



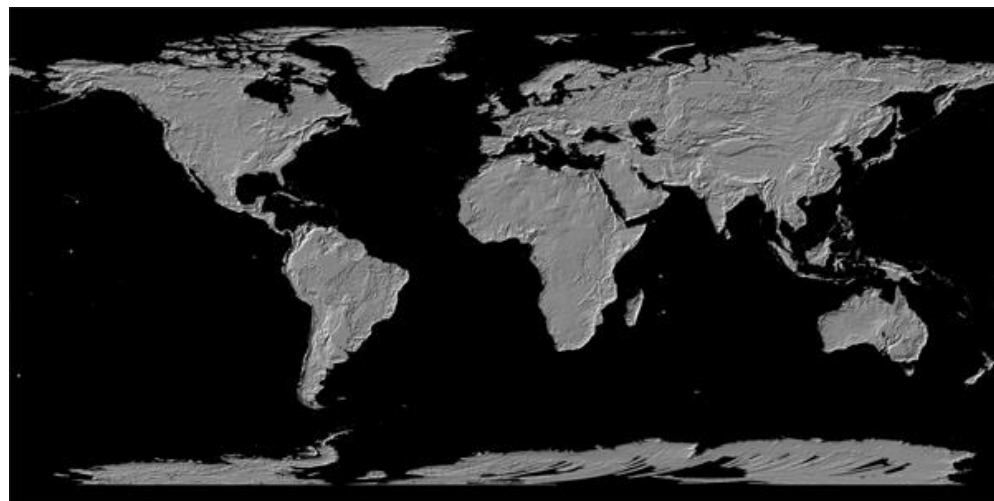
Dick, C., Krüger, J. H., & Westermann, R. (2009, March). GPU Ray-Casting for Scalable Terrain Rendering. In *Eurographics (Areas Papers)* (pp. 43-50).

- ◆ Terrain Visualization
- ◆ Data Structures
- ◆ Terrain Rendering
- ◆ Implementation
- ◆ Batched Methods
- ◆ Terrain Shading
- ◆ References

Terrain

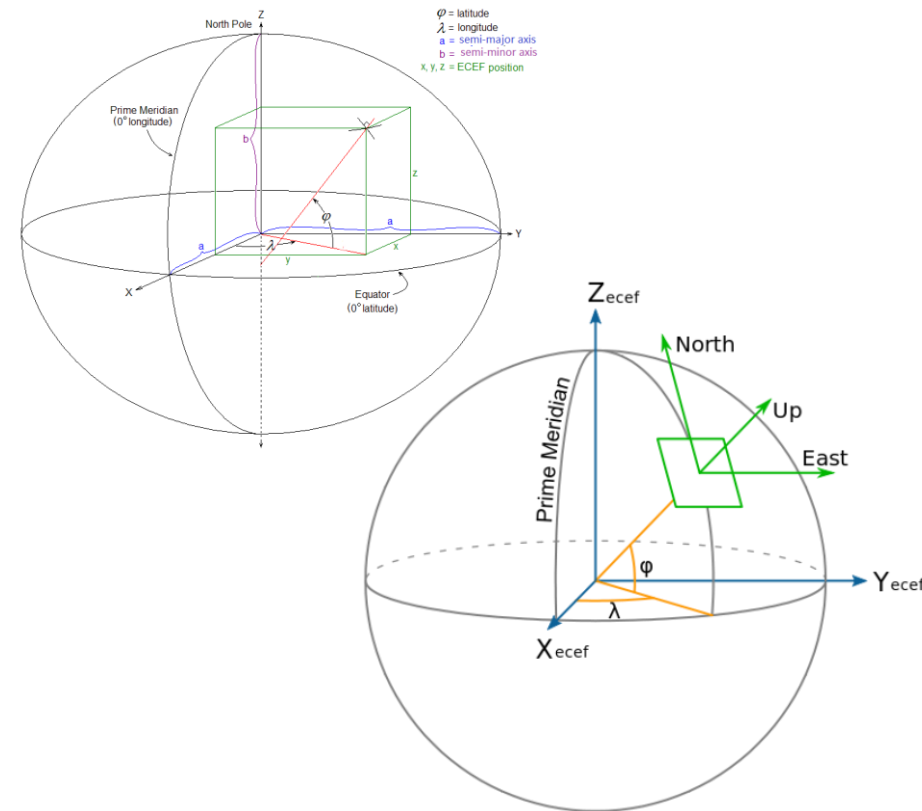
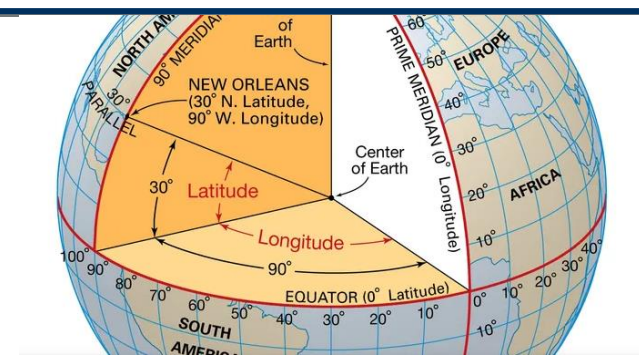
# VISUALIZATION

- ◆ **DEM** ... Digital elevation Model
  - ◆ DEM data is usually stored in **16-bit GeoTiff images** and projected with Mercator projection
  - ◆ Acquisition is done with **laser scanners** in airplanes or high-resolution **line cameras** in satellites
  - ◆ processing is costly and error-prone in each case
- ◆ Example for ASTER GDEM satellite data:
    - ◆ ASTER GDEM version 2 (G = global, i.e. the whole earth surface is covered)
    - ◆ Resolution: hor/ver 15/20m
    - ◆ 22600 tiles of  $4000^2$  pixels each (approx. 750 GB)



# Geographic Coordinate Systems

- **geodetic datum / system** is composed of **horizontal datum** (typically Latitude & Longitude) and a **vertical datum** (typically Height in meters above mean sea level – MSL)
- **World Geodetic System (WGS)** in current Version **84** is used in for satellite based navigation. It uses reference ellipsoid
- **Earth-centered, Earth-fixed coordinate system (ECEF)** is 3D coordinate system with earth center in origin
- **East-North coordinates** are 3D coordinates relative to geo location aligned with geographic directions



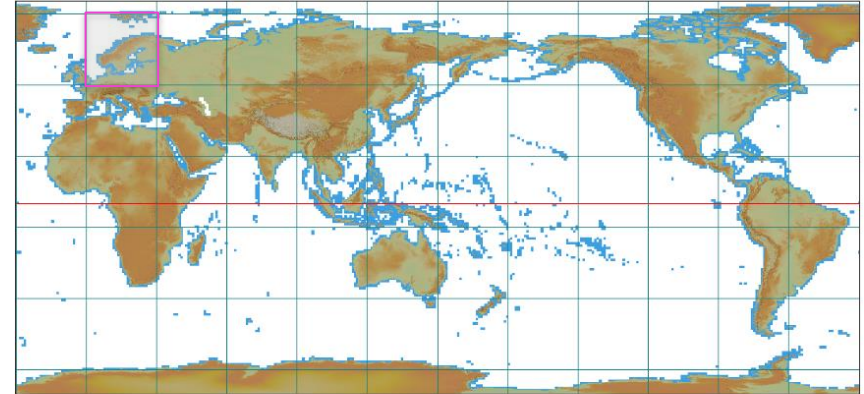
# JAXA's global ALOS-2 3D World

- ◆ Global dem is provided in tiles that correspond to geodetic datum ranges
- ◆ Preview in the JAXA browser is based on a color palette and non uniform image extends with respect to pixels



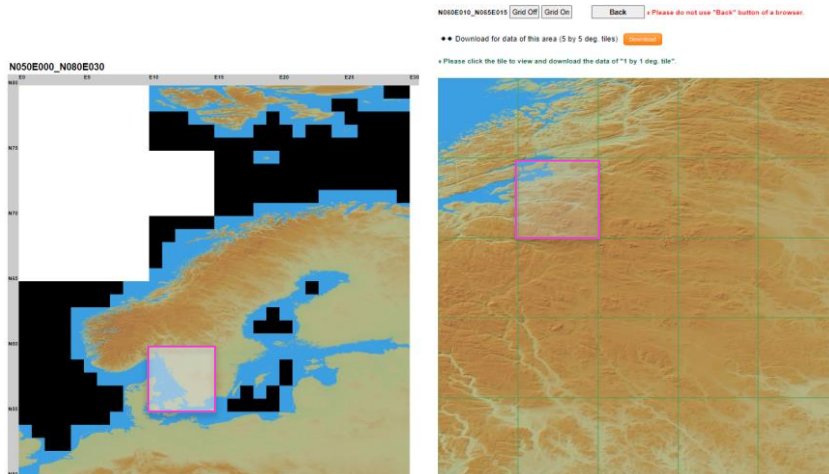
AW3D30 DSM data map

Please click the area which downloads a mosaic.



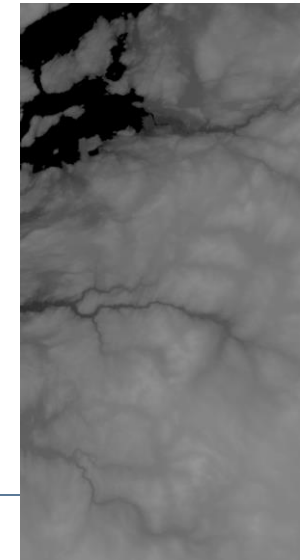
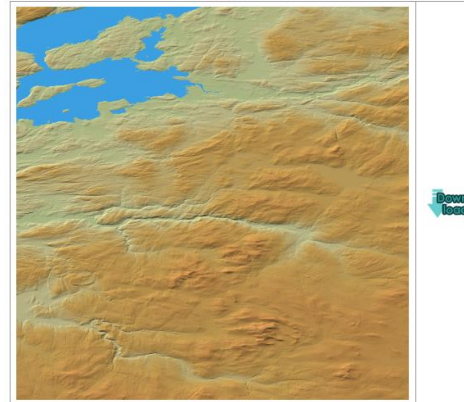
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Japan Aerospace Exploration Agency Earth Observation Research Center  
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Thumbnail

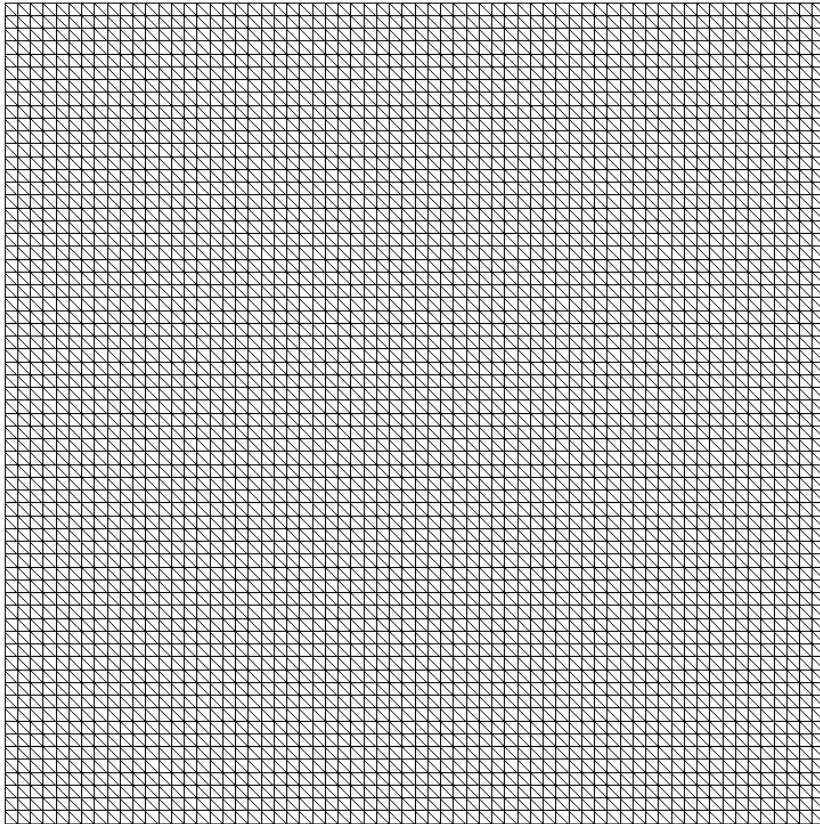


Free data sources ([gisgeography.com/free-global-dem-data-sources](https://gisgeography.com/free-global-dem-data-sources))

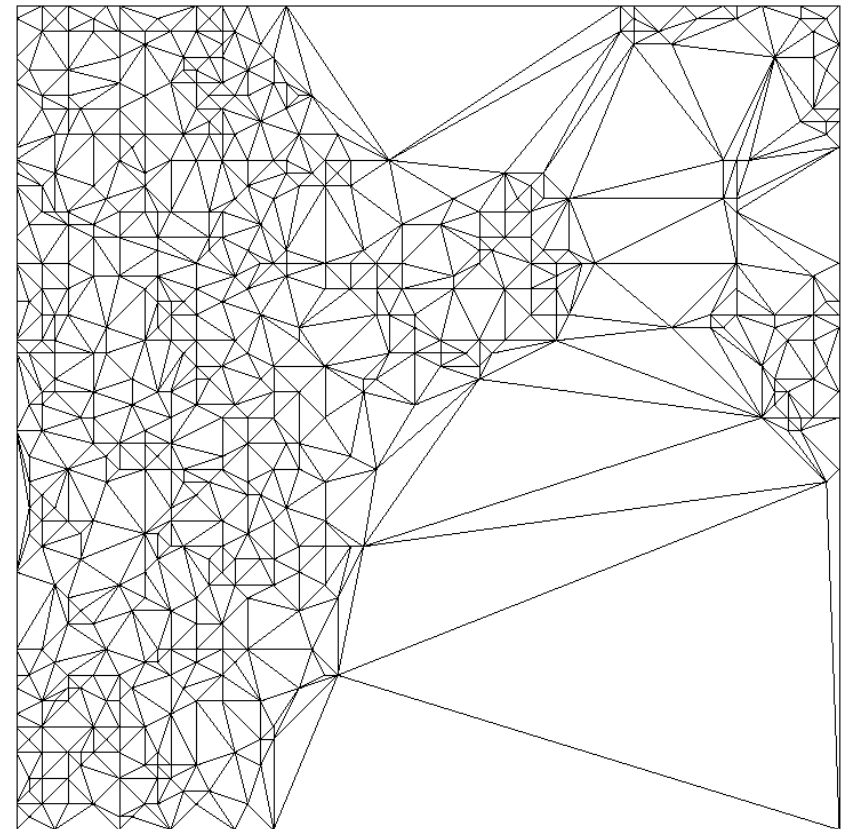
- ◆ **Space Shuttle Radar Topography Mission (SRTM)**  
(30m spacing, 16m height accuracy)
- ◆ **ASTER global Digital elevation Model**  
(90m spacing, 30m height accuracy)
- ◆ **JAXA's global ALOS-2 3D World**  
(30m spacing, 5-17m height accuracy)
- ◆ **Mars Orbiter Laser altimeter (MOLA)**  
(100m spacing, 3m height accuracy)

## Commercial Sources

- ◆ **WorldDEM™**  
(12m spacing, 2m accuracy)



- render as 2.5D height map
- tessellation of regular grid yields far too many triangles

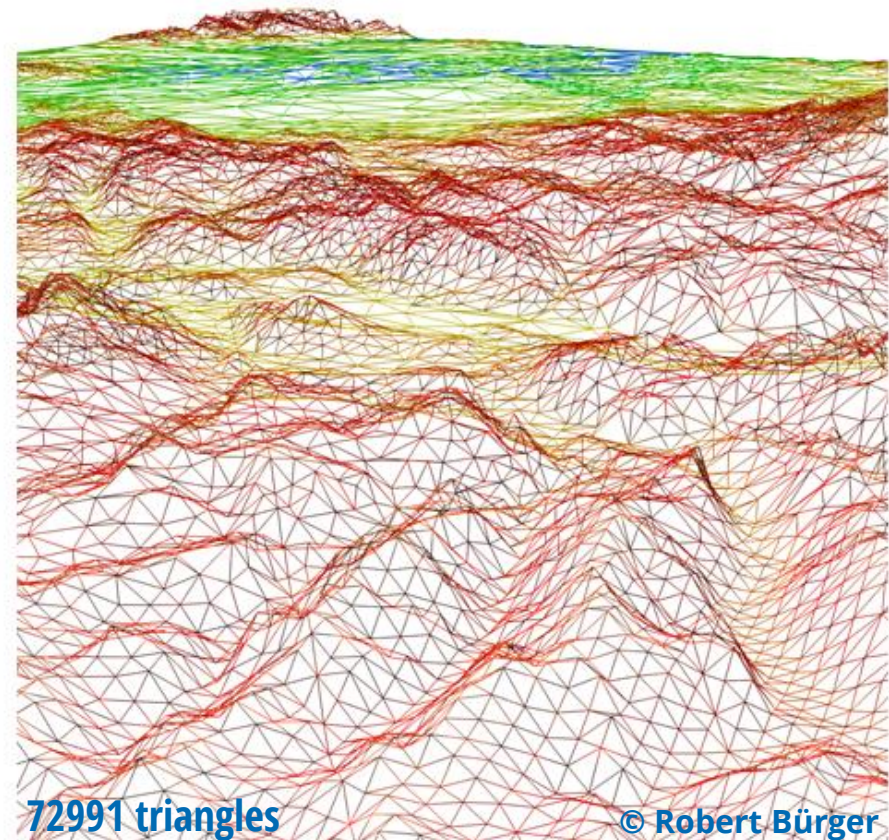
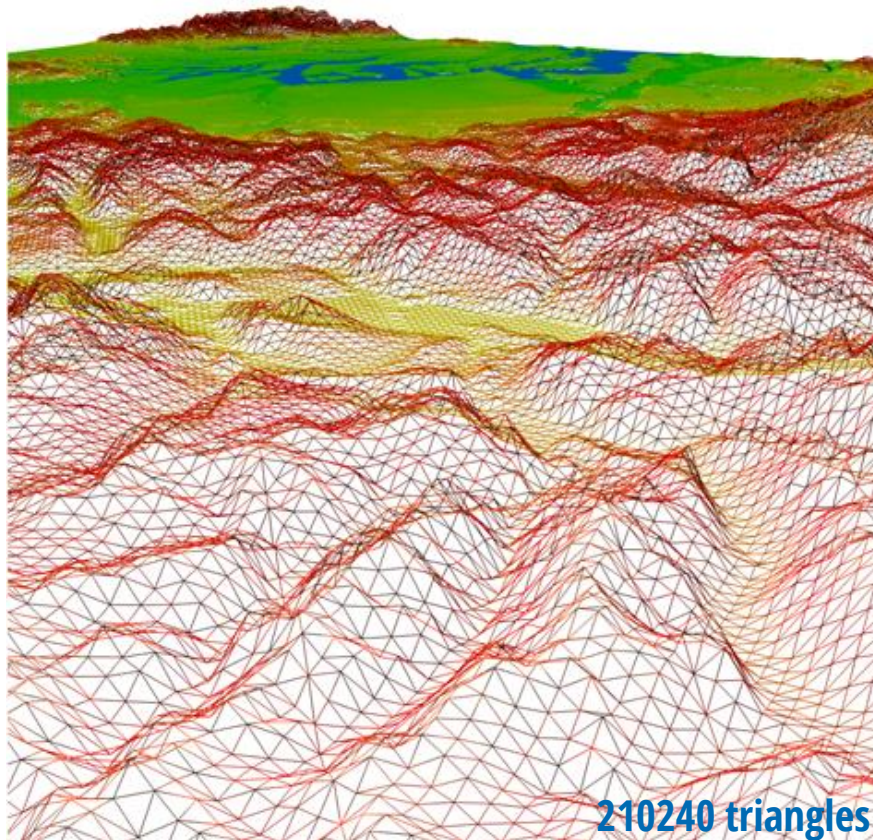


- TIN ... triangular irregular networks adapt to data
- but demand for additional storage of x and y coordinates



# View-Dependent Tessellation

- ◆ Significant performance boost by adaptive refinement with respect to view-dependent approximation error



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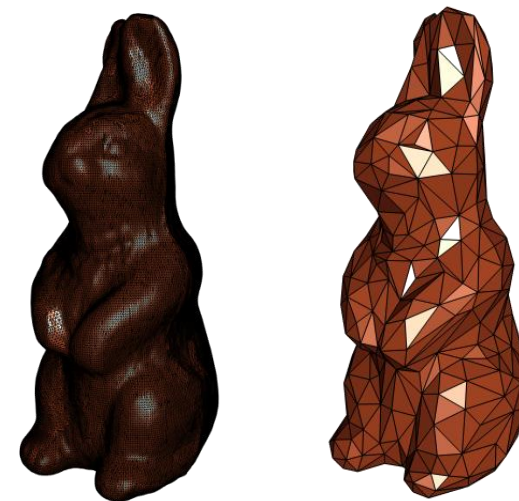
left: no adaptation, right: adaptation with respect to 1-pixel threshold of projected error

## terrain data is simple

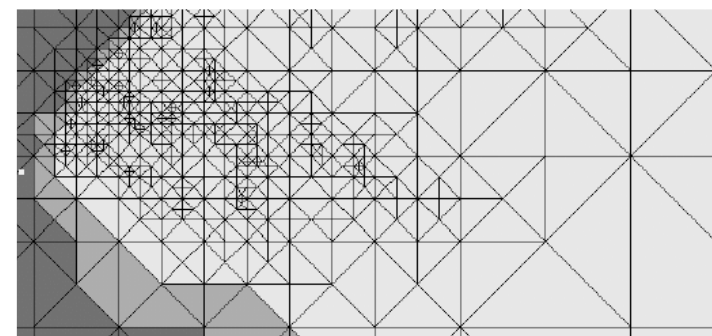
- ◆ 2 ½ Data
- ◆ more specialized simple algorithms

## terrain rendering is difficult

- ◆ very large contiguous models
- ◆ very close and very far away at the same time
- ◆ LOD dependent on point of view necessary
- ◆ out-of-core



for compact 3D models one simply selects to be rendered mesh from collection of approximations with decreasing complexity



in terrain rendering resolution needs to be varied over model in a view-dependent manner

S. Hahmann and D. Burghardt. How much information is geospatially referenced? networks and cognition. *International Journal of Geographical Information Science*, 27(6):1171–1189, 2013.

- ◆ Study on how often geospatial references are used
- ◆ Examined wikipedia articles and asked people
- ◆ They found that 57% of the information in German wikipedia articles are geospatially referenced

# 2D or 3D Viewing?

- Tory, M., Kirkpatrick, A. E., Atkins, M. S., & Moller, T. (2006). Visualization task performance with 2D, 3D, and combination displays. *IEEE transactions on visualization and computer graphics*, 12 (1), 2-13.
- Tory et al. conducted experiments of 2D, 3D, and combined visualizations for estimation of relative positioning and orientation as well as region selection.
- their results show that 3D can be effective for approximate navigation and relative positioning, but 2D is more suitable for precise measurement
- 3D views should support
  - lighting cues like shadows
  - dedicated measurement tools
  - view point optimization
- Combined 2D/3D views are suggested

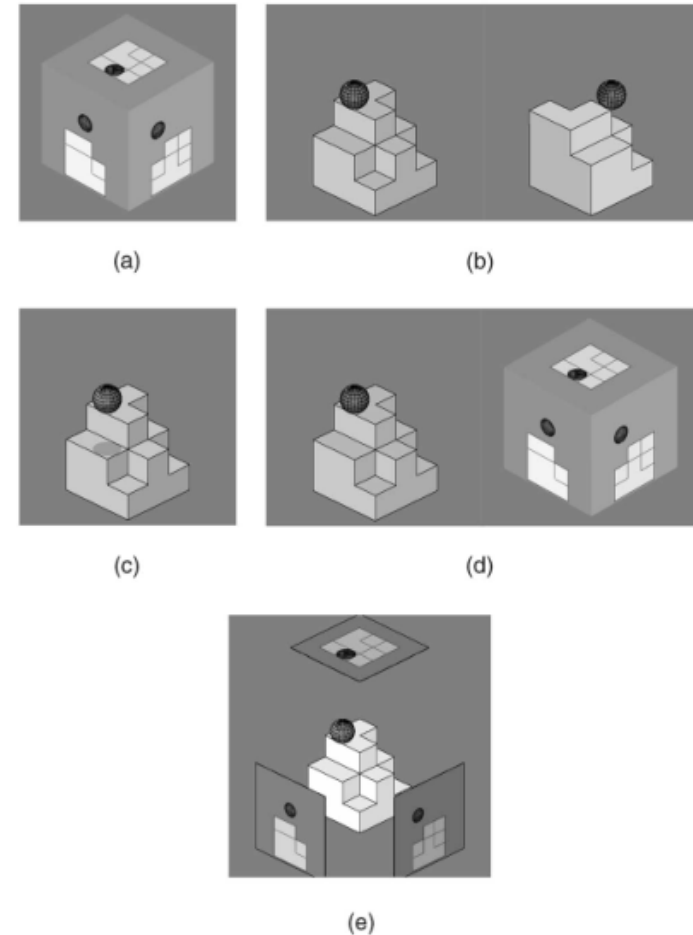
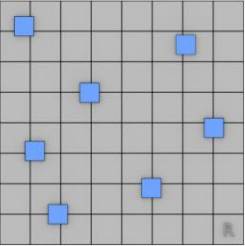
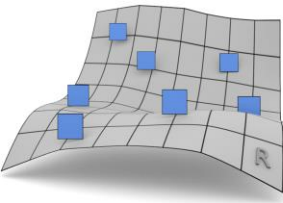
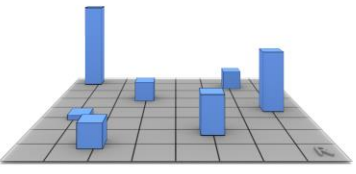
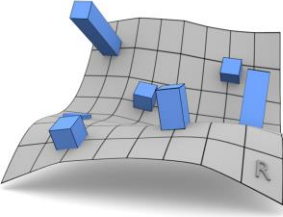
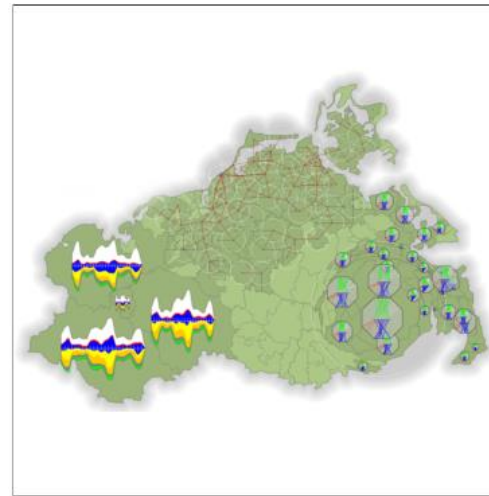


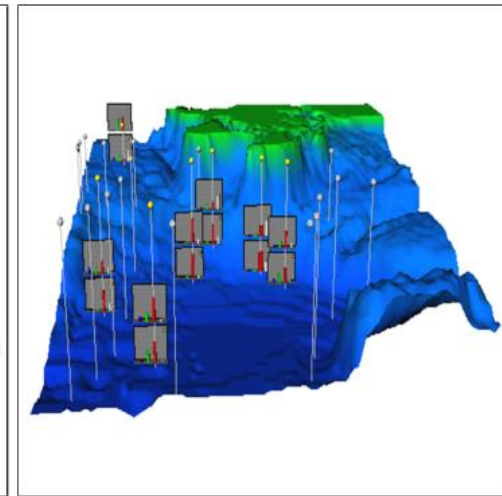
Fig. 1. (a) 2D, (b) 3D rotated, (c) 3D shadow, (d) orientation icon, and (e) ExoVis displays used in Experiment 1 (position estimation). Participants estimated the height of the ball relative to the block shape. In this example, the ball is at height 1.5 diameters above the block shape.

# classification of GeoVis Techniques

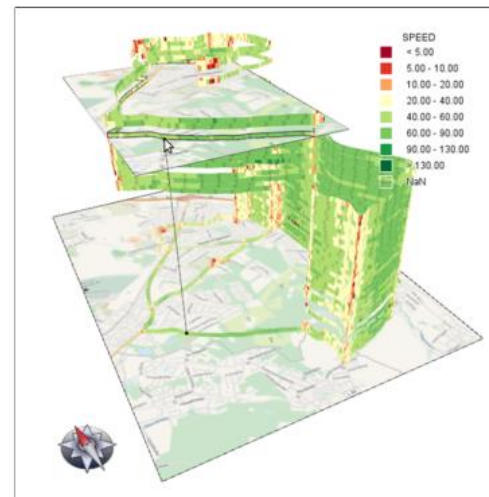
Space		Dimensionality of Reference Space	
		2D	3D
Dimensionality of Attribute Space	2D	 $(A^2 \oplus R^2)$	 $(A^2 \oplus R^3)$
	3D	 $(A^3 \oplus R^2)$	 $(A^3 \oplus R^3)$



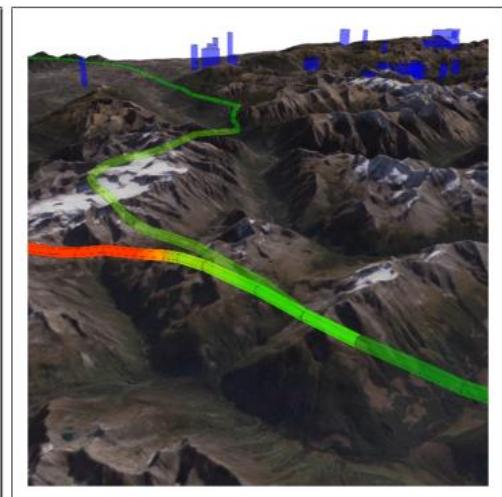
(a)  $(A^2 \oplus R^2)$



(b)  $(A^2 \oplus R^3)$



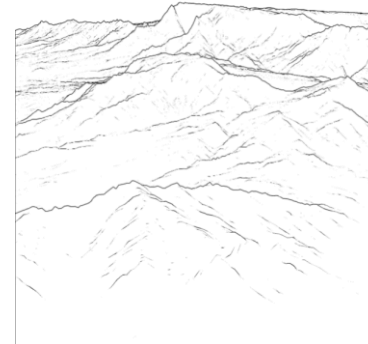
(c)  $(A^3 \oplus R^2)$



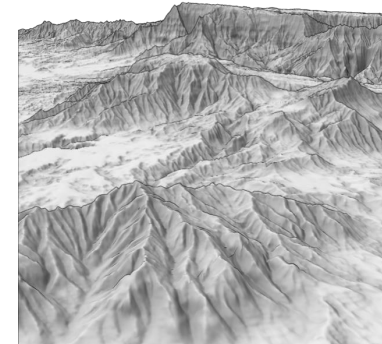
(d)  $(A^3 \oplus R^3)$

- Dübel, S., Röhlig, M., Schumann, H., & Trapp, M. (2014, November). [2D and 3D presentation of spatial data: A systematic review](#). In *2014 IEEE VIS International Workshop on 3DVis (3DVis)* (pp. 11-18). IEEE.
- $A^i$  : selected attributes are visualized using i-dimensional graphical elements,
- $R^j$  : the reference space is visualized using j-dimensional graphical elements.

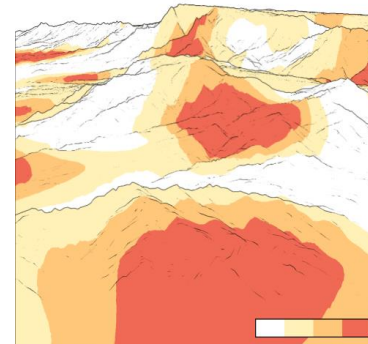
- ◆ Dübel, S., Röhlig, M., Tominski, C., & Schumann, H. (2017, March). Visualizing 3D terrain, geo-spatial data, and uncertainty. In *Informatics* (Vol. 4, No. 1, p. 6).
- ◆ Joint visualization of *T*errain, *D*ata, and *U*ncertainty
- ◆ Propose different viewing scenarios like take-off or overflight
- ◆ Refined focus & context techniques by prioritizing (+) or deemphasizing (-) terrain ( $T^-$  vs  $T^+$ ), data ( $D^-$  vs  $D^+$ ) and uncertainty ( $U^-$  vs  $U^+$ ) individually



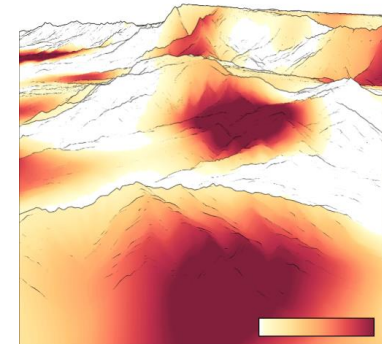
$T^-$ : line drawing



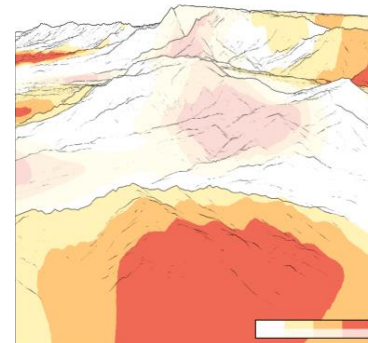
$T^+$ : shading



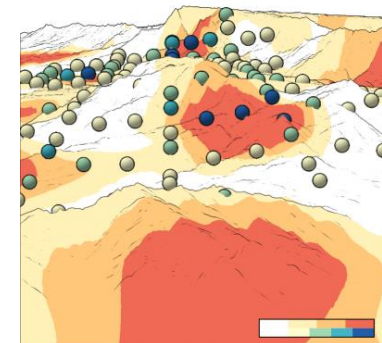
$D^-$ : aggregation



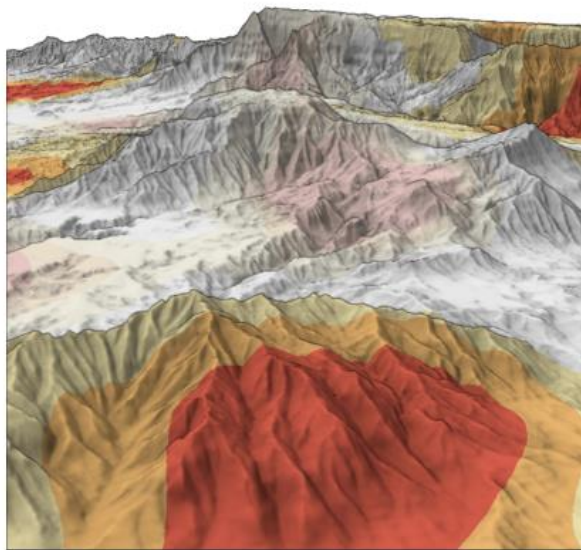
$D^+$ : individual values



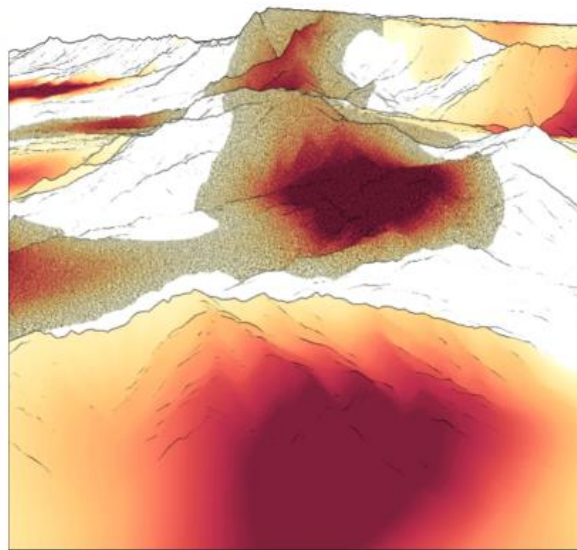
$U^-$ : intrinsic encoding



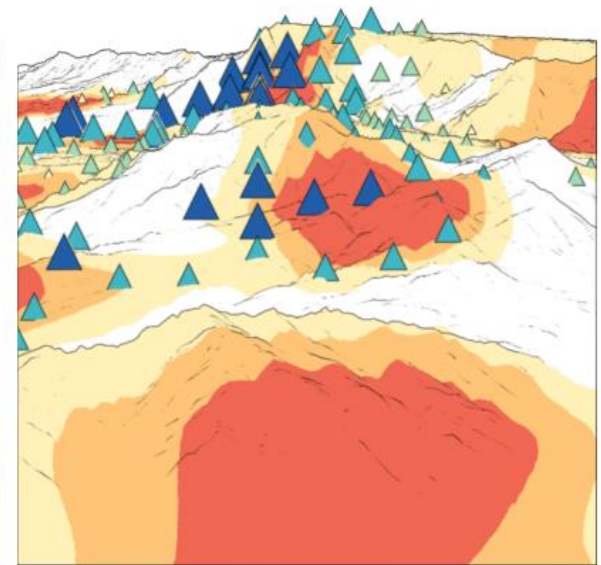
$U^+$ : extrinsic encoding



(a) Prioritizing terrain



(b) Prioritizing data



(c) Prioritizing uncertainty

**Figure 5.** Differently prioritized visual representations of the same terrain, data, and uncertainty. (a) The terrain is prioritized and rendered as a detailed shaded surface. The data are visualized in an aggregated fashion by a segmented color scale, and uncertainty is depicted intrinsically by using transparency; (b) The data are prioritized and individual values are visualized with a continuous color scale. The terrain is rendered at less detail by drawing only lines, and uncertainty is depicted intrinsically by using noise; (c) Uncertainty is prioritized and depicted extrinsically via colored triangular glyphs. The terrain is rendered at less detail as in (b), and the data are visualized in an aggregated fashion as in (a).

Terrain Rendering

# DATA STRUCTURES



- ◆ DEM ... digital elevation models (images stored in tiles)
- ◆ TIN ... **T**riangular **I**rregular **N**etworks (typically stored as triangle mesh in indexed format)
- ◆ bintree ... used to store hierarchy over triangles
- ◆ quadtree ... used to store color texture information
- ◆ HRT ... **H**ierarchy of **R**ight[-angled] **T**riangles

# Implicit Binary and Quad Trees

- Complete binary trees can be stored implicitly by indexing the nodes in a breadth first traversal

- access to parent:

$$p(c) = c/2$$

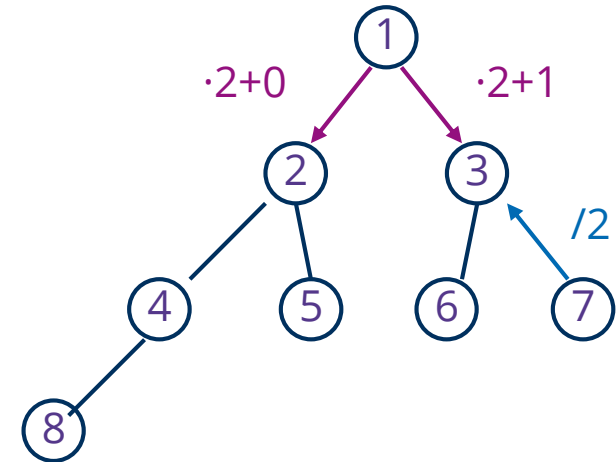
- access to children:

$$c(p, k) = 2p + k$$

- one can **interleave** two binary trees with indices 2 and 3 as roots, by adding tree specific offset  $m$ :

$$p(c) = (c - m)/2$$

$$c(p, k) = 2p + k + m$$



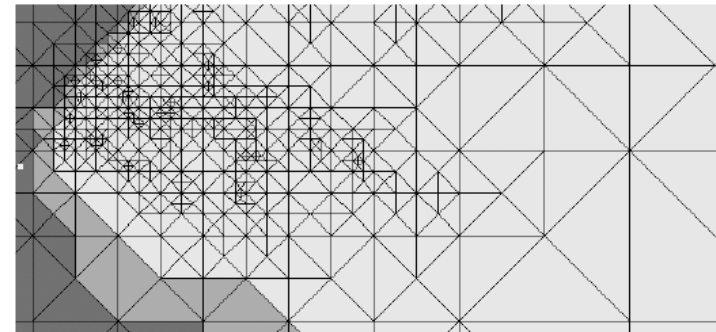
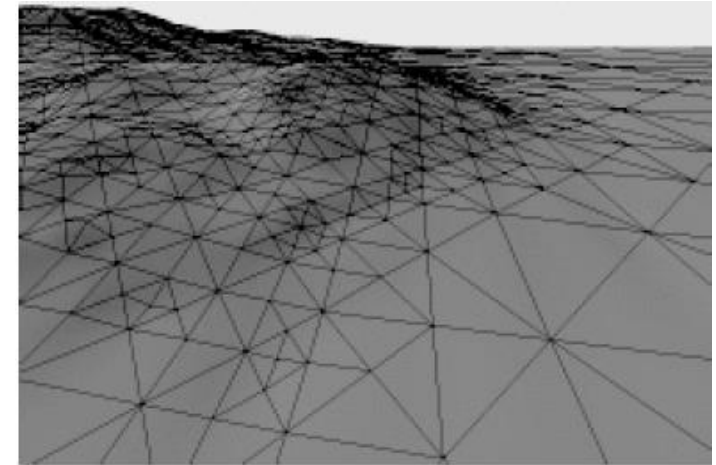
- This also works for quad trees:

$$p(c) = (c - m)/4$$

$$c(p, k) = 4p + k + m$$

# Hierarchy of Right Triangles

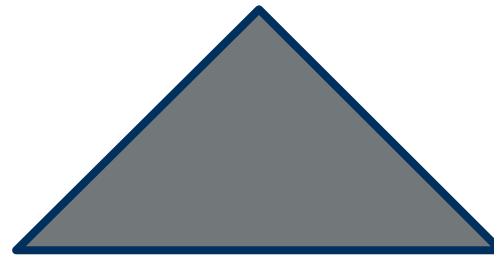
- ◆ ROAM is one of the first and most important publications in terrain rendering
- ◆ defines adaptive tessellation
- ◆ optimal refinement algorithm
- ◆ preprocessing independent of point of view
- ◆ view-dependent refinement criterion



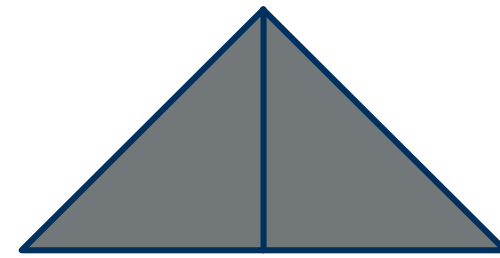
Mark Duchaineau et al., *ROAMing terrain: real-time optimally adapting meshes*,  
IEEE Visualization, pp 81-88, 1997  
[http://www.cognigraph.com/ROAM\\_homepage](http://www.cognigraph.com/ROAM_homepage)

# Hierarchy of Right Triangles

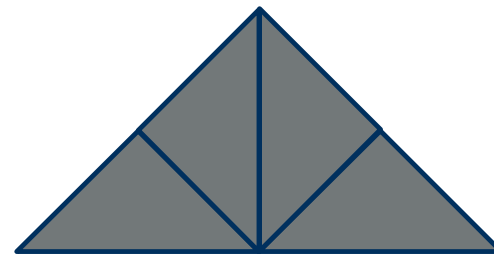
- ◆ HRT is created by iterated subdivision of a right-angled triangle along the longest side
- ◆ combination of two HRTs results in the tessellation of a square
- ◆ **Idea:** vary subdivision depth to achieve adaptive triangulation



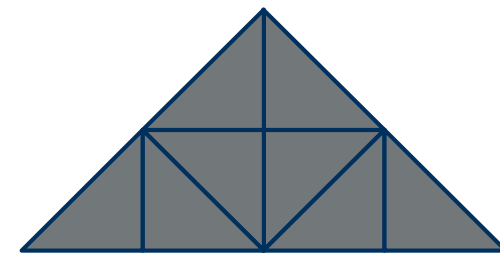
$l = 0$



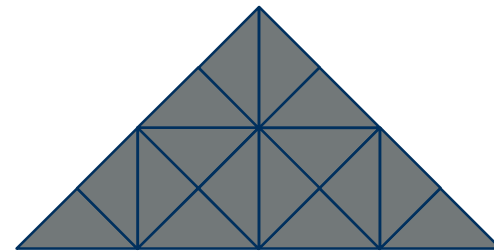
$l = 1$



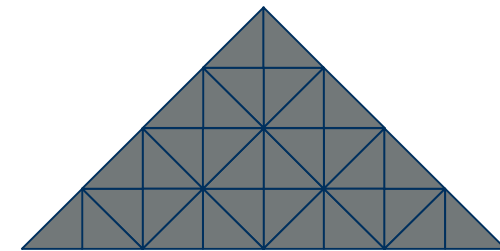
$l = 2$



$l = 3$



$l = 5$

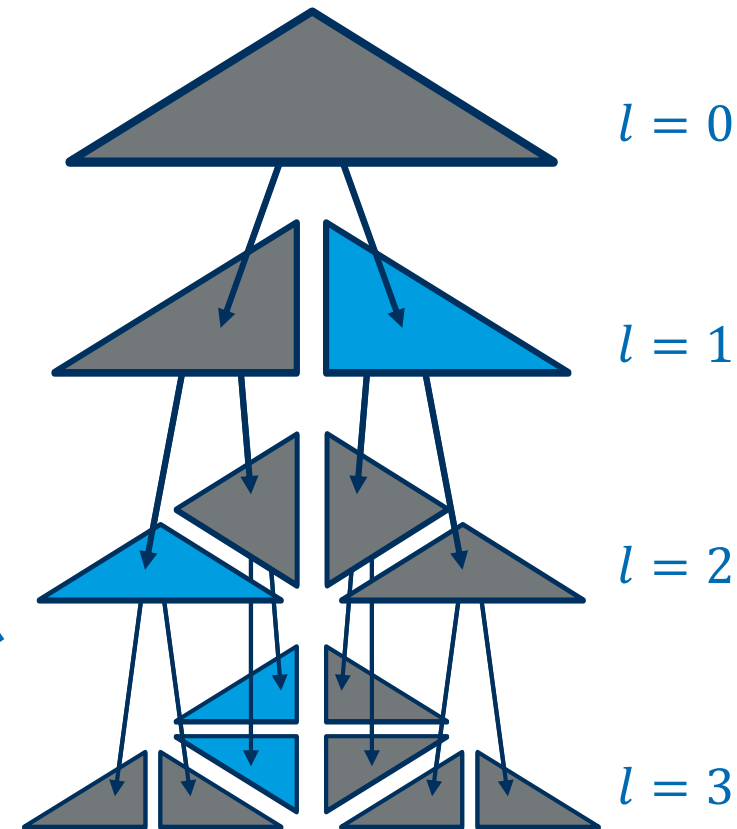
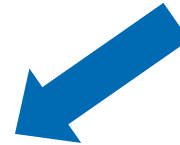
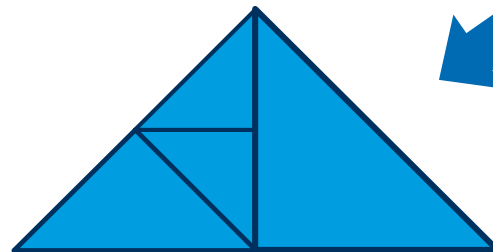


$l = 6$

# Hierarchy of Right Triangles

- ◆ A **front** is a subset of nodes, such that each path from the root to a leaf contains exactly one node.
- ◆ Each front defines a partition of the root triangle,

for example:



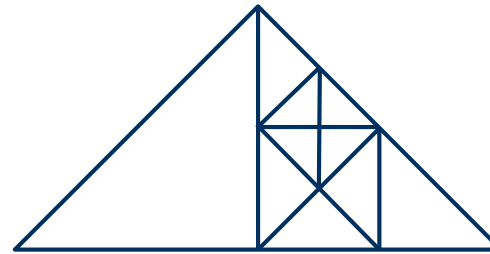
# Hierarchy of Right Triangles

◆ A triangulation is **valid** if the intersection of two triangles is

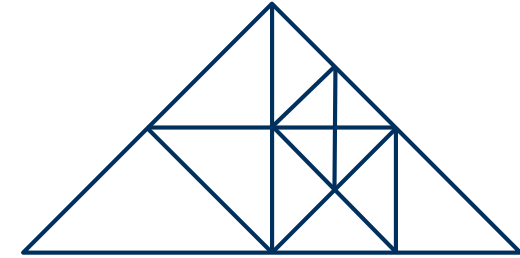
- ◆ empty,
- ◆ common corner or
- ◆ common edge

➔ no cracks

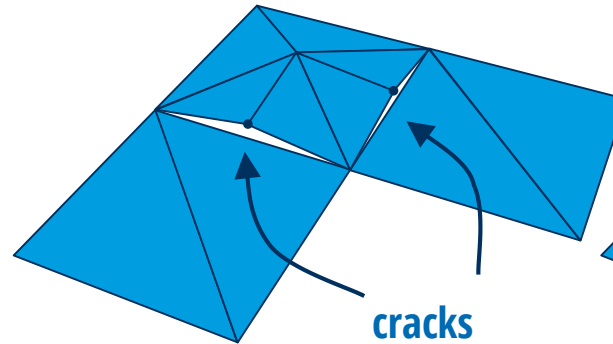
➔ no T-junctions



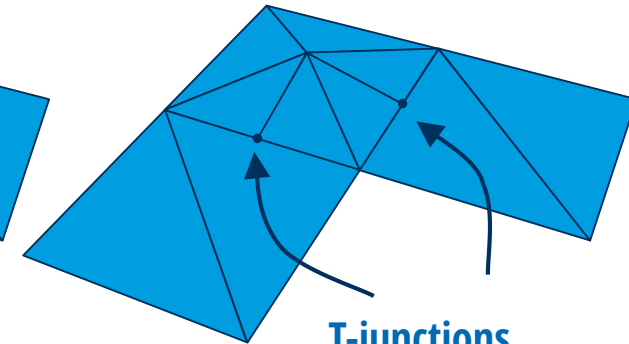
invalid



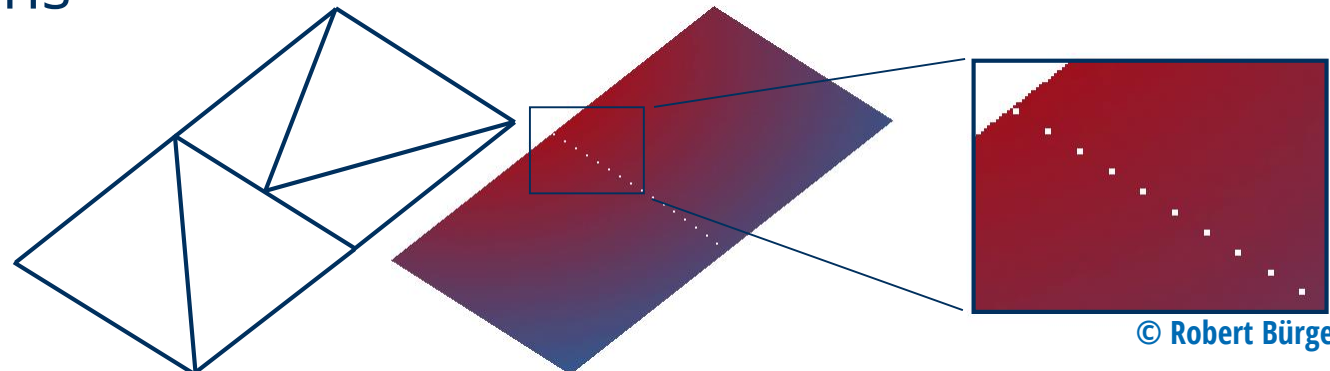
valid



cracks



T-junctions

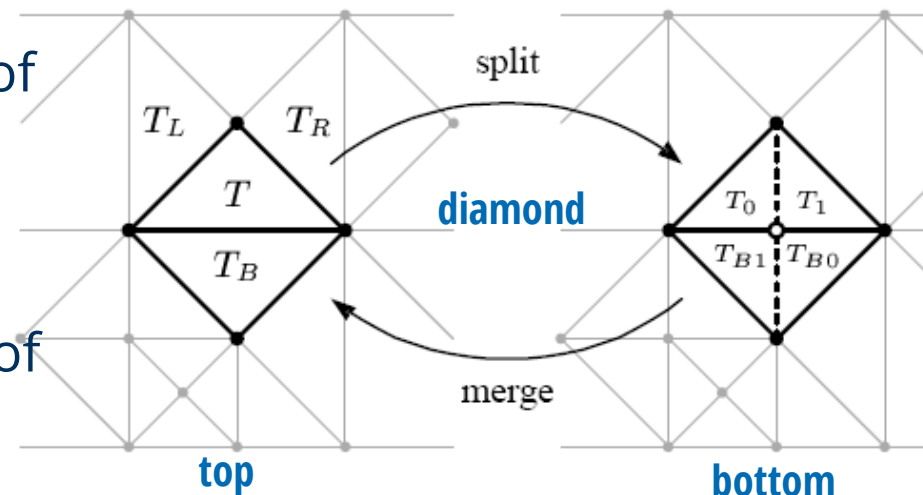
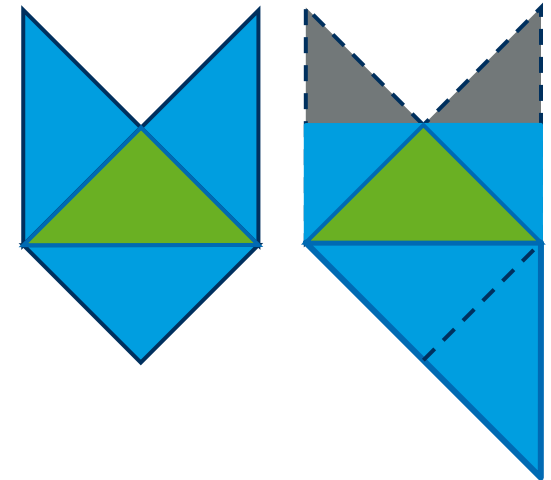


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numeric imprecisions at T-junctions lead to holes in rendering even if T-vertex is on edge

## Conditions for valid triangulation

- For each triangle of a valid triangulation it must hold
  - on the short side there can be triangles of the same or a finer level
  - the long side can have a triangle of the same or a coarser level
- two triangles  $T, T_B$  adjacent over long edge form **top** of a **diamond**
- **bottom** of diamond is composed of 4 triangles  $T_0, T_1, T_{B0}, T_{B1}$
- **Theorem:** Any valid triangulation can be transformed to any other valid triangulation by a sequence of diamond **split** and **merge** operations



Terrain  
**RENDERING**



## Refinement Criterion

- ◆ **binary predicate** that maps each triangle / diamond to binary value {accurate,inaccurate}
- ◆ It is based on
  - ◆ user defined threshold  $\tau$  on **geometric screen space error** measured in pixels
  - ◆ user defined threshold  $\gamma$  on color difference caused by **texture approximation** or difference in **lighting calculations**

## Priority

- ◆ per triangle / diamond measure of **importance** for further refinement
- ◆ Used for adaptive refinement with **limited time budget**
- ◆ It can be based on
  - ◆ screen space error
  - ◆ view-frustum visibility
  - ◆ silhouette edges
  - ◆ occlusion culling
  - ◆ backface culling
  - ◆ objects placed in terrain

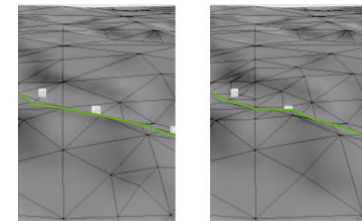


Figure 9: Left side is before LOS correction, right is after.

## Top Down

- ◆ The starting point is the coarsest diamond, which covers the entire terrain
- ◆ refinement
  - ◆ Split operations are performed recursively until all triangles meet an adaptation criterion, or
  - ◆ unprocessed triangles / diamonds are processed in order of decreasing priority with the help of a priority queue

## Incremental

- ◆ Starting point is the adaptive tessellation of the previous time step
- ◆ two [priority] queues are necessary and all previous triangles / diamonds are inserted into both:
  - ◆ Split queue performs refinement as in top down approach
  - ◆ Merge queue performs coarsening where less resolution is necessary

## Top Down Refinement on Triangles

T := mesh with base triangles  
insert triangles of T into fifo /  
priority queue F

```
while not F.empty()  
  t = F.extract()  
  if not accurate(t)  
    force_split(t, F, T)
```

output triangles in T

`force_split(t, F, T)`

`t_n := long_side_neighbor(t, T)`

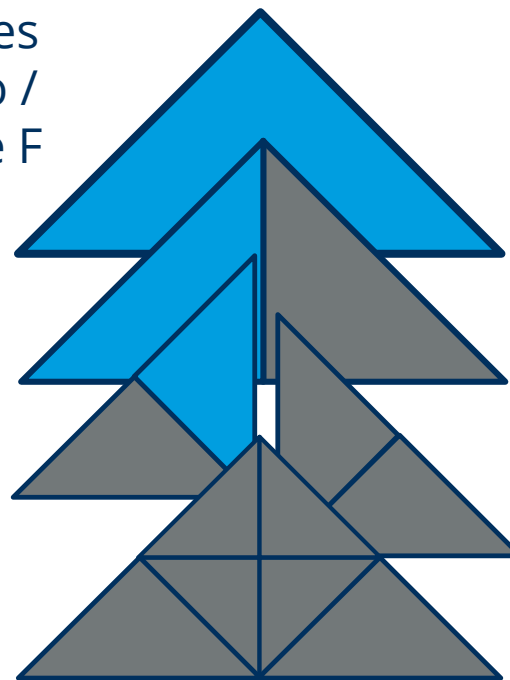
`if level(t_n, T) < level(t, T)`

`force_split(t_n, F, T)`

`remove diamond top from F`

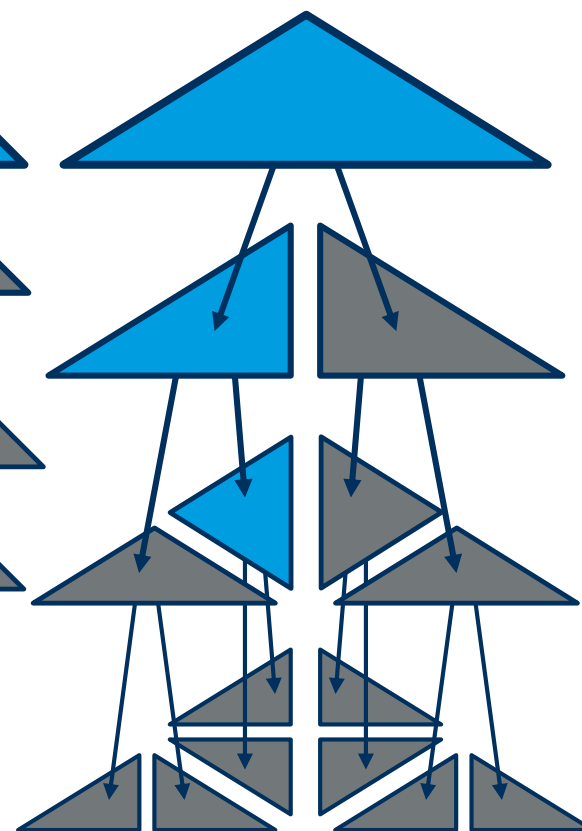
`split diamond of t in T`

`insert diamond bottom into F`



accurate

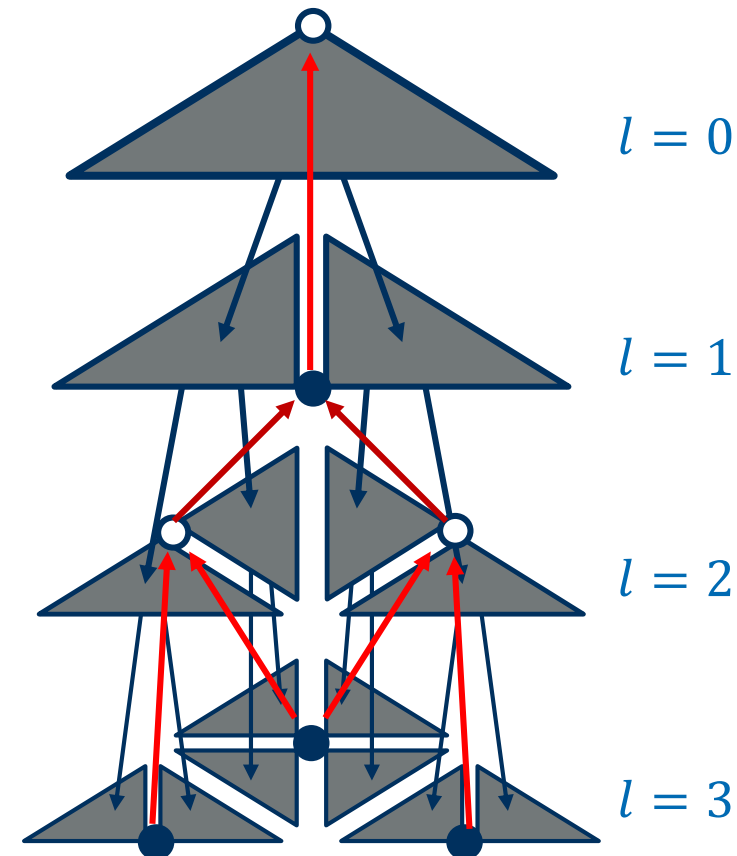
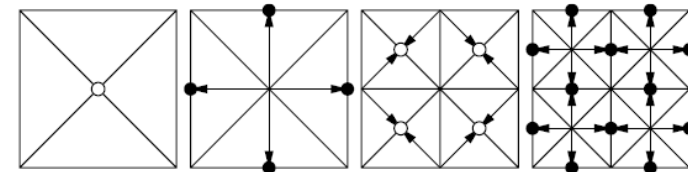
inaccurate



to implement this either long side neighbor  
or triangle selection status need to be stored  
and updated during adaptive refinement

# Diamond DAG - Definition

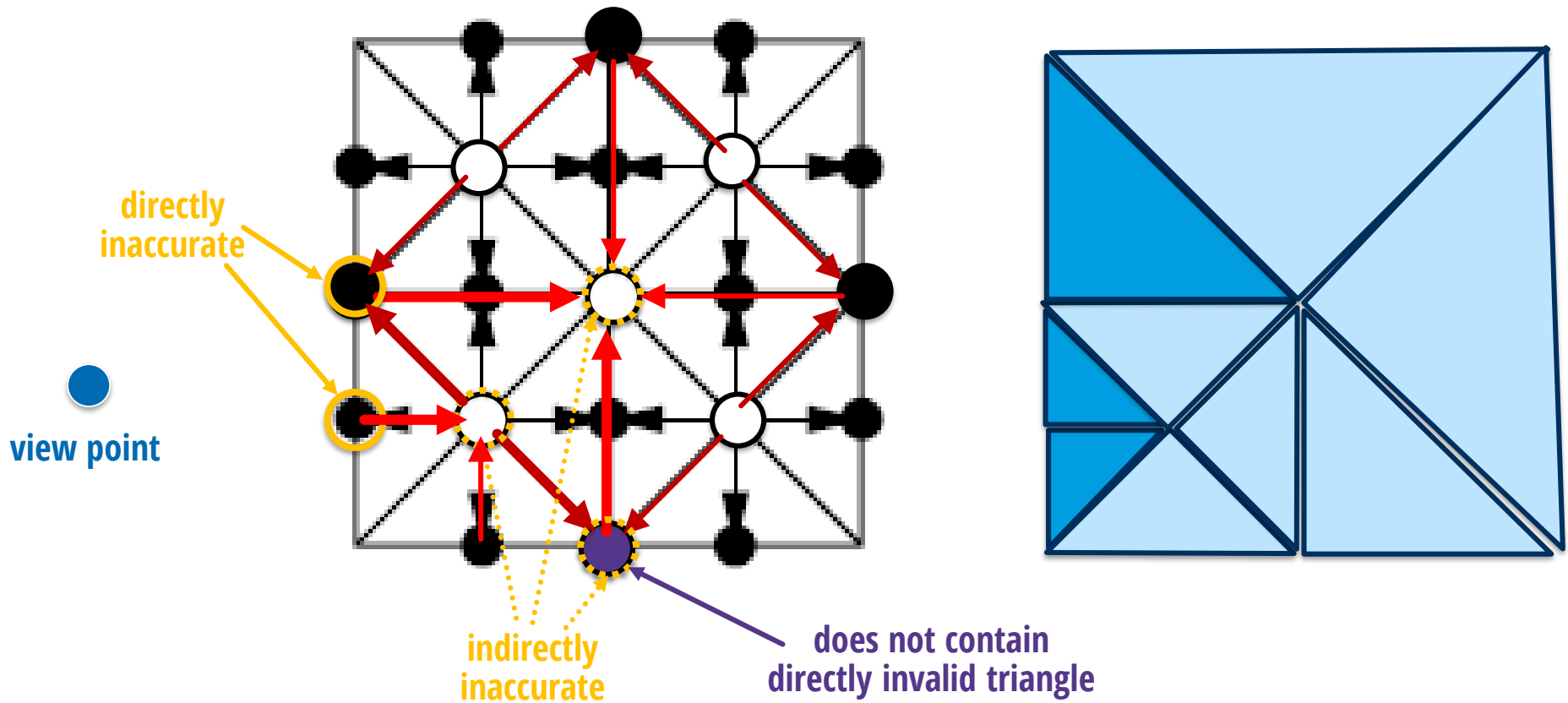
- ◆ Diamonds can be identified with the vertex index  $i$  inserted in their bottom tessellation
- ◆ In top figure on the right the **even** level vertices are filled white and the **odd** leveled vertices black
- ◆ Red arrows show **dependencies** among diamonds: in a valid triangulation a diamond vertex can only be inserted if the dependent vertices on coarser levels are also present
- ◆ Dependencies form a **directed acyclic graph (DAG)**



dependency arrows point in opposite direction as child arrows

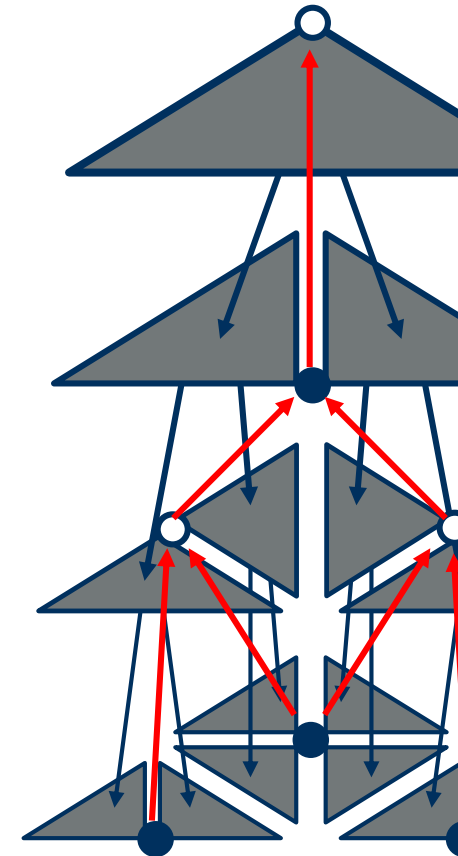
# Diamond DAG - Example

- ◆ DAG dependencies enforce presence of diamonds not containing any inaccurate triangle



## Diamond Monotonicity

- A refinement criterion  $\rho(i)$  is **diamond monotonous** if  $\rho(i) = \textit{inaccurate}$  implies  $\rho(j) = \textit{inaccurate}$  for all parents  $j$  of which  $i$  is a child ( $i \in C_j$ ).
- A priority  $\pi(i)$  is **diamond monotonous** if  $\pi(i) < \min \pi(j)$  for all parents  $j$  of which  $i$  is a child:  $i \in C_j$ .
- One can extend diamond monotonous refinement criteria and priorities to right triangles by mapping both top triangles to the value of the diamond
- Diamond monotonicity yields top down and incremental refinement algorithms without the need to check or store DAG dependencies. Implementations can be done **without storing state** of diamonds.



dependency arrows point in  
opposite direction as child arrows

## State-Less Refinement with Diamond Monotonicity

$T := \emptyset$

insert base triangles into **fifo**

or priority queue  $F$

while not  $F.empty()$

$t = F.extract()$

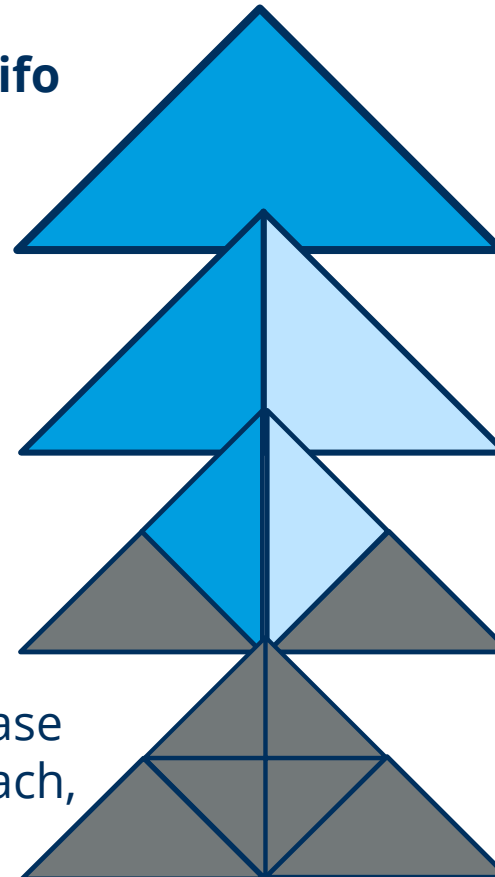
if  $accurate(t)$

$T := T \cup t$

else

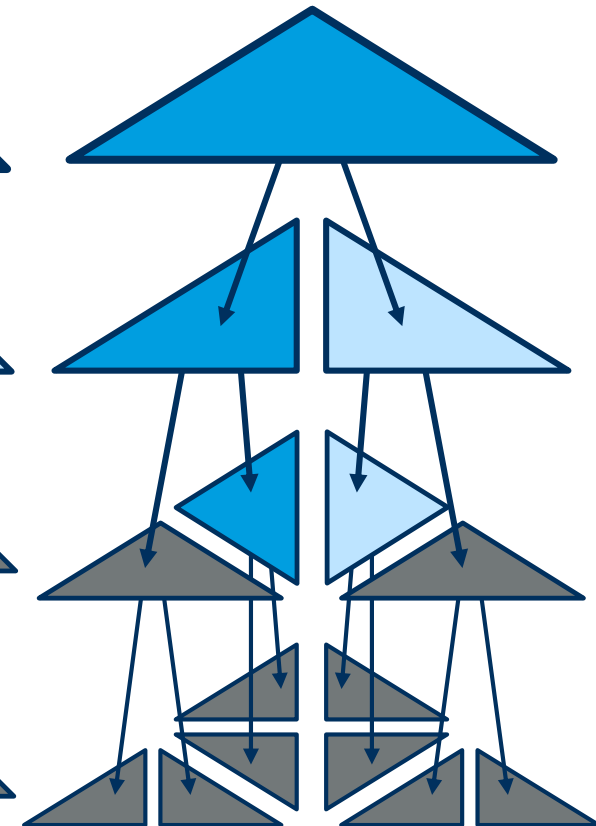
$F.add(children(t))$

- Before termination in case of priority based approach, process top element in queue if it is long edge neighbor of last processed triangle



accurate

inaccurate



$l = 0$

$l = 1$

$l = 2$

$l = 3$



## Preprocessing

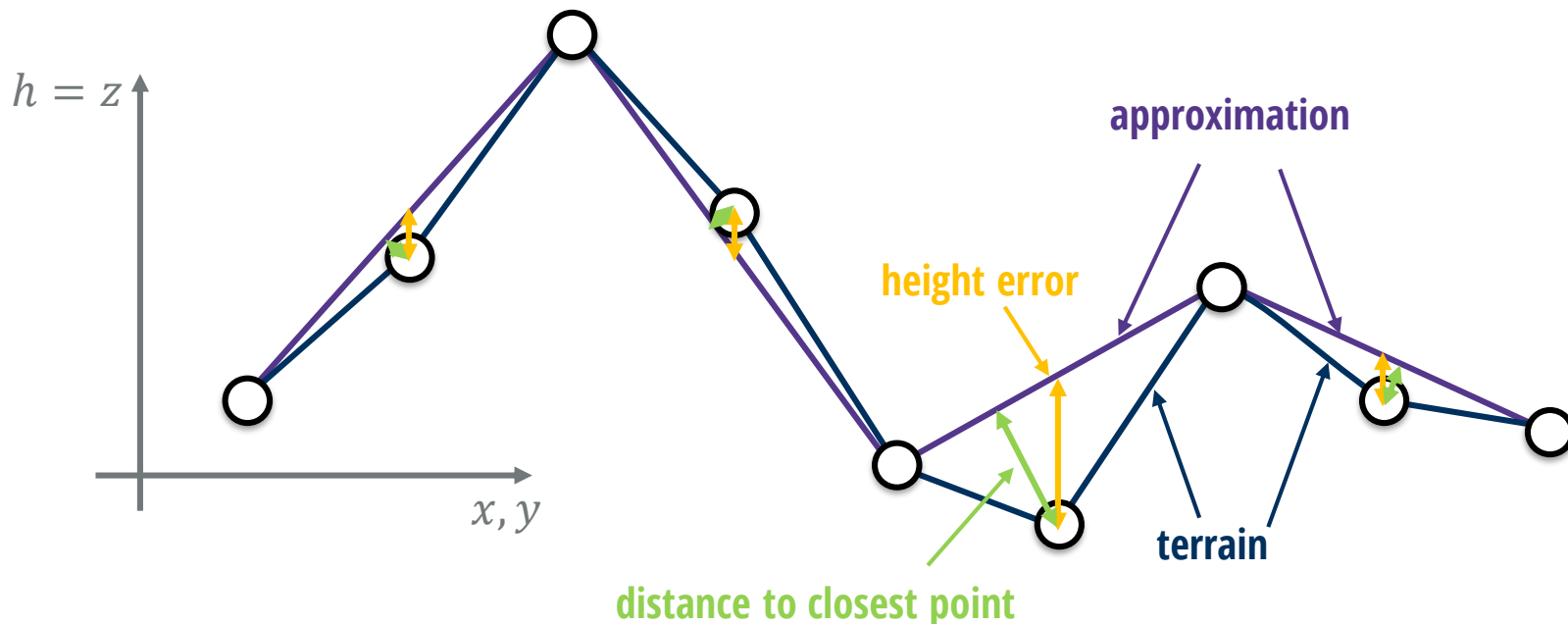
- ◆ determine **geometric error** introduced by removing diamond vertex (typically directed along height field direction)
- ◆ define **bounding volume** (extrusion of right triangle or sphere around diamond center point)
- ◆ post process error and bounding volume to **ensure monotonicity** (to avoid storing the Diamond DAG)

## Adaptive refinement

- ◆ Estimate screen space error conservatively by maximizing screen space projection of geometric error over bounding volume



- Geometric Error can be measured by finding for each point on the terrain the closest point on the approximation and maximizing over all terrain points (one-sided Hausdorff-distance), or
- maximizing the error on the height value, which is typically used for terrain adaptation as it also provides a single direction of the error.



## According to ROAM 1997 Paper:

- The geometric errors are computed from bottom to top
- Leaf triangles are assigned 0 error
- for each triangle further up in the hierarchy compute height/z-error of vertex splitting long edge and its projection onto the edge:  $|z(v_c) -$

$$e_T = \max\{e_{T_0}, e_{T_1}\} + |z(v_c) - z_T(v_c)|$$

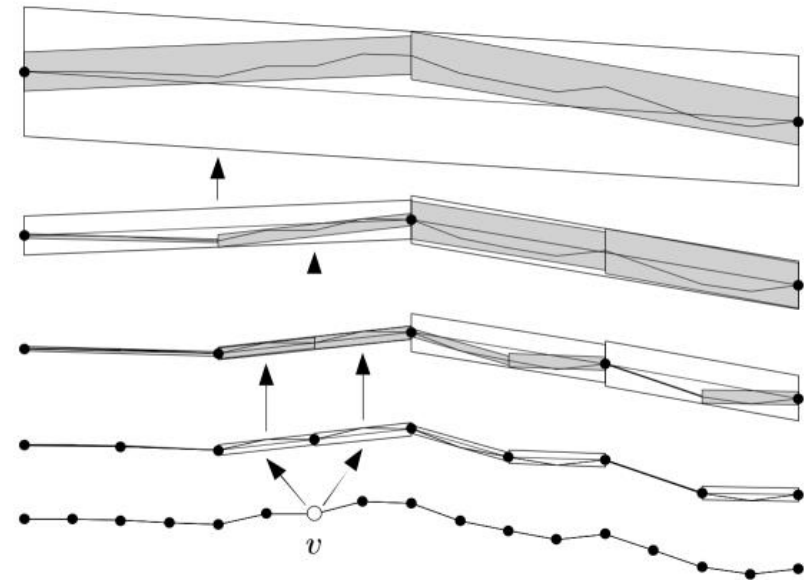
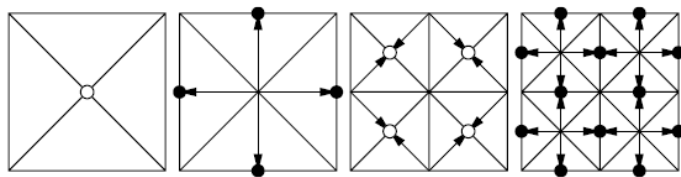
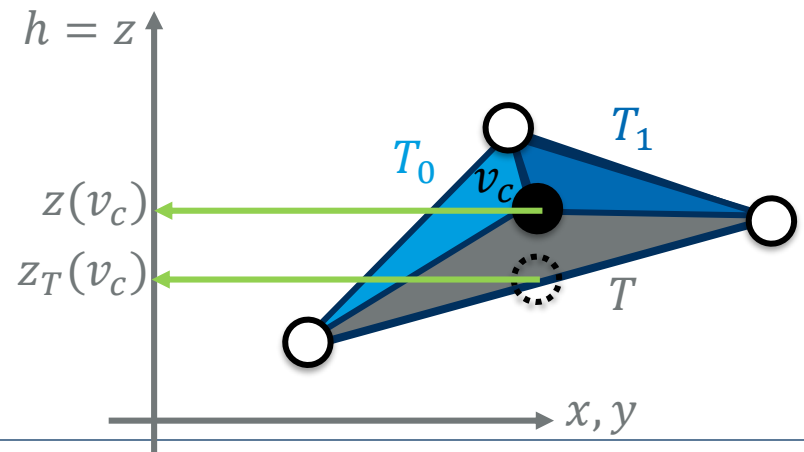


Figure 6: Nested wedgies for 1-D domain with dependents of  $v$ .

Duchaineau et al.: ROAM 1997



## Maximum Project Length Error Vectors (ROAM 1997)

- extruding right triangles along the z-direction by the nested error value yields **wedgies** that can be used to estimate screen space error:
- For a wedge of a triangle  $T$  all error vectors can be spanned by the point pairs  $\underline{w}_{\pm}$  with

$$\underline{w}_{\pm} = \underline{v} \pm e_T \hat{z}$$

for triangle point  $\underline{v}$  and z-axis  $\hat{z}$ .

- Next transform to clip space:

$$MVP \cdot \tilde{v} \pm MVP \cdot e_T \tilde{z}$$

$$\text{with } \tilde{z} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

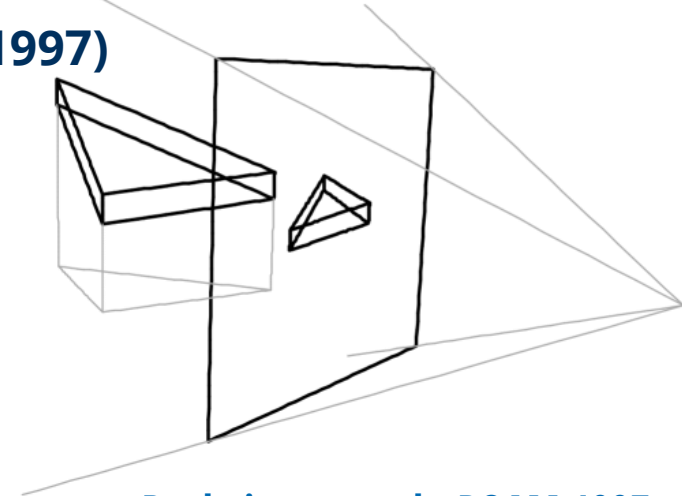
- Error only depends on clip x, y, w components.
- We delay w-clip, apply viewport transform to x and y to get homo. screen coords of  $\underline{v}$  and  $e_T \hat{z}$ :

$$\tilde{w}_{\text{screen}, \pm} = (p, q, r) \pm (a, b, c)$$

- Pixel error is length of difference vector after w-clip:

$$\widehat{\text{dist}}(v) = \left\| \frac{p+a}{r+c} - \frac{p-a}{r-c}, \frac{q+b}{r+c} - \frac{q-b}{r-c} \right\|_2$$

$$= \frac{2}{r^2 - c^2} \left( (ar - cp)^2 + (br - cq)^2 \right)^{1/2}$$



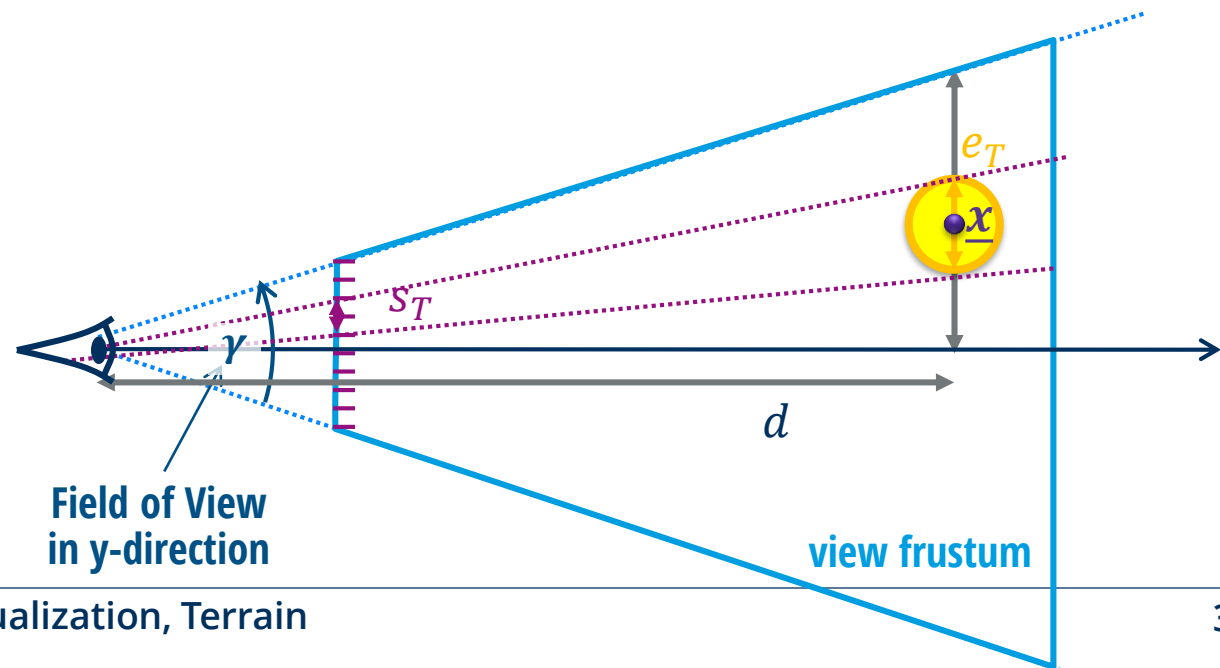
Duchaineau et al.: ROAM 1997

- Duchaineau showed that minimum of  $r^2 - c^2$  and maximum of  $(ar - cp)^2 + (br - cq)^2$  are achieved at tgl corners
- plugging corner min & max into  $\widehat{\text{dist}}$  yields upper bound on screen space error

## Screen Space Error from Isotropic Object Space Error (Blow 2000)

- In 2000 Blow proposed to use  $e_T$  as an isotropic object space error independent of current view direction.
- The screen space error  $s_T$  in pixels at point  $\underline{x}$  can be estimated from eye location  $\underline{e}$ , screen height  $h$  in pixels, and field of view angle  $\gamma$  in y-direction:  $s_T = c \cdot \frac{e_T}{\|\underline{x} - \underline{e}\|}$ , with  $c = \frac{h}{2 \tan \gamma/2}$
- This can be used as priority or to threshold against pixel error  $\tau$

$$\frac{s_T}{\frac{h}{2}} = \frac{e_T}{d \cdot \tan \frac{\gamma}{2}}$$



# Thresholding Screen Space Error – Blow

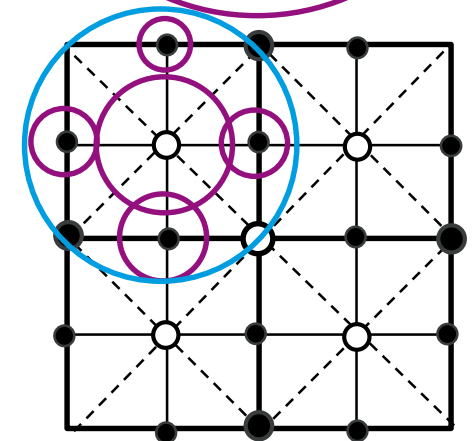
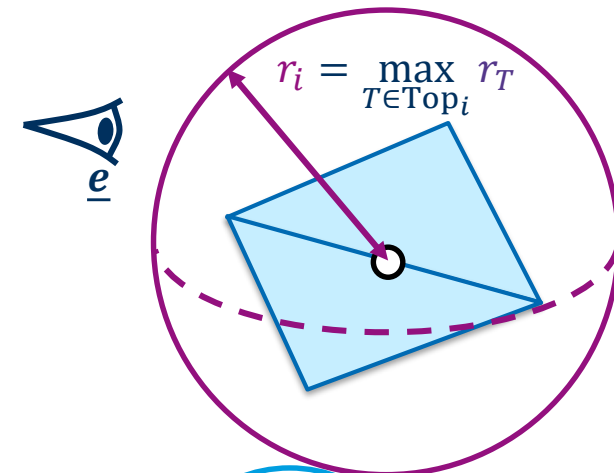
- Screen space threshold  $\tau$  can be translated per triangle  $T$  to sphere radius  $r_T$  defining a **threshold sphere**, when entered by view-point triangle becomes inaccurate
- Per **diamond radii**  $r_i$  are computed as max over diamond top triangle radii.
- For **monotonicity** parent spheres are increased bottom up to enclose child threshold spheres by computing final radius  $\hat{r}_i$ :

$$\hat{r}_i := \begin{cases} 0 & \text{if } i \text{ is Leaf} \\ \max \left\{ r_i(\tau), \max_{j \in C_i} \left\{ \|\underline{p}_i - \underline{p}_j\| + \hat{r}_j \right\} \right\} & \text{otherwise} \end{cases}$$

where the  $\underline{p}_i$  are 3D points of diamonds

$$s_T = c \cdot \frac{e_T}{\|\underline{x} - \underline{e}\|} < \tau \quad c := \frac{h}{2 \tan \frac{\gamma}{2}}$$

$$\|\underline{x} - \underline{e}\| > c \cdot \frac{e_T}{\tau} =: r_T$$

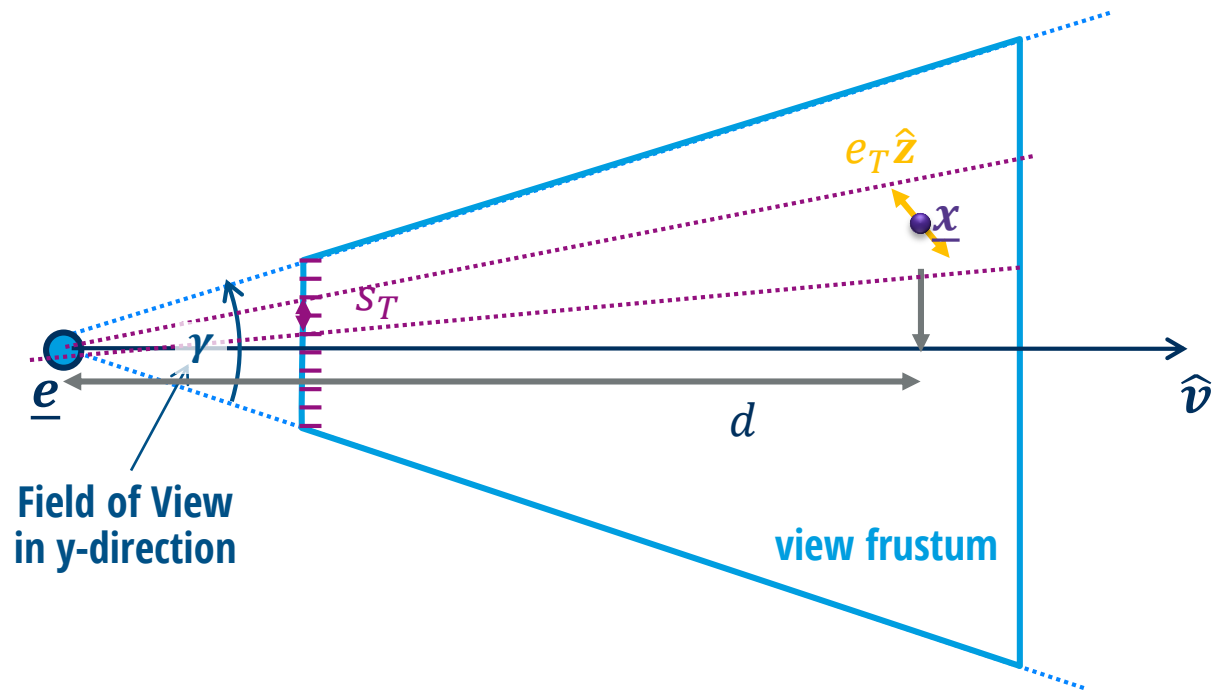


- In case of an **anisotropic error**  $e_i$  directed along  $\hat{z}$  located at point  $\underline{x}$  viewed along direction  $\hat{v}$ , the screen space error  $s_i$  in pixels is reduced by the cosine between z and v:

$$s_i = c \cdot (1 - |\langle \hat{z}, \hat{v} \rangle|) \frac{e_i}{\|\underline{x} - \underline{e}\|}, \text{ with } c = \frac{h}{2 \tan \gamma/2}$$

- Now threshold sphere depends on view angle

$$\frac{s_T}{\frac{h}{2}} = \frac{e_T}{d \cdot \tan \frac{\gamma}{2}}$$



- Lindstrom 2001 proposed to build nested diamond balls  $\mathcal{B}_i$  (spheres of radius  $r_i$  around diamond points  $\underline{p}_i$ ) independent of error:

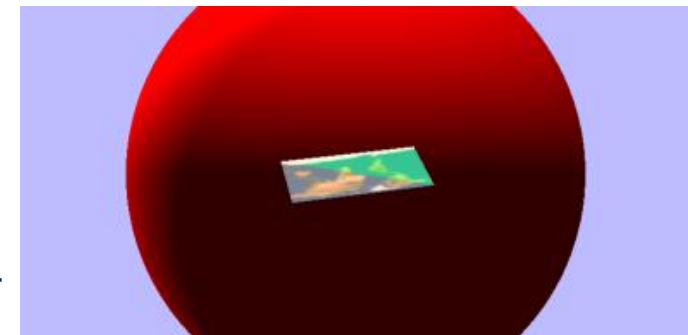
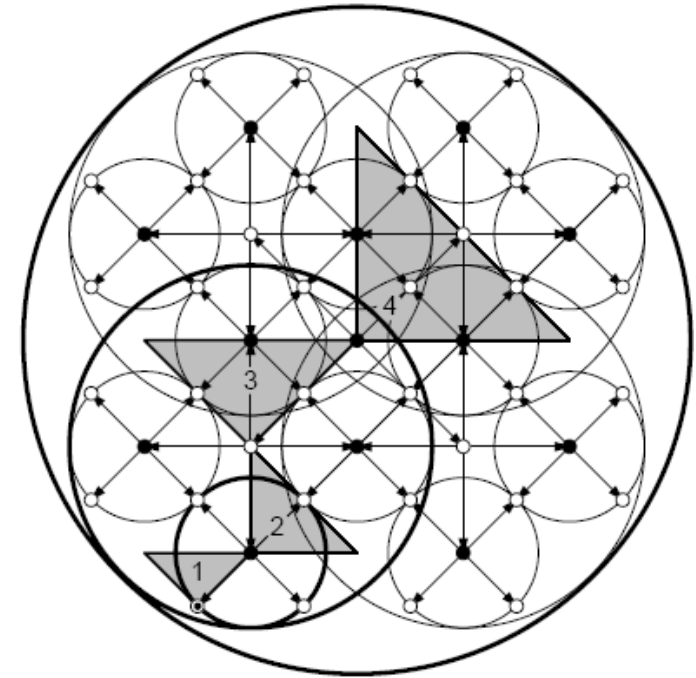
$$r_i = \begin{cases} 0 & \text{if } i \text{ is a leaf node} \\ \max_{j \in C_i} \{ \|\underline{p}_i - \underline{p}_j\| + r_j \} & \text{otherwise} \end{cases}$$

- And to maximize screen space error over nested diamond balls

$$s_i(e_i, \mathcal{B}_i, \underline{e}) = \max_{\underline{x} \in \mathcal{B}_i} c \cdot (1 - |\langle \hat{\underline{z}}, \hat{\underline{v}} \rangle|) \frac{e_i}{\|\underline{x} - \underline{e}\|}$$

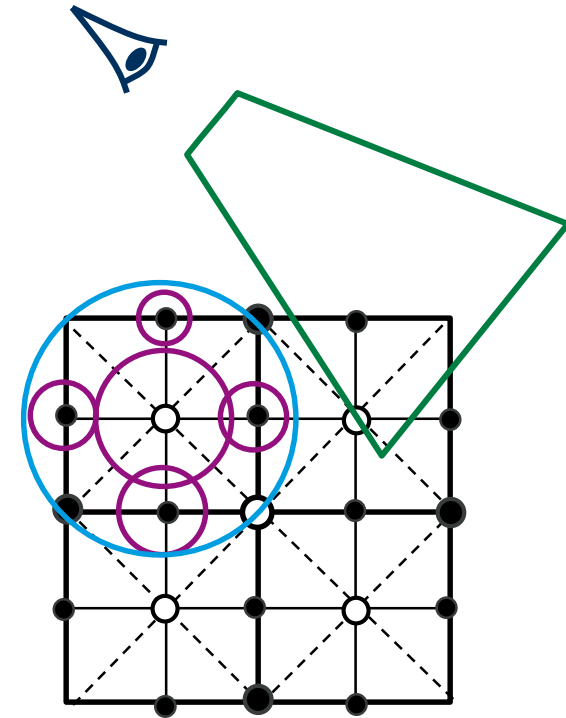
- If eye is inside  $\mathcal{B}_i$  diamond is to be split, otherwise screen maximum is achieved at point closest to eye:

$$s_i(e_i, \mathcal{B}_i, \underline{e}) = c \cdot (1 - |\langle \hat{\underline{z}}, \hat{\underline{v}} \rangle|) \frac{e_i}{\|\underline{p}_i - \underline{e}\| - r_i}$$



observation: nested balls become quite huge

- View frustum culling gives significant speed up
- Lindstrom et al. 2001 propose to exploit the diamond balls for this
- The six view frustum planes are passed to the refinement algorithm
- If a ball is **completely outside** of one of the clipping planes, the whole subtree can be clipped away
- If a ball is **completely inside** of one clipping planes, all descendants are also inside and therefore no more view frustum culling test need to be performed anymore



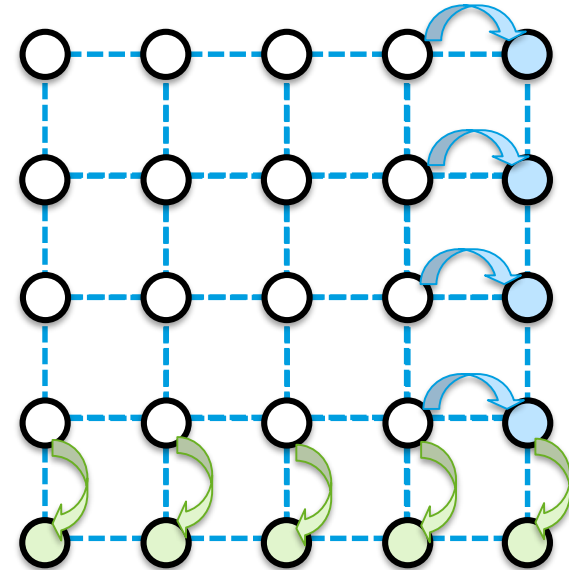


Terrain

# IMPLEMENTATION

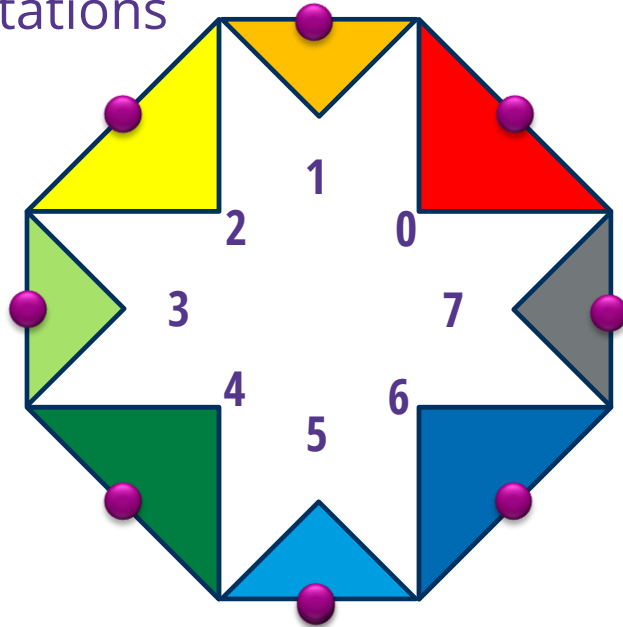
# Implementation Details

- ◆ Texture resolution is assumed to be squared with  $N \times N$  texels, with  $N$  being a power of 2, i.e. 4096
- ◆ To make HRT work, texture needs to be padded to dimensions  $(N + 1) \times (N + 1)$  typically by replicating last row and column

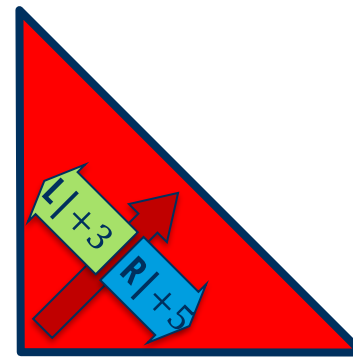


# Implementation – Triangle Representation

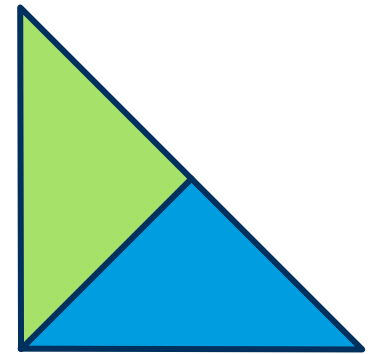
- enumerate 8 different triangle orientations



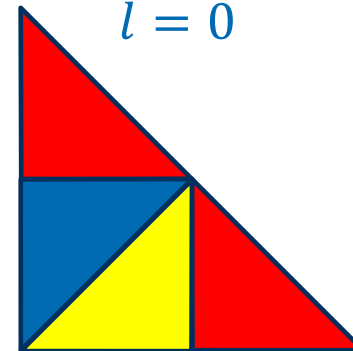
- $(\underline{l}, L, \omega)$  represents triangle node
  - 2D diamond point  $\underline{l}$  on longest edge measured in texels (i.e. ranging from 0...4096)
  - Edge length  $L$  along x or y coordinate axis (i.e. 1, 2, 4, 8, ..., 4096)
  - triangle orientation  $\omega$  (i.e. 0 ... 7)



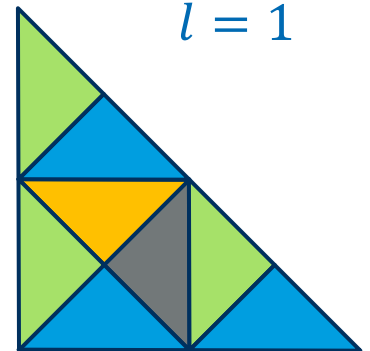
$l = 0$



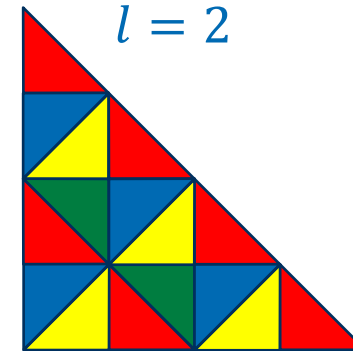
$l = 1$



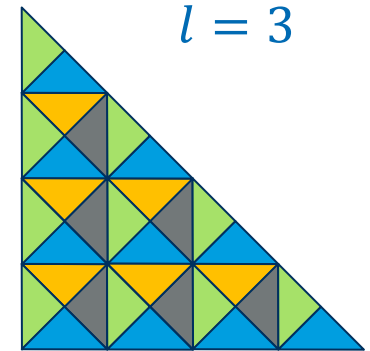
$l = 2$



$l = 3$



$l = 5$

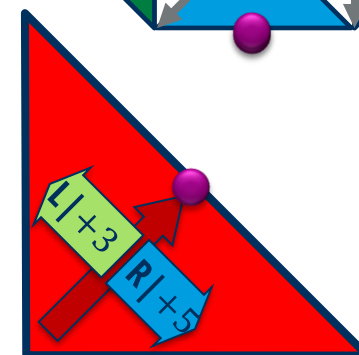
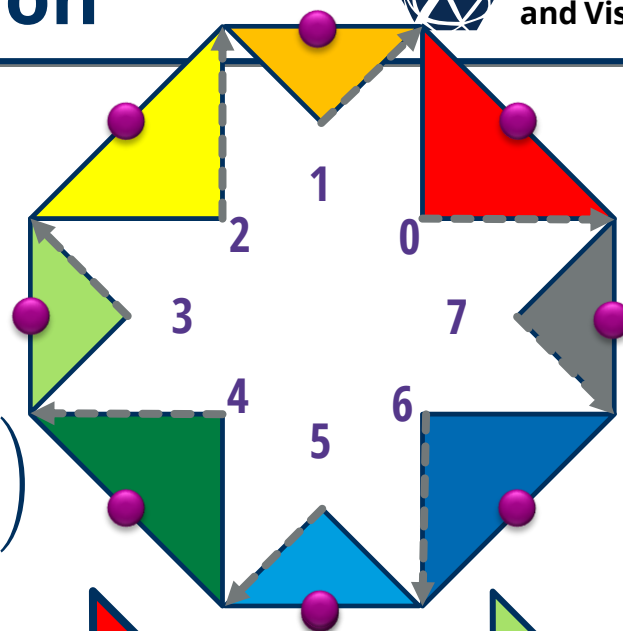


$l = 6$

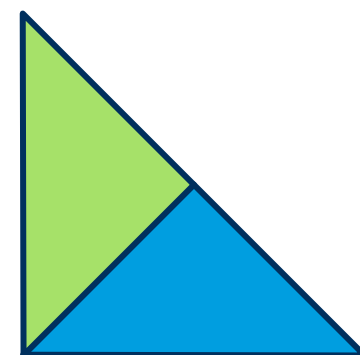
# Implementation - Navigation



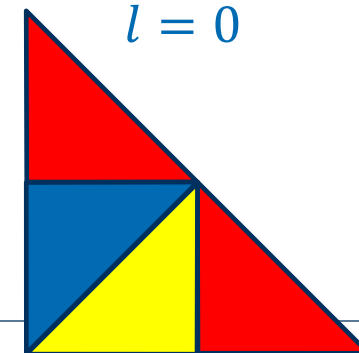
- $\vec{v}(L, \omega) = \left[ \begin{pmatrix} L \\ 0 \end{pmatrix}, \begin{pmatrix} L/2 \\ L/2 \end{pmatrix}, \begin{pmatrix} 0 \\ L \end{pmatrix}, \begin{pmatrix} -L/2 \\ L/2 \end{pmatrix}, \dots \right]$
- $\text{leftChild}(\underline{l}, L, \omega)$   
 $\left( \underline{l} + \vec{v}(L/2, \omega + 4), \begin{cases} L & \omega \text{ even} \\ L/2 & \omega \text{ odd} \end{cases}, \omega + 3 \right)$
- $\text{rightChild}(\underline{l}, L, \omega)$   
 $\left( \underline{l} + \vec{v}(L/2, \omega + 6), \begin{cases} L & \omega \text{ even} \\ L/2 & \omega \text{ odd} \end{cases}, \omega + 5 \right)$
- $\text{diamondTopSibling}(\underline{l}, L, \omega)$   
 $\begin{cases} \text{undef} & l_x = 0 \vee l_y = 0 \vee \\ & l_x = N \vee l_y = N \\ (\underline{l}, L, \omega + 4) & \text{otherwise} \end{cases}$



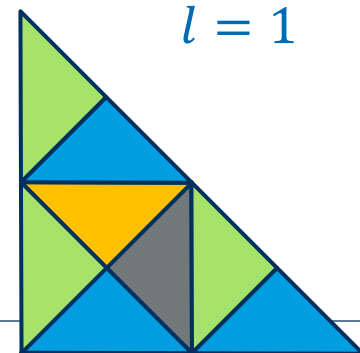
$l = 0$



$l = 1$



$l = 2$



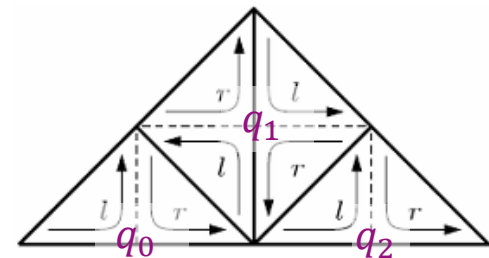
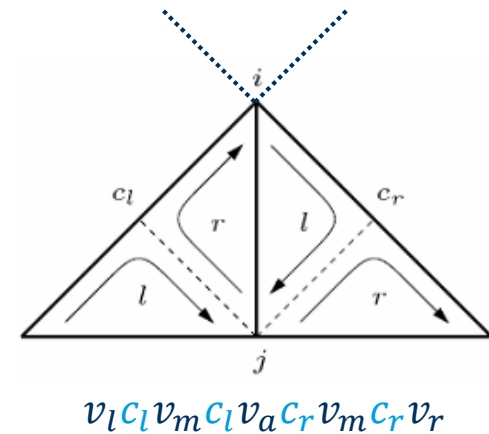
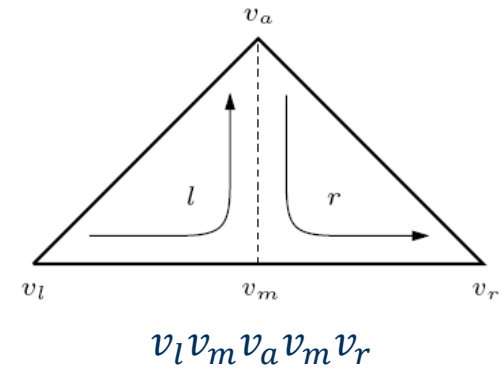
$l = 3$

operations on  $\omega$  are always modulo 8

# Implementation - Stripification



- ◆ Lindstrom et al. 2001 propose to directly generate single **triangle strip** during adaptive refinement
- ◆ No data structure is used
- ◆ For hierarchical navigation, the triangles are defined by a pair  $(i, j)$  of nodes on level  $l$  and  $l - 1$ .
- ◆ When descending to the left/right partial triangle, the nodes  $c_l$  and  $c_r$  are needed
- ◆ meaning of "left" / "right" alternates from level to level



$$v_l q_0 c_l q_0 v_m q_1 c_l q_1 v_a q_1 c_r q_1 v_m q_2 c_r q_2 v_r$$

# Implementation – Stripification

tstrip-append( $V, v, p$ )

- 1 if  $v \neq v_{n-1}$  and  $v \neq v_n$  then
- 2   if  $p \neq \text{parity}(V)$  then
- 3      $\text{parity}(V) \leftarrow p$
- 4   else
- 5      $V \leftarrow (V, v_{n-1})$
- 6      $V \leftarrow (V, v)$

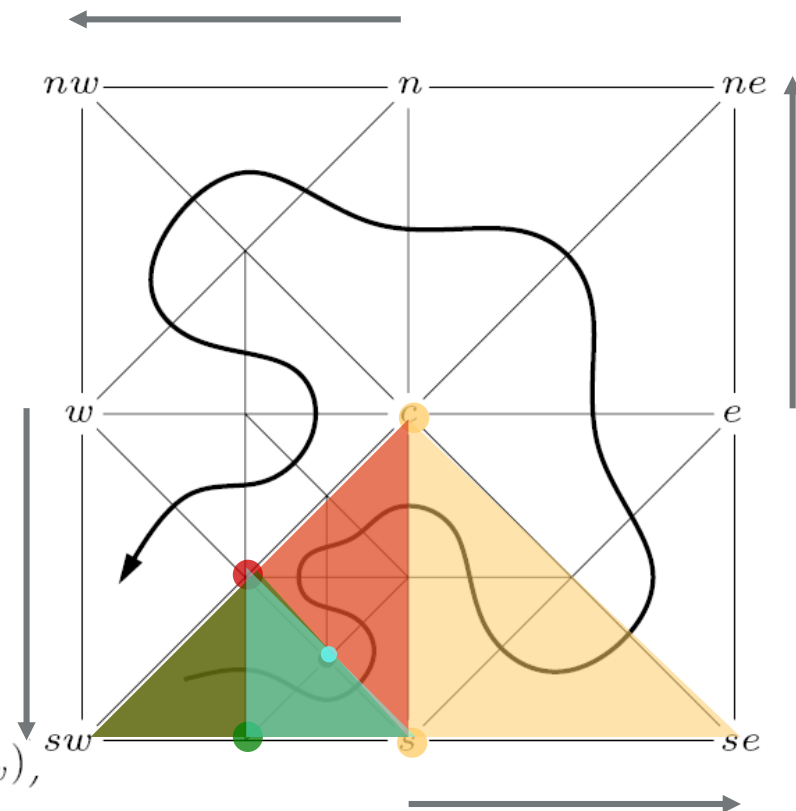
level  $l$  with 0 for leaf  
and  $n$  for root

submesh-refine( $V, i, j, l$ ) → inaccurate

- 1 if  $l > 0$  and active( $i$ ) then
- 2   submesh-refine( $V, j, c_l(i, j), l - 1$ )
- 3   tstrip-append( $V, i, l \bmod 2$ )
- 4   submesh-refine( $V, j, c_r(i, j), l - 1$ )

mesh-refine( $V, n$ )

- 1  $V \leftarrow (i_{sw}, i_{sw})$
- 2  $\text{parity}(V) \leftarrow 0$
- 3 for each  $(j, k) \in ((i_s, i_{se}), (i_e, i_{ne}), (i_n, i_{nw}), (i_w, i_{sw}))$ ,
- 4   submesh-refine( $V, i_c, j, n$ )
- 5   tstrip-append( $V, k, 1$ )



SOAR paper:

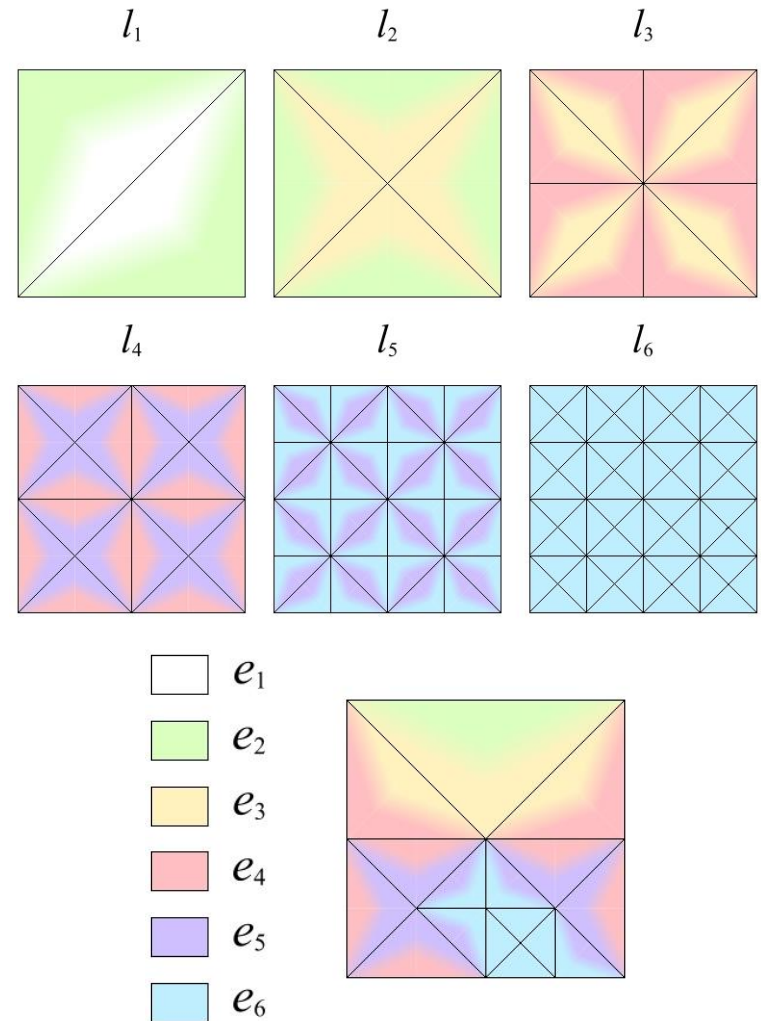
Peter Lindstrom and Valerio Pascucci, *Visualization of Large Terrains Made Easy*  
Proceedings of IEEE Visualization 2001, pp. 63-370, 574, October 2001

Terrain

# **BATCHED METHODS**

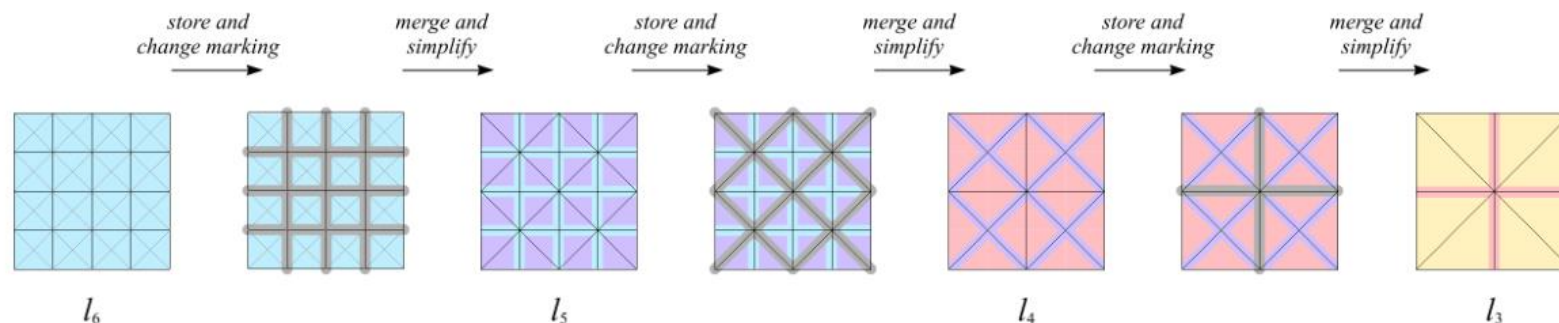
# Batched HRT

- ◆ Per triangle adaptation on CPU cannot cope with modern GPU speed
- ◆ Cignoni et al. propose in BDAM paper to use HRT on triangular batches
- ◆ Each triangle in the HRT is replaced by a TIN which is optimized with respect to underlying geometry by a **mesh simplification** algorithm and for fast rendering by generating **triangle strips**

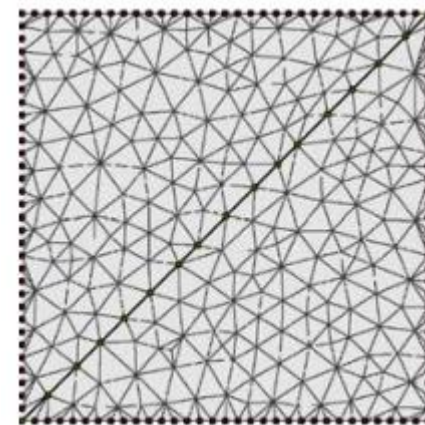


Cignoni et al., **BDAM – Batched Dynamic Adaptive Meshes for High Performance Terrain Visualization**, EG 2003



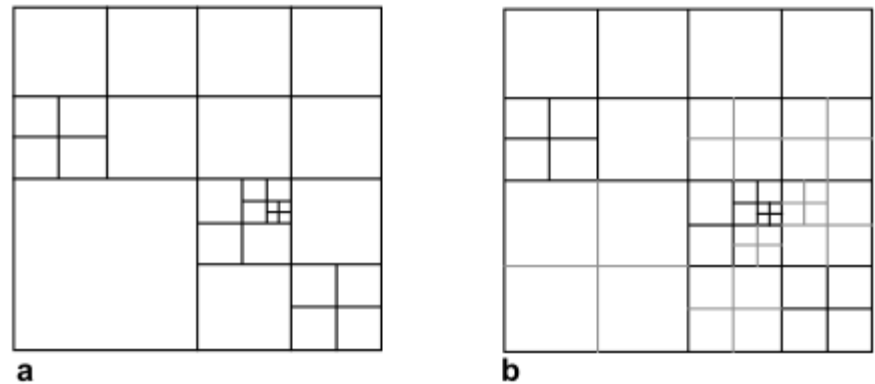


- ◆ Batches are built bottom up
- ◆ per diamond a simplification algorithm is used to half the triangle count
  - ◆ vertices along diamond boundaries (long edges of finer level) are marked to stay untouched
  - ◆ vertices along long edges of coarser level are restricted to move on edge only
- ◆ simplification errors are used in combination with nested spheres for view-dependent refinement

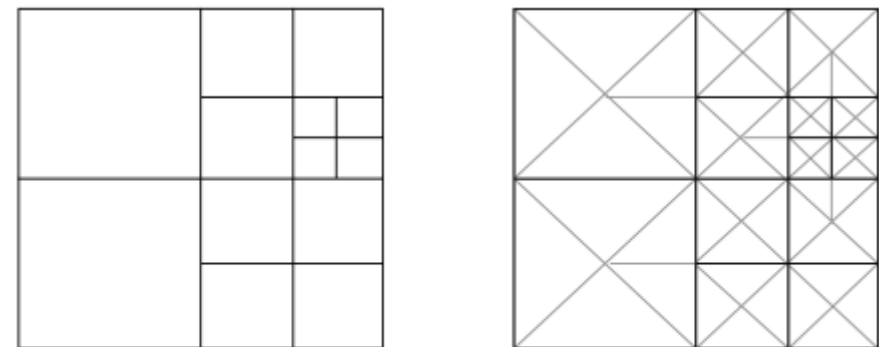


zoomed in view into a coarsened diamond

- Color texture is typically organized in quadtree
- Quadtree can also be used for geometry refinement
- Typically adaptation is restricted in a way to ensure that resolution change is nowhere more than one
- tessellation is extracted by splitting quadtree cells into triangle fans according to vertices on cell boundary



**Fig. 4.** **a** Example of an unrestricted quadtree subdivision in parameter space. **b** The restricted subdivision



**Fig. 7.** Improved conforming triangulation of a restricted quadtree subdivision as in [58]

# Quadtree based refinement

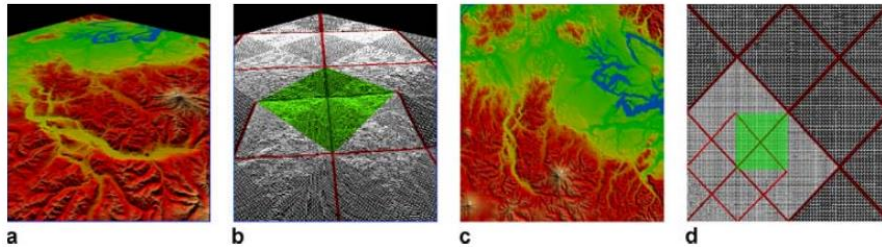


Fig. 1a-d. Terrain rendering using seamless patches. **a** A selected view. **b** The wire-frame of **a**, where the *green* region marks one patch. **c** Top view of **a**. **d** The wire-frame of **c** with the same marked patch

- Again a batched version can better balance between CPU and GPU speed
- Livny et al. propose to use a patch per quadtree cell that can contain triangles of different resolution
- Specialized triangle strips are used to adapt resolution changes

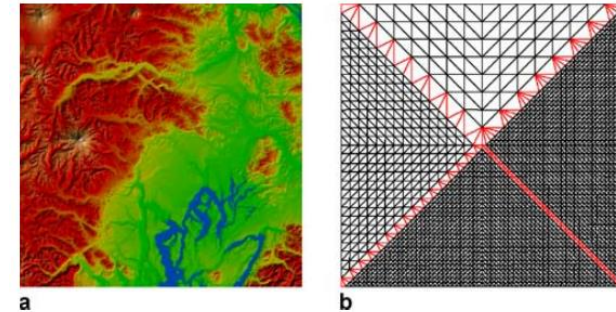


Fig. 2. **a** An image of one patch. **b** The wire-frame of the same patch which shows the stitching of different resolution tiles

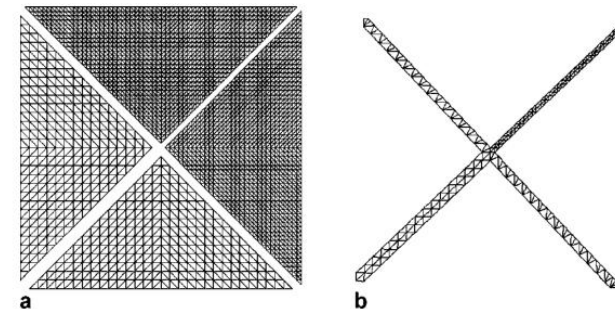
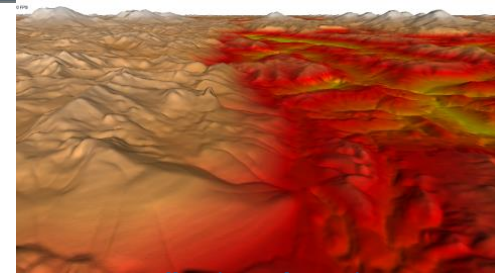


Fig. 3a,b. The components of one patch. **a** The image of four triangular tiles. **b** The image of four strips

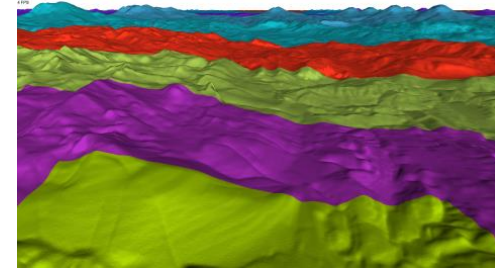
Livny, Y., Kogan, Z., & el-Sana, J. (2009). Seamless patches for GPU-based terrain rendering. *The Visual Computer*, 25(3), 197-208.

# Geometry Clipmaps

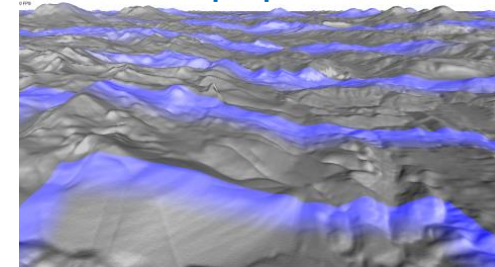
- Idea: use mipmap hierarchy to store color and elevation textures such that texture extent increases with decreasing texture level
- Tessellate with resolution adapting triangle strips
- Upload data incrementally when changing the view point
- Blend geometry and color close to level transition
- Supports compressed images
- Suited for GPU implementation
- Combined with texture synthesis
  - **F. Losasso, H. Hoppe, Geometry clipmaps: Terrain rendering using nested regular grids. SIGGRAPH 2004.**  
Geometry clipmap framework with compressed and fractal terrains
  - **A. Asirvatham, H. Hoppe, Terrain rendering using GPU-based geometry clipmaps. GPU Gems 2, 2005.**  
Efficient GPU implementation
  - **S. Lefebvre, H. Hoppe, Parallel controllable texture synthesis. SIGGRAPH 2005**  
Parallel texture synthesis algorithm



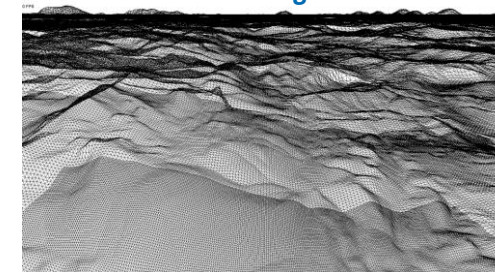
Illuminated terrain



Mipmap levels



Blend regions



triangulation

Terrain  
**SHADING**

- For local lighting we need to compute a the terrain normal, which can be computed from the height field gradient according to

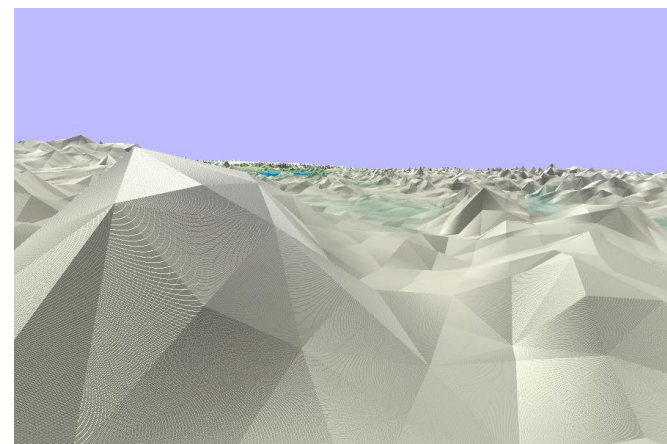
$$\hat{\mathbf{n}} = \text{normalize}(-h_x, -h_y, 1)^T$$

- The normal can either be stored in a normal texture, or (see exer.)
- to avoid additional storage, one can also compute the normal in the fragment shader exploiting the derivative functions `dFdx` and `dFdy` of glsl that compute the derivatives in screen space:

```
vec3 estimate_normal(in vec2 tc)
{
    vec2 tx = dFdx(tc);
    vec2 ty = dFdy(tc);
    vec2 hg = vec2(
        texture(dem_tex, tc + 0.5*tx).x - texture(dem_tex, tc - 0.5*tx).x,
        texture(dem_tex, tc + 0.5*ty).x - texture(dem_tex, tc - 0.5*ty).x);

    hg /= (extent.xy*vec2(length(tx), length(ty)));
    hg *= extent.z;
    return normalize(get_normal_matrix()*vec3(-hg, 1.0));
}
```

- When zooming into the normals computed in the shader, one can see artefacts on the bilinear interpolation of the GPU



linear texture magnification imprecision is visible in the lighting artefacts

# Ambient Occlusion

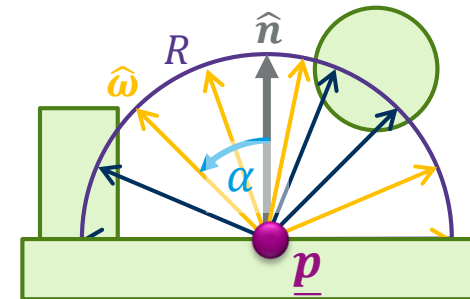
Zhukov, S., Iones, A., & Kronin, G. (1998). An ambient light illumination model. In *Rendering Techniques' 98* (pp. 45-55). Springer, Vienna.



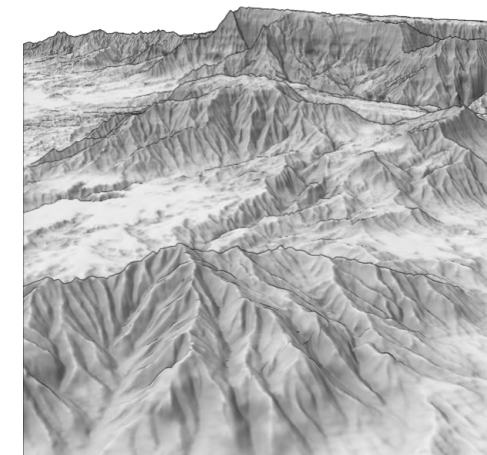
- Ambient occlusion (AO) assumes ambient light everywhere in the scene or coming from the sky.
- At every surface point  $\underline{p}$  a scalar factor  $A_{\underline{p}} \in [0,1]$  measures the percentage of the arriving light by integrating over all directions  $\hat{\omega}$  of the hemisphere around the surface normal  $\hat{n}$ :

$$A_{\underline{p}} = \frac{1}{\pi} \iint_{\hat{\omega} \in \Omega} \rho \left( L(\underline{p}, \hat{\omega}) \right) \frac{\langle \hat{\omega}, \hat{n} \rangle}{\cos \alpha} \cdot d\hat{\omega}$$

- With the length  $L(\underline{p}, \hat{\omega})$  of the free path length to the next occluder clamped to a maximum radius  $R$  (typically estimation of  $L$  starts at  $\underline{p} + \epsilon \hat{\omega}$ )
- $\rho$  can be used to emulate different AO models:
  - step function:  $\rho(L) = \begin{cases} 0 & L < R \\ 1 & L = R \end{cases}$  used for sky illumination
  - linear function:  $\rho(L) = L/R$  used for volumetric illumination
  - Other monotonous function mapping  $[0, R]$  to  $[0,1]$  possible



AO in 2D hillshading techniques



AO in 3D terrain rendering

# Ambient Occlusion

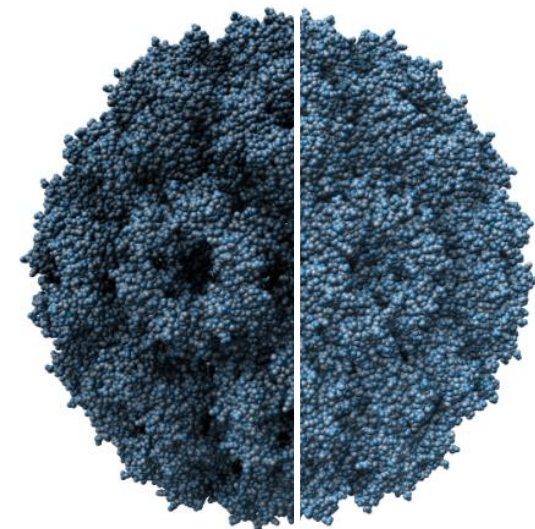
Zhukov, S., Iones, A., & Kronin, G. (1998). An ambient light illumination model. In *Rendering Techniques' 98* (pp. 45-55). Springer, Vienna.



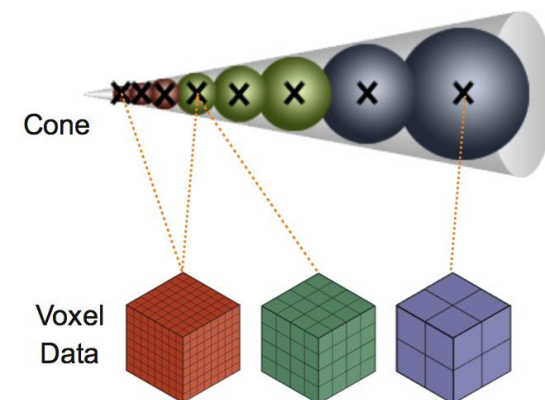
- Ambient occlusion can be evaluated in screen space (SSAO) as a post processing step or in object space (OSAO)
- Object space methods use  $N$  sample directions  $\hat{\omega}_i$  sampled with probability  $p(\hat{\omega}_i)$  and do a Monte Carlo estimate:

$$A_{\underline{p}} = \frac{1}{N} \cdot \frac{1}{\pi} \sum_i \frac{\rho(L(\underline{p}, \hat{\omega}_i)) \langle \hat{\omega}_i, \hat{n} \rangle}{p(\hat{\omega}_i)}$$

- For a given direction  $\hat{\omega}_i$ , **shadow mapping** can be used to compute  $L$  for all scene points in one rendering pass in parallel
- With the help of a voxelization of the scene, **voxel cone tracing** can be used to trace a direction cone very efficiently
- In terrain rendering the terrain is typically static and AO can be “**baked**” into the color texture in a preprocessing step



OSAO vs SSAO

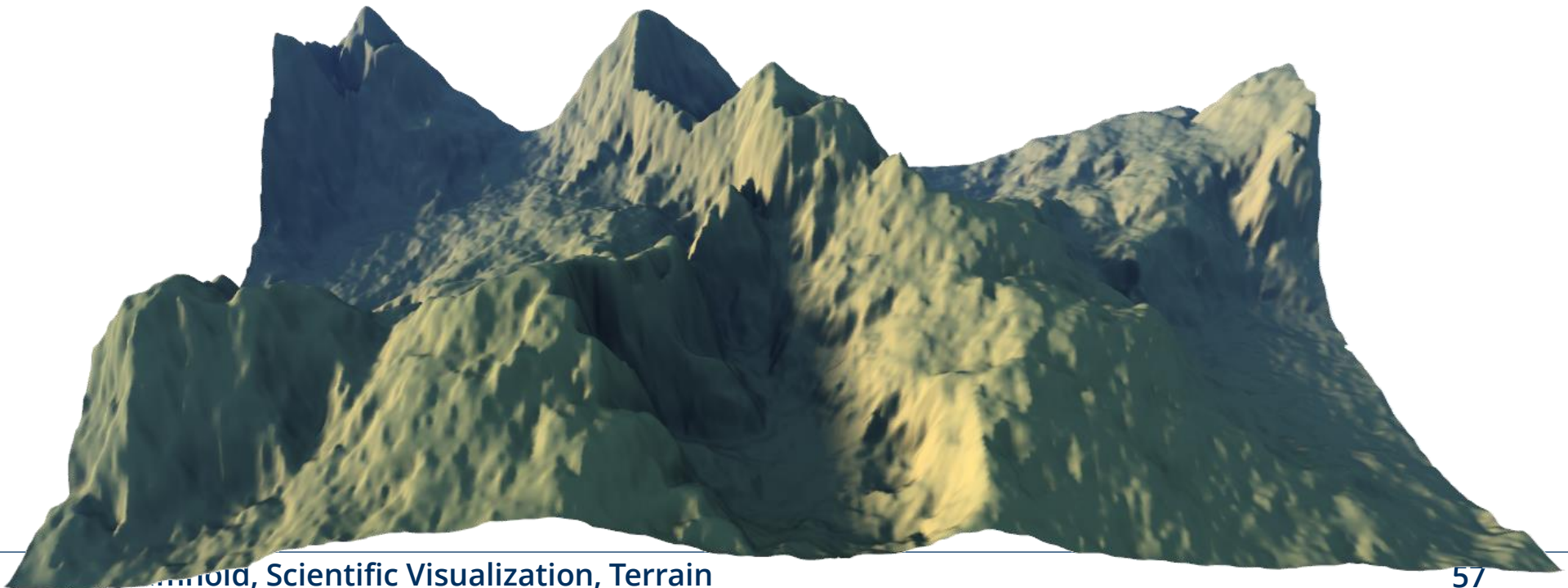


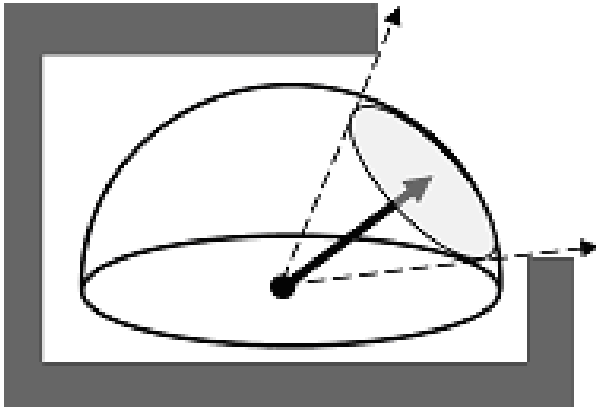


# Ambient Aperture Lighting

Oat, C., & Sander, P. V. (2007, April). Ambient aperture lighting. In *Proceedings of the 2007 symposium on Interactive 3D graphics and games* (pp. 61-64). ACM.

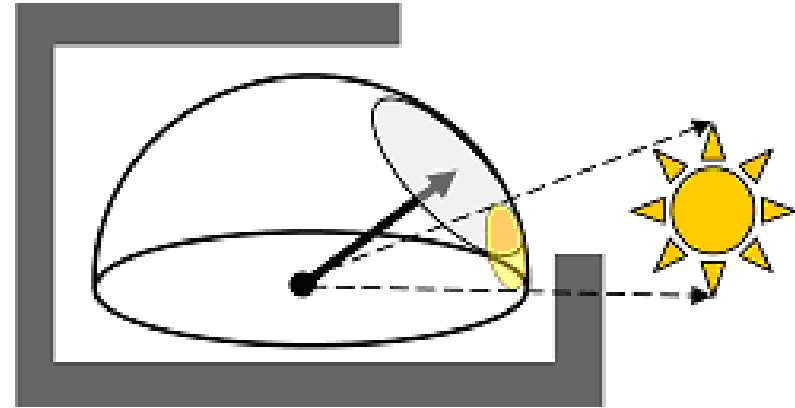
- ◆ Assumption: terrain (optionally with buildings) is static
- ◆ Features: dynamic point & spherical area Light sources with hard & soft shadows
- ◆ Approach: pre-compute visibility





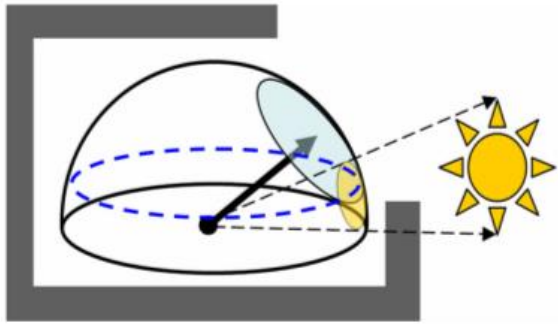
## Precomputation Stage

- ◆ at every mesh vertex / height field texel compute directional visibility
- ◆ approximate visibility function with spherical cap stored as direction and radius

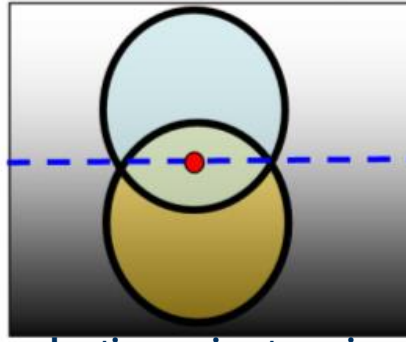


## Rendering Stage

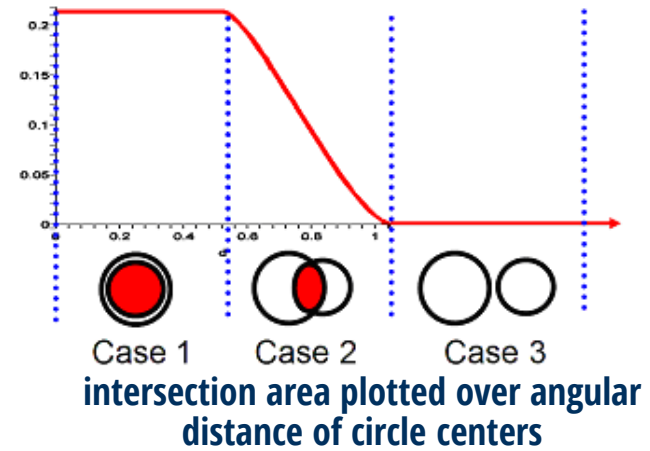
- ◆ Spherical cap acts as an aperture
- ◆ Light sources are projected to sphere and clipped against aperture
- ◆ remainder determines incoming light



circular caps of aperture and area light



Lambertian cosine term is evaluated at center of intersection

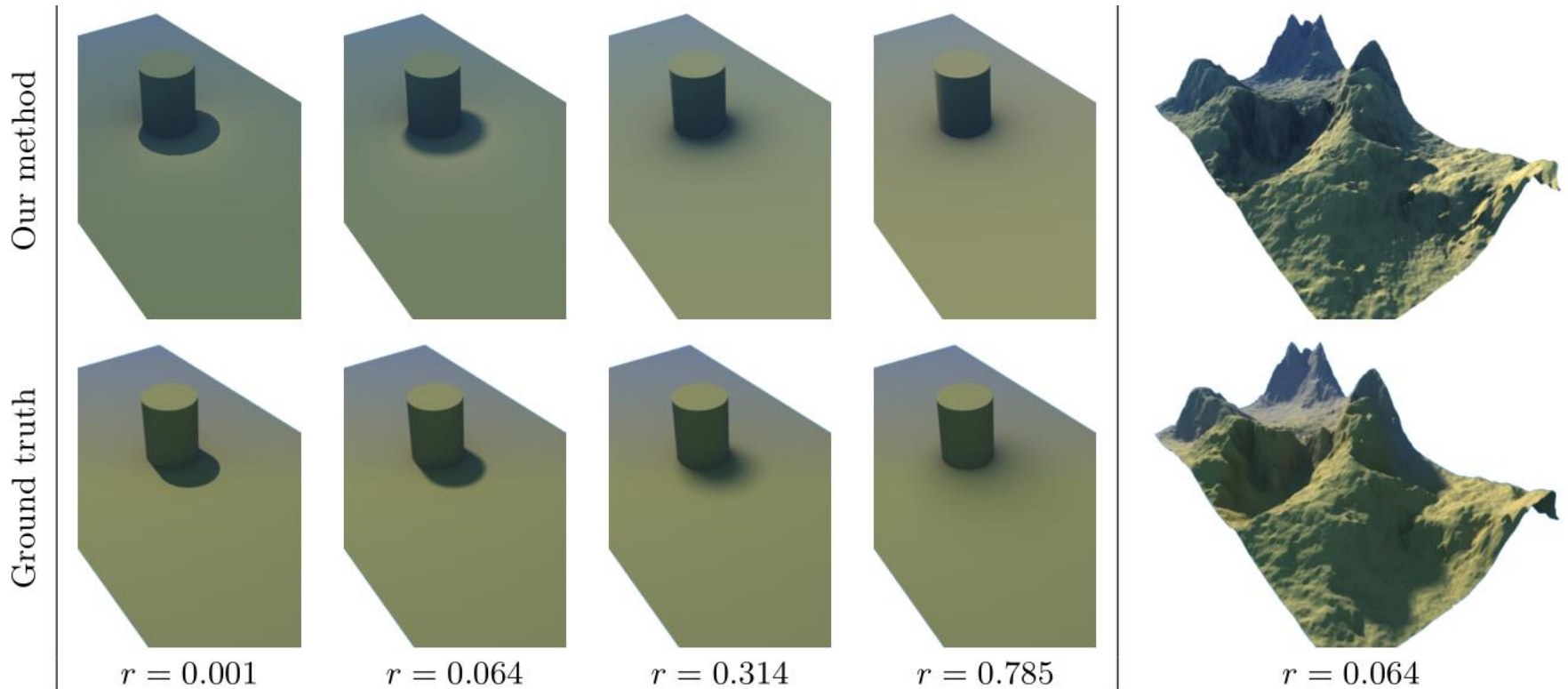


## Lighting in Fragment Shader

- ◆ area light is assumed to be circular too
- ◆ intersect aperture and light circles
  1. one is included in other → result is smaller circle
  2. approximate intersection of circles with smoothstep function
  3. no intersection (full shadow)
- ◆ influence of cosine term in diffuse illumination is approximated to be constant and equal to cosine term at intersection center

# Ambient Aperture Lighting

- ◆ Point light sources are individually clipped at aperture
- ◆ Ambient illumination can be computed with an area light source that covers the whole hemisphere



Terrain

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