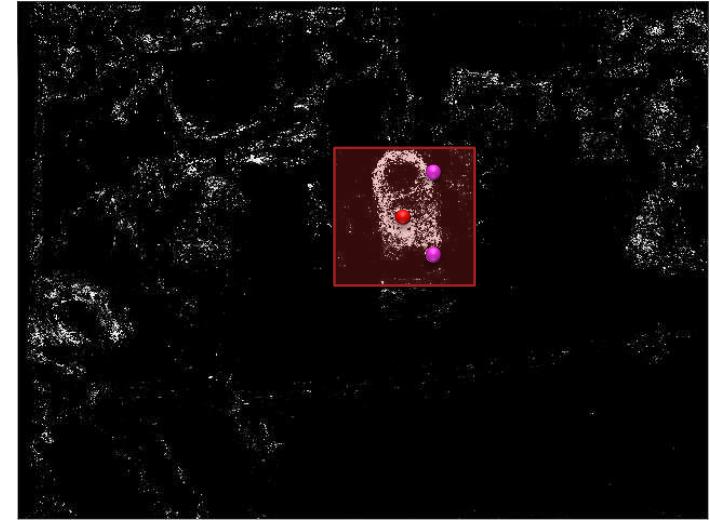
# Object Recognition & Pose Estimation

## RANSAC based optimization

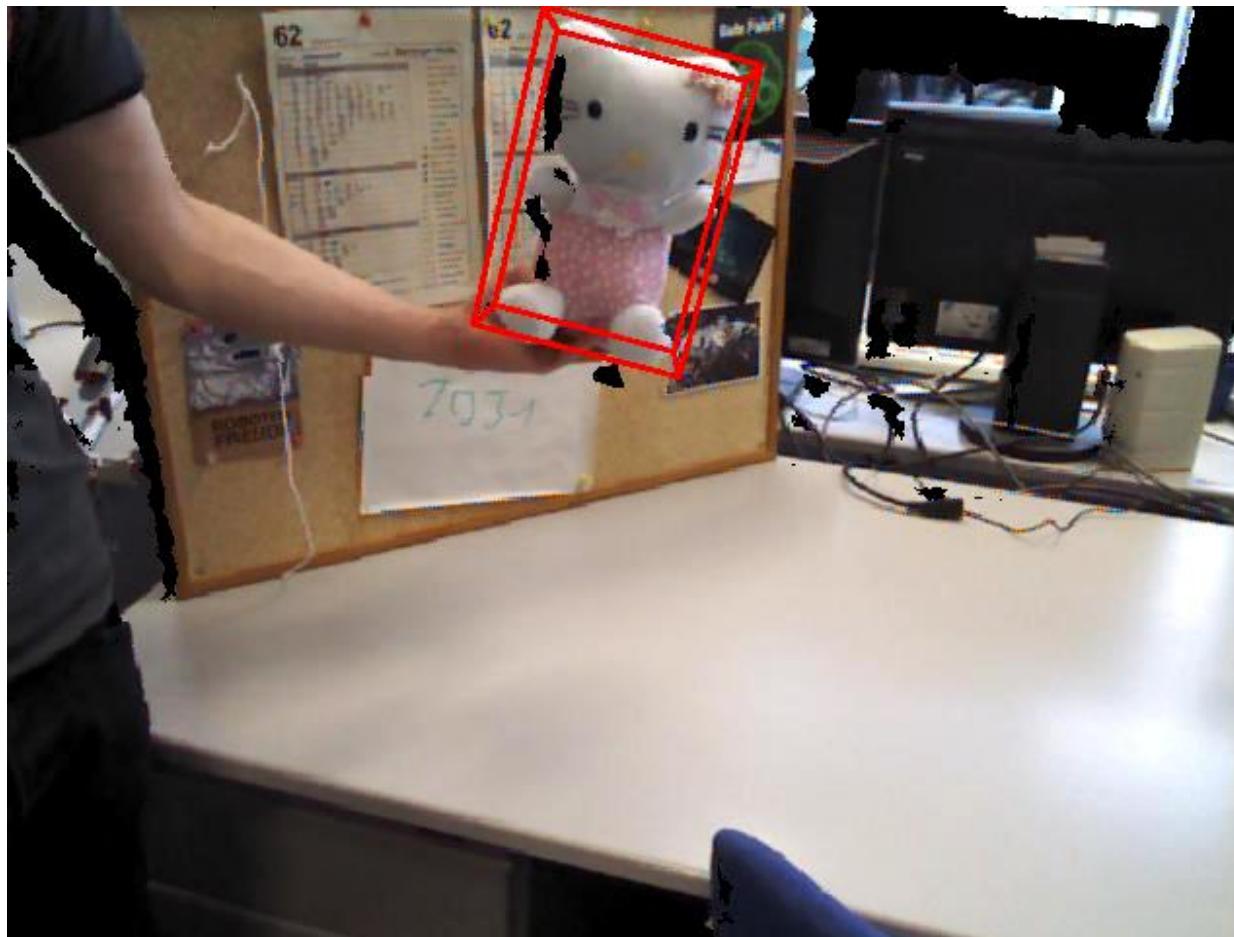


**input: per pixel object  
probability and coordinate**

- ➊ draw pixel sample proportional to object probability (red)
- ➋ determine size of object in image from pixel depth
- ➌ sample two more pixels
- ➍ construct hypothesis and discard inconsistent ones
- ➎ evaluate energy & discard all but 25 best ones
- ➏ do final refinement



# Object Recognition & Pose Estimation Results





# Oral Exam

- Based on these Slides with details only for
  - Normal Estimation for 3D Scanning
  - Efficient Particle Rendering for Visualization
- The following publications should be read
  - [Consistent Propagation of Normal Orientations in Point Clouds](#), König, S.; and Gumhold, S. In *Proceedings of Workshop on Vision, Modeling and Visualization*, pages 83–92, 2009.
  - [Robust Surface Reconstruction from Point Clouds](#), König, S.; and Gumhold, S. Technical Report TUD-FI13-03, TU Dresden, 2013.
  - [Splatting Illuminated Ellipsoids with Depth Correction](#), Gumhold, S. In *Proceedings of International Workshop on Vision, Modeling, and Visualization*, pages 245–252, November 2003.
  - [Incremental Raycasting of Piecewise Quadratic Surfaces on the GPU](#), Stoll, C.; Gumhold, S.; and Seidel, H. In *Proceedings of IEEE Symposium on Interactive Raytracing*, pages 141–150, 2006.