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Security and Cryptography 1

Module 2: A little history class...

Disclaimer: Thanks to Dan Boneh, Mark Manulis, Günter Schäfer

Dresden, WS 18

Reprise from Module 1

You know what to expect from the lecture

You have seen some trends that are happening

You have been introduced to some ***typical actors***

You understand what ***threats*** are ... and what this means

You can tell ***security goals*** (CIA!) from ***security services***

You know ***adversary models*** and which aspects they define

Module Outline

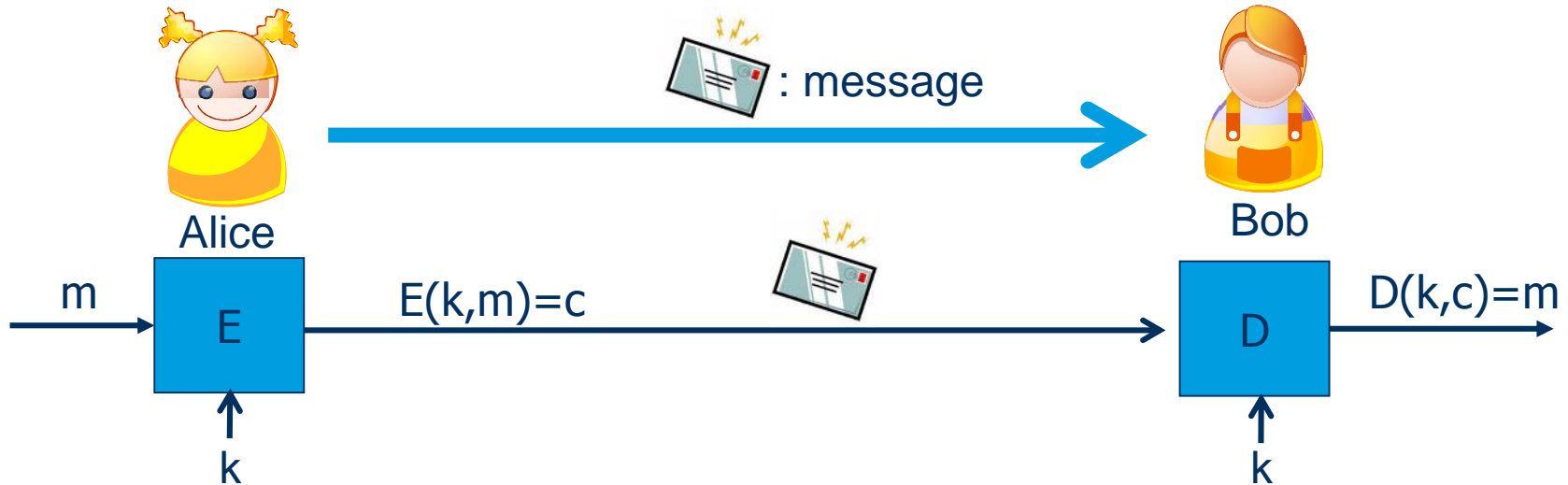
Two words on „Crypto“

A little history of crypto

- Transposition
- Substitution
- Cryptanalysis
- Cesar Cipher
- Vigenère Cipher
- Enigma
- Vernam Cipher – The One Time Pad

Secure Communication

Alice sends Bob a private (any!) message...



m : message (plaintext) $\in M$ (message space, sometimes P)

k : key $\in K$ (key space)

c : ciphertext $\in C$ (ciphertext space)

A cipher is a triple of algorithms: $\mathbf{E}, \mathbf{D}, keygen$ (sometimes: Enc, Dec)

Correctness: for all $k \in \mathcal{K}, m \in \mathcal{M}$: $Dec(k, Enc(k, m)) = m$

Terminology: Cryptology



Cryptology:

—Science concerned with communications in secure and usually secret form

—Derived from the Greek

- *kryptós* (hidden) and
- *lógos* (word)

—Cryptology encompasses:

- **Cryptography** (*gráphein* = to write): principles and techniques by which information can be concealed in *ciphertext* and later revealed by legitimate users employing a secret key
- **Cryptanalysis** (*analýein* = to loosen, to untie): recovering information from ciphers without knowledge of the key

Terminology: Cipher

Cipher:

- Method of transforming a message (plaintext) to conceal its meaning (and to transform it back)
- Ciphers are one class of cryptographic algorithms (E,D)
- The transformation usually takes the message and a (*secret*) key as input
- Unfortunately:*** sometimes also used as synonym for the concealed *ciphertext*

Crypto Basics

Encrypt written communication:

\mathcal{M} : language over

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

 \mathcal{C} : language over

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

\mathcal{K} is determined by a *bijective* mapping

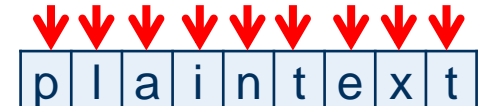
$$f : \mathcal{M} \rightarrow \mathcal{C} \text{ for Enc} \quad \text{and} \quad f^{-1} : \mathcal{C} \rightarrow \mathcal{M} \text{ for Dec}$$

Classification



Transposition permute letters according to some scheme

Substitution substitute letters by other letters (or symbols)



A simple Substitution Cipher

Key Generation

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

choose a *shift value* $k \in [0, 25]$

Encryption

Let $m = m_0 \dots m_n$ and

Let $\#m_i$ denote the position of m_i in the alphabet.

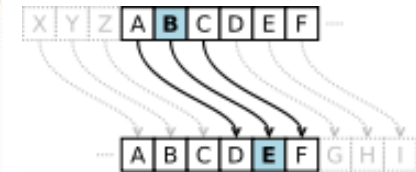
$\text{Enc}(k, m) = c_0 \dots c_n$ where for each $c_i : \#c_i = \#m_i + k \pmod{26}$

The Caesar Cipher (or: ROT3)

Shift by 3,

$\text{Enc}(m_i) :=$

a	→	D
b	→	E
c	→	F
...		
y	→	B
z	→	C



m = the die has been cast

c = WKHGLHKDVEHHQFDVW

What is the size of the key space?



Slightly better Substitution Cipher

Use arbitrary permutation:

$k :=$

a → X
b → C
c → Y
...
y → V
z → B

More formally:

Key Generation

choose k as a permutation of the alphabet/an l -tuple of distinct symbols

Encryption

Let $m = m_0 \dots m_n$.

$\text{Enc}(k, m) = c_0 \dots c_n$ where each $c_i = f(k, m_i)$.

What is the size of the key space in this case?

How would you break it?

Breaking a Substitution Cipher

Brute force:

Assuming a shift cipher, try all 26 possible keys

Assuming permutation: 2^{88} (too large)

More intelligent:

1. Use frequency of letters in the expected language
„e“:12.7%, „t“: 9.1%, „a“: 8.1%, ...

Letter Frequencies for Some Languages

Letter	French ^[3]	German ^[4]	Spanish ^[5]	Portuguese ^[6]	Esperanto ^[7]	Italian ^[8]	Turkish	Swedish ^[9]	Polish ^[10]	Toki Pona ^[11]	Dutch ^[12]
a	7.636%	6.51%	12.53%	14.63%	12.12%	11.74%	11.68%	9.3%	8.0%	17.2%	7.49%
b	0.901%	1.89%	1.42%	1.04%	0.98%	0.92%	2.95%	1.3%	1.3%	0.0%	1.58%
c	3.260%	3.06%	4.68%	3.88%	0.78%	4.5%	0.97%	1.3%	3.8%	0.0%	1.24%
d	3.669%	5.08%	5.86%	4.99%	3.04%	3.73%	4.87%	4.5%	3.0%	0.0%	5.93%
e	14.715%	17.40%	13.68%	12.57%	8.99%	11.79%	9.01%	9.9%	6.9%	7.4%	18.91%
f	1.066%	1.66%	0.69%	1.02%	1.03%	0.95%	0.44%	2.0%	0.1%	0.0%	0.81%
g	0.866%	3.01%	1.01%	1.30%	1.17%	1.64%	1.34%	3.3%	1.0%	0.0%	3.40%
h	0.737%	4.76%	0.70%	1.28%	0.38%	1.54%	1.14%	2.1%	1.0%	0.0%	2.38%
i	7.529%	7.55%	6.25%	6.18%	10.01%	11.28%	8.27%*	5.1%	7.0%	14.8%	6.50%
j	0.545%	0.27%	0.44%	0.40%	3.50%	0.00%	0.01%	0.7%	1.9%	3.0%	1.46%
k	0.049%	1.21%	0.01%	0.02%	4.16%	0.00%	4.71%	3.2%	2.7%	5.1%	2.25%
l	5.456%	3.44%	4.97%	2.78%	6.14%	6.51%	5.75%	5.2%	3.1%	10.2%	3.57%
m	2.968%	2.53%	3.15%	4.74%	2.99%	2.51%	3.74%	3.5%	2.4%	4.4%	2.21%
n	7.095%	9.78%	6.71%	5.05%	7.96%	6.88%	7.23%	8.8%	4.7%	11.6%	10.03%
o	5.378%	2.51%	8.68%	10.73%	8.78%	9.83%	2.45%	4.1%	7.1%	7.7%	6.06%
p	3.021%	0.79%	2.51%	2.52%	2.74%	3.05%	0.79%	1.7%	2.4%	3.7%	1.57%
q	1.362%	0.02%	0.88%	1.20%	0.00%	0.51%	0	0.007%	-	0.0%	0.009%
r	6.553%	7.00%	6.87%	6.53%	5.91%	6.37%	6.95%	8.3%	3.5%	0.0%	6.41%
s	7.948%	7.27%	7.98%	7.81%	6.09%	4.98%	2.95%	6.3%	3.8%	4.1%	3.73%
t	7.244%	6.15%	4.63%	4.74%	5.27%	5.62%	3.09%	8.7%	2.4%	4.6%	6.79%
u	6.311%	4.35%	3.93%	4.63%	3.18%	3.01%	3.43%	1.8%	1.8%	3.2%	1.99%
v	1.628%	0.67%	0.90%	1.67%	1.90%	2.10%	0.98%	2.4%	-	0.0%	2.85%
w	0.114%	1.89%	0.02%	0.01%	0.00%	0.00%	0	0.03%	3.6%	2.8%	1.52%
x	0.387%	0.03%	0.22%	0.21%	0.00%	0.00%	0	0.1%	-	0.0%	0.04%
y	0.308%	0.04%	0.90%	0.01%	0.00%	0.00%	3.37%	0.6%	3.2%	0.0%	0.035%
z	0.136%	1.13%	0.52%	0.47%	0.50%	0.49%	1.50%	0.02%	5.1%	0.0%	1.39%

 most frequent

 2nd most frequent



Di- and Trigram Frequencies

Digram (Bigram) Frequencies

— frequency for a combination of two letters

English: th, he, in, en, nt, re, er, an, ti, es, on, at, se, nd, or, ar, al,...

German: er, en, ch, te, nd, ei, de, ie, in, es, ge, ne, un, ic, st, an,...

Trigram Frequencies

— frequency for a combination of three letters

English: the, and, tha, ent, ing, ion, tio, for, nde, has, nce, edt, tis,...

German: ein, ich, der, sch, und, die, nde, cht, ine, den, end, che, ens,...

A small Example

Given the ciphertext C:

LIHKDGDQBWKLQJFRQILGHQWLDOWRVDBKHZURWHLWLQFLSKHUWKD
WLVEBVRFKDQJLQJWKHRUGHURIWKHOHWWHUVRIWKHDOSKDEHWWKD
WQRWDZRUGFRXOGEHPDGHRXWLIDQBRQHZLVKHVWRGHFLSKHUWKHVH
DQGJHWDWWKHLUPHDQLQJKHPXVWVXEVWLWXWHWKHIRXUWKOHWWH
URIWKHDOSKDEHWQDPHOBGIRUDDQGVRZLWKWKHRWKHUV

35 W	e,t,a?
33 H	e,t,a?
24 K	e,t,a?
20 D	e,t,a?
18 R	
16 L	
15 Q	

10 KH	he,..,th?
5 WK	he,..,th?
4 KD	
3 WW	

bigrams

2 WWK	
2 WKH	the,and?
2 RIW	the,and?
2 HWW	
1 ZRU	the,and?

trigrams

```
$ grep -o . file.txt | sort -f | uniq -ic | sort -rg
```



A small Example – solved

```
$ rotix -f enc_cesar.txt -r23
```

if he had anything confidential to say he wrote it in cipher that is by so changing the order of the letters of the alphabet that not a word could be made out if any one wishes to decipher these and get at their meaning he must substitute the fourth letter of the alphabet namely D for A and so with the others

If he had anything confidential to say, he wrote it in cipher, that is, by so changing the order of the letters of the alphabet, that not a word could be made out. If anyone wishes to decipher these, and get at their meaning, he must substitute the fourth letter of the alphabet, namely D, for A, and so with the others.

- Suetonius, Life of Julius Caesar

**Ciphertext-only
attack!**

Breaking a Substitution Cipher

Brute force:

Assuming a shift cipher, try all 26 possible keys

Assuming permutation: 2^{88} (too large)

More intelligent:

1. Use frequency of letters in the expected language
 „e“:12.7%, „t“: 9.1%, „a“: 8.1%, ...
2. Use frequency of bi-grams, tri-grams...

Would you know an immediate remedy to make cipher more secure?

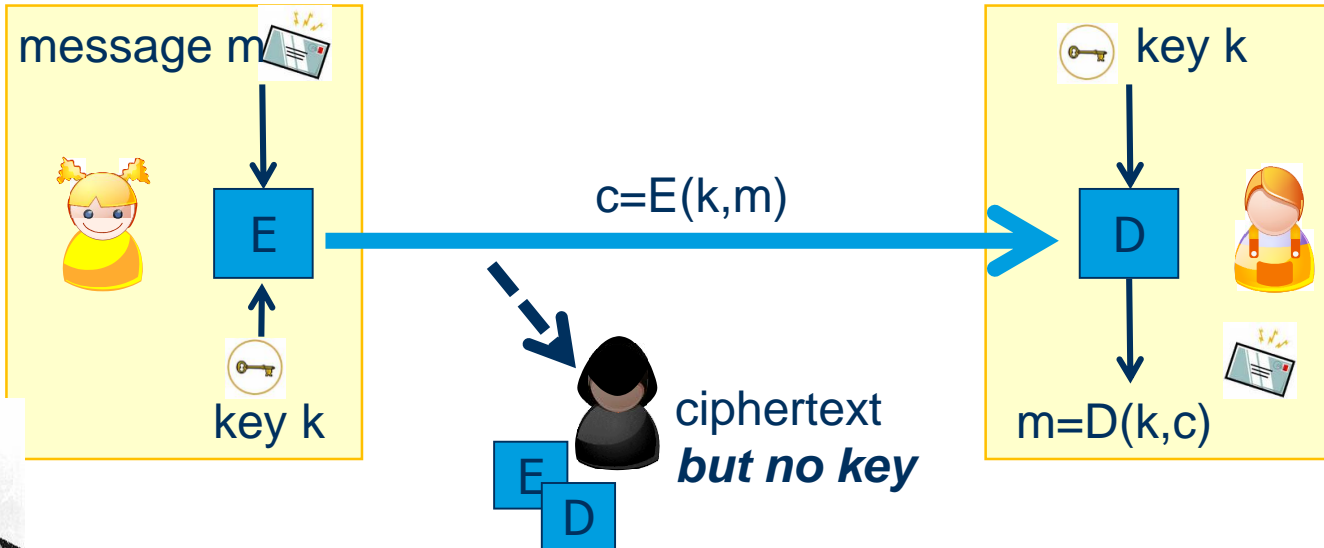
Auguste Kerckhoffs (1835 – 1903)



1. *The system should be, if not theoretically unbreakable, unbreakable in practice.*
2. *The design of a system should not require secrecy, and compromise of the system should not inconvenience the correspondents.*
(Kerckhoffs' principle)
3. *The key should be memorable without notes and should be easily changeable.*
4. *The cryptograms should be transmittable by telegraph.*
5. *The apparatus or documents should be portable and operable by a single person.*
6. *The system should be easy, neither requiring knowledge of a long list of rules nor involving mental strain.*

Auguste Kerckhoffs: „La Cryptographie Militaire“ in „le Journal des Sciences Militaires“, 1883

The Communication Model and Kerckhoff



“The cipher method must not be required to be secret, and it must be able to fall into the hands of the enemy without inconvenience.”

i.o.w: KGen, E, and D will inevitably be discovered at some stage

→ All algorithms should be public

→ security must rely on secrecy of the key only



Polyalphabetic Substitution (Vigenère)

Monoalphabetic cipher easily broken by statistics

Goal: decrease impact of language statistics

What could you do?

Concept:

1. use periodic, variable substitution
2. define (and communicate) periods by key

Key Generation

choose key $k = (k_1, \dots, k_d)$ where each k_i is defined through *shift value*

Encryption

Let $m = m_0 \dots m_n$.

$\text{Enc}(k, m) = c_0 \dots c_n$ where $c_i = f(k_{i+1}, m_i)$ and index of k is taken mod d .

Vigenère Poly-alphabetic Table

2: Substitute as usual..

1: Chose alphabet from keyword

Recta transpositionis tabula.

a	b	c	d	e	f	g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	
b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a
c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b
d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c
e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d
f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e
g	b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g
b	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g
i	k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b
k	l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i
l	m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k
m	n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l
n	o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m
o	p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n
p	q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o
q	r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	
r	s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	
s	t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	
t	u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	
u	x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	
x	y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	
y	z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	
z	w	a	b	c	d	e	f	g	g	b	i	k	l	m	n	o	p	q	r	s	t	u	v	
w	a	b	c	d	e	f	g	b	i	k	l	m	n	o	p	q	r	s	t	u	v	w	x	

In hac tabula literarū canonica siue recta tot ex uno & usuali
 latinarum literarum ipsarum per mutationem seu transpositionem
 alphabeta, quot in ea per torum sunt monogrammata, uidelicet
 & uigentes quatuor & uiginti, quae faciunt in numero D. lxxvi.
 tūc multiplicata, paulo efficiunt minus q̄ quatuordecē milia.



Vigenère Cipher

k = C R Y P T O C R Y P T O C R Y P T (+ mod 26)

m = W H A T A N I C E D A Y T O D A Y

c = Z Z Z J U C L U D T U N W G C Q S

How would you break it?

Suppose most common $|k|^{\text{th}}$ letter = „D“

Test: first letter of keyword = „D“ – „E“ = Y...

Try incremental keyword length

Kasiski's Differences

Assuming a ciphertext:

T I G I M K Z O T I G V M C Z O A O F W L J Z D E M X H X M X L G J O H R Y C P L M C W X F I R T I G F M Y E H T U Y H P A K

Find repeating patterns and their distance:

T I G : 1,9,49 (-> distance 8, 40)

Z O : 7,15 (-> distance 8)

Distance	Potential key length	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
----------	----------------------	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----

T I G	40		X		X			X												X
-------	----	--	---	--	---	--	--	---	--	--	--	--	--	--	--	--	--	--	--	---

Z O	8		X		X			X												
-----	---	--	---	--	---	--	--	---	--	--	--	--	--	--	--	--	--	--	--	--

(subsequently: analyse frequencies...)

T I G I M K Z O T I G V M C Z O A O F W L J Z D E M X H X M X L G J O H R Y C P L M C W X F I R T I G F M Y E H T U Y H P A K
 a b c d e f g h a b c d e f g h a b c d e f g h a b c d e f g h a b c d e f g h a b c d e f g h a b c d e f g h a b c d e f g h a b c d e f g
 t h e f i f t h t h e s i x t h a n d t h e t w e l v e t h r e g i m e n t w i l l a t t a c k t h e c i t y a t t w e l v e

Observation

Statistics of language and periodicity cause weakness

Goal: Make predictions again harder

Product Ciphers:

combine different transformation and/or substitution ciphers:

let F_1, \dots, F_d be different ciphers (with same spaces $\mathcal{K}, \mathcal{M}, \mathcal{C}$)

$$\text{Enc}(k, m) = F_d \circ \dots \circ F_2 \circ F_1(m)$$

Rotor Machines

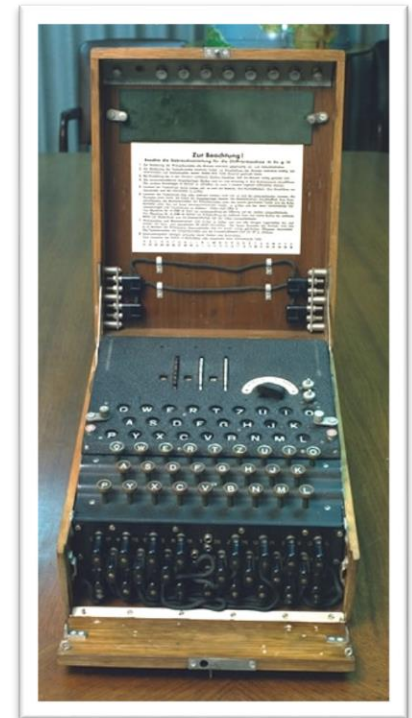
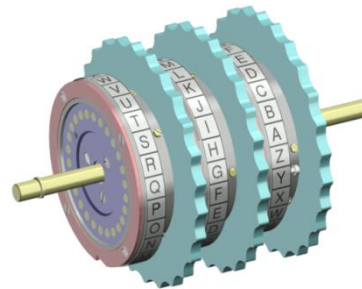
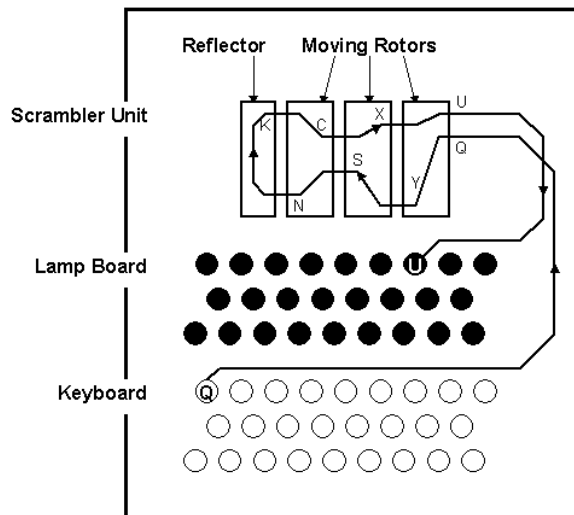
Idea: change alphabet independent of key (extend periods)

rotors R_1, \dots, R_d ; each R_i performs simple substitution

Rotors rotate incrementally after each encrypted character

wirings ensure *polyalphabetic* substitution ($\sim 26^d$ keys)

reflector allows Enc & Dec to be the same algorithms



The Enigma
Used in WWII; Visit Bletcheley Park!

Breaking Enigma

Brute force, ciphertext only: too many possible keys

Observation: No letter is ever encoded to itself

C	O	H	J	Y	P	D	O	M	Q	N	J	C	O	S	G	A	W	H	L	E	I	H	Y	S	O	P	J	S	M	N	U	
Pos 1			K	E	I	N	E	B	E	S	O	N	D	E	R	E	N	E	R	E	I	G	N	I	S	S	E					
Pos 2			K	E	I	N	E	B	E	S	O	N	D	E	R	E	N	E	R	E	I	G	N	I	S	S	E					
Pos 3			K	E	I	N	E	B	E	S	O	N	D	E	R	E	N	E	R	E	I	G	N	I	S	S	E					

Positions 1 and 3 for the possible plaintext are impossible because of matching letters. The red cells represent these *crashes*. Position 2 is a possibility.

© Wikipedia

Goal: reduce possible solutions

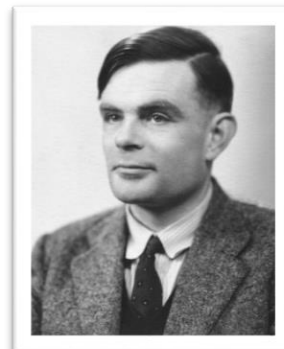
Guesses:

- codeword changed infrequently
- frequent similar plaintexts

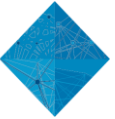
Idea: guess plaintext, vary position



Marian Rejewski
(1905-1980)



Alan Turing
(1912-1954)



Perfect Secrecy

Observation: Patterns are your enemy!

Concept:

- Long key (long/no periodicity)
- No recognizable pattern



Gilbert Vernam
(1890-1960)

Key Generation

choose $k = (k_1, \dots, k_n)$ where each k_i is truly random permutation

Encryption

Let $m = m_0 \dots m_n$.

$\text{Enc}(k, m) = c_0 \dots c_n$ where $c_i = f(k_i, m_i)$

The One-Time-Pad (Vernam cipher)

Cannot be broken with CTO, not even brute force attack...

Truly random key, as long as the message:

$$\begin{array}{l}
 m_0 = \boxed{A} \boxed{T} \boxed{T} \boxed{A} \boxed{C} \boxed{K} \boxed{T} \boxed{H} \boxed{E} \boxed{C} \boxed{I} \boxed{T} \boxed{Y} \boxed{A} \boxed{T} \boxed{T} \boxed{W} \boxed{E} \boxed{L} \boxed{V} \boxed{E} \\
 k = \boxed{P} \boxed{S} \boxed{P} \boxed{I} \boxed{U} \boxed{H} \boxed{G} \boxed{D} \boxed{S} \boxed{P} \boxed{H} \boxed{G} \boxed{D} \boxed{S} \boxed{P} \boxed{I} \boxed{W} \boxed{E} \boxed{E} \boxed{W} \boxed{O}
 \end{array}
 \quad (+ \text{ mod } 26)$$

$$c = \boxed{P} \boxed{L} \boxed{I} \boxed{I} \boxed{W} \boxed{R} \boxed{Z} \boxed{K} \boxed{W} \boxed{R} \boxed{P} \boxed{Z} \boxed{B} \boxed{S} \boxed{I} \boxed{B} \boxed{S} \boxed{I} \boxed{P} \boxed{R} \boxed{S}$$

$$\begin{array}{l}
 k = \boxed{Y} \boxed{H} \boxed{P} \boxed{R} \boxed{S} \boxed{R} \boxed{G} \boxed{F} \boxed{F} \boxed{D} \boxed{D} \boxed{X} \boxed{N} \boxed{S} \boxed{Q} \boxed{I} \boxed{S} \boxed{P} \boxed{W} \boxed{N} \boxed{F} \\
 m_1 = \boxed{R} \boxed{E} \boxed{T} \boxed{R} \boxed{E} \boxed{A} \boxed{T} \boxed{F} \boxed{R} \boxed{O} \boxed{M} \boxed{C} \boxed{O} \boxed{A} \boxed{S} \boxed{T} \boxed{A} \boxed{T} \boxed{T} \boxed{E} \boxed{N}
 \end{array}
 \quad (+ \text{ mod } 26)$$

... or any other message of the same length, for that matter

Now, what are the two problems with this method?

Summary

You've learned the tuple (M,C,K,E,D)

You can tell the difference of

- Cryptology
- Cryptography
- Cryptanalysis

You know basic cryptanalysis (statistics/combinatorics)

You know transposition and substitution

You have learned the shift cipher

... Mono- and Polyalphabetic substitution ciphers

... Product ciphers

You know CTO attacks

You understand why we model security as a game!

And you have heard of the legends of Enigma and Alan Turing...