Advanced Materials

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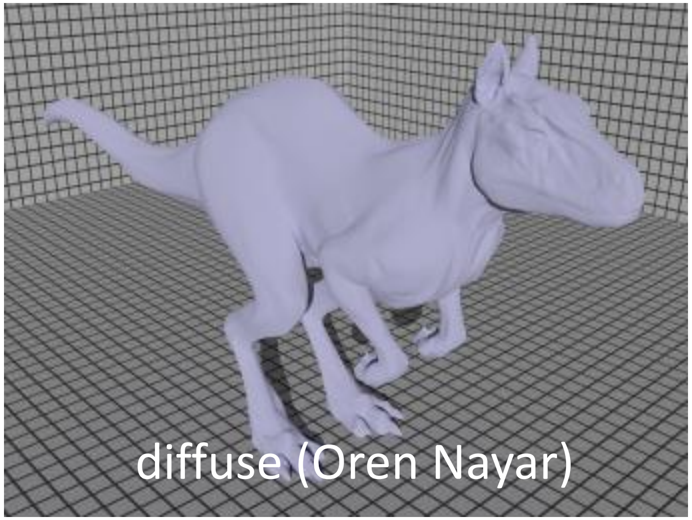
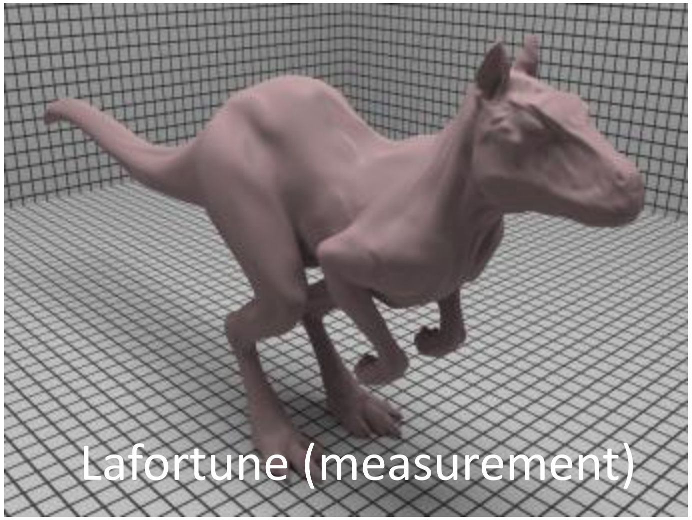
Motivation


diffuse (Oren Nayar)


Torrance Sparrow


Lafortune (measurement)


plastic (Pharr/Humphreys)

Materials and BRDF

- engineers classify materials in main classes:
  - Metals
  - Ceramics (e.g. glass, porcelain)
  - Polymers (nylon, plastic, rubber, etc.)
  - Composites (wood, semiconductors...)
  - Biomaterials (used for body part replacement)
- optic relevant distinction
  - conductor (metals)
  - insulator (dielectric)

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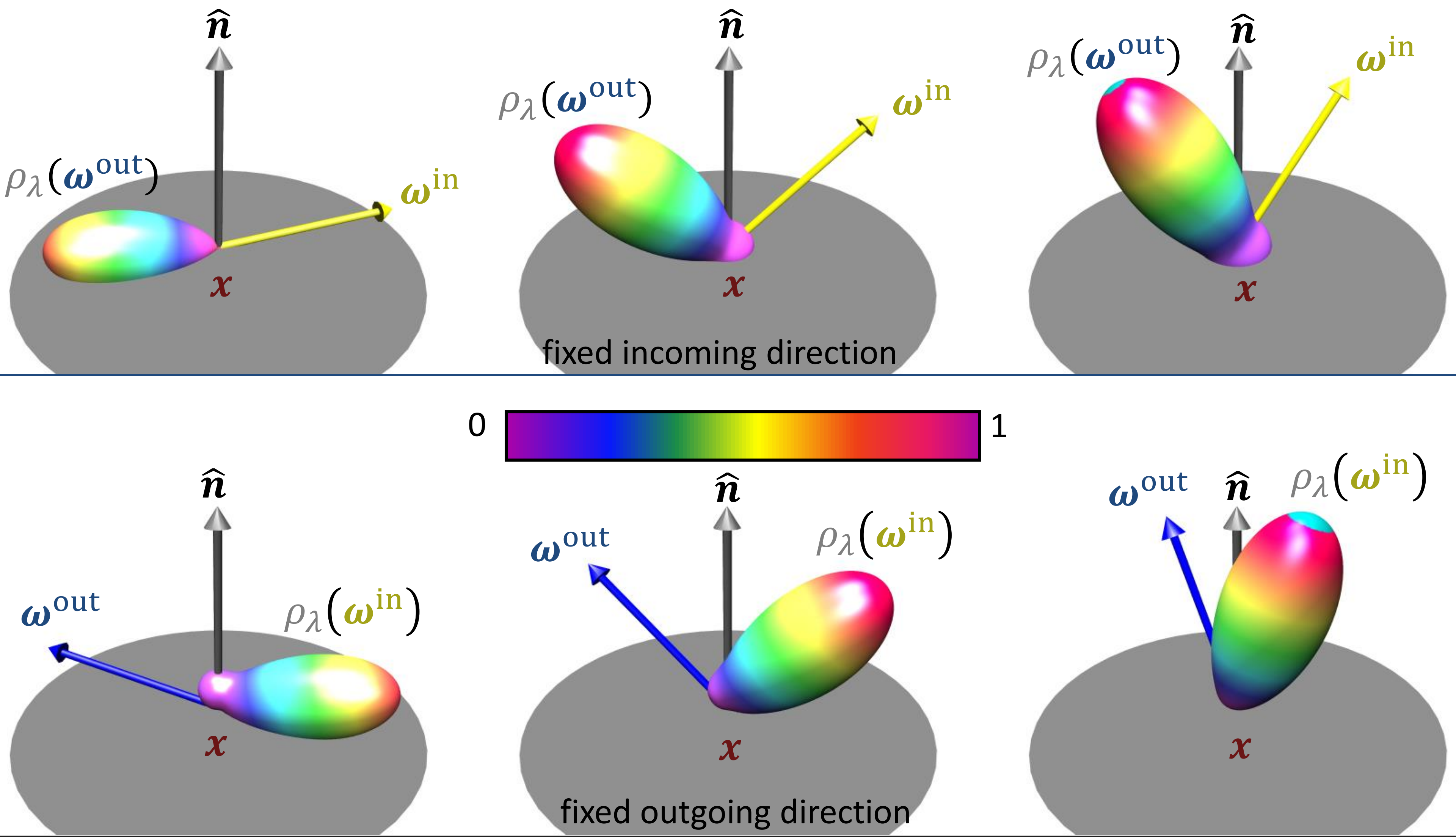
$$\rho_\lambda(\boldsymbol{x}, \boldsymbol{\omega}^{\text{in}}, \boldsymbol{\omega}^{\text{out}}) = \frac{dL_\lambda^{\text{reflect}}(\boldsymbol{x}, \boldsymbol{\omega}^{\text{out}})}{dH_\lambda(\boldsymbol{x}, \boldsymbol{\omega}^{\text{in}})}$$

- Surfaces absorb and reflect light
- The reflection type can vary a lot from purely diffuse to mirror reflection and is characterizes real materials

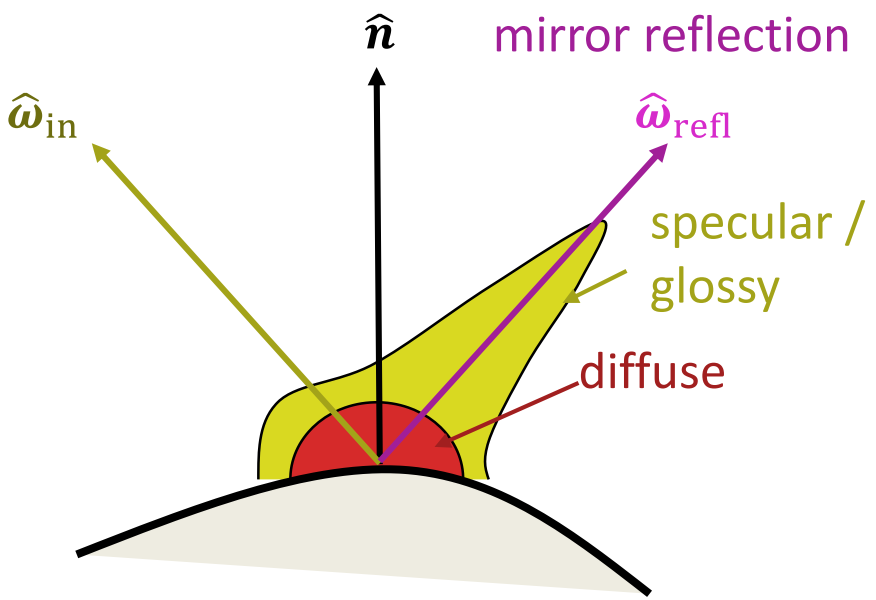
light from behind light from front

BRDF – Visualization



BRDF – Decomposition

- real materials can be approximated by splitting the BRDFs into the sum
  $\dddot{\rho} = \dddot{\rho}_{\mathrm{diff}} + \dddot{\rho}_{\mathrm{spec}} + \dddot{\rho}_{\mathrm{mirror}}$
  - ideal diffuse reflection,
  - specular / glossy reflection
  - ideal mirror reflection, and
- the dependency on wavelength can be modeled through a spectral coefficient $\dddot{\boldsymbol{r}}_*$ times a scalar BRDF f_*:
  $\dddot{\rho} = f_{\mathrm{d}} \cdot \dddot{\boldsymbol{r}}_{\mathrm{d}} + f_{\mathrm{s}} \cdot \dddot{\boldsymbol{r}}_{\mathrm{s}} + f_{\mathrm{m}} \cdot \dddot{\boldsymbol{r}}_{\mathrm{m}}$
- for most insulators the spectral coefficients $\dddot{\boldsymbol{r}}_{\mathrm{s}}$ and $\dddot{\boldsymbol{r}}_{\mathrm{m}}$ are set to $\dddot{\mathbf{1}}$, i.e. the specular reflection is mirror like and not color selective



content

- Empirical vs Physical Plausible
- Microfacet Models
  - Roughness and Anisotropy
  - Cook Torrance BRDF
    - Microfacet Distributions
    - Geometry Terms
    - Fresnel Equations
  - Oren-Nayar BRDF
  - Anisotropic BRDF
- The BRDF Zoo
- BRDF Measurement
- BTFs
- BSSRDF

Literatur

- Andrew S. Glassner, Principles of Digital Image Synthesis, chapters 11 and 15 (download)

Empirical Shading Models

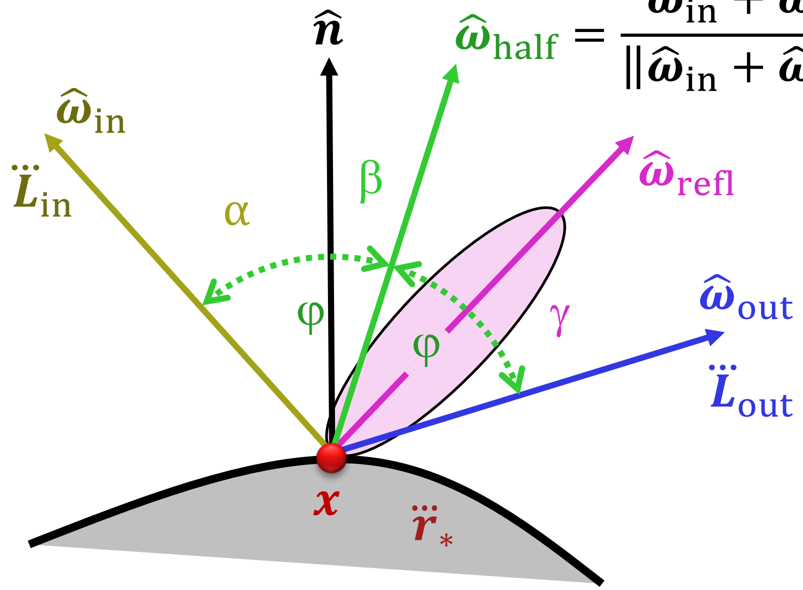
- empirical shading models have been developed for point and directional light sources
- the reflection integral is approximated by a sum over point / directional light sources

$$\ddot{L}_{\mathrm{reflect}} = \sum_{i}^{n} \ddot{L}_{\mathrm{reflect,i}}\left(\hat{\omega}_{\mathrm{in}}, \hat{\omega}_{\mathrm{out}}, \ddot{L}_{\mathrm{in},i}\right)$$

Prominent examples are

- Lambertian model (diffuse only)
- Phong model (diffuse + specular)
- Blinn-Phong: similar to Phong with variant in specular term

$$\hat{\omega}_{\mathrm{half}} = \frac{\hat{\omega}_{\mathrm{in}} + \hat{\omega}_{\mathrm{out}}}{\|\hat{\omega}_{\mathrm{in}} + \hat{\omega}_{\mathrm{out}}\|}$$



$$\ddot{L}_{\mathrm{diff}} = \ddot{r}_{\mathrm{diff}} \otimes \ddot{L}_{\mathrm{in}} \cdot \langle \hat{\omega}_{\mathrm{in}}, \hat{n} \rangle_+$$

shininess

$$\ddot{L}_{\mathrm{spec,Phong}} = \nu \cdot \langle \hat{\omega}_{\mathrm{out}}, \hat{\omega}_{\mathrm{refl}} \rangle_+^m \cdot \ddot{r}_{\mathrm{spec}} \otimes \ddot{L}_{\mathrm{in}}$$

$$\ddot{L}_{\mathrm{spec,Blinn-Phong}} = \nu \cdot \langle \hat{\omega}_{\mathrm{half}}, \hat{n} \rangle_+^m \cdot \ddot{r}_{\mathrm{spec}} \otimes \ddot{L}_{\mathrm{in}}$$

$$\nu = \begin{cases} 1 \ldots \langle \hat{\omega}_{\mathrm{in}}, \hat{n} \rangle \geq 0 \\ 0 \ldots \text{otherwise} \end{cases}$$

Empirical Shading Models

Computergraphik und Visualisierung

- scalar BRDFs are defined implicitly:

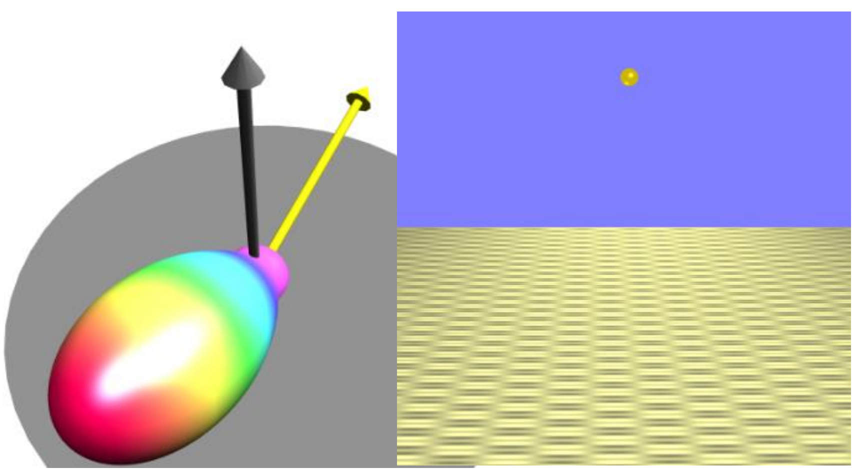
$$\nu = \begin{cases} 1 \ldots \langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle \geq 0 \\ 0 \ldots \text{otherwise} \end{cases}$$

$$\ddot{\boldsymbol{L}}_{\text{diff}} = \ddot{\boldsymbol{r}}_{\text{diff}} \otimes \ddot{\boldsymbol{L}}_{\text{in}} \cdot \langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle_+ \quad \Rightarrow \quad f_{\text{d}} = 1$$

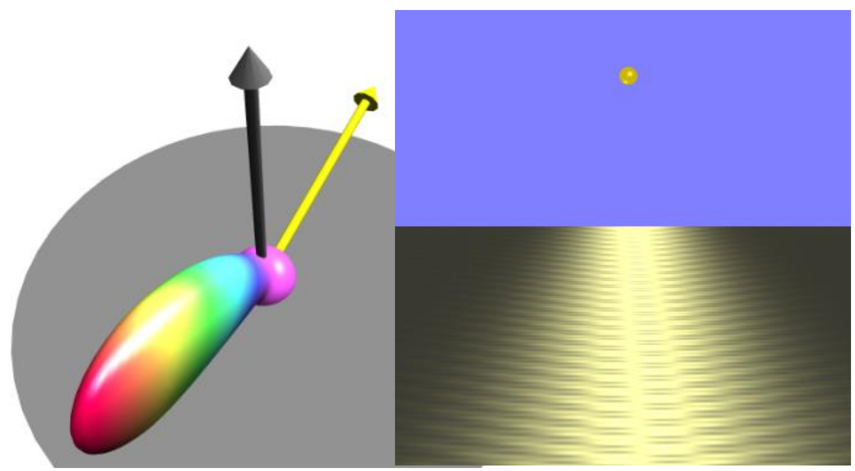
$$\ddot{\boldsymbol{L}}_{\text{spec,Phong}} = \nu \cdot \langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{refl}} \rangle_+^m \cdot \ddot{\boldsymbol{r}}_{\text{spec}} \otimes \ddot{\boldsymbol{L}}_{\text{in}} \Rightarrow f_{\text{s,Phong}} = \nu \cdot \frac{\langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{refl}} \rangle_+^m}{\langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle_+}$$

$$\ddot{\boldsymbol{L}}_{\text{spec,Blinn-Phong}} = \nu \cdot \langle \hat{\boldsymbol{\omega}}_{\text{half}}, \hat{\boldsymbol{n}} \rangle_+^m \cdot \ddot{\boldsymbol{r}}_{\text{spec}} \otimes \ddot{\boldsymbol{L}}_{\text{in}} \Rightarrow f_{\text{s,Blinn-Phong}} = \nu \cdot \frac{\langle \hat{\boldsymbol{\omega}}_{\text{half}}, \hat{\boldsymbol{n}} \rangle_+^m}{\langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle_+}$$

- When looking towards light source above a reflecting sources, Phong model is unrealistically:



Phong



Blinn-Phong

Empirical Ambient Shading

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- For ambient shading one assumes a homogeneous irrandiance H_λ
- reflected radiance $L_{\lambda,\mathrm{amb}}$ needs to be integrated over $\boldsymbol{\omega}^{\mathrm{in}}$:

$$L_{\lambda,\mathrm{amb}}(\boldsymbol{\omega}^{\mathrm{out}}) = \iint_{\Omega^{\mathrm{in}}} \rho_\lambda(\boldsymbol{\omega}^{\mathrm{in}}, \boldsymbol{\omega}^{\mathrm{out}}) \frac{H_\lambda}{\pi} \cos\theta^{\mathrm{in}} d\Omega^{\mathrm{in}} = \frac{H_\lambda}{\pi} \cdot B_\lambda(\boldsymbol{\omega}^{\mathrm{out}})$$

- with the bi-hemispherical reflectance B_λ

$$B_\lambda(\boldsymbol{\omega}^{\mathrm{out}}) = \iint_{\Omega^{\mathrm{in}}} \rho_\lambda(\boldsymbol{\omega}^{\mathrm{in}}, \boldsymbol{\omega}^{\mathrm{out}}) \cos\theta^{\mathrm{in}} d\Omega^{\mathrm{in}}$$

- $B_\lambda(\boldsymbol{\omega}^{\mathrm{out}})$ is also called white-sky albedo and dual to directional hemispherical reflectance $R_\lambda(\boldsymbol{\omega}^{\mathrm{in}})$, which is also called black-sky albedo
- Empirical ambient shading assumes B_λ to be independent of $\boldsymbol{\omega}^{\mathrm{out}}$:

$$\dddot{\boldsymbol{L}}_{\mathrm{amb}} = f_{\mathrm{a}} \cdot \ddot{\boldsymbol{r}}_{\mathrm{amb}} \otimes \frac{\dddot{\boldsymbol{H}}_{\mathrm{in}}}{\pi} \quad \text{with } f_{\mathrm{a}} \equiv 1.$$

Physical Plausibility

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- Any physical plausible BRDF-modell must fulfill the following two properties

Helmholtz-Reciprocity (HR):
$\rho_\lambda(\boldsymbol{x}, \boldsymbol{\omega}^{\text{in}}, \boldsymbol{\omega}^{\text{out}}) = \rho_\lambda(\boldsymbol{x}, \boldsymbol{\omega}^{\text{out}}, \boldsymbol{\omega}^{\text{in}})$

Energy Preservation (EP):
$\forall \boldsymbol{\omega}^{\text{in}}: R_\lambda(\boldsymbol{\omega}^{\text{in}}) \leq 1$

- HR is typically enforced for each individual scalar model:

$$f_*(\boldsymbol{\omega}^{\text{in}}, \boldsymbol{\omega}^{\text{out}}) = f_*(\boldsymbol{\omega}^{\text{out}}, \boldsymbol{\omega}^{\text{in}})$$

- EP can be enforced by limiting the coefficient sums of the $\dddot{\boldsymbol{r}}_*$ to $[0,1]$ and by scaling the f_* appropriately, e.g. $f_d = 1/\pi$.

Physical Plausibility for Specular Models

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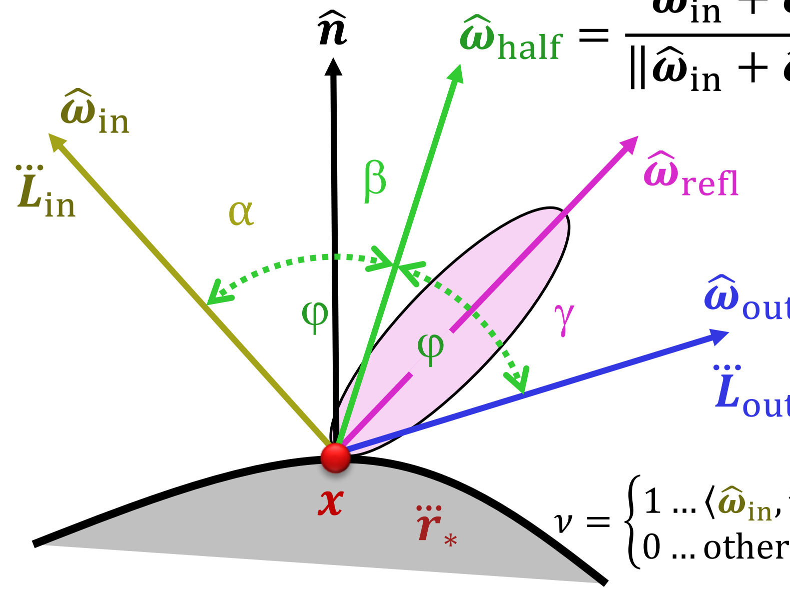
Helmholtz Reciprocity

- analyzing the constitutes:
  - ν is asymmetric only when the hemispherical domain is left
  - $\hat{\boldsymbol{\omega}}_{\text{half}}$ is symmetric with respect to switching $\hat{\boldsymbol{\omega}}_{\text{in}}$ and $\hat{\boldsymbol{\omega}}_{\text{out}}$.
  - $\langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{refl}} \rangle$ is also symmetric
- Thus we just need to skip the denominator terms, yielding:

$$f_{\text{s,Ph,mod}} = \nu \cdot \eta_{Ph} \cdot \langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{refl}} \rangle_+^m$$

$$f_{\text{s,Bl-Ph,mod}} = \nu \cdot \eta_{BP} \cdot \langle \hat{\boldsymbol{\omega}}_{\text{half}}, \hat{\boldsymbol{n}} \rangle_+^m$$

$$\hat{\boldsymbol{\omega}}_{\text{half}} = \frac{\hat{\boldsymbol{\omega}}_{\text{in}} + \hat{\boldsymbol{\omega}}_{\text{out}}}{\|\hat{\boldsymbol{\omega}}_{\text{in}} + \hat{\boldsymbol{\omega}}_{\text{out}}\|}$$



$$\nu = \begin{cases} 1 \ldots \langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle \geq 0 \\ 0 \ldots \text{otherwise} \end{cases}$$

$$f_{\text{s,Phong}} = \nu \cdot \frac{\langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{refl}} \rangle_+^m}{\langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle_+}$$

$$f_{\text{s,Blinn-Phong}} = \nu \cdot \frac{\langle \hat{\boldsymbol{\omega}}_{\text{half}}, \hat{\boldsymbol{n}} \rangle_+^m}{\langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle_+}$$

Energy Preservation

(http://www.thetenthplanet.de/archives/255)

- The normalization constants η_* can be computed from $1/R_\lambda(\hat{\boldsymbol{n}})$:

$$\hat{\boldsymbol{\omega}}_{\text{refl}} = 2\langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle \hat{\boldsymbol{n}} - \hat{\boldsymbol{\omega}}_{\text{in}}$$

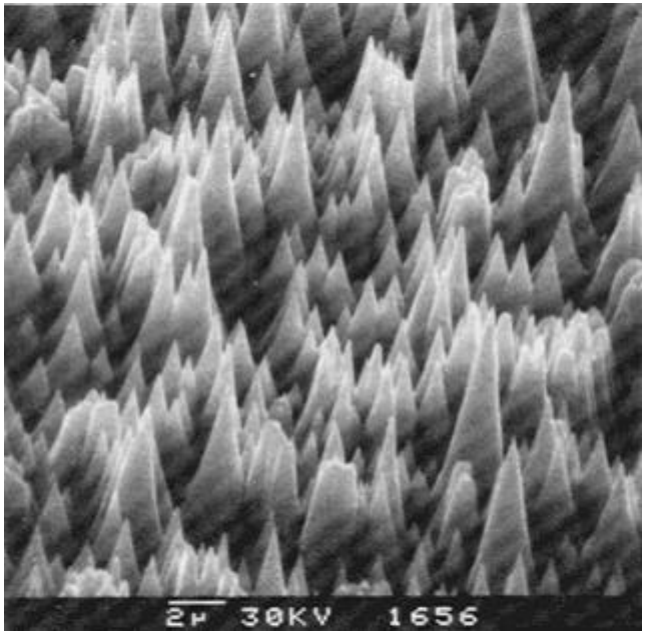
$$\eta_{Ph} = \frac{m+2}{2\pi}, \quad \eta_{BP} = \frac{(m+2)(m+4)}{8\pi(2^{-m/2}+m)}$$

$$\langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{refl}} \rangle = 2\langle \hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{n}} \rangle \langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{n}} \rangle - \langle \hat{\boldsymbol{\omega}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{in}} \rangle$$

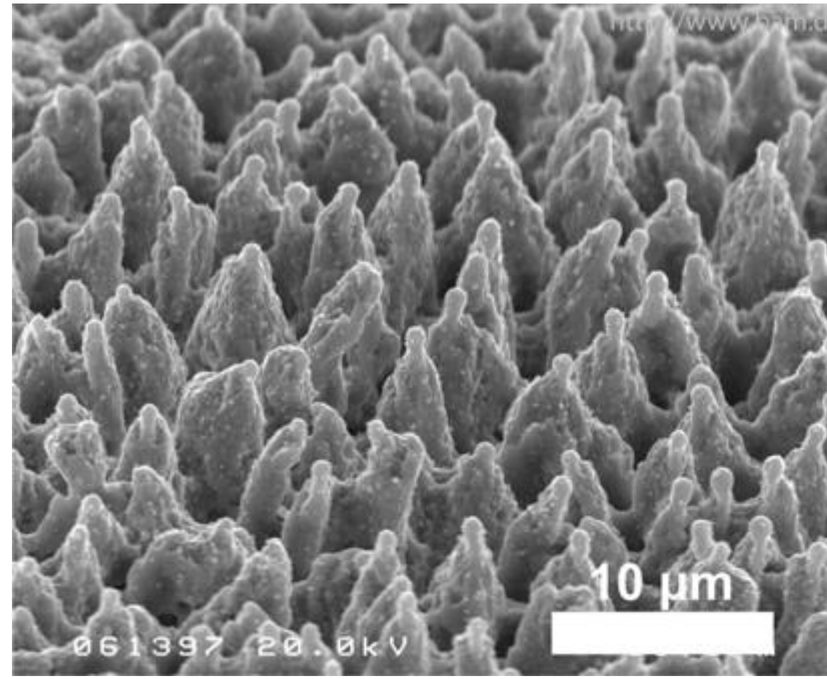
MICROFACET BRDF MODELS

Microstructure & Roughness

Computergraphik und Visualisierung



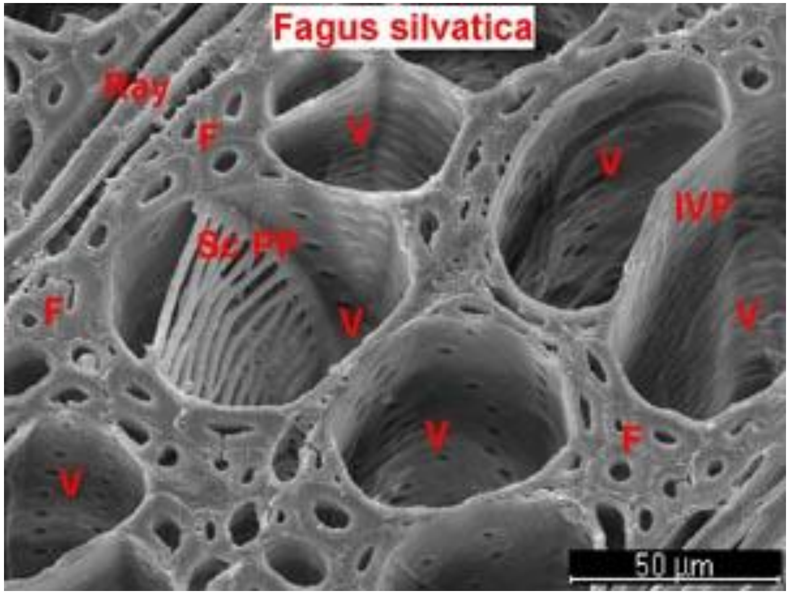
Scanning electron microscope 2: Image of a silver (11,3,1) single crystal surface after bombardment with argon ions. (from: Bergmann/Schäfer, Textbook of Experimental Physics vol. 3)



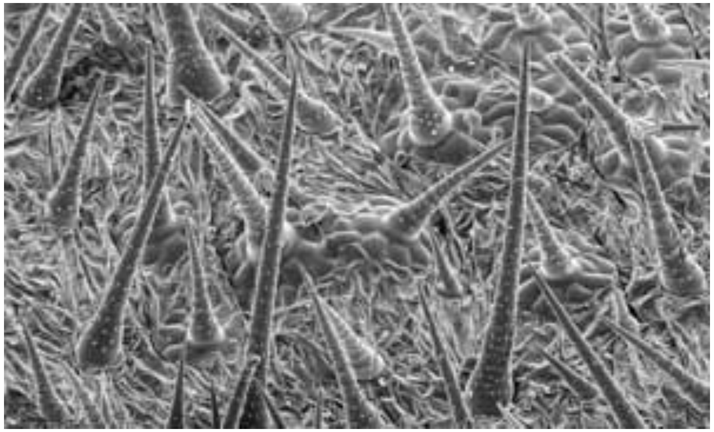
Femtosecond laser microstructured steel surface (scanning electron microscope image)

Microstructure & Roughness

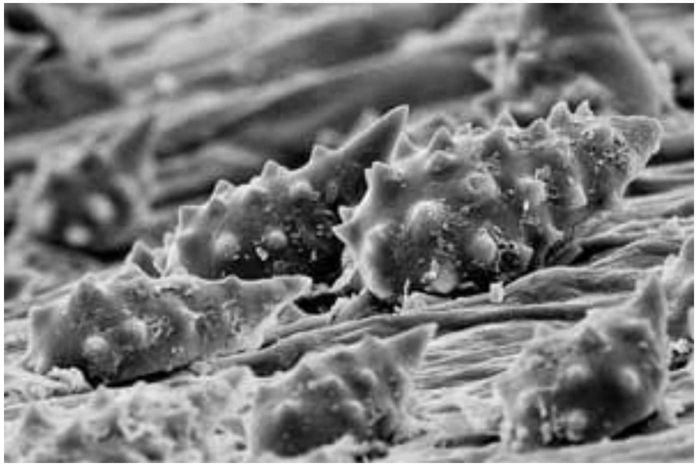
Computergraphik
und Visualisierung



red beech wood, SEM micrograph, cross section (F wood fibre, Ray wood beam, ScPP conductor-shaped vessel opening, V vessel, JVP vessel wall pit)

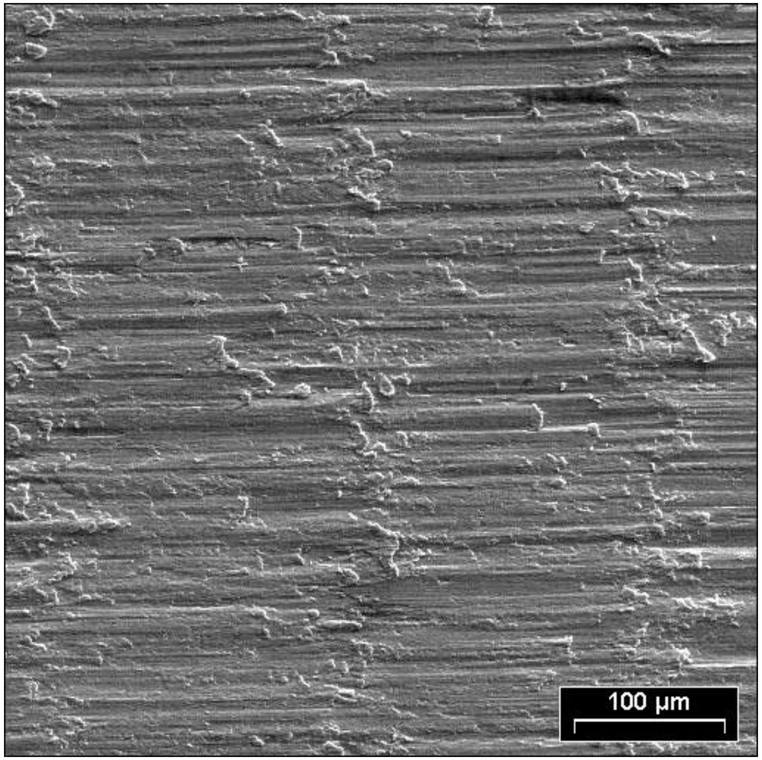


Rough hair from the top of the leaf. The long hairs are about 0.5 mm long.

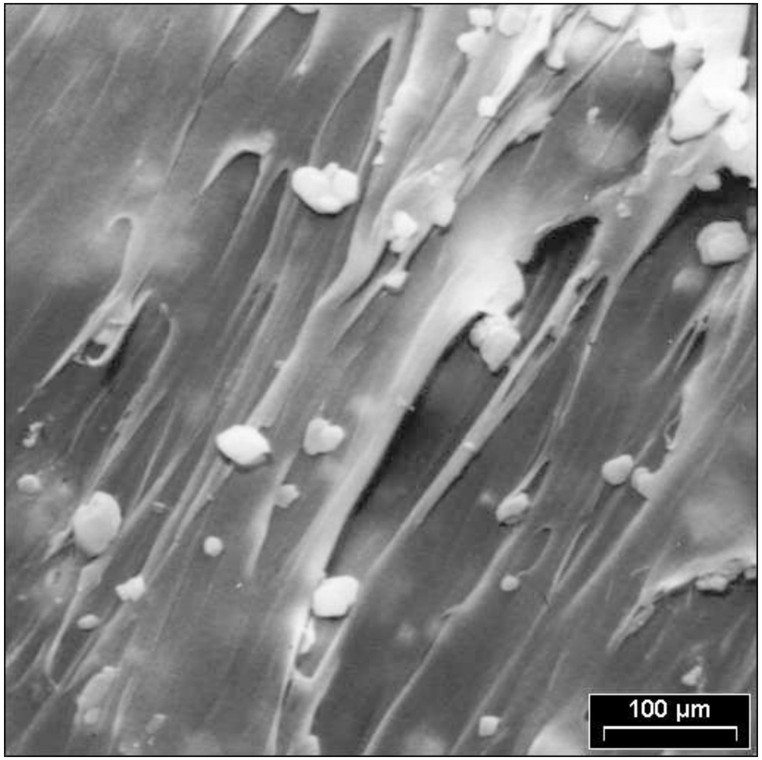


Strongly reduced rough hair on the underside of the leaf of a species from the ever-wet cloud forest. The entire hair is 0.02 to 0.03 mm long.

Microstructure & Anisotropy

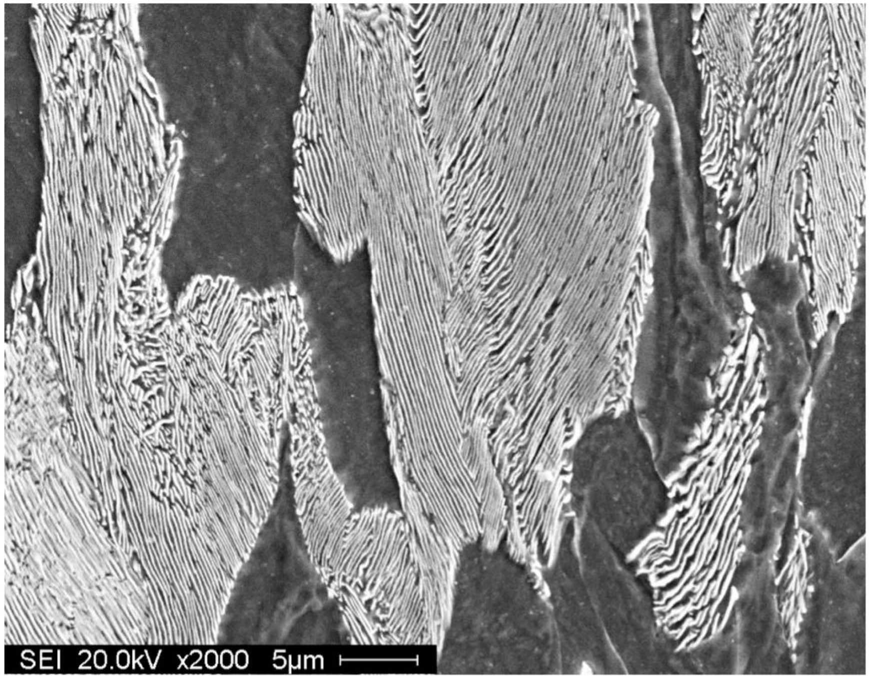


Milled surface



Wear surface of chalk-filled polypropylene

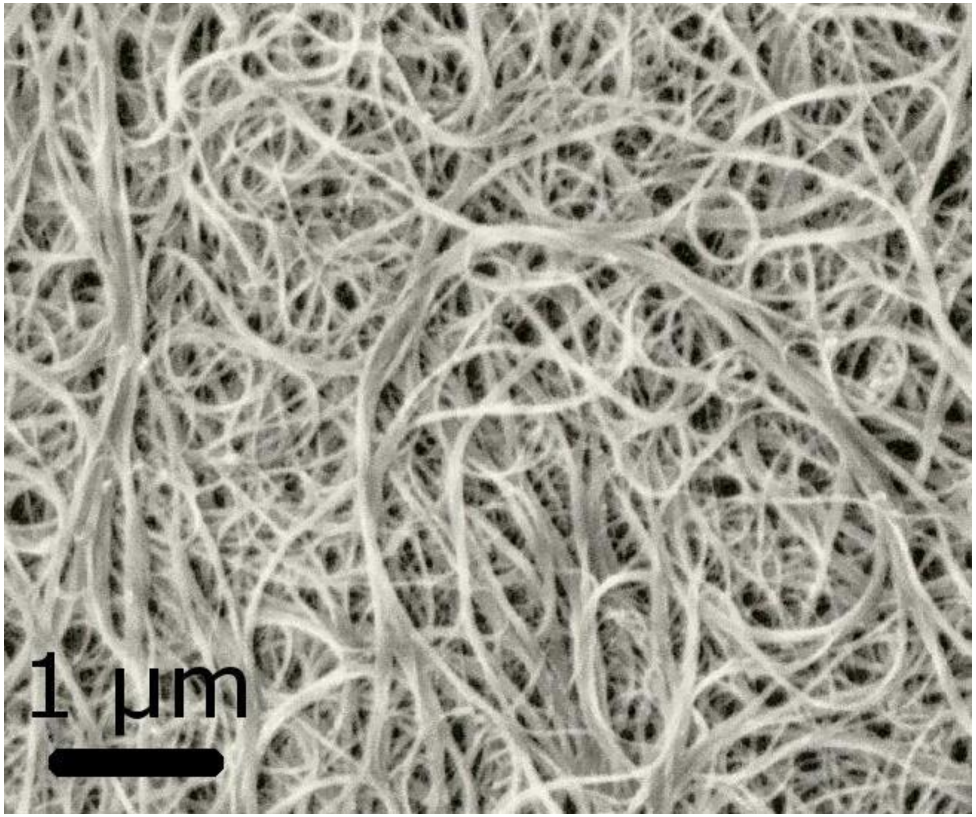
Microstructure Anisotropy



Microstructure of pearlite

Perlite (volcanic glass) with white, powdery appearance

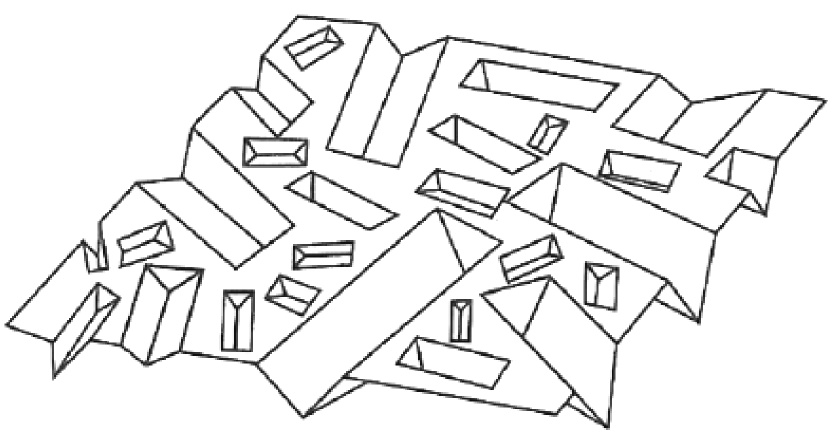
Nanowires

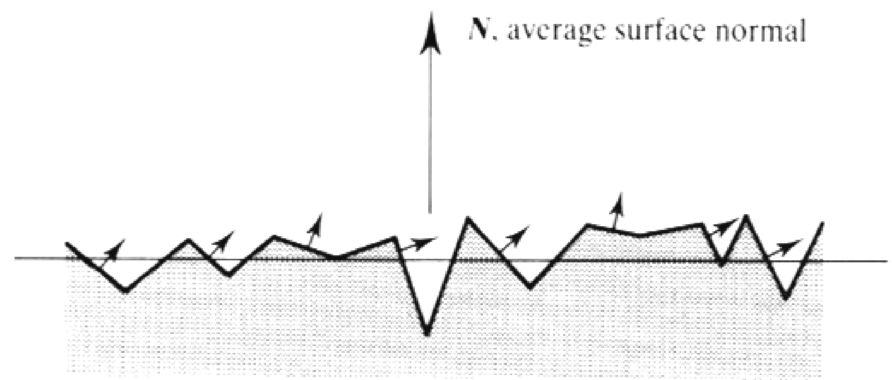


A scanning electron microscopy image of carbon nanotubes bundles

Microfacet Models

- Idea: The surface consists of equally distributed microfacets modeled as planar reflectors.
- The BRDF results from
  - Distribution of the orientation of the microfacets
  - Properties of Planar Reflection
  - Self-occlusion and self-shadowing



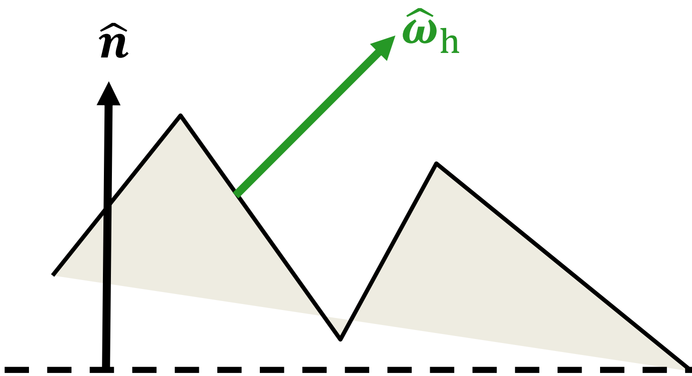


Microfacet Models

- Cook and Torrance consider V-shaped grooves with a mirror reflecting cover and derive the following brdf

$$f_{s,\text{Cook Torrance}} = \frac{F_r(\hat{\boldsymbol{\omega}}_{\text{out}})G(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})D(\hat{\boldsymbol{\omega}}_{\text{h}})}{\pi \cdot \cos\theta_{\text{out}} \cos\theta_{\text{in}}}$$

- with the components
  - $F_r(\hat{\boldsymbol{\omega}}_{\text{out}})$ … reflection on micro facets
  - $G(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})$… geometry term covering self occlusion and self shadowing of V-shaped grooves.
  - $D(\hat{\boldsymbol{\omega}}_{\text{h}})$ … distribution of microfacet normals that correspond to half vectors in Blinn-Phong model
    Normalization: $1 = \int \langle \hat{\boldsymbol{n}}, \hat{\boldsymbol{\omega}}_{\text{h}} \rangle D(\hat{\boldsymbol{\omega}}_{\text{h}}) d\Omega_h$,



- Cook, Robert L., and Kenneth E. Torrance. "A reflectance model for computer graphics." *ACM Transactions on Graphics (TOG)* 1.1 (1982): 7-24. (pdf)

Comparison Phong vs. Torrance

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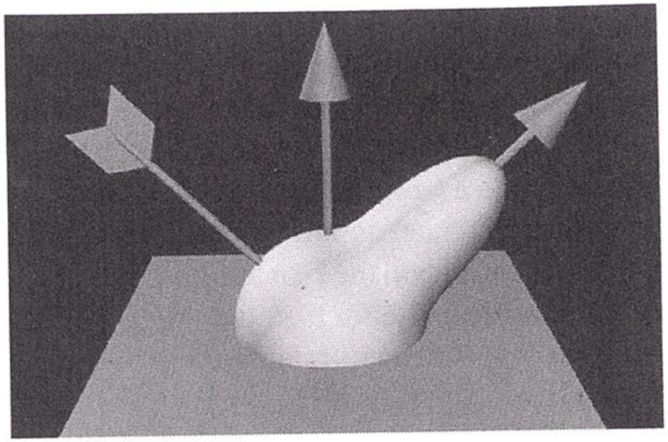
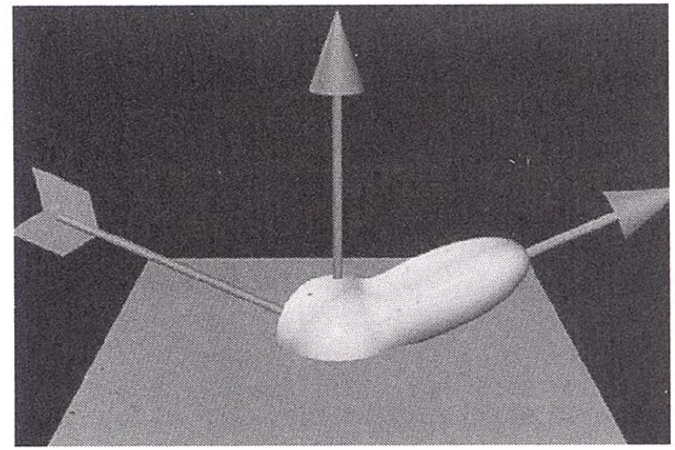
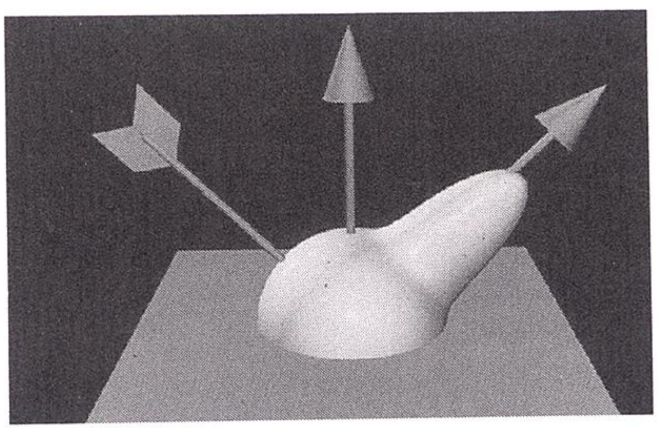


(a)

Phong

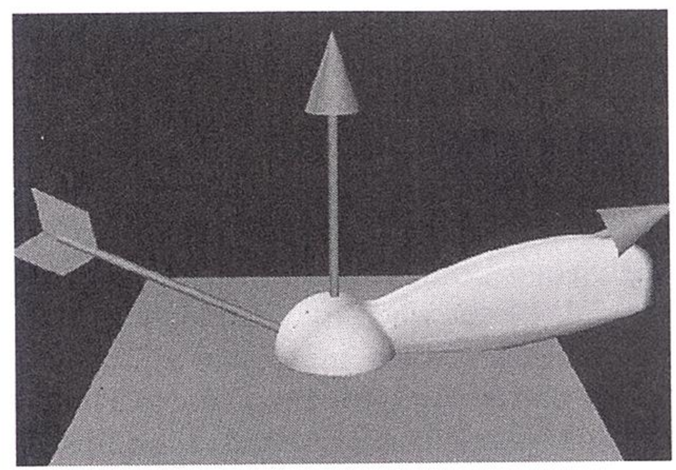


(b)



(c)

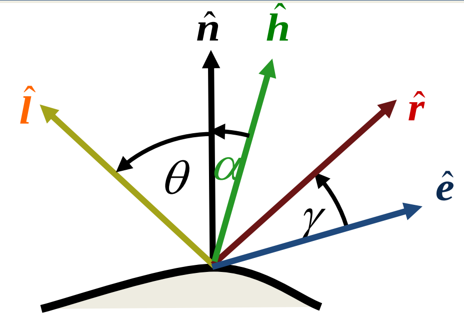
Cook Torrance



(d)

Microfacet Models

- For isotropic distributions $D(\hat{\boldsymbol{\omega}}_{\mathrm{h}})$ parameterization over angle α.



Distributions for Roughness

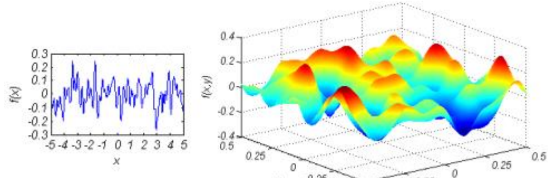
- Blinn
  - exponential with normalization constant c (from Torrance Sparrow)
  - cosine to the power of shininess s

$$D(\alpha) = c \cdot \exp\left(-\frac{\alpha^2}{m^2}\right)$$

$$D(\alpha) = \frac{s+2}{2\pi} \cos^s \alpha$$

- Cook Torrance (Beckmann Theory)
  - $m \in [0,1]$ … roughness measure
  - example of a Gaussian random surface:

$$D(\alpha) = \frac{\exp\left(-\frac{\tan^2 \alpha}{m^2}\right)}{m^2 \cos^4 \alpha}$$



http://www.mysimlabs.com/surface_generation.html

- affine combinations of several distributions allow to approximate materials with multiple layers:

$$D(\alpha) = \sum_i \lambda_i \cdot D_i(\alpha), \text{with } \sum_i \lambda_i = 1$$

Mikrofacetten Modelle

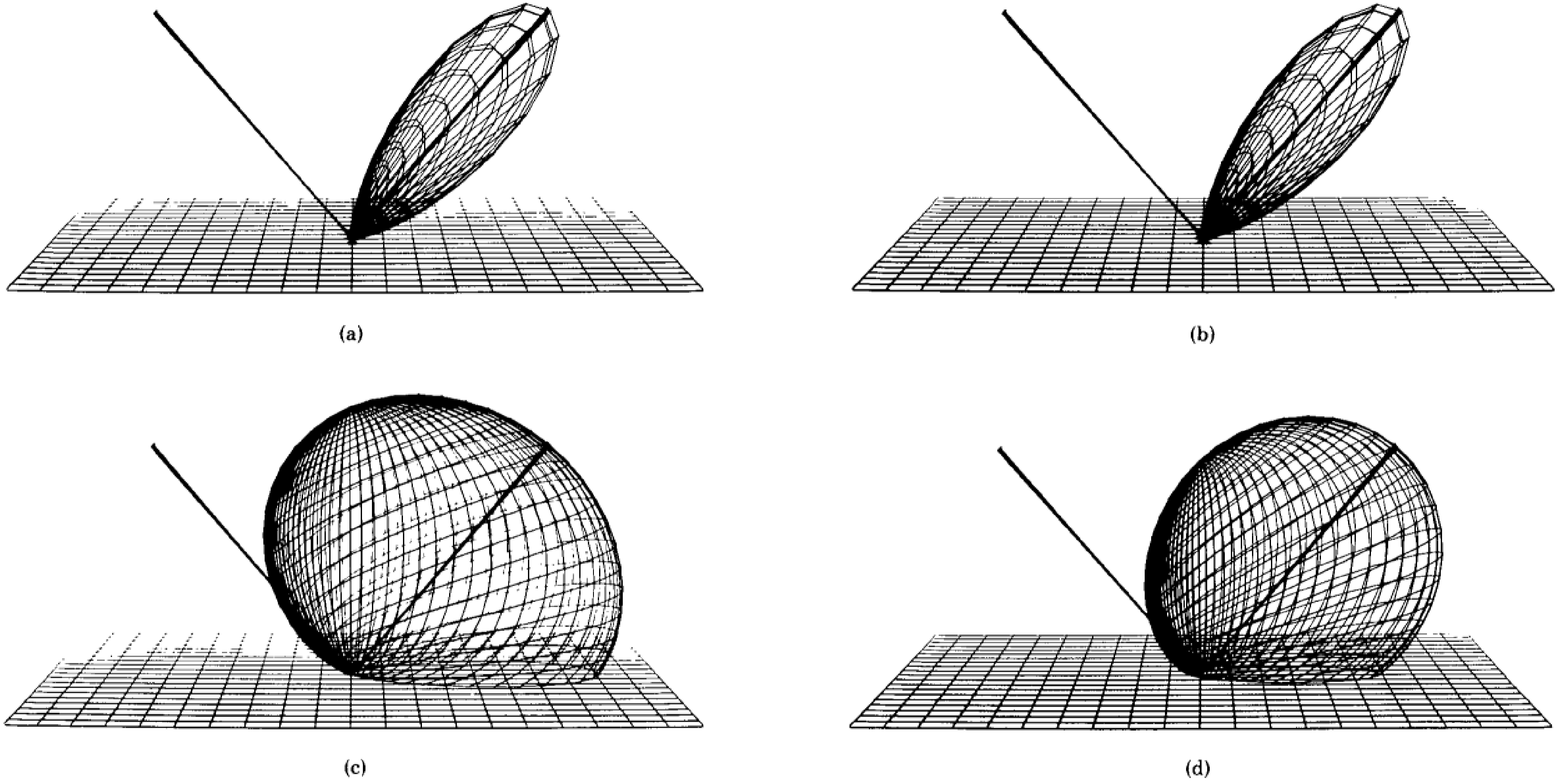
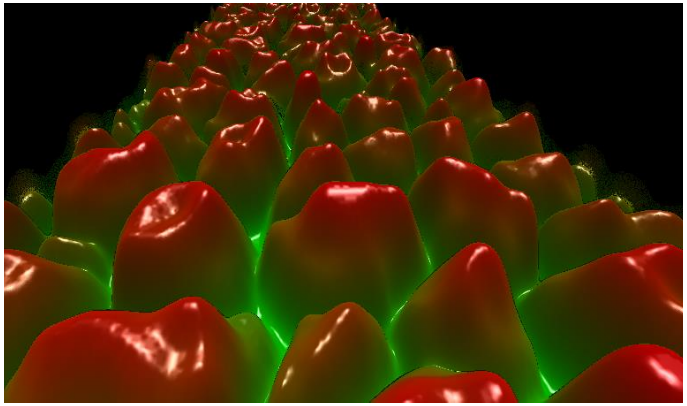
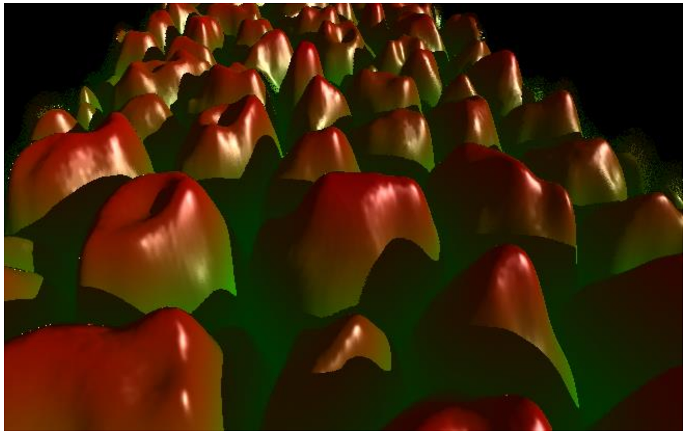


Fig. 3. (a) Beckmann distribution for $m = 0.2$, (b) Gaussian distribution for $m = 0.2$, (c) Beckmann distribution for $m = 0.6$, (d) Gaussian distribution for $m = 0.6$.

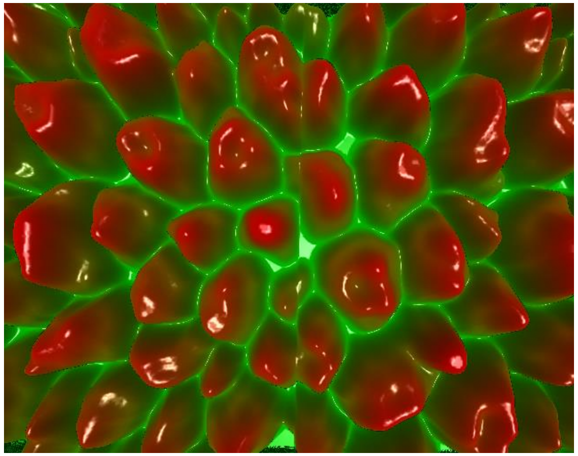
Geometry Term

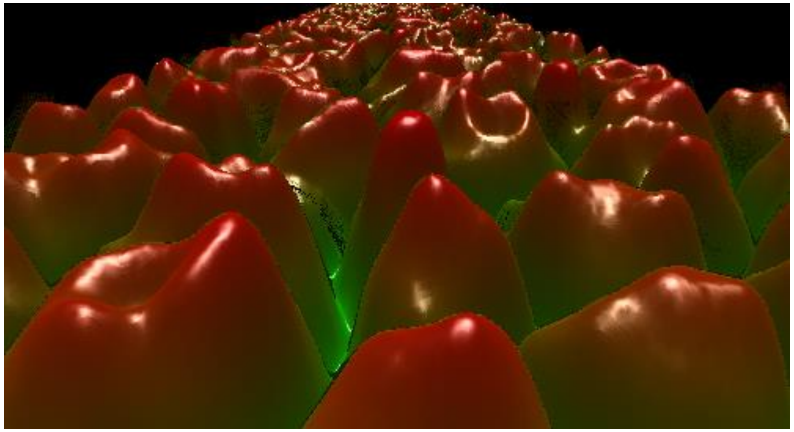
self shadowing





self occlusion





Geometry Term

- Geometry term G models self-shadowing and masking, where minimum is taken:
- Filter approach: $G = \min\left\{g_{\max}, g_{\text{mask}}, g_{\text{shadow}}\right\}$
  - fully illuminated and visible

    $$g_{\max} = 1$$

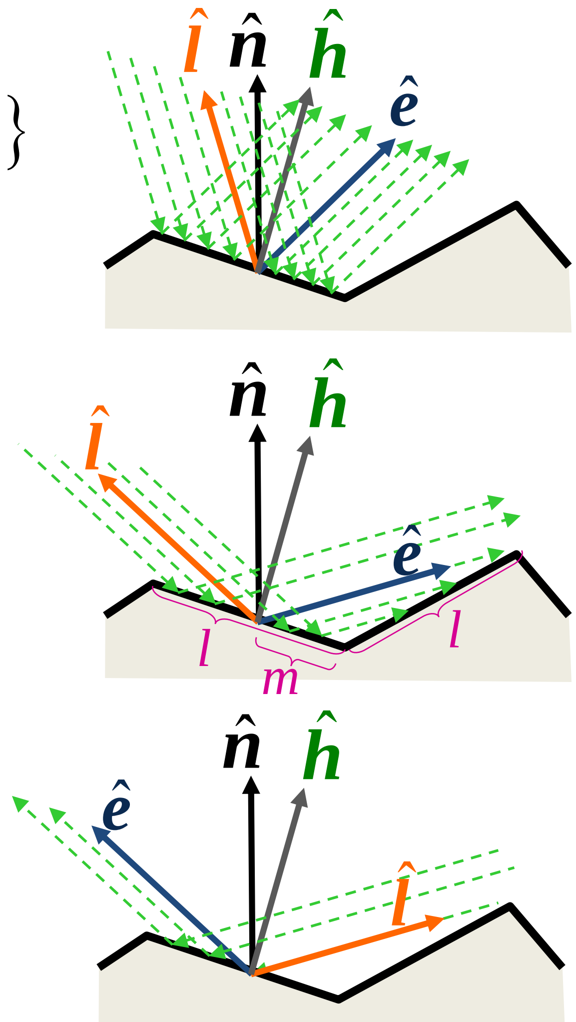
  - occlusion of reflected light

    $$g_{mask} = 1 - \frac{m}{l} = \frac{2(\hat{\boldsymbol{n}}^T\hat{\boldsymbol{h}})(\hat{\boldsymbol{n}}^T\hat{\boldsymbol{e}})}{\hat{\boldsymbol{e}}^T\hat{\boldsymbol{h}}}$$

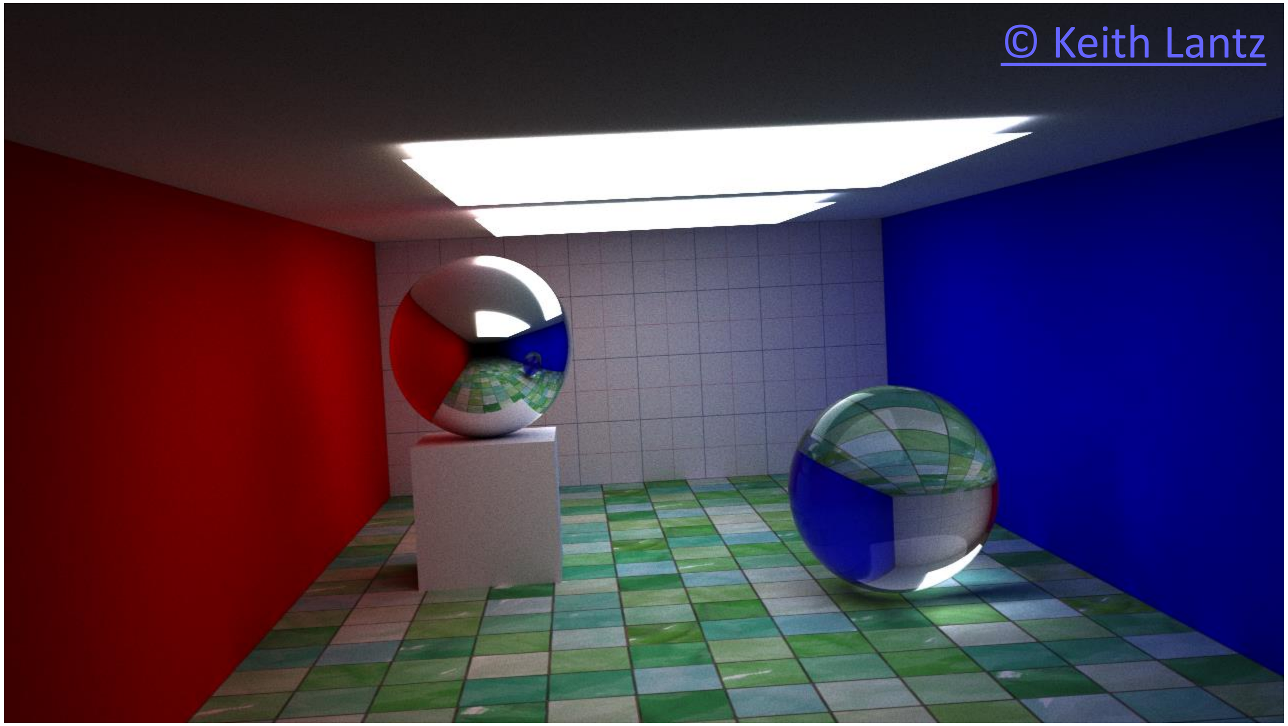
    (details in https://www.microsoft.com/en-us/research/wp-content/uploads/1977/01/p192-blinn.pdf)
  - shadowing of incoming light

    $$g_{\text{shadow}} = \frac{2\left(\hat{\boldsymbol{n}}^T\hat{\boldsymbol{h}}\right)\left(\hat{\boldsymbol{n}}^T\hat{\boldsymbol{l}}\right)}{\hat{\boldsymbol{e}}^T\hat{\boldsymbol{h}}}$$

V-shaped grooves



Fresnel Equations



mixing of Reflection & Refraction via Fresnel Equations on spheres

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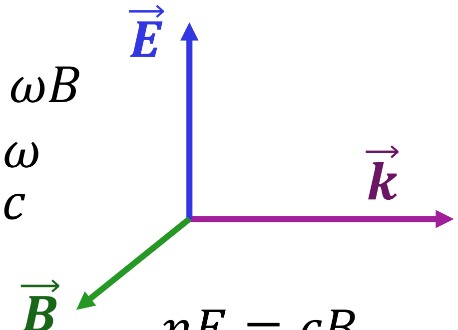
- Wikipedia
- Optics I Script
- R. Cook, K. E. Torrance, A Reflectance Model for Computer Graphics, 1981
- C. Schlick, An inexpensive BRDF model for physically-based rendering, 1994
- I. Lazányi, L. S. Szirmay-Kalos, Fresnel Term Approximation for Metals, 2005

Derivation of Frensnel Equations

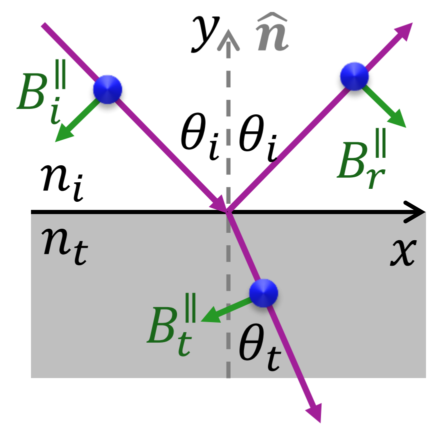
electromagnetic wave

$$kE = \omega B$$
$$kv = \omega$$
$$nv = c$$

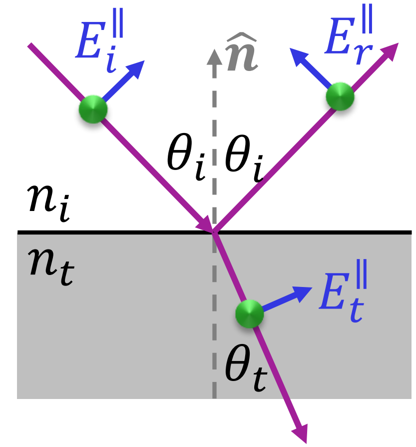


$$nE = cB$$

orthogonal E-component



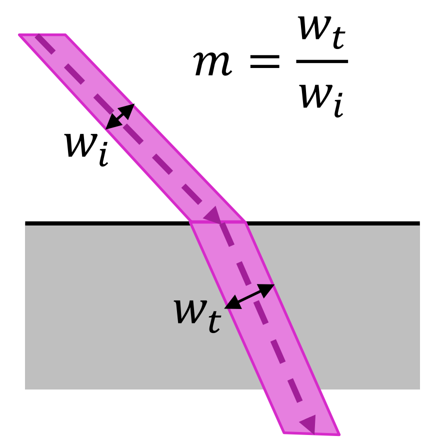
parallel E-component



- E-field continuity: $E_i^{\perp} + E_r^{\perp} = E_t^{\perp}$ $\quad\left(E_i^{\parallel} - E_r^{\parallel}\right)\cos\theta_i = E_t^{\parallel}\cos\theta_t$
- B-field x-continuity: $\left(B_i^{\parallel} - B_r^{\parallel}\right)\cos\theta_i = B_t^{\parallel}\cos\theta_t$ $\quad B_i^{\perp} + B_r^{\perp} = B_t^{\perp}$
- substitute B with E: $n_i\left(E_i^{\perp} - E_r^{\perp}\right)\cos\theta_i = n_t E_t^{\perp}\cos\theta_t$
- substitute $E_t^{\perp}$: $n_i\left(E_i^{\perp} - E_r^{\perp}\right)\cos\theta_i = n_t\left(E_i^{\perp} + E_r^{\perp}\right)\cos\theta_t$
- rearrange: $E_i^{\perp}(n_i\cos\theta_i - n_t\cos\theta_t) = E_r^{\perp}(n_i\cos\theta_i + n_t\cos\theta_t)$
- reflection factor: $r_{\perp} = \frac{E_r^{\perp}}{E_i^{\perp}} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$ $\quad r_{\parallel} = \frac{E_r^{\parallel}}{E_i^{\parallel}} = \frac{n_i\cos\theta_t - n_t\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$
- transmittance factor: $t_{\perp} = \frac{E_t^{\perp}}{E_i^{\perp}} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t}$ $\quad t_{\parallel} = \frac{E_t^{\parallel}}{E_i^{\parallel}} = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i}$

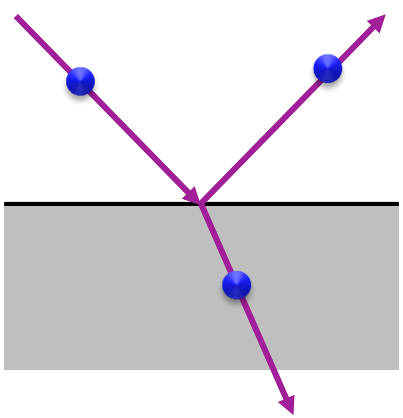
Fresnel Equations for Dielectric

- define propagation slow down $\rho = \frac{n_t}{n_i}$
- and magnification of ray width $m = \frac{\cos\theta_t}{\cos\theta_i}$
- Snell's law $n_i \sin\theta_i = n_t \sin\theta_t$ allows to compute



$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \left(\frac{n_i}{n_t}\sin\theta_i\right)^2}$$

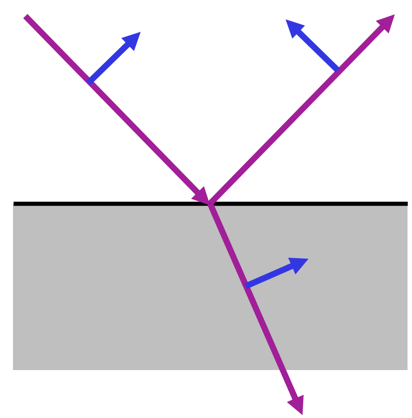
- the definitions simplify Fresnel Equations significantly:



$$r_\perp = \frac{1 - \rho m}{1 + \rho m}$$

orthogonal

$$t_\perp = \frac{2}{1 + \rho m}$$



$$r_\parallel = \frac{m - \rho}{m + \rho}$$

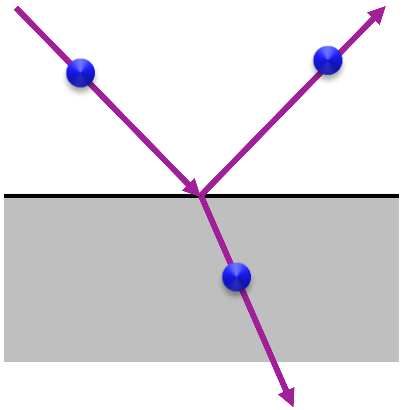
parallel

$$t_\parallel = \frac{2}{m + \rho}$$

Fresnel Equations for Dielectric

$$m = \frac{\cos\theta_t}{\cos\theta_i}$$

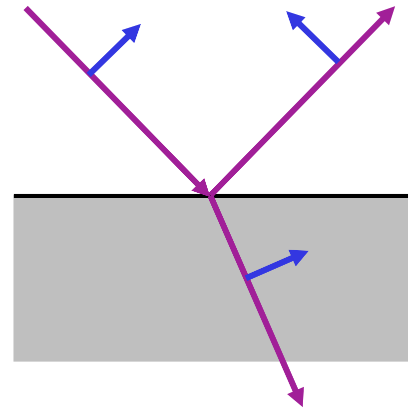
$$\rho = \frac{n_t}{n_i}$$



$$r_\perp = \frac{1-\rho m}{1+\rho m}$$

orthogonal

$$t_\perp = \frac{2}{1+\rho m}$$



$$r_\parallel = \frac{m-\rho}{m+\rho}$$

parallel

$$t_\parallel = \frac{2}{m+\rho}$$

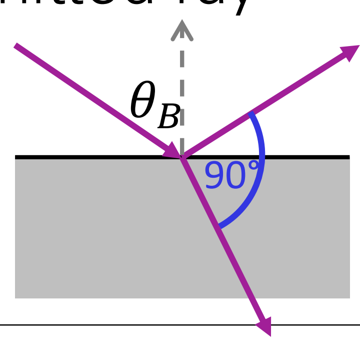
- fraction of reflected light (reflectance) is computed as
$$F_{r,*} = r_*^2$$
- fraction of transmitted light (transmittance) computes to
$$F_{t,*} = \rho m t_*^2$$
- for both components we have energy preservation
$$F_{r,*} + F_{t,*} = 1$$
- when ignoring polarization one combines by averaging
$$F_r = \tfrac{1}{2}\left(F_{r,\parallel} + F_{r,\perp}\right) \;\wedge\; F_t = \tfrac{1}{2}\left(F_{t,\parallel} + F_{t,\perp}\right)$$

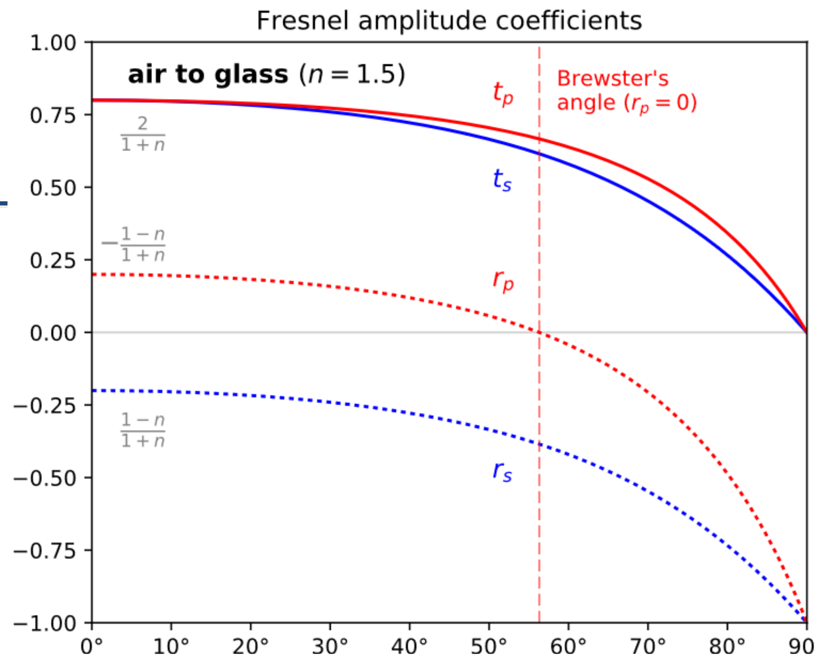
Air to Glass example

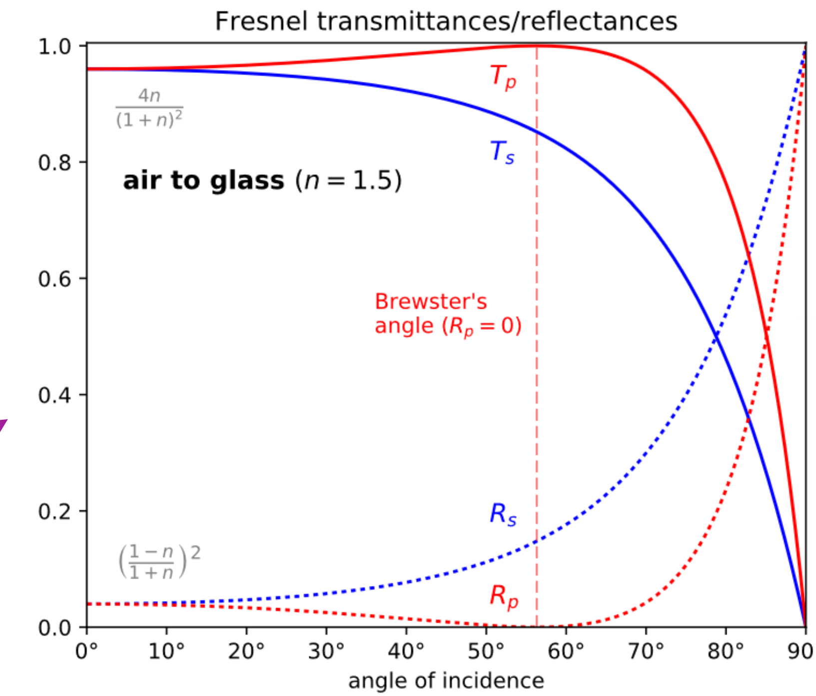
- notation:
  - p … parallel (∥)
  - s … orthogonal [*senkrecht*] (⊥)
  - $R_* = F_{r,*}$ and $T_* = F_{t,*}$
- example: $n_{\text{air}}\approx 1 \ < \ n_{\text{glass}}\approx 1.5$

Note:

- Total reflection at $\theta = 90°$ for both polarizations
- $r_{\parallel} = 0$ when reflected ray is orthogonal to transmitted ray "Brewster's angle":
  $\theta_B = \arctan \rho$
  (56.3° for $\rho = 1.5$)



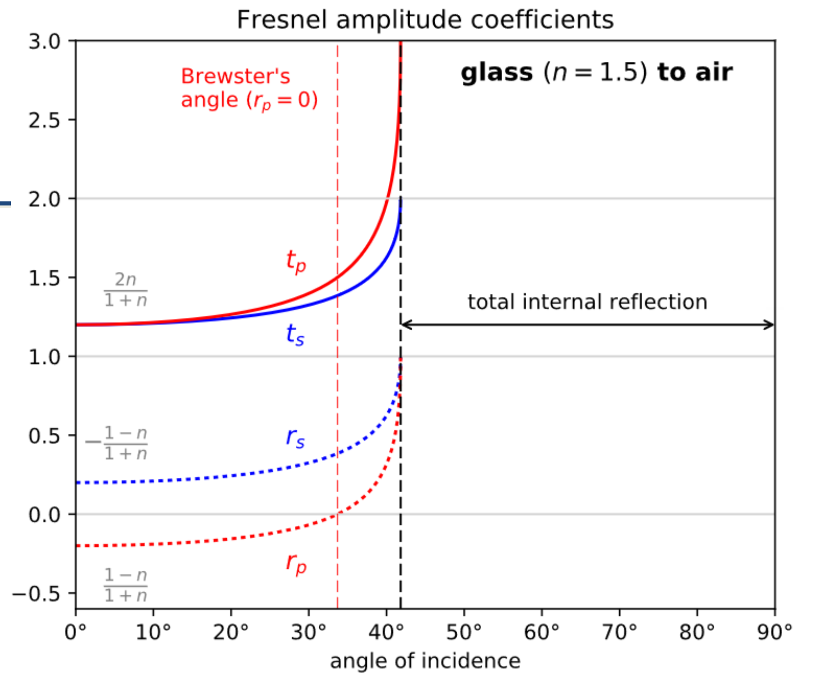


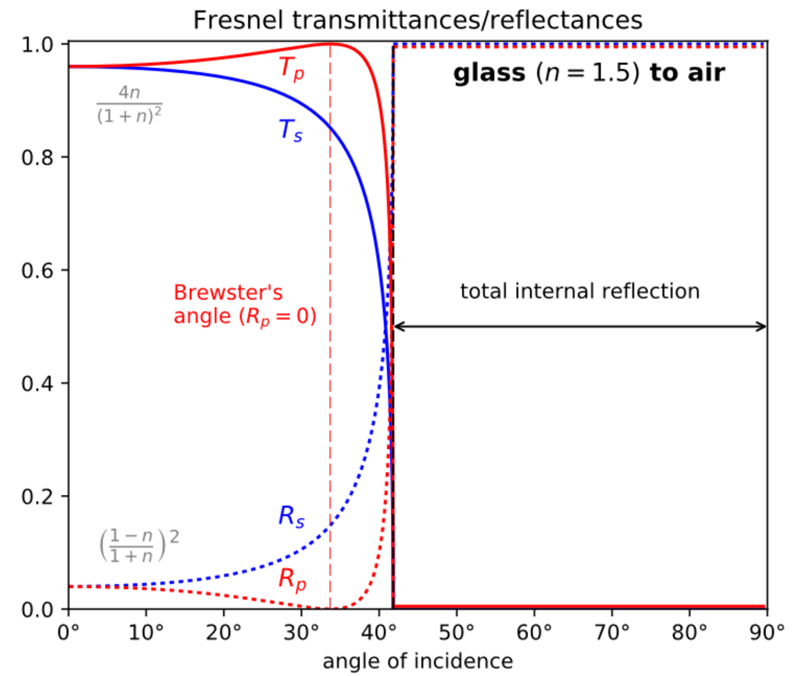


Glass to Air example

Note:

- amplitude transmission factors can be larger than 1
- transmitted power factor is $F_{t,*} = \rho m t_*^2$ with $m \to 0$ at total reflection
- Total internal reflection from Snell's law when $\theta_t = 90°$: $n_i \sin\theta_T = n_t$ yields
$$\theta_T = \arcsin \rho$$





©Wikipedia

Fresnel Equations for Metals

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- for metals no transmitted ray is needed
- refraction index of the metal is complex: $n_t = \eta_t + i\kappa_t$, where κ_t is called extinction coefficient.
- we assume real refraction index of exterior material n_i such that the propagation slow down is: $\rho = \frac{\eta_t}{n_i} + i\,\frac{\kappa_t}{n_i}$
- Snell's law holds also for complex case:
$$n_i \sin\theta_i = \eta_t \sin\theta_t$$
and allows to eliminate θ_t. Computing $F_* = r_* \bar{r}_*$ results in:

$$F_\perp = \frac{(a-\cos\theta_i)^2 + b^2}{(a+\cos\theta_i)^2 + b^2} \qquad a = \tfrac{1}{2}\sqrt{c+d} \qquad c = \sqrt{d^2 + 4n^2\kappa^2}$$

$$F_\parallel = F_\perp \cdot \frac{(a-\sin\theta_i\tan\theta_i)^2 + b^2}{(a+\sin\theta_i\tan\theta_i)^2 + b^2} \qquad b = \tfrac{1}{2}\sqrt{c-d} \qquad d = n^2 - \kappa^2 - \sin^2\theta_i$$

Fresnel Approximations for Metals

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- Cook and Torrance approximate n and κ for metals from one reflectance measurement $F_r(\theta_i = 0)$ and assume $n = 1$ when computing κ and $\kappa = 0$ when computing n:

$$n \approx \frac{1 + \sqrt{F_r(0)}}{1 - \sqrt{F_r(0)}} \quad \wedge \quad \kappa \approx 2\sqrt{\frac{F_r(0)}{1 - F_r(0)}}$$

- Schlick's approximation assumes $\kappa \approx 0$ and $1.4 \leq n \leq 2.2$

$$F_{Schlick} = \frac{(n-1)^2 + 2n(1-\cos\theta_i)^5}{(n+1)^2}$$

- Lazányi's approximation incorporates also κ:

„rescaling"

$$F_{Lazányi} = \frac{(n-1)^2 + 4n(1-\cos\theta_i)^5 + \kappa^2}{(n+1)^2 + \kappa^2}$$

„compensated"

optionally the correction $a\cos\theta_i\,(1-\cos\theta_i)^\alpha$ is subtracted with material specific parameters a and α.

Metal Fresnel Term Approximations

computation time:	28%	33%	54%	100%

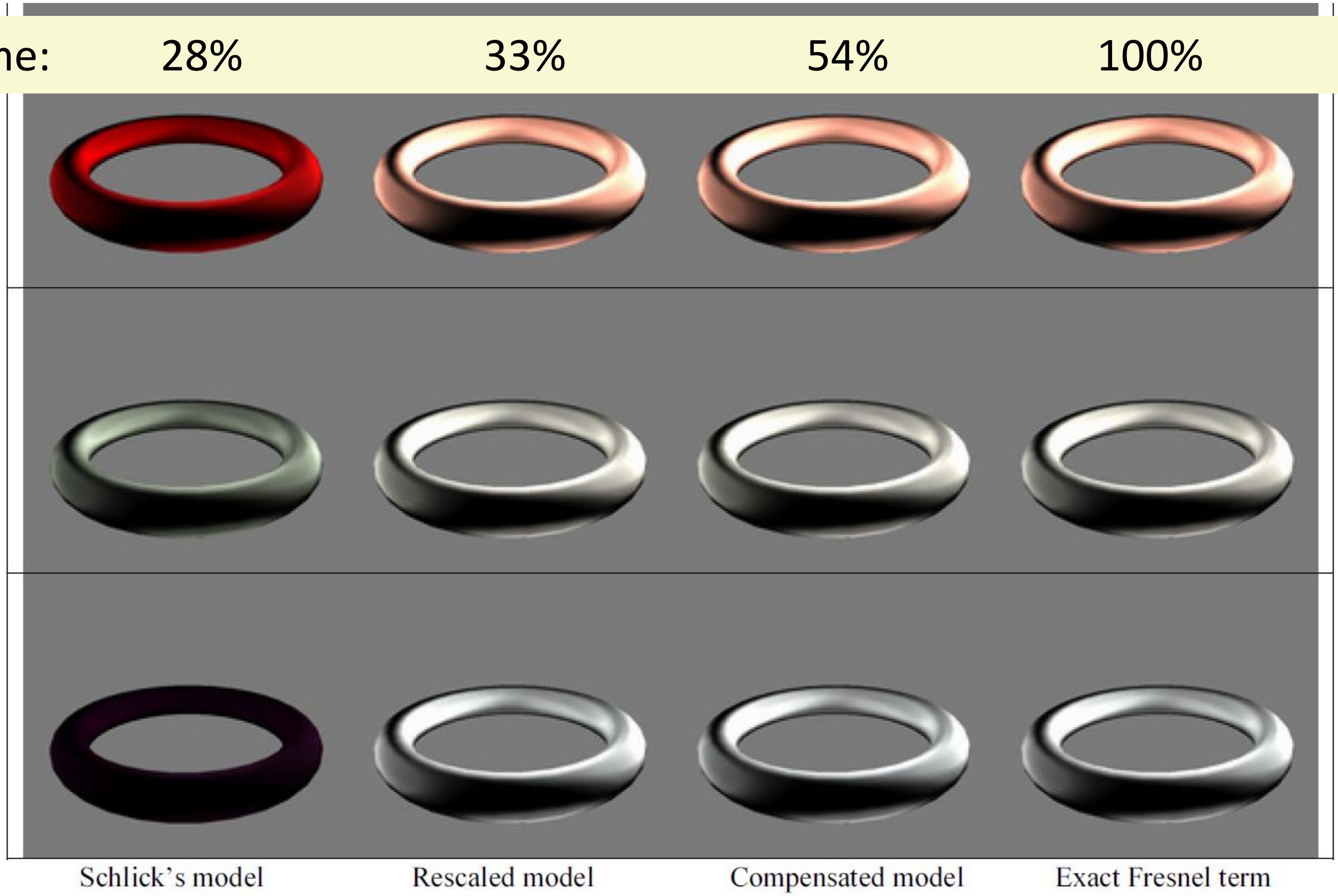
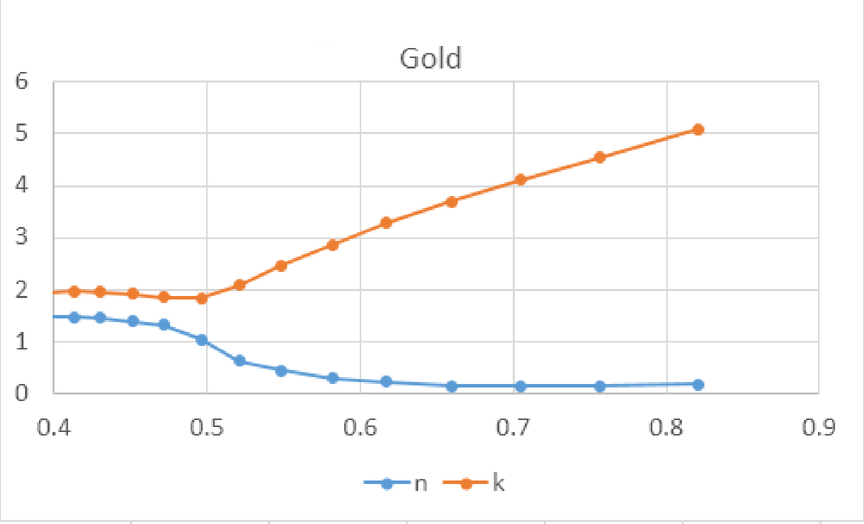


Figure 7. Copper, silver and aluminum rings rendered with different Fresnel approximations.

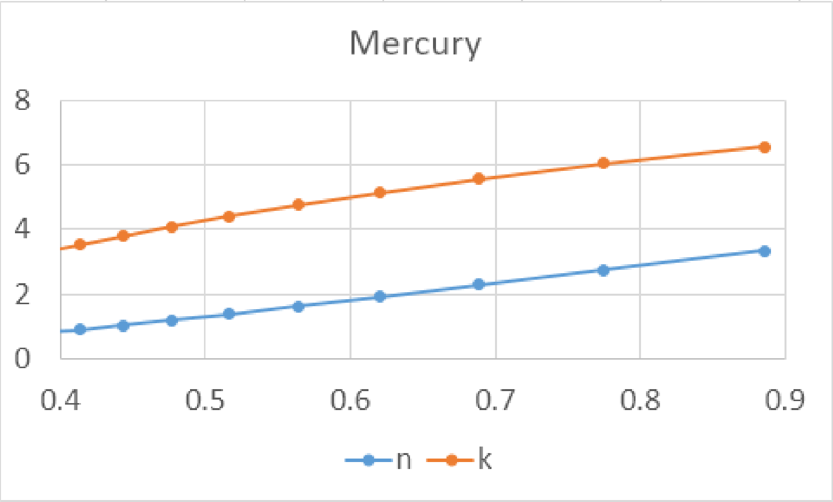
- Lazániy, István, and László Szirmay-Kalos. "Fresnel term approximations for metals.„ (2005) pdf

Spectral Complex Refractive Indices

mu	n	k
0.3974	1.47	1.952
0.4133	1.46	1.958
0.4305	1.45	1.948
0.4509	1.38	1.914
0.4714	1.31	1.849
0.4959	1.04	1.833
0.5209	0.62	2.081
0.5486	0.43	2.455
0.5821	0.29	2.863
0.6168	0.21	3.272
0.6595	0.14	3.697
0.7045	0.13	4.103
0.756	0.14	4.542
0.8211	0.16	5.083



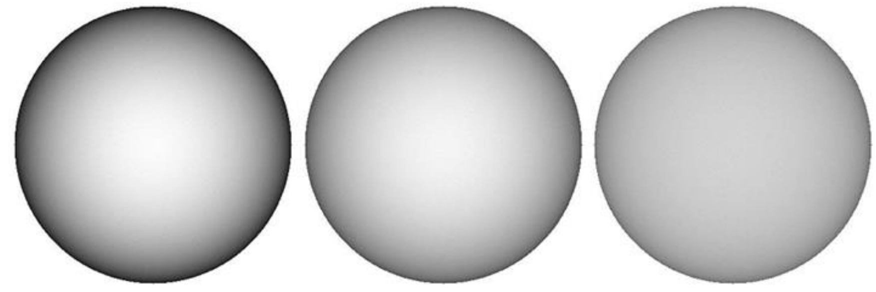
mu	n	k
0.38745	0.79824	3.29351
0.41328	0.898	3.53785
0.4428	1.02698	3.80193
0.47686	1.186	4.08981
0.5166	1.38332	4.40608
0.56356	1.62088	4.7505
0.61992	1.90988	5.14953
0.6888	2.28417	5.58189
0.7749	2.74611	6.05402
0.8856	3.3238	6.55726



- online source: https://refractiveindex.info

Oren-Nayar

Photograph of a matte vase and its renderings with the Lambertian model and the Oren-Nayar model. ©Wikipedia



Influence of roughness parameter. © Wikipedia

- Oren-Nayar use Micro-Facette model with diffuse V-shaped grooves distributed according to Gaussian with standard deviation $\sigma \in [0,1]$ to model retro-reflective materials.
- They approximate direct and indirect reflection and geometry term and provide a simple but coarse approximation:

$$f_{\mathrm{d,Oren-Nayar}}(\hat{\boldsymbol{\omega}}_{\mathrm{out}}, \hat{\boldsymbol{\omega}}_{\mathrm{in}}) = \frac{1}{\pi}(A + B\cos_{+}(\phi_{\mathrm{in}} - \phi_{\mathrm{out}})\sin\alpha\tan\beta)$$

$$A = 1 - 0.5\frac{\sigma^2}{\sigma^2+0.33}, B = 0.45\frac{\sigma^2}{\sigma^2+0.09}, [\alpha|\beta] = [\max|\min]\{\theta_{\mathrm{in}}, \theta_{\mathrm{out}}\}$$

Anisotropic APS-BRDF

Computergraphik und Visualisierung

- Reference: Ashikmin, Premoze, Shirley, A Microfacet-based BRDF Generator, 2000
- specular reflection is modeled through

$$f_{s,\mathrm{APS}}(\hat{\boldsymbol{\omega}}_{\mathrm{out}}, \hat{\boldsymbol{\omega}}_{\mathrm{in}}) = \frac{F_r(\langle \hat{\boldsymbol{\omega}}_{\mathrm{in}}, \hat{\boldsymbol{\omega}}_{\mathrm{h}} \rangle) \cdot f \cdot D(\hat{\boldsymbol{\omega}}_{\mathrm{h}})}{4 \cdot g(\hat{\boldsymbol{\omega}}_{\mathrm{in}}) \cdot g(\hat{\boldsymbol{\omega}}_{\mathrm{out}})}$$

- where f is a normalization constant extracted from the distribution $D(\hat{\boldsymbol{\omega}}_{\mathrm{h}})$, which can be varied:

$$f = \int \langle \hat{\boldsymbol{n}}, \hat{\boldsymbol{\omega}}_{\mathrm{h}} \rangle \, D(\hat{\boldsymbol{\omega}}_{\mathrm{h}}) d\Omega_h,$$

- shadow and self-occlusion is implemented with the following pre-compute and tabulated function

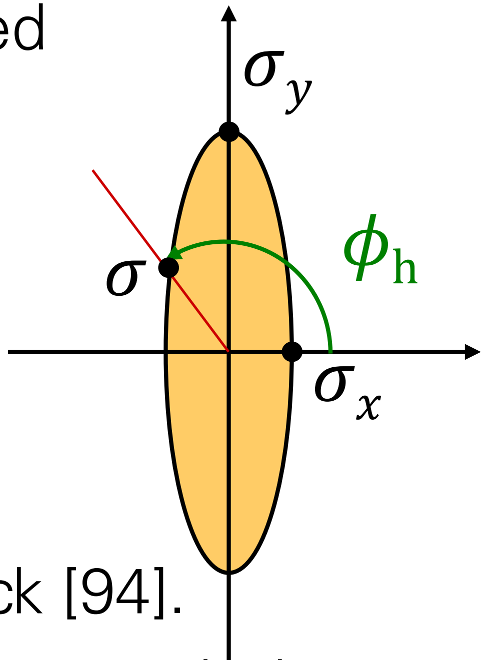
$$g(\hat{\boldsymbol{\omega}}) = \int \langle \hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\omega}}_{\mathrm{h}} \rangle_+ \cdot D(\hat{\boldsymbol{\omega}}_{\mathrm{h}}) d\Omega_h$$

with two underlying assumptions: shadow and self-occlusion is uncorrelated and microfacet orientation is independent of its visibility.

Anisotropic APS-BRDF

- one example anisotropic distribution is based on Gaussian with dependence to ϕ_{h}:

$$D(\widehat{\boldsymbol{\omega}}_{\mathrm{h}}) = c_1 \cdot \exp\left(-\tan^2\theta_h\left(\frac{\cos^2\phi_h}{\sigma_x^2} + \frac{\sin^2\phi_h}{\sigma_y^2}\right)\right)$$



- To support anisotropy, one needs a tangent vector $\vec{\boldsymbol{t}}$ pointing in x-direction within tangent space.
- Fresnel term is approximated through Schlick [94].
- Specular term becomes quite large for $\theta_{in} \to 0$ such that together with diffuse reflection, energy preservation is not given anymore. For this diffuse BRDF is corrected to

$$f_{\mathrm{d,APS}}(\widehat{\boldsymbol{\omega}}_{\mathrm{out}}, \widehat{\boldsymbol{\omega}}_{\mathrm{in}}) = c_2 \cdot \left(1 - R_s(\widehat{\boldsymbol{\omega}}_{\mathrm{in}})\right) \cdot \left(1 - B_s(\widehat{\boldsymbol{\omega}}_{\mathrm{out}})\right)$$

with [bi-]hemispherical reflectance R_s/B_s of specular term $f_{\mathrm{s,APS}}$ only.

Anisotropic APS-BRDF – Results

Computergraphik und Visualisierung

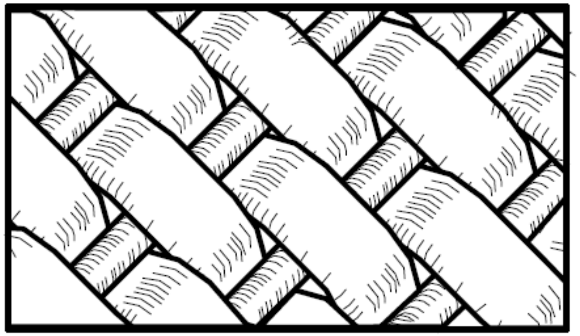


Figure 10: *Microgeometry of our sample of satin.*

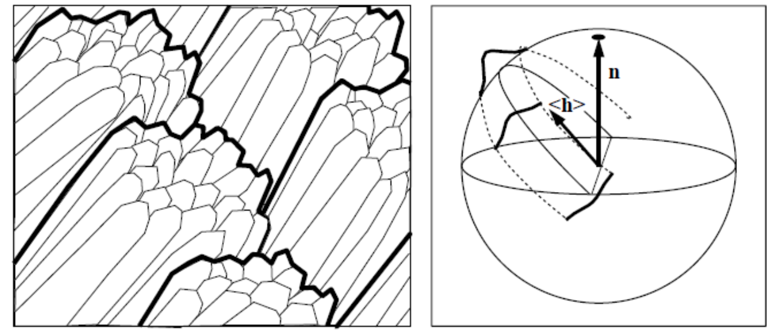


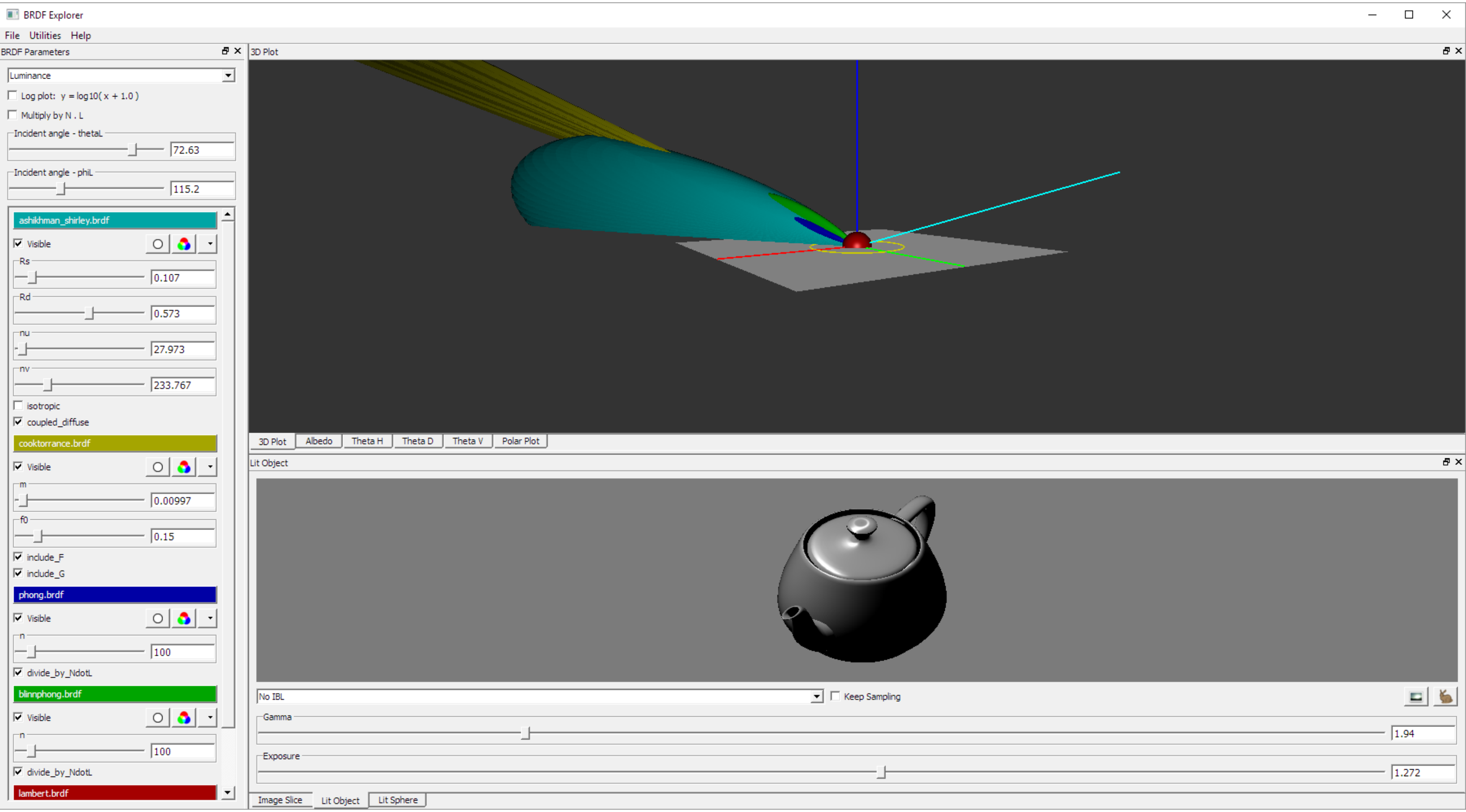
Figure 12: *Microgeometry of velvet (left) and p(***h***) used to model it (right).*

Figure 11: *Synthetic satin (left)*

Figure 13: *A tablecloth made of two different colors of slanted fiber velvets.*

THE BRDF ZOO

BRDF Explorer



Overview over BRDF models

Computergraphik und Visualisierung

- Rosana Montes, Carlos Ureña, An Overview of BRDF Models, Technical Report LSI-2012-001

Column Explanations

- physical … derived from laws of physics
- plausible … non negative, symmetric, energy conservation
- sampling … efficient importance sampling possible

Models	Physical	Plausible	Fresnel Eq.	Anisotrotopic	Sampling	Rel.Cost (cycles)	Material Type
Ideal Specular	★	★	▼	▼	★	x	perfect specular
Ideal Diffuse	★	★	▼	▼	★	x	perfect diffuse
Minnaert	▼	· · ·	▼	▼	▼	$5.35x$	Moon surf.
Torrance-Sparrow	★	▼	★	★	▼	· · ·	rough surf.
Beard-Maxwell	★	▼	★	▼	▼	$397x$	painted surf.
Blinn-Phong	▼	▼	▼	▼	★	$9.18x$	rough surf.
Cook-Torrance	★	★	★	▼	▼	$16.9x$	metal,plastic
Kajiya	★	▼	★	★	▼	· · ·	metal,plastic
Poulin-Fournier	★	▼	▼	★	▼	$67x$	clothes
Strauss	▼	· · ·	★	▼	▼	$14.88x$	metal,plastic
He et al.	★	★	★	▼	▼	$120x$	metal
Ward	▼	▼	▼	★	★	$7.9x$	wood
Westin	★	· · ·	★	★	▼	· · ·	metal
Lewis	▼	★	▼	▼	★	$10.73x$	mats
Schlick	▼	★	★	★	▼	$26.95x$	heterogeneous
Hanrahan	★	· · ·	★	▼	▼	· · ·	human skin
Oren-Nayar	★	★	▼	▼	★	$10.98x$	matte, dirty.
Neumann	▼	★	▼	★	★	· · ·	metal,plastic
Lafortune	▼	★	▼	★	★	$5.43x$	rough surf.
Coupled	★	★	★	▼	★	$17.65x$	polished surf.
Ashikhmin-Shirley	★	▼	★	★	★	$79.6x$	polished surf.
Granier-Heidrich	★	· · ·	★	▼	▼	· · ·	old-dirty metal

Table 1: *Brief summary of the properties exhibited by the reviewed BRDFs. Legend: (★) if the BRDF has this property; (▼) if the BRDF does not; (· · ·) unknown value.*

Graphical overview of BRDF models

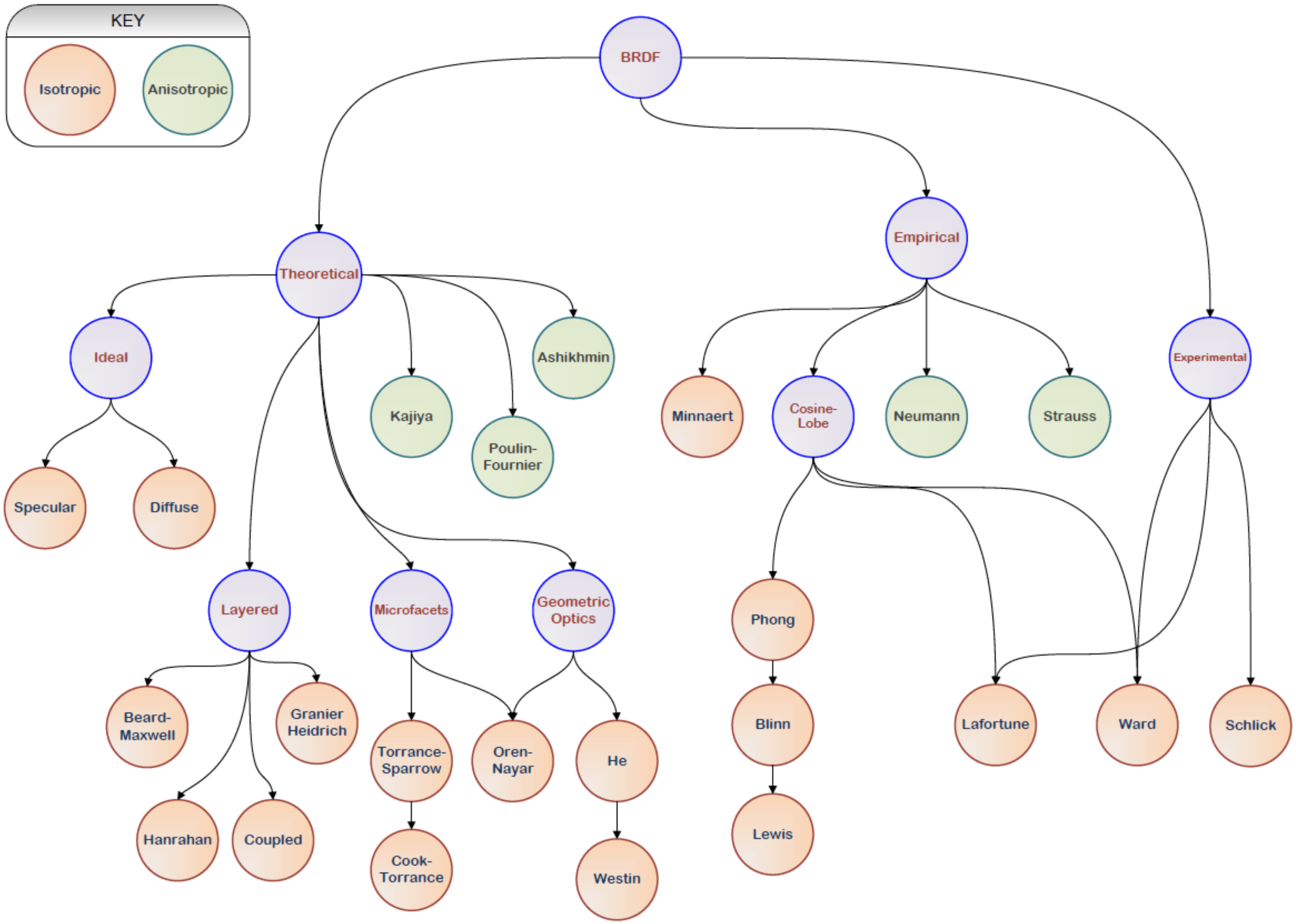
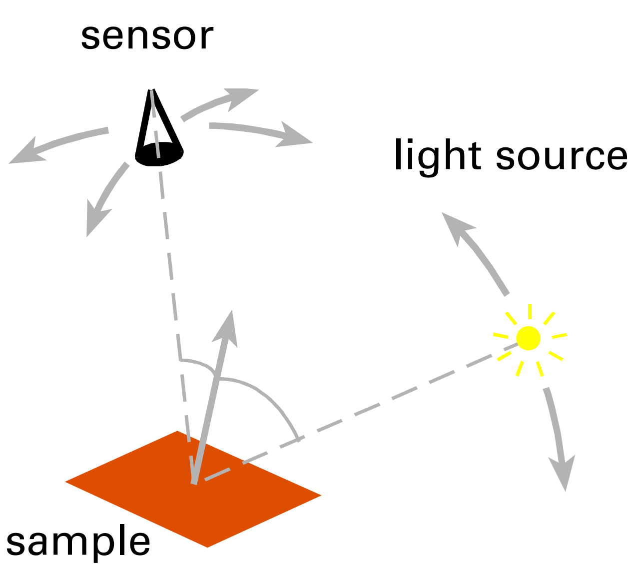


Figure 2: *A graphical classification of the BRDFs cited in this paper. Some BRDFs are built on previous ones.*

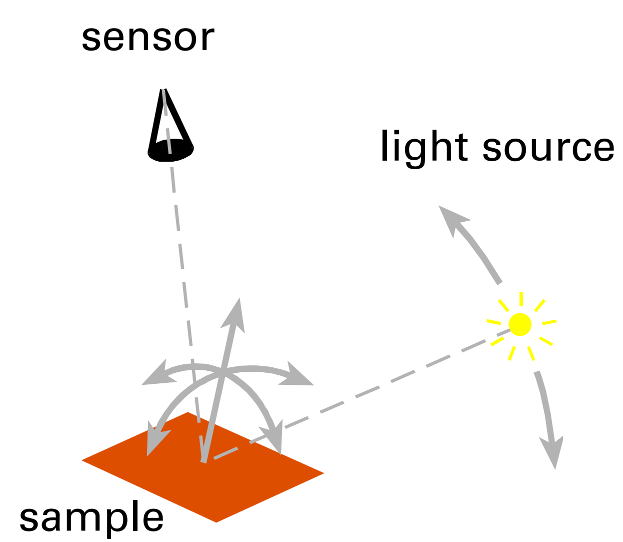
BRDF MEASUREMENT

Gonioreflectometer

- measures reflectance for combinations of sensor ($\hat{\boldsymbol{\omega}}_{\mathrm{out}}$) and light source position ($\hat{\boldsymbol{\omega}}_{\mathrm{in}}$)
- Helmholtz reciprocity and isotropy of BRDF help to reduce number of necessary measurements
- one can move sensor or rotate sample



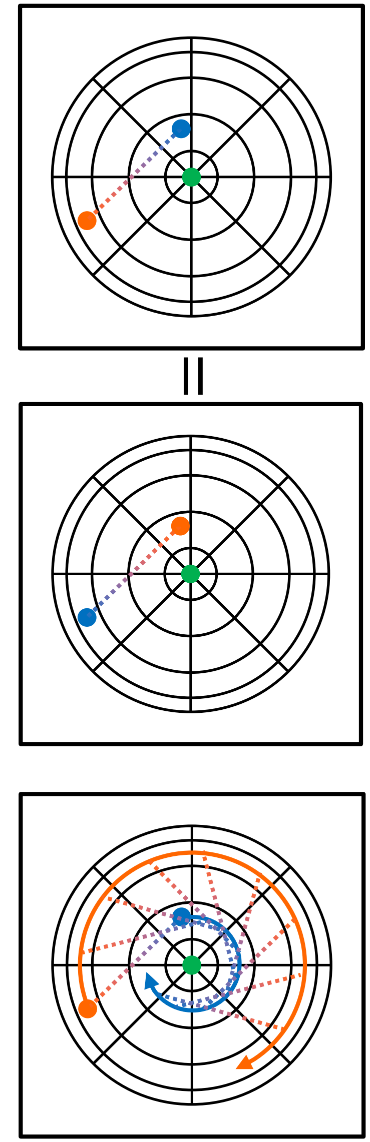
move sensor



move sample

BRDF Measurement Samples

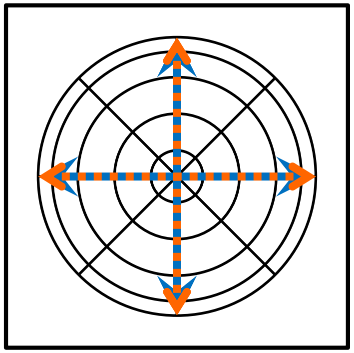
- a single BRDF sample can be illustated by two dots (orange for light and blue for sensor) on a hemi-sphere drawn from above with the normal direction in the center
- Helmholz Reciprocity implies that measurement of interchanged dots yields the same value
- For isotropic BRDFs all measurements with the dots rotated around the normal direction yield the same value



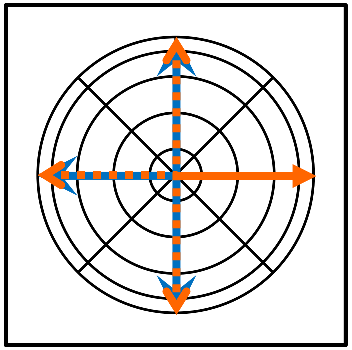
isotropic

BRDF Samples

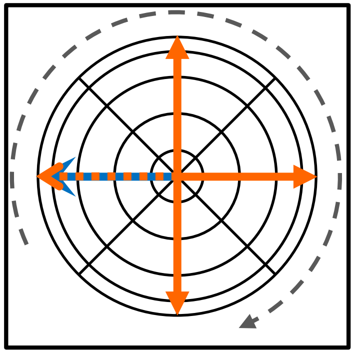
- to completely measure a BRDF we need to sample all combinations of light and sensor locations.



- Helmholz Reciprocity allows to reduce sampling of light or sensor location to half of the hemi-sphere



- For isotropic BRDFs or when rotating the material sample with a turn table, sensor locations can be restricted to a 1D half arc



isotropic or turn table

Image Based BRDF Measurement

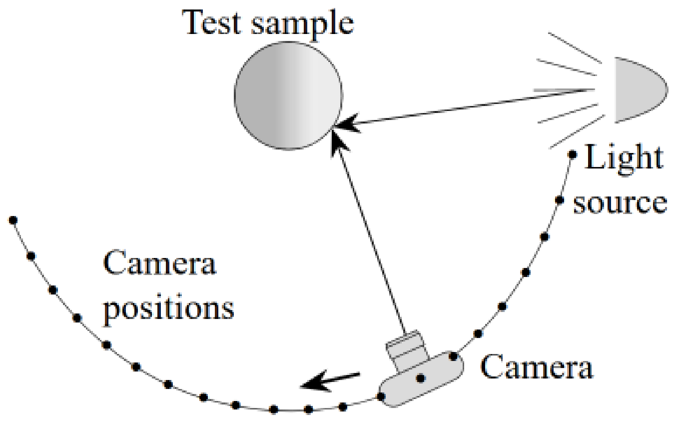


Fig. 3. Schematic of measurement setup.

idea:

- capture many samples from curved surface with one image from a CCD camera
- store samples in large table and interpolate / extrapolate

preparation:

- determine surface geometry from known geometry or 3D scan
- calibration of setup and object

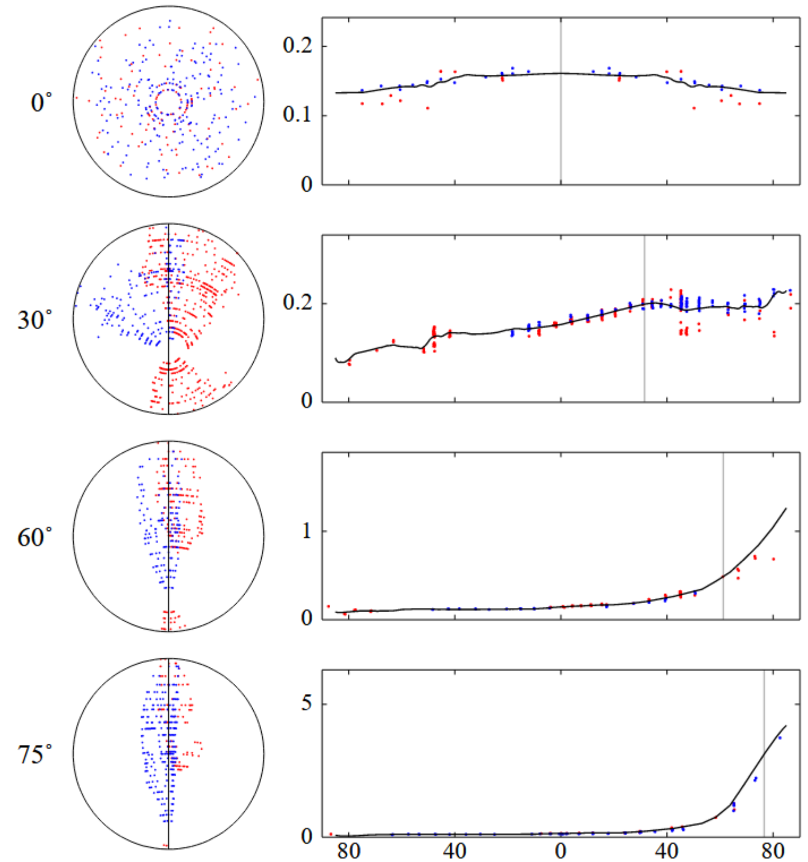
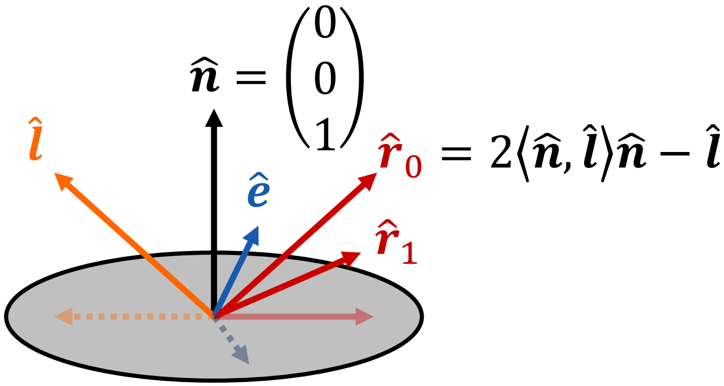


Fig. 9. BRDF of typical skin, showing coverage and scatter in raw data

Marschner, Stephen R., et al. "Image-based BRDF measurement including human skin." *Eurographics Workshop on Rendering Techniques*. Springer, Vienna, 1999.

Lafortune Model

- Idea: extended Phong model by further lobes for non-ideal reflection components
- In the coordinate system with surface normals as z-axis, the reflected vector can be calculated from the incoming direction by multiplying with $\mathrm{diag}(-1 \quad -1 \quad 1)$.
- Extension by adding several lobes, which are defined by different diagonal matrices $\boldsymbol{D}_i$.



$$\begin{pmatrix}r_x\\r_y\\r_z\end{pmatrix} = \begin{pmatrix}-l_x\\-l_y\\l_z\end{pmatrix} = \begin{pmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}l_x\\l_y\\l_z\end{pmatrix}$$

$$f_{\text{spec,Lafortune},i} = \langle\hat{\boldsymbol{e}},\boldsymbol{D}_i\hat{\boldsymbol{l}}\rangle_+^{s_i} \Rightarrow$$

$$\ddot{\boldsymbol{f}}_{\text{Lafortune}} = f_{\text{diff}}\ddot{\boldsymbol{r}}_d + \sum_i f_{\text{spec,Lafortune},i}\ddot{\boldsymbol{r}}_{s,i}$$

Fitting of Lafortune Parameters

- reduce number of parameters by assuming that BRDF is isotropic:

$$\dddot{\boldsymbol{f}}_{\text{Lafortune}}(\hat{\boldsymbol{l}}, \hat{\boldsymbol{e}}) = \dddot{\boldsymbol{\rho}}_{\text{diff}} + \sum_i \left(\dddot{\boldsymbol{C}}_{t,i}\left(l_x e_x + l_y e_y\right) + \dddot{\boldsymbol{C}}_{n,i} l_z e_z\right)^{\dddot{\boldsymbol{s}}_i}$$

- diffuse component plus several Phong-lobes
- total of $3(1 + 3i)$ parameters (e.g. 12 for 1-lobe model)
- use non linear fitting approach to estimate parameters from set of samples

Examples – MPII Saarbrücken

Computergraphik und Visualisierung

fit of single BRDF

Setup with

- 3D scanner (structured light)
- digital camera (HDR)
- point-light source
- dark room - array
- calibration targets (checkerboard)

Cluster extraction and fit of one BRDF per cluster

Examples – MPII Saarbrücken

Photo

Rekonstruktion

Examples – MPII Saarbrücken

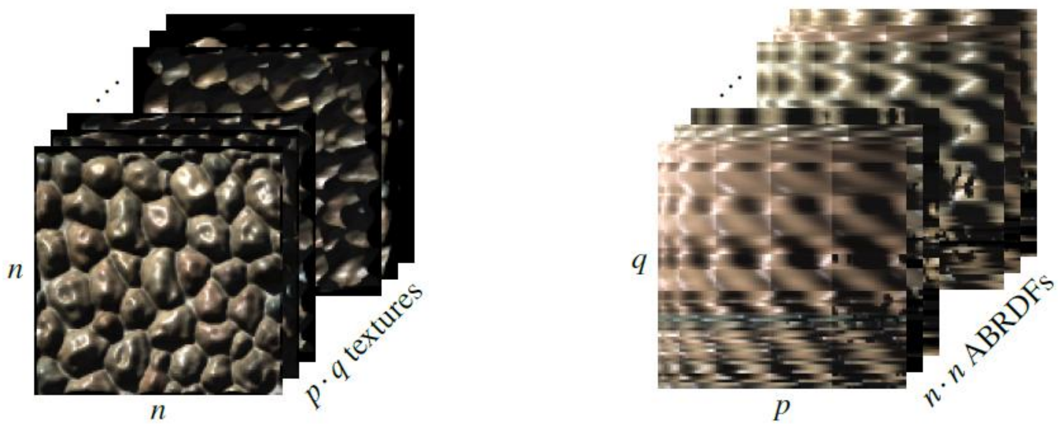
Max Planck
Geometry

Max Planck
BRDF

Minerva
BRDF

Bidirectional texture function (BTF)

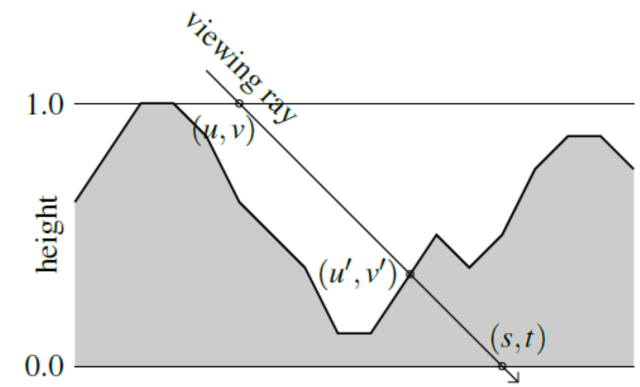
Bidirectional texture function (BTF)



(a) Texture representation

(b) ABRDF representation

Figure 2.8: Two different BTF representations of a BTF



BTF

- given extended material sample
- sample hemisphere with p camera and q light directions
- for each pair of directions acquire image of $n \times n$ reflectance samples

Apparent BTF

- rearrange BTF into image of $n \times n$ apparent BRDFs sampled on $p \times q$ camera-light pairs
- it is called "apparent" as the surface has significant bumps and the camera / light ray intersect the surface at different texture locations

BTF Measurement in Bonn (slides)

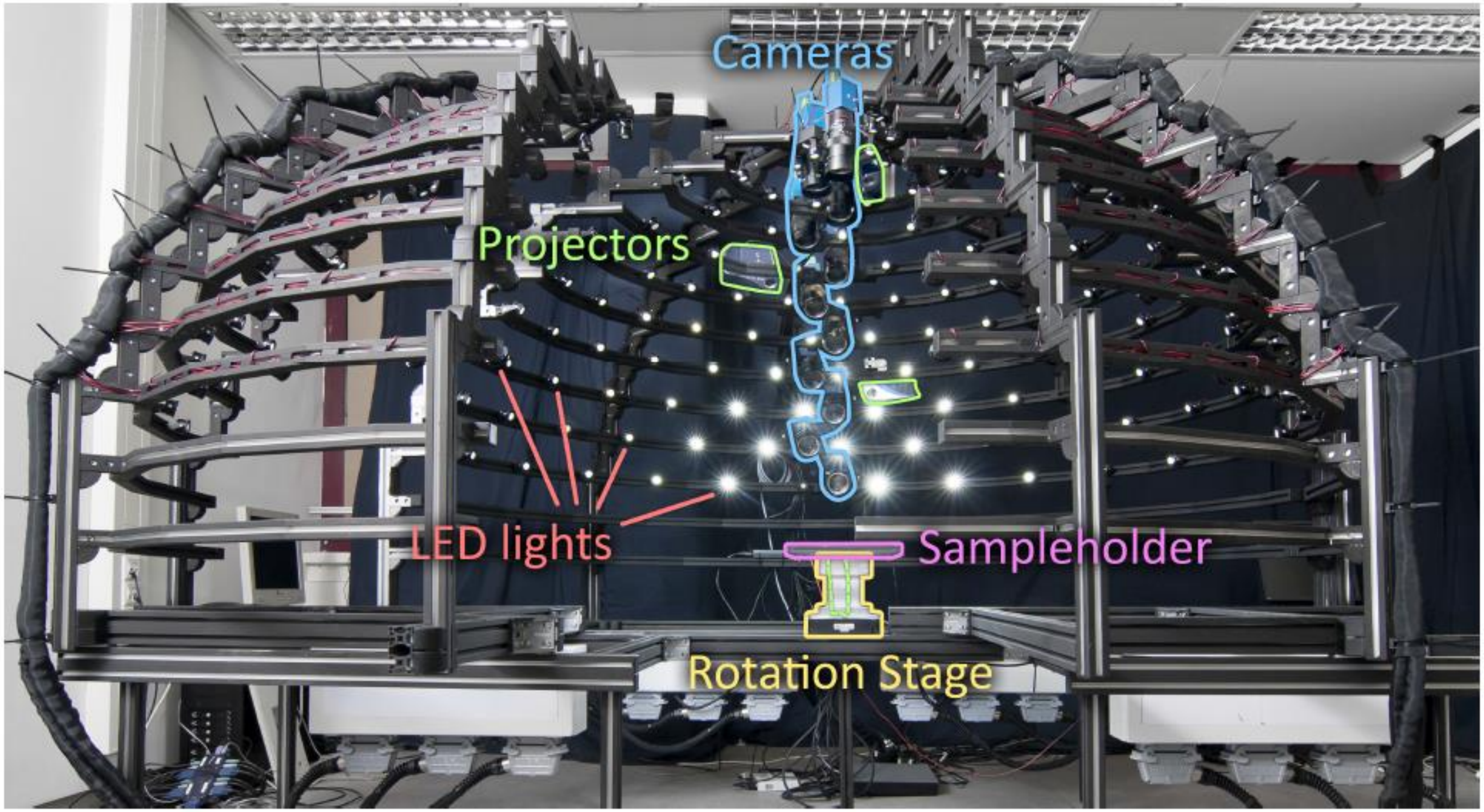


Figure 1: The DOME II BTF acquisition setup. One quarter has been slid open to expose the view on the inside.

Databases

- CURET: Columbia-Utrecht Reflectance and Texture Database
  - 61 samples with 205 measurements per sample and 205 additional samples for anisotropy of BRDF, fitted BRDF models & BTFs
- BTFDBB: BTF Datenbank Bonn und Messlabor
  - UBO2003 Datasets … 6 Samples with 81x81x256x256 resolution
  - ATRIUM Datasets … 4 Samples with 81x81x800x800 resolution
  - OBJECTS2011 Datasets … 4 Objects with BTF HDR-Textures (100-300GB), WebGLViewer
  - Spectral Datasets … 4 Samples with multichannel spectral images
  - OBJECTS2012 Datasets … 12 Objects with compressed BTFs
  - UBO2014 Datasets … 7x12 samples with 151x151x512x512 resolution

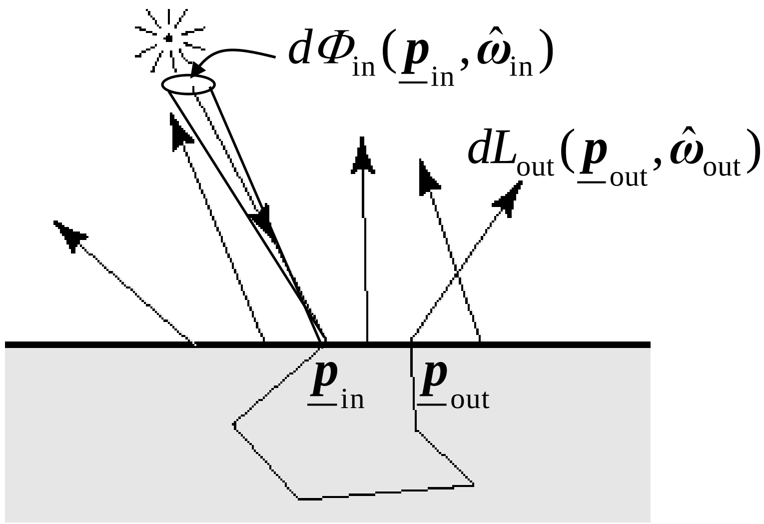
OBJECTS2012 Datasets

UBO2014 Datasets

Bidirectional Subsurface Scattering Reflection Distribution Function

Bidirectional Subsurface Scattering Reflection Distributior Function

- Split incoming light at surface into pa reflected via BRDF and part that enters surface and gets scattered inside of the surface before it exists a different location
- The internal scattering process is called subsurface scattering and can be modelled by a BSSRDF parameterized additionally over point $\underline{\boldsymbol{p}}_{out}$ where light leaves the surface.
- The subsurface reflection process has to be integrated additionally over area such that BSSRDF is derivative with respect to light power:



$$f_{SS}(\underline{\boldsymbol{p}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{in}}, \underline{\boldsymbol{p}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{out}}) = \frac{dL_{\text{out}}(\underline{\boldsymbol{p}}_{\text{out}}, \hat{\boldsymbol{\omega}}_{\text{out}})}{d\Phi_{\text{in}}(\underline{\boldsymbol{p}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{in}})}$$

Bidirectional Subsurface Scattering Reflection Distribution Function

- unit: $1/(m^2 \cdot sr)$
- 8-dimensional parameter space

Literature:

- F. E. Nicodemus, J.C. Richmond, J.J. Hsia, I.W. Ginsberg and T. Limperis, Geometric considerations and nomenclature for reflectance. Monograph 161, National Bureau of Standards (US), October 1977

Examples for BRDF and BSSRDF

Left: BRDF „hard“ light distribution
right: BSSRDF describes light transport and scattering inside of material.

Beispiele für BRDF und BSSRDF

left: BRDF „hard“ light distribution.
right: BSSRDF much more natural light distribution on skin.
additionally: internal color bleeding in shadowed region under nose.

Examples for BSSRDF

Photography

Simulation

diffus

low fat

full fat

organic?