



Security and Cryptography 1

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Module 6: Integrity

Disclaimer: large parts from Mark Manulis, Dan Boneh, Stefan Katzenbeisser

Dresden, WS 17/18

You have an overview of cryptography and cryptology

You know different adversary models and their corresponding games

You know what symmetric cryptography is

You recall the difference of stream and block ciphers

You can explain the OTP and constructions for stream ciphers

You can prove that the OTP has perfect secrecy

You can tell PRFs and PRP apart and you know constructions for block ciphers

You can explain different modes of operation and their properties

Verification of message integrity as a goal

Adversary and security models

Hashes and cryptographic hash functions

Collisions and how to create them (also: the birthday paradox)

The Merkle-Damgard construction and some real hash functions (MD5, SHA-1)

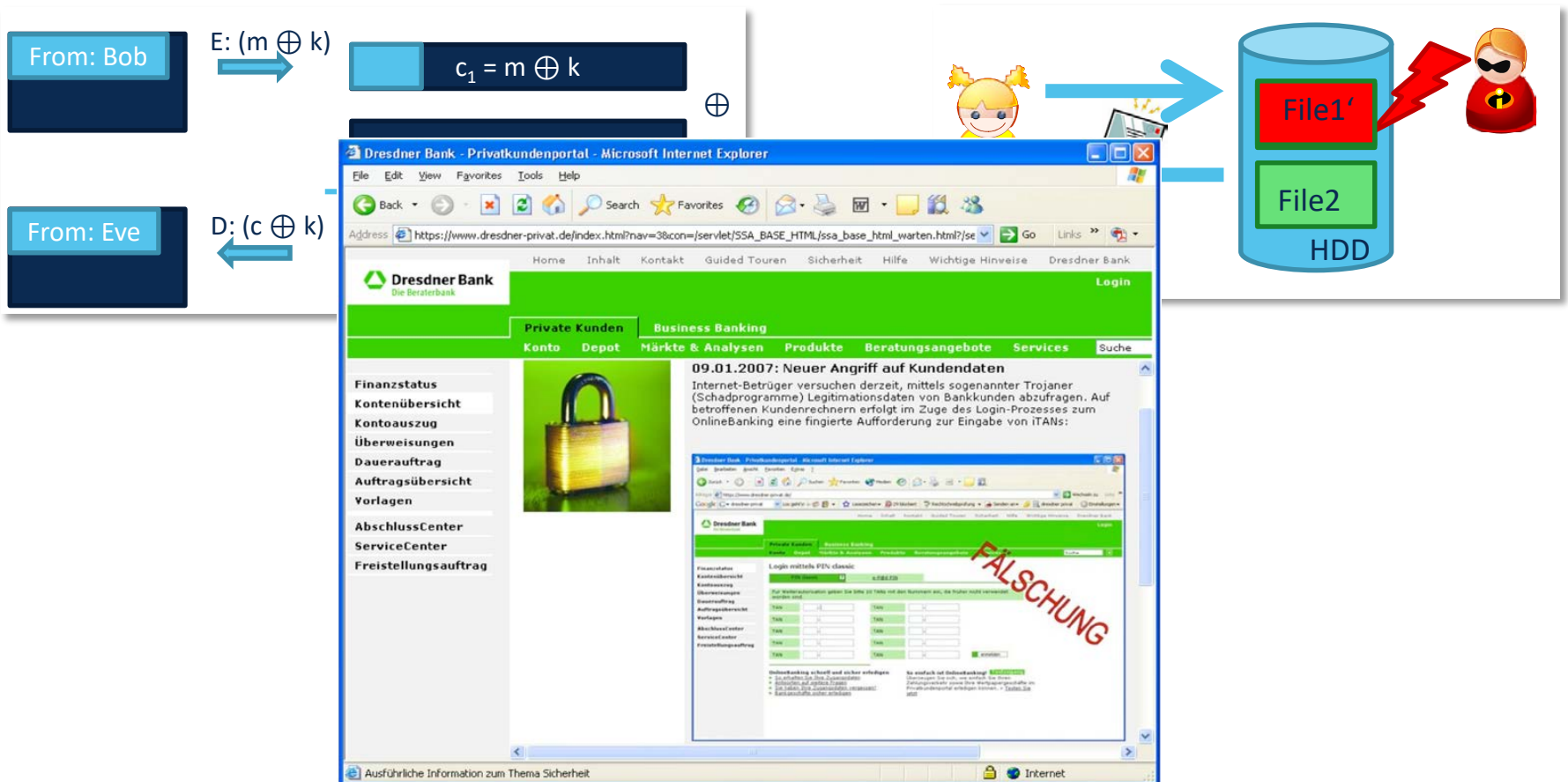
Block ciphers as compression functions

Secure MACs from hash functions and PRFs

MACs using block ciphers (CBC, NMAC, HMAC)

So far messages can be kept confidential

Integrity of messages not given





Algorithms:

Tag $S: M \rightarrow T$ $M = \{0,1\}^n ; S = \{0,1\}^t$ with $n \gg t$

Verify $V: M \times T \rightarrow \{\text{yes}, \text{no}\}$

Cross sum:

$f(x)$: calculate the cross sum of all bytes in the message

Is this secure? Why (not)?

$$f(5\ 23) = f(23\ 5)$$

CRC:

$\text{tag} \leftarrow \text{CRC}(m)$;

Verify tag: return $\text{CRC}(m) == \text{tag}$

Is this secure? Why (not)?

Integrity requires a secret

Adversary can create new message and recompute CRC

Simple Encryption:

$\text{tag} \leftarrow \text{Enc}(k,m) \mid_{1,\dots,6}$

Verify tag: return $\text{tag} == \text{Enc}(k,m) \mid_{1,\dots,6}$

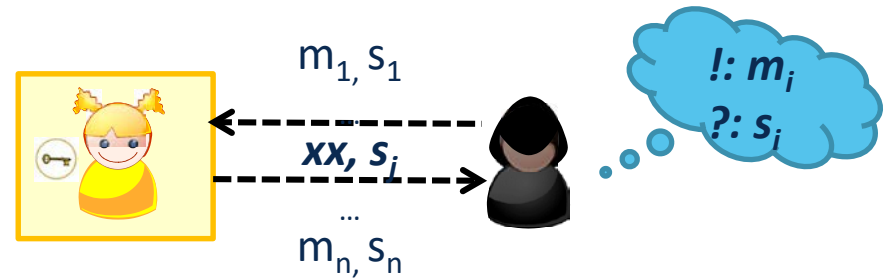
Is this secure? Why (not)?

Security depends on $|S|$

Adversary can guess tag for a message in 2^6

Chosen Message Attack:

- given s_1, s_2, \dots, s_n for chosen m_i



(variations are: known key / known signature attacks)

Existential Forgery:

Produce **some** new valid tuple (m,s) (any message, even gibberish)

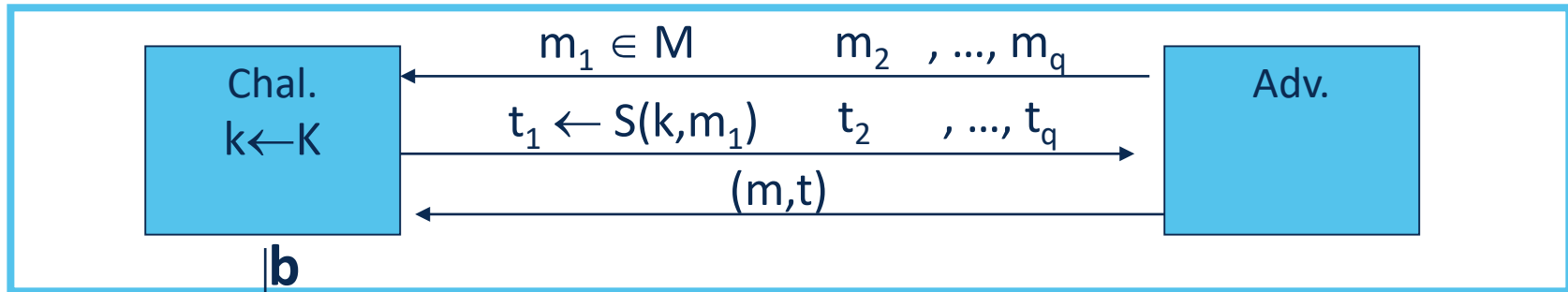
⇒ adversary cannot produce a valid tag for a new message

⇒ adversary cannot even produce (m,t') for (m,t) and $t' \neq t$

Breaches in general:

Exist. forgery < *selective forgery* < *universal forgery* < *total break*

For a MAC $I=(S,V)$ and adversary A , where (S,V) additionally take k :



$$\begin{cases} \mathbf{b}=1 & \text{if } V(k,m,t) = \text{'yes'} \text{ and } (m,t) \notin \{(m_1,t_1), \dots, (m_q,t_q)\} \\ \mathbf{b}=0 & \text{otherwise} \end{cases}$$

Def: $I=(S,V)$ is a **secure MAC** if for all “efficient” A :

$$\text{Adv}_{\text{MAC}}[A,I] = \Pr[\text{Chal. outputs } 1] \leq \epsilon$$

So how can we build a secure MAC?

Variations:

Existential forgery may forge tag on a message that seems gibberish (like, for instance, a random-looking bitstring like a ciphertext or a key...)

Assume a PRF $F: K \times X \rightarrow Y$

Define integrity scheme $I_F = (S, V)$:

- $S(k, m) := F(k, m)$
- $V(k, m, t) := \text{test } t == F(k, m)$



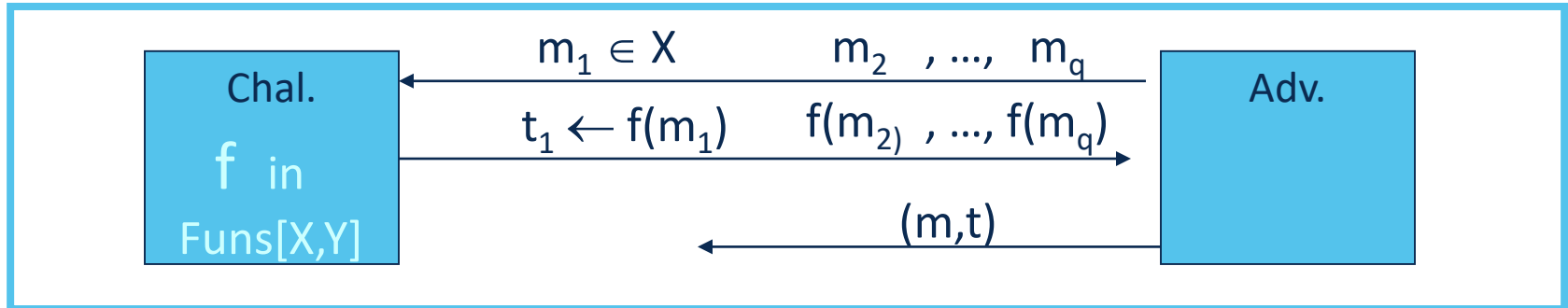
- *Is this a secure integrity scheme?*

suppose F is PRF, m given, what is the chance of the adv. to guess s ?

$$\text{Adv}_{\text{MAC}}[A, I_F] = \text{[redacted]}$$

- *What are its downsides?*

Assume $f: X \rightarrow Y$ is a random function and $1/|Y|$ is negligible



Assume $F: K \times X \rightarrow Y$ is a secure PRF, construct MAC I_F :

For every efficient MAC adversary A attacking I_F , there must be an efficient PRF adversary B attacking F, s.t.:

$$\text{Adv}_{\text{MAC}}[A, I_F] \leq \text{Adv}_{\text{PRF}}[B, F] + 1/|Y|$$

$\Rightarrow I_F$ is secure as long as:

- F is a secure PRF (given), and $|Y|$ is large ($|Y| \geq 2^{160}$)

What happens if we truncate the output?

Goal:

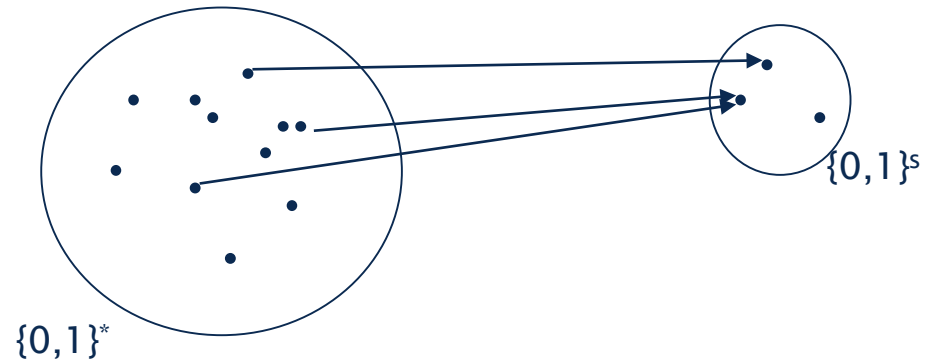
Map a message of arbitrary length to a characteristic digest (fingerprint)

Hash $H: M \rightarrow S$ with $M = \{0,1\}^*$ and $S = \{0,1\}^s$

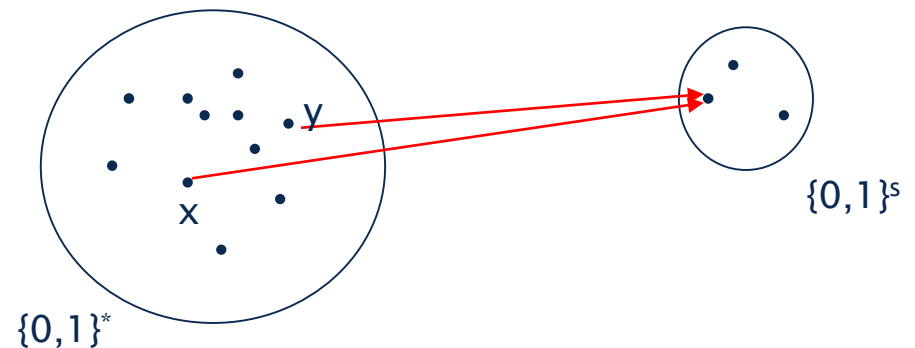
- has an efficient algorithm to evaluate $H(x)$
- is an „onto“ function (surjective, $\text{Im}(H) = S$)
- avoids collision (\rightarrow maps uniformly to S)
- creates chaos (slight changes in m yield large differences in s)

Further properties / requirements of hash functions for security:

- Compression is irreversible

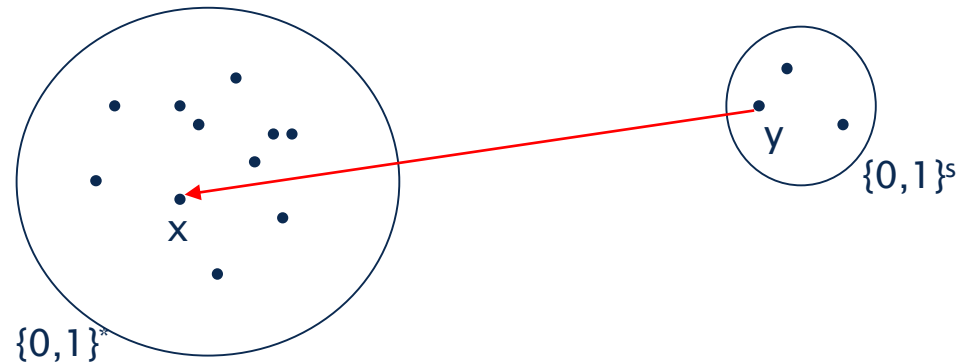


- Collision resistance

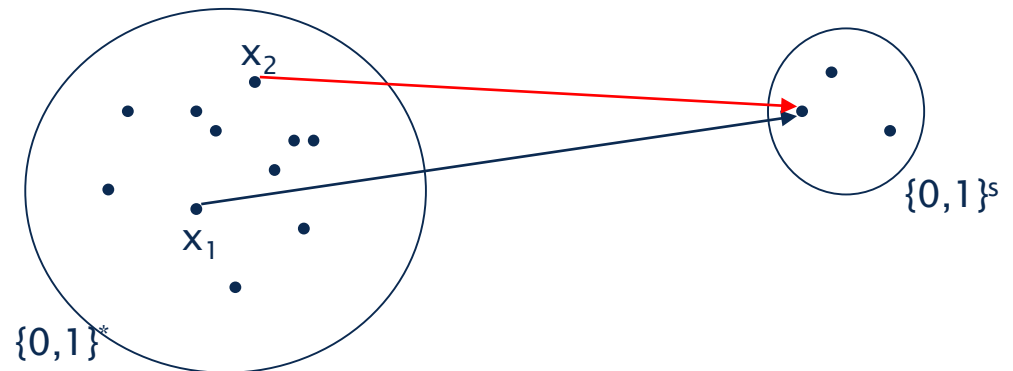


Further requirements to hash functions for security:

- Pre-image resistance



- 2nd pre-image resistance



Let $H: M \rightarrow S$ be a hash function ($|M| \gg |S|$)

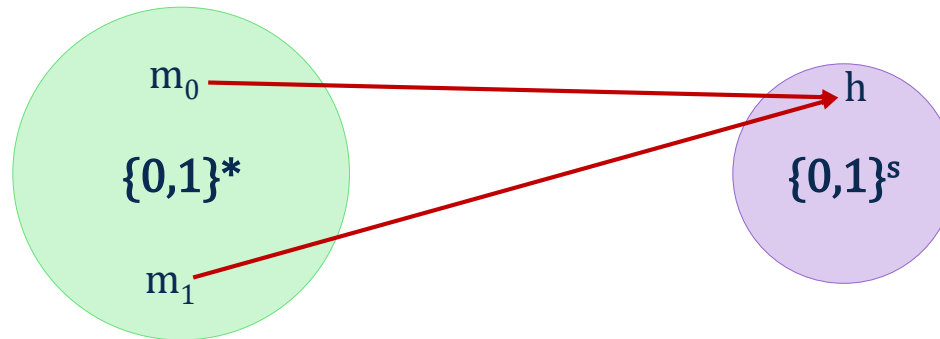
A **collision** for H is a pair $m_0, m_1 \in M$ such that:

$$H(m_0) = H(m_1) \quad \text{and} \quad m_0 \neq m_1$$

A function H is **collision resistant** if for all (explicit) “eff” algs. A :

$$\text{Adv}_{\text{CR}}[A, H] = \Pr[A \text{ outputs collision for } H] \leq \epsilon$$

Do collisions exist?



Yes.

but it should be hard to find collisions (in polynomial time)

Trivial Collision-Finder (Brute Force)

compute $H \stackrel{\text{def}}{=} \{H(m) \mid \text{for all } m \in \{0,1\}^s\}$

if no collision found compute $H(m^*)$ for any $m^* \notin \{0,1\}^s$

there must be at least one m with $H(m) \in H$ such that $H(m) = H(m^*)$

s must be sufficiently large

time needed
 $O(2^s)$

Let $H: M \rightarrow \{0,1\}^s$ be a hash function ($|M| \gg 2^s$)

Generic alg. to find a collision **in time** $O(2^{s/2})$ hashes

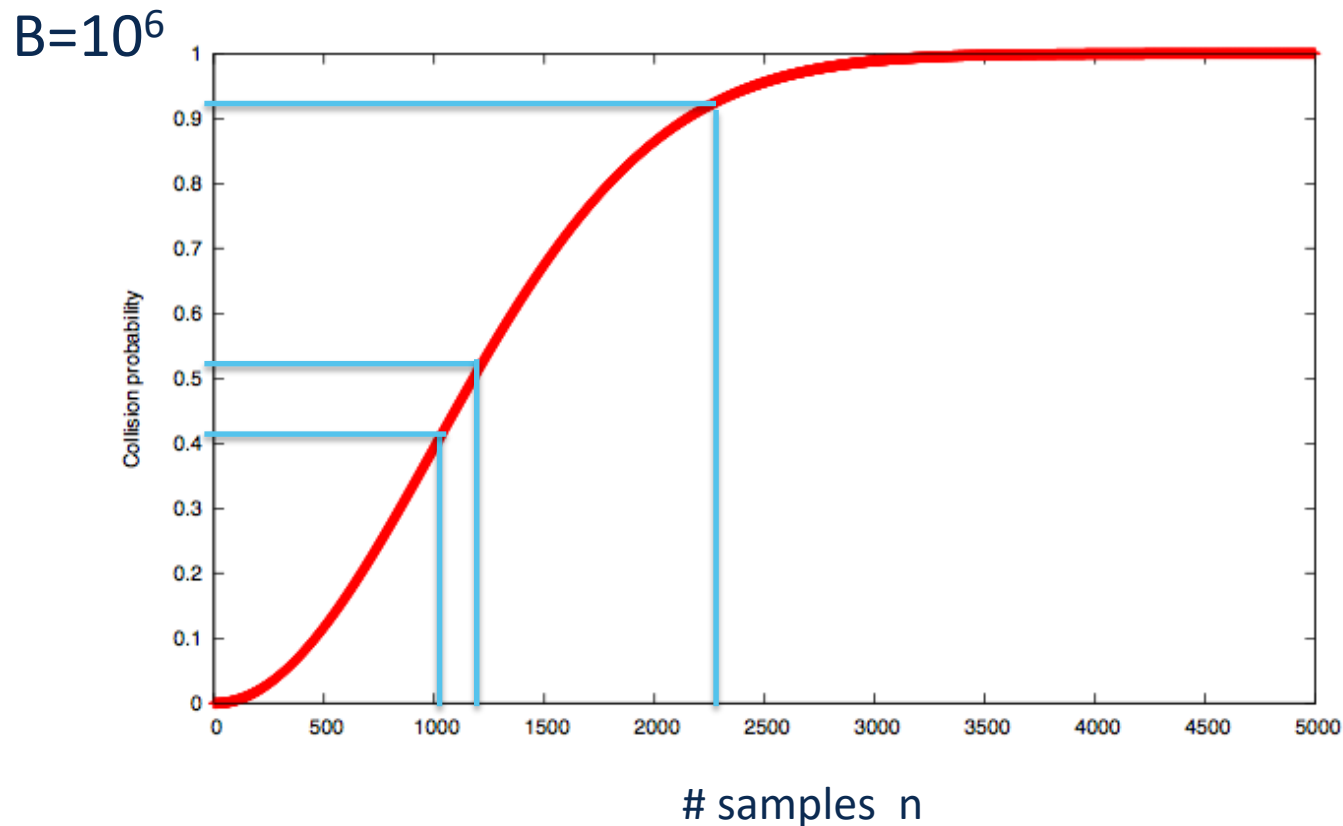
Algorithm:

1. Choose $2^{s/2}$ random messages in M : $m_1, \dots, m_{(2^{s/2})}$ (distinct w.h.p.)
2. For $i = 1, \dots, 2^{s/2}$ compute $t_i = H(m_i) \in \{0,1\}^s$
3. Look for a collision ($t_i = t_j$). If not found, got back to step 1.

How well will this work?

Let $r_1, \dots, r_n \in \{1, \dots, B\}$ be random integers, chosen iid.

BP states: when $n = 1.2 \times B^{1/2}$ then $\Pr[\exists i \neq j: r_i = r_j] \geq \frac{1}{2}$



$H: M \rightarrow \{0,1\}^s$. Collision finding algorithm:

1. Choose $2^{s/2}$ random elements in M : $m_1, \dots, m_{2^{s/2}}$
2. For $i = 1, \dots, 2^{s/2}$ compute $t_i = H(m_i) \in \{0,1\}^s$
3. Look for a collision ($t_i = t_j$). If not found, got back to step 1.

Expected number of iterations until success \approx 

Running time: $O(2^{n/2})$ (space $O(2^{n/2})$)

Cryptographic hash functions:

Hash $h: M \rightarrow S$ with $M = \{0,1\}^*$ and $S = \{0,1\}^s$

Three core security requirements:

- **Collision** resistance

- Collision: Given $h(\cdot)$, find m_1, m_2 with $m_1 \neq m_2$ such that $h(m_1) = h(m_2)$
- $Adv_{CR}(A, h(\cdot))$: $Pr[\text{Collision}] \leq \min\{|M|, |S|\}^{\frac{1}{2}}$
 - Common assumption: $|M| \gg |S|$, hence $O(2^{s/2})$ (General attack due to BP)

- **Pre-image, second pre-image** resistance

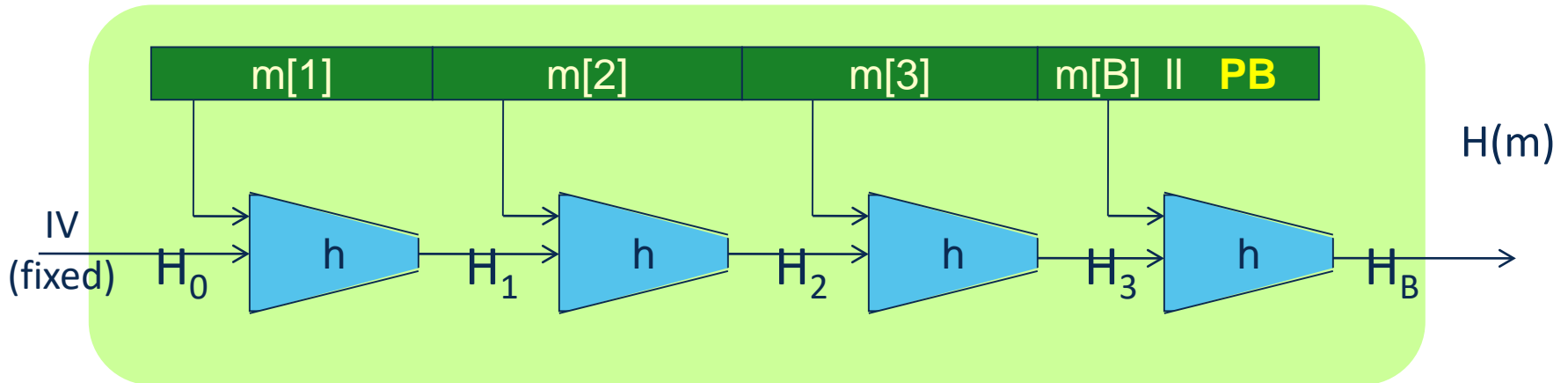
- Pre-image: given $h(m)$ find $h^{-1}(h(m)) = m$
- 2nd pre-image: given m_1 find m_2 such that $m_1 \neq m_2$ and $h(m_1) = h(m_2)$
- $Adv_{(S)PR}(A, h(\cdot)) \leq |S| + \epsilon$, (cf. assumption above, hence $O(2^s)$)

From *short message blocks* to *arbitrarily long messages*...

Given a compression function $h : \{0,1\}^{2s} \rightarrow \{0,1\}^s$ and

Input $m \in \{0,1\}^*$ of length L and PB: = 1000...0 || L
64 bits

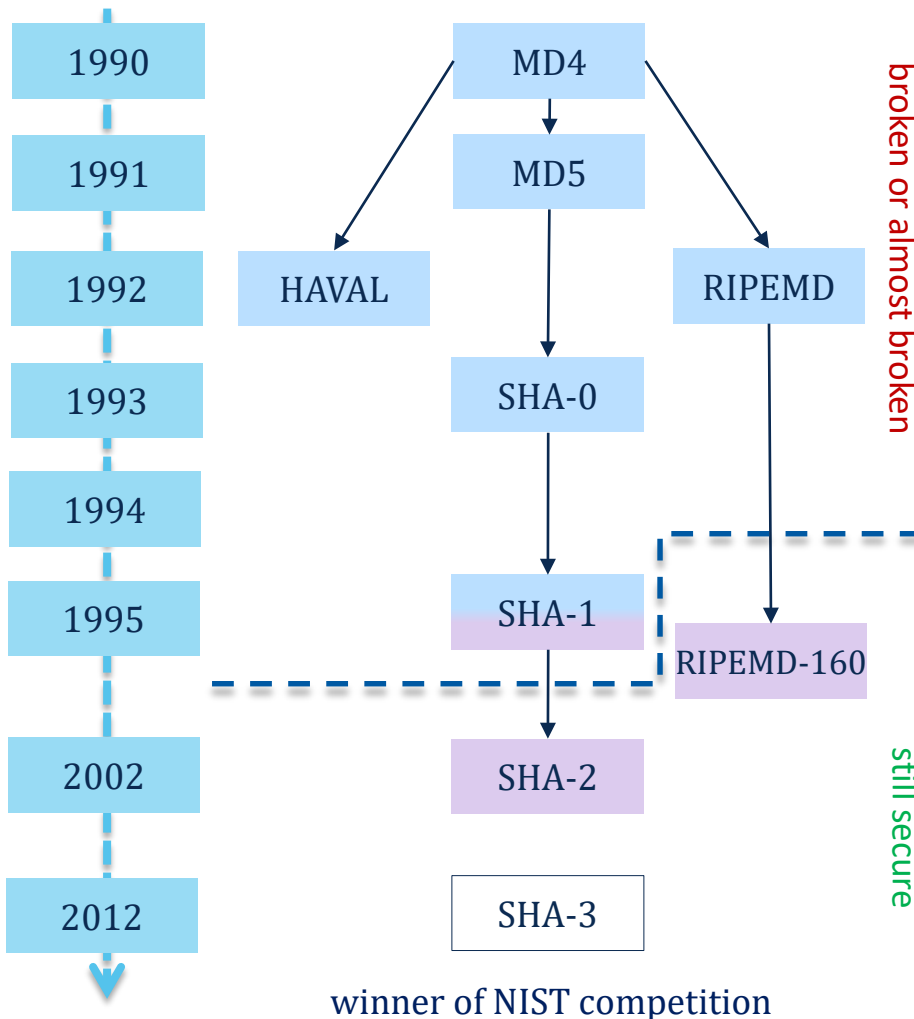
Construct H of $B = \lceil L/s \rceil$ iterations of h :



If h is a fixed length CRHF, then H is an arbitrary length CRHF

Proof: either $M=M'$, or $H_{B-i}(m[B-i])=H_{B-i}(m'[B-i])$
no collision collision on h

A brief history of Hash Functions



MD4 $s = 128$ bits
collisions in $O(2^8)$, preimages in $O(2^{102})$

MD5 $s = 128$ bits
collisions in $O(2^{32})$
known colliding documents, certificates

HAVAL $s = 128, 160, 192, 224, 256$ bits
collisions on HAVAL-128 in $O(2^6)$

RIPEMD $s = 128$ bits
collisions are known

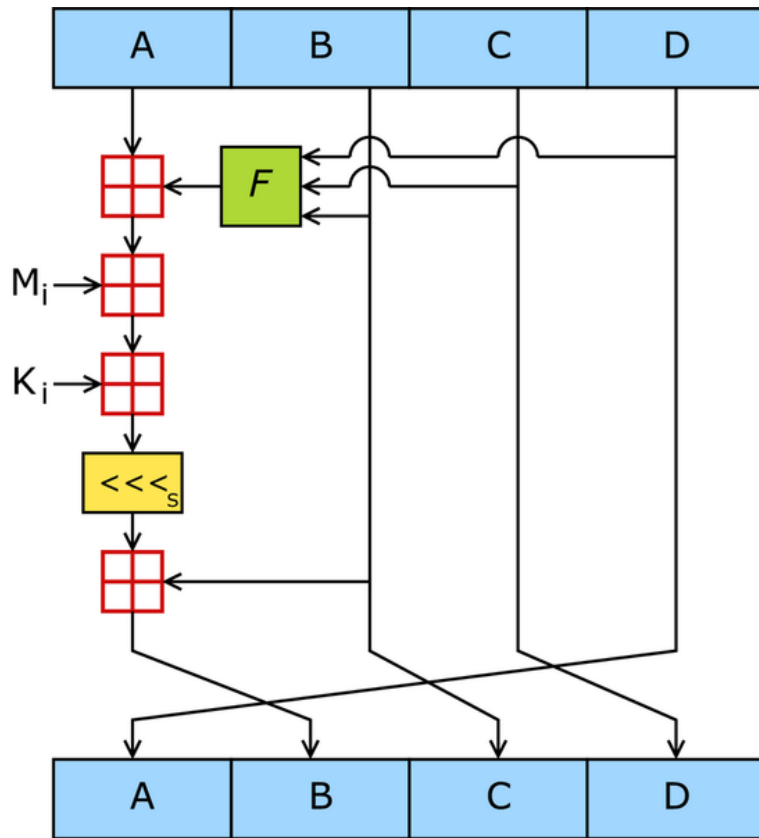
SHA-0 $s = 160$ bits
collisions in $O(2^{39})$, replaced by SHA-1 in '95
meanwhile collisions in 1 hour

SHA-1 $s = 160$ bits
collisions in $O(2^{63}) - O(2^{69})$
still secure in practice

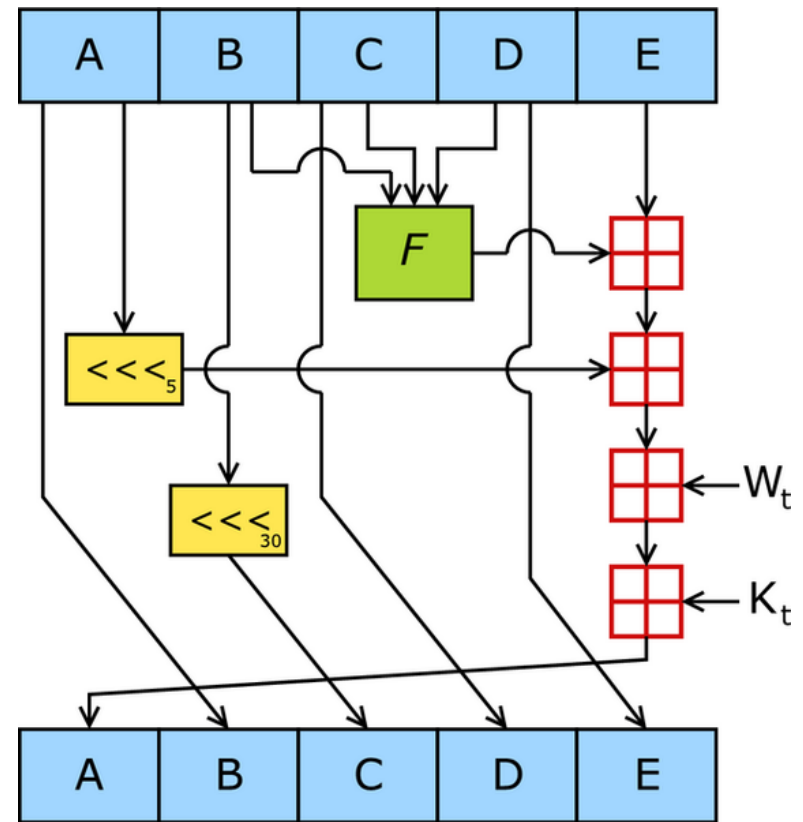
SHA-2 supports $s = 224, 256, 384, 512$ bits

Fixed length h (for arbitrary length m , as MD):

While SHA-2 is also a MD construction, SHA-3 isn't...



MD5 (128 bit)

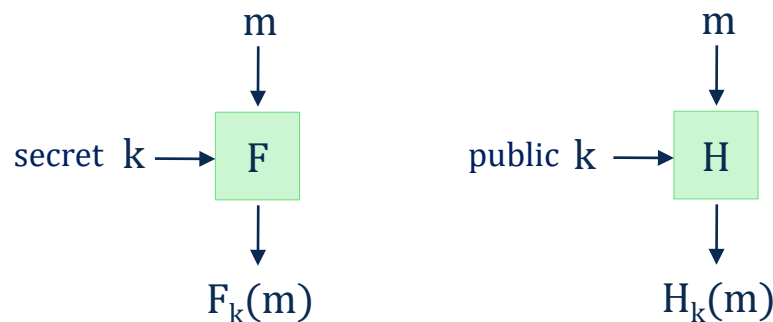


SHA-1 (160 bit)

Can we build compression functions from block ciphers?

- block ciphers $F : \{0,1\}^k \times \{0,1\}^b \rightarrow \{0,1\}^b$ behave like PRPs
- the outputs of F are close to uniform
- collision-resistance
- provable security: reduction to the security of a block cipher
- block ciphers used in practice are efficient
- implementation of a block cipher immediate hash function

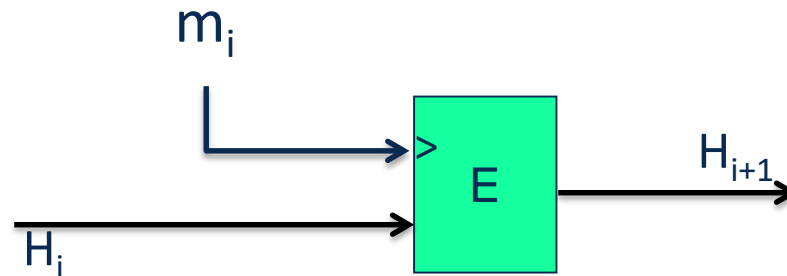
but what about the keys?



However, block ciphers are reversible!

$E: K \times \{0,1\}^n \rightarrow \{0,1\}^n$ a block cipher.

Construct cascade, for compression encrypt message blocks:



What's wrong with that?

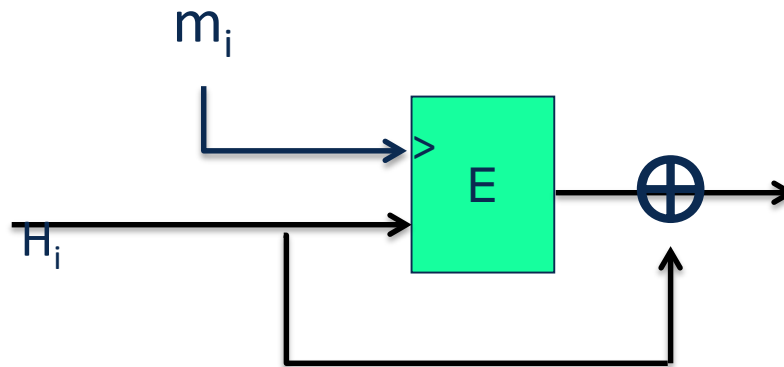
$$H_{i+1} = E(m_i, H_i)$$

Can you find a collision on this compression function?

$$H_{i+1} = E(m', D(m', H_{i+1}))$$

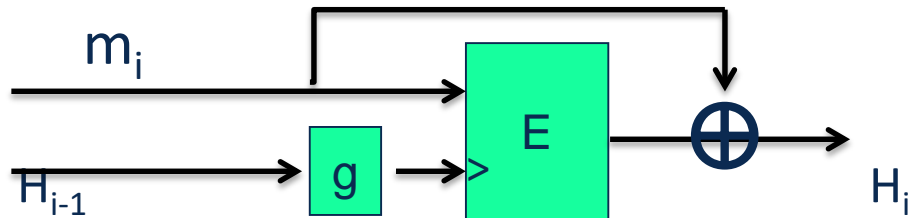
$E: K \times \{0,1\}^n \rightarrow \{0,1\}^n$ a block cipher.

The **Davies-Meyer** compression function: $h(H, m) = E(m, H) \oplus H$

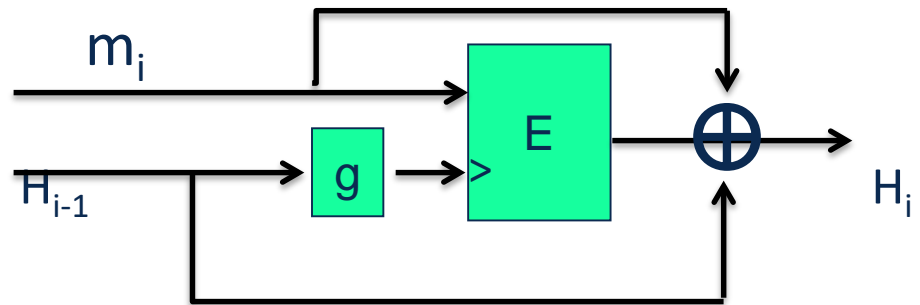


Thm: Suppose E is ideal cipher (collection of $|K|$ random perms.).
 To find collision $h(H, m) = h(H', m')$ takes $O(2^{n/2})$ evaluations of (E, D) .
 Equivalent to birthday attack \rightarrow as good as possible.

Matyas-Meyer-Oseas



Miyaguchi-Preneel



... and many insecure one's..

The efficiency of Davies-Meyer compression function in the Merkle-Damgård transformation depends on the efficiency of the chosen block cipher

Example

AES-128 has key length $\kappa = 128$ bits, block length $b = 128$ bits

AES-192 has key length $\kappa = 192$ bits, block length $b = 128$ bits

AES-256 has key length $\kappa = 256$ bits, block length $b = 128$ bits

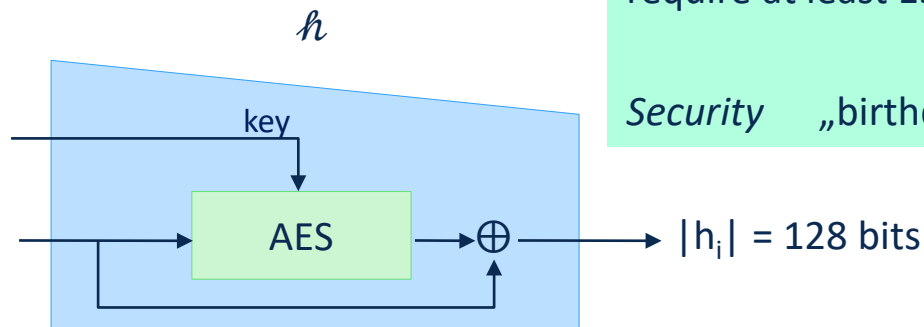
Davies-Meyer with AES

$|m_i| = 128$ bits

$|m_i| = 192$ bits

$|m_i| = 256$ bits

$|h_{i-1}| = 128$ bits



Efficiency hashing of a 5MB file would require at least $156250 \approx 2^{17}$ AES executions

Security „birthday attack“ $O(2^{64})$

block ciphers with much larger key lengths and block sizes needed...

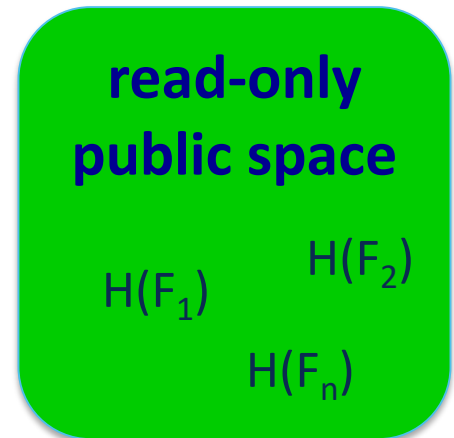
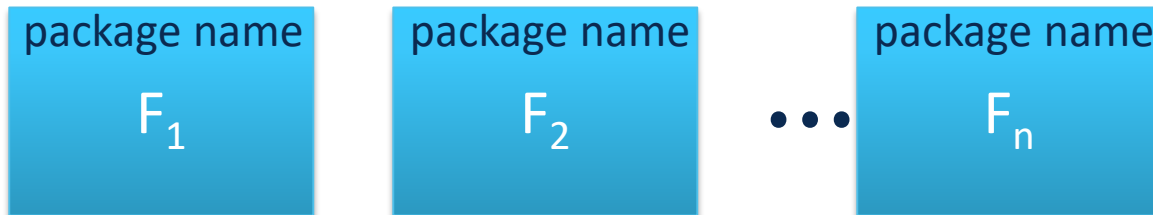
So can we use these hash functions directly as a MAC?

Quick answer: *no, we need some secret (recall: CRC)!*

Intermediate answer is „*for special cases, yes*“:

Assume a public repository of files

and a **public read-only space** with their hashes



User can verify validity of the contents of a packet

Adversary cannot forge packets for given $H(F_i)$ (collision!)

MAC: signing alg. $S(k,m) \rightarrow t$ and verification alg. $V(k,m,t) \rightarrow \{0,1\}$

First Idea: keyed hash functions, MAC: $H(k || m)$

Recall: Secure hash is collision resistant

But a secure MAC needs to be **unforgeable** under **chosen message attack**
no existential forgery: no valid tag forged for arbitrary message

Consider Merkle-Damgard construction: $s = H(m || PB)$

Can you forge a valid tag?

Feasible chosen message attack:

$$A - [m] \rightarrow C \qquad s = h(k || m || PB)$$

$$C \leftarrow [s] - A$$

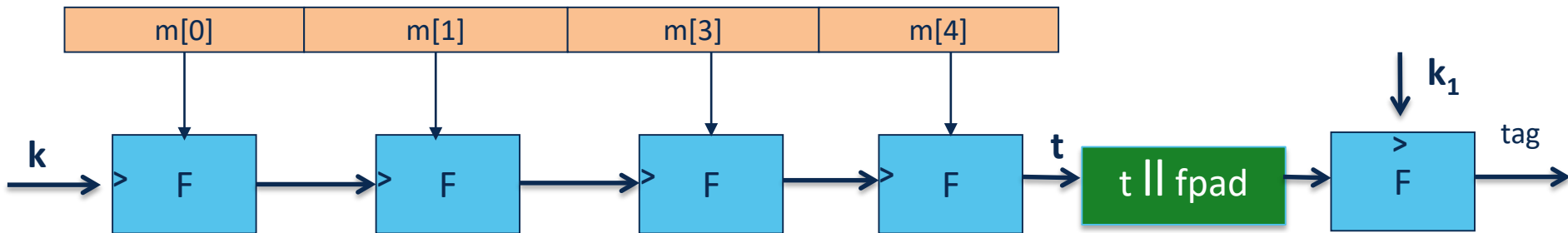
$$s' = h(m || m') = h(s || m' || PB')$$

$$A - (m || m', s') \rightarrow C \text{ and wins the game!}$$

Recall Merkle-Damgard, k instead of fixed IV:

Let $F: K \times X \rightarrow X$ be a PRF, define new PRF $F_{\text{NMAC}}: K^2 \times X^{\leq L} \rightarrow X$

Cascade:



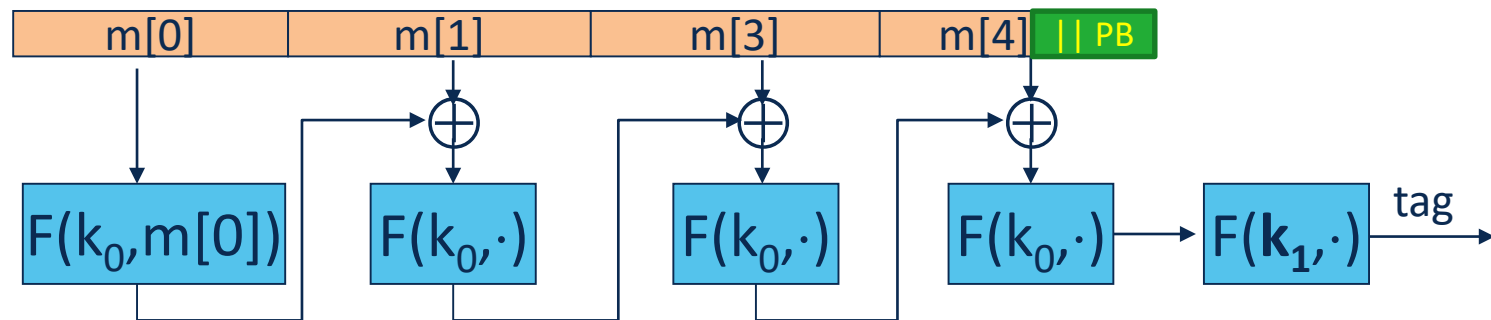
Why the last encryption with second key?

Otherwise: $\text{cascade}(k, m \parallel m') = \text{cascade}(\text{cascade}(k, m) \parallel m')$

Recall Merkle-Damgard, PRFs & mode of operation

Let's create a **MAC** from AES (a PRP for small messages)!

Let $F: K \times X \rightarrow X$ be a PRF, define new PRF $F_{ECBC}: K^2 \times X^{\leq L} \rightarrow X$
 CBC-MAC (ECBC: Encrypted Last Block CBC)



CBC-MAC insecure if $|m| \neq$ multiple of block size

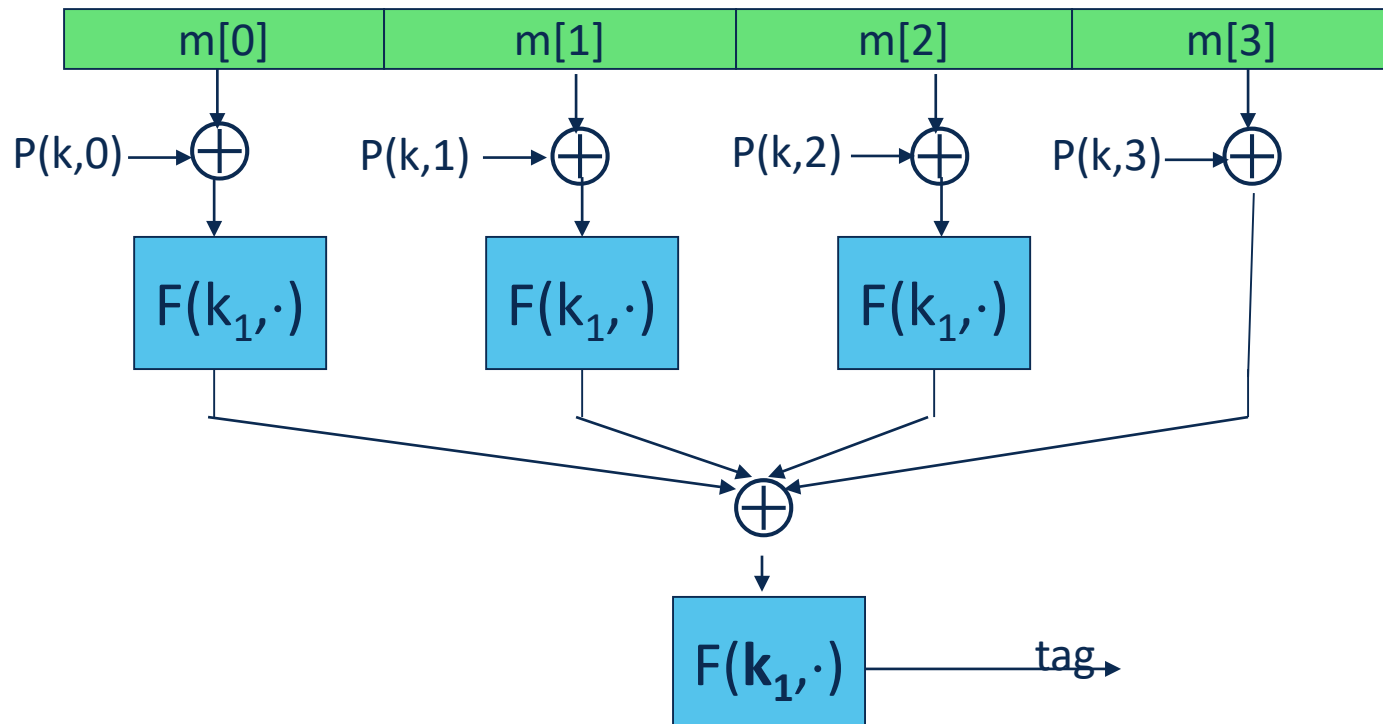
Solve by padding (cmp. hash): $PB = 10\dots0$ (Why not “ $0\dots0$ ”)?

What if $|m| =$ multiple of block size?

Let $F: K \times X \rightarrow X$ be a PRF

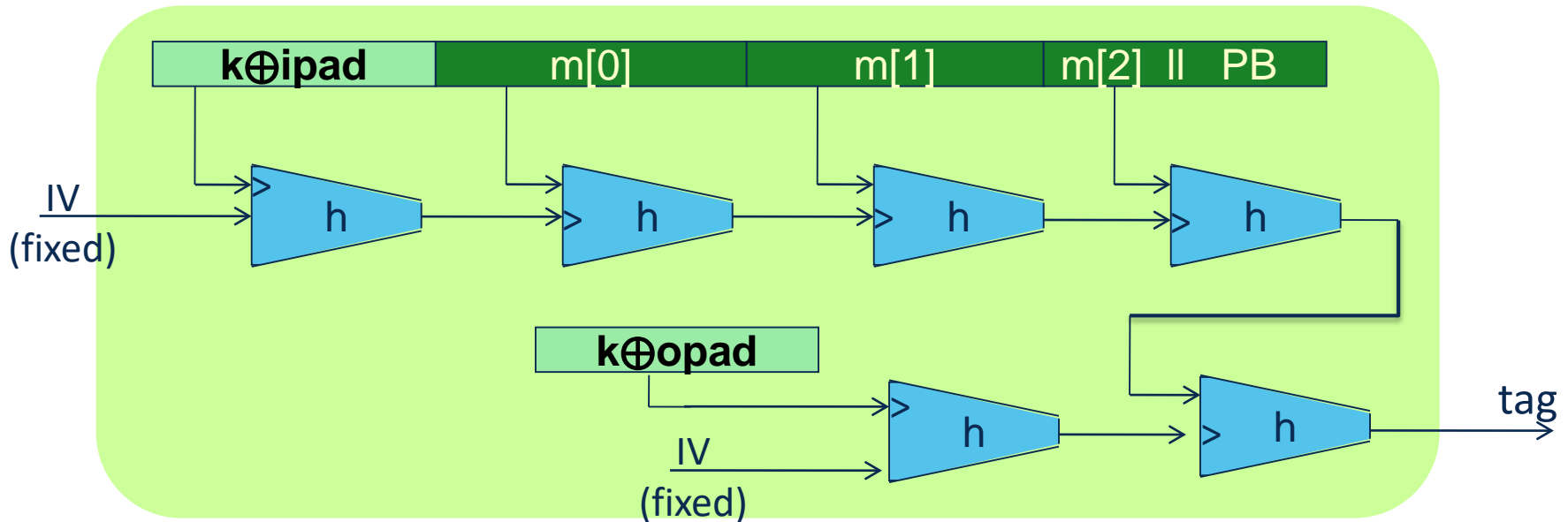
Given $P(k,i)$ being a function that is easy to compute, and key: (k, k_1)

Define new PRF $F_{\text{PMAC}}: K^2 \times X^{\leq L} \rightarrow X$



Hashing is fast, but $H(k || m)$ insecure

Solution: encase message with keys! (Essentially double NMAC)



HMAC: $S(k, m) = H(k \oplus opad || H(k \oplus ipad || m))$

(used in TLS, IPsec,...)

Merkle-Damgard on compression functions

- Create an H for arbitrary sized input to fixed size output
- Using „small“ compression functions
- We already know (and have implemented) some small compression functions, which we seem to „understand“ well (we have a good feeling about AES, and it's already implemented in hardware)
- Given the above, we can prove the security of H

However: other approaches to create secure arbitrary length hash functions do exist...

Recall the definition of $h: M \rightarrow S$ with $M = \{0,1\}^*$ and $S = \{0,1\}^s$

Proving security: the output is indistinguishable from an output of a really *random oracle*

$$RO: M \rightarrow Y \text{ with } Y = \{0,1\}^s$$

Now what happens if

$$F: M \rightarrow Y \text{ with } Y = \{0,1\}^*$$

The security in the random oracle model depends on $|S|$, does the security of the latter increase with the length of the output?

Probably not, rather with the size of internal state...

- 2005: NIST discusses new standard for Secure Hash Algorithm
rationale: all prior standards are based on MD, consider what happened if there was a systematic break...
- 2007: Call for proposals; requirements (replacement for SHA-2):
224, 256, 384, and 512 bit output length
- 2008: 64 submissions
- 2009, 2010, 2012: Candidate conferences (14 candidates, 5 finalists:
BLAKE (Aumasson, Henzen, Meier, Phan, based on ChaCha20), **Grøstl** (TU Graz, DTU, based on AES), **JH** (Hongjun Wu), **Keccak** (Bertoni, Daemen, Peeters, van Assche), **Skein** (“Team World” including Schneier, Lucks, based on Threefish).
- 2012: Announcement of Keccak as SHA-3

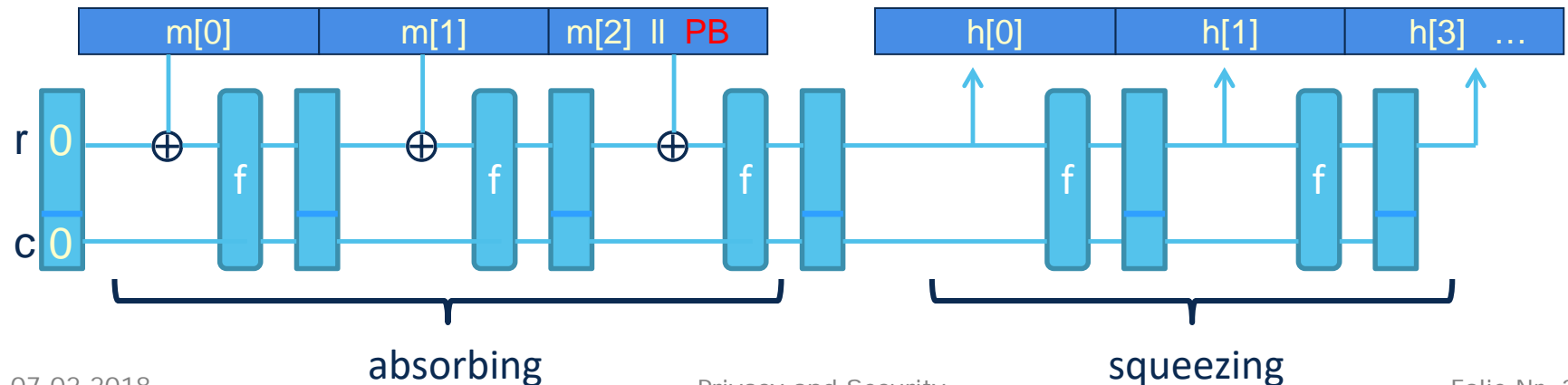


Specification of two modes:

- SHA-2 replacement: fixed length output of 224, 256, 384, 512 bits
- Variable length output: output of arbitrary length
- Keccak[r,c] with internal permutation Keccak-f[b]

Construction of two phases

- Absorb: block-wise input of message
- Squeeze: output of required bits as hash value



Keccak[r,c] parameters:

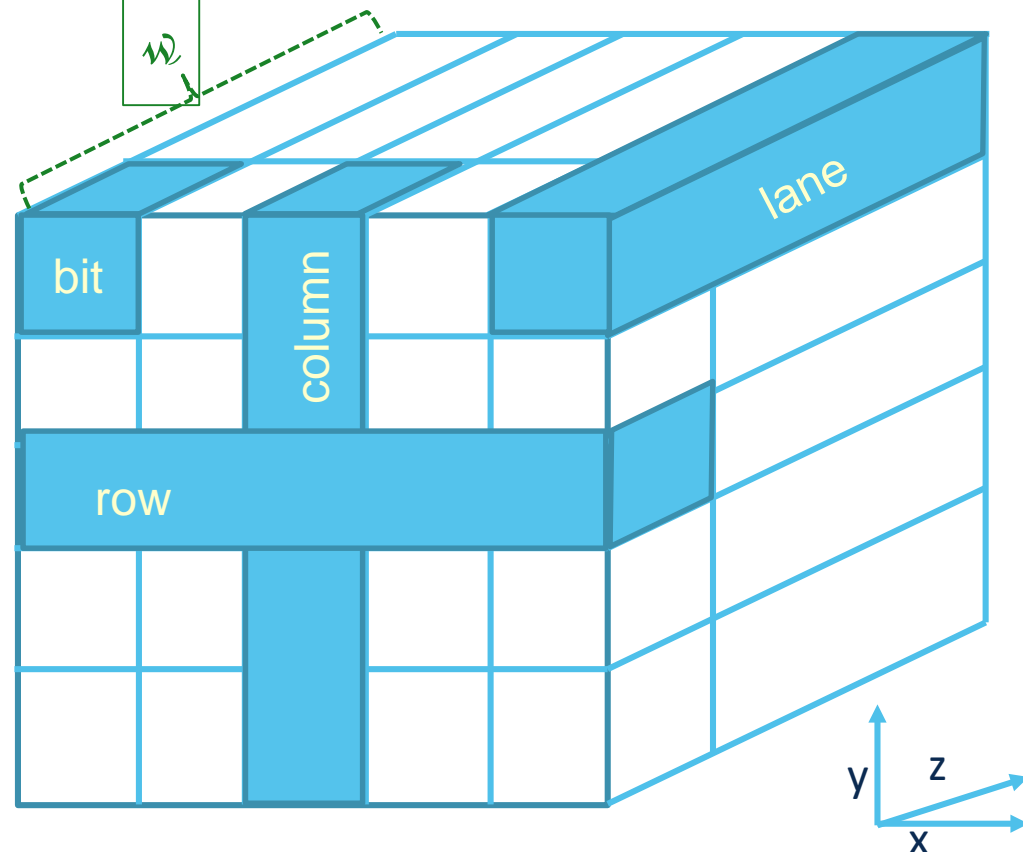
- bit rate r, capacity c (c = 256,512)
- word length $w = 2^l \quad l=0,1,\dots,6$ (6 for 64 bit systems)

$$b = r+c = 5 \times 5 \times w$$

$$= 25, 50, 100$$

$$100, 200, 400$$

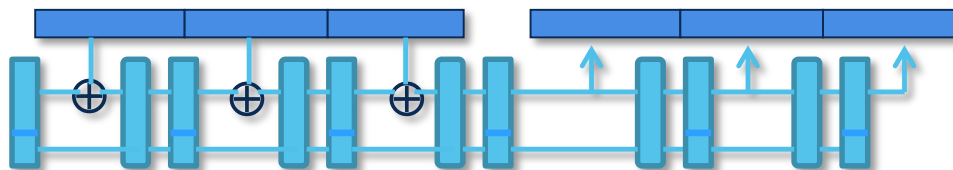
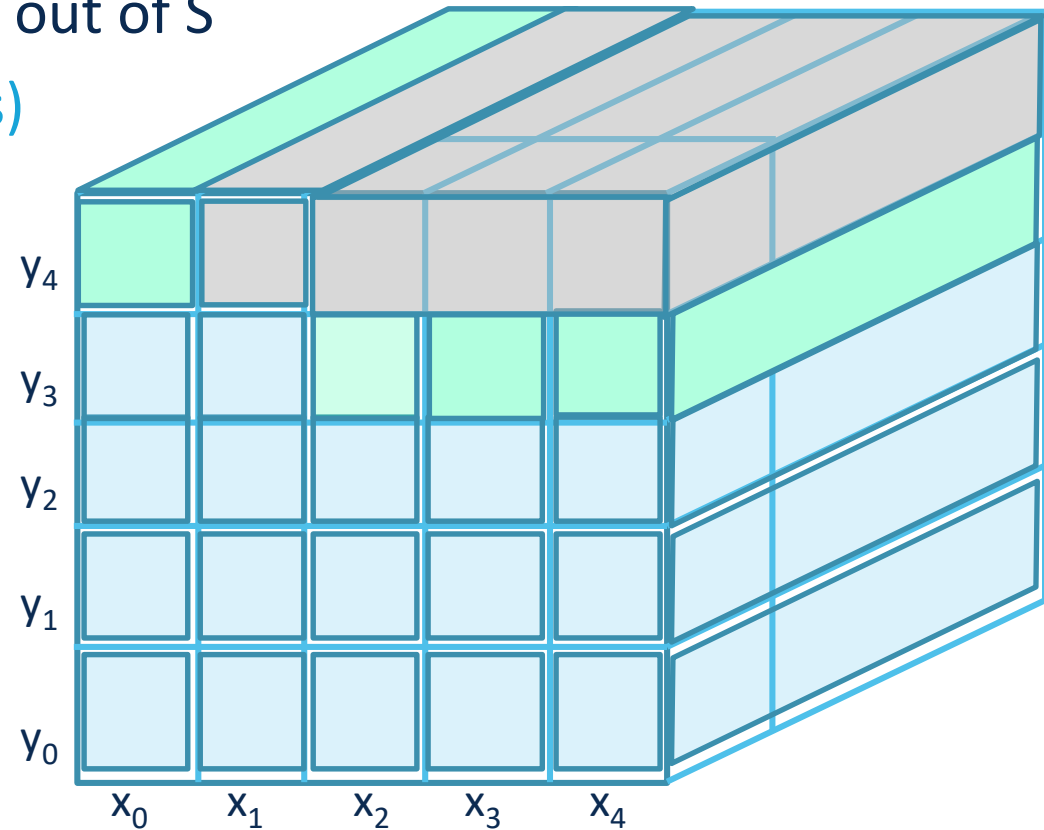
$$800, 1600$$



Absorbing into and squeezing out of S

- 256 bit security (c=512 bits)
- 128 bit security

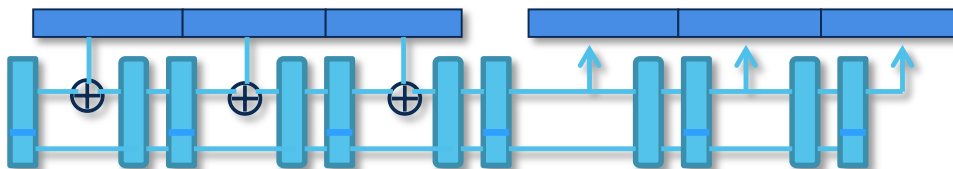
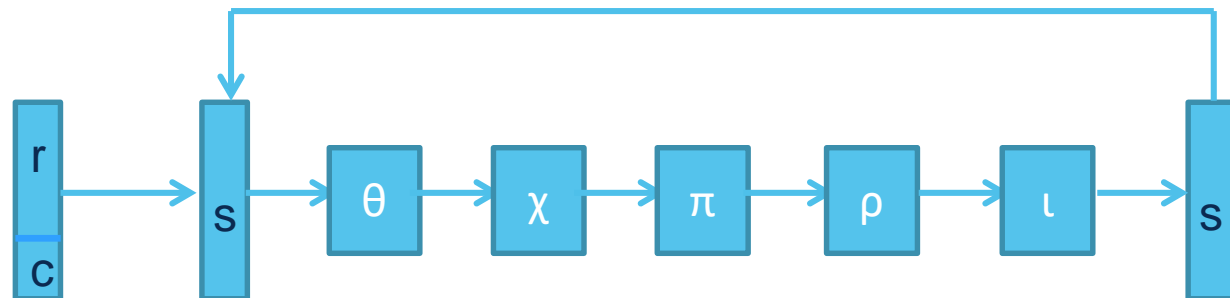
Bits in c never in- nor output



Keccak permutes state in 12 + 2l rounds

- 32 bit processor -> Keccak-f[800] -> 22 rounds
- 64 bit processor -> Keccak-f[1600] -> 24 rounds
- (Keccak-f[25] -> 12 rounds)

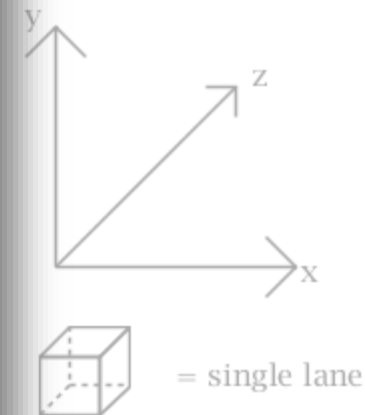
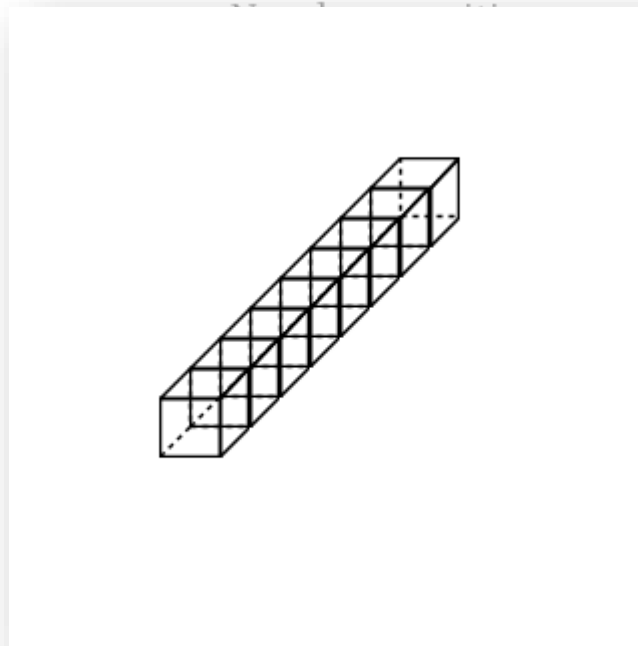
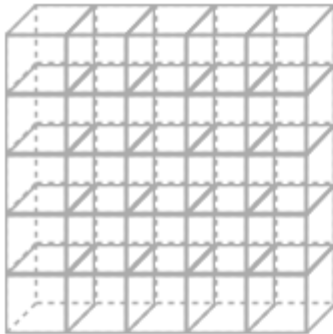
Five operations for each round:



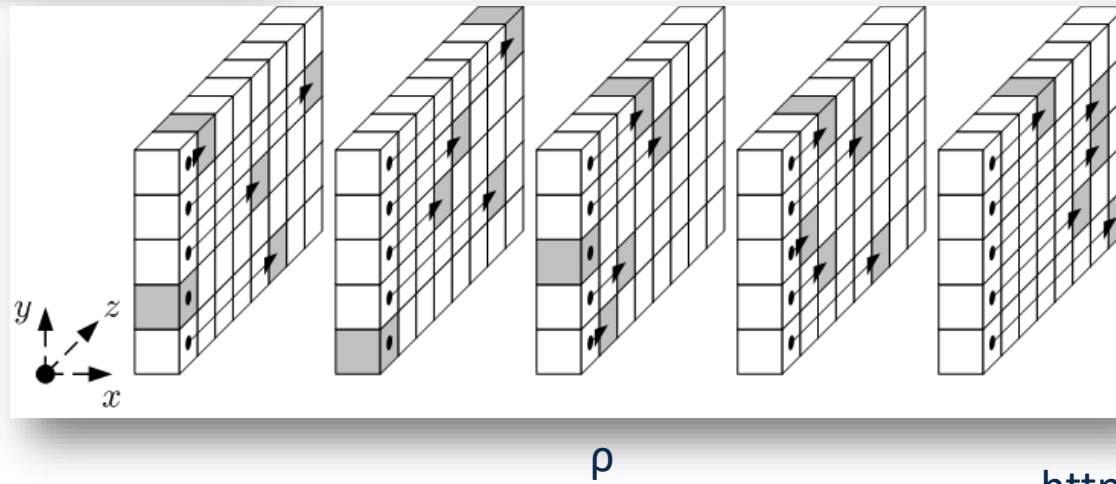
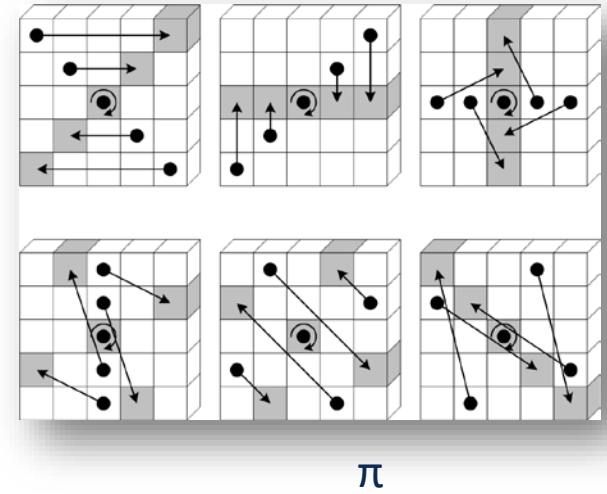
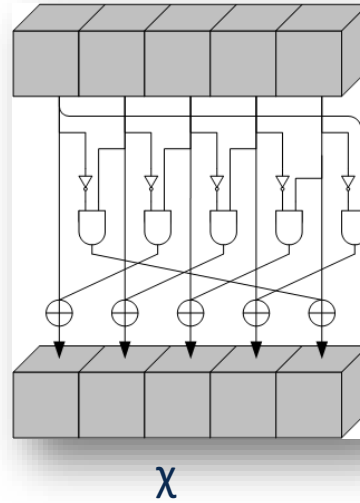
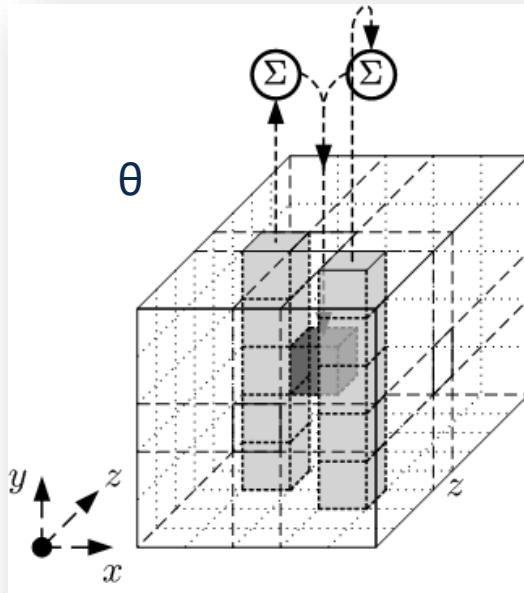
Internal function defined as permutation Keccak-f[b]:

$$S \leftarrow \iota \circ \rho \circ \pi \circ \chi \circ \theta (S)$$

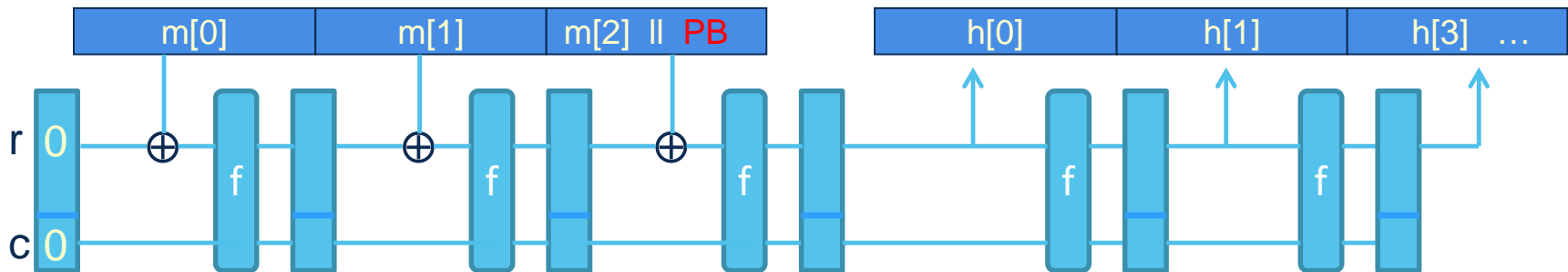
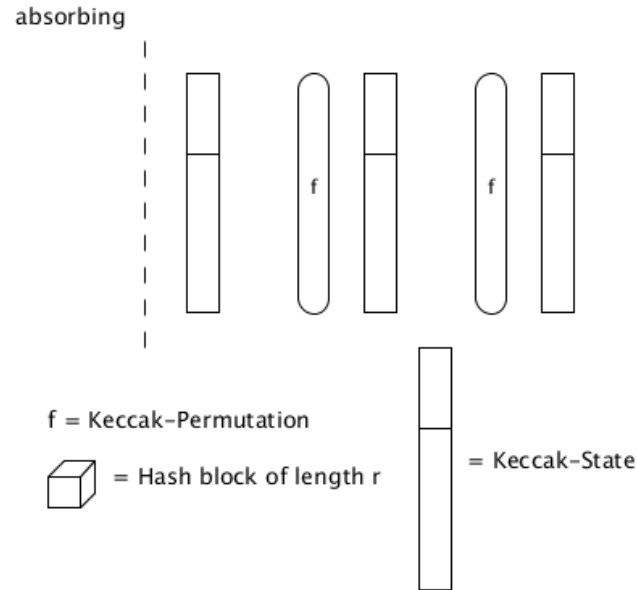
Old lane positions



- Iota adds predefined constant to lane [0,0]



Hashausgabe z:



Keccak[r,c] (l=6 -> Keccak-f[1600]) standardized as SHA-3:

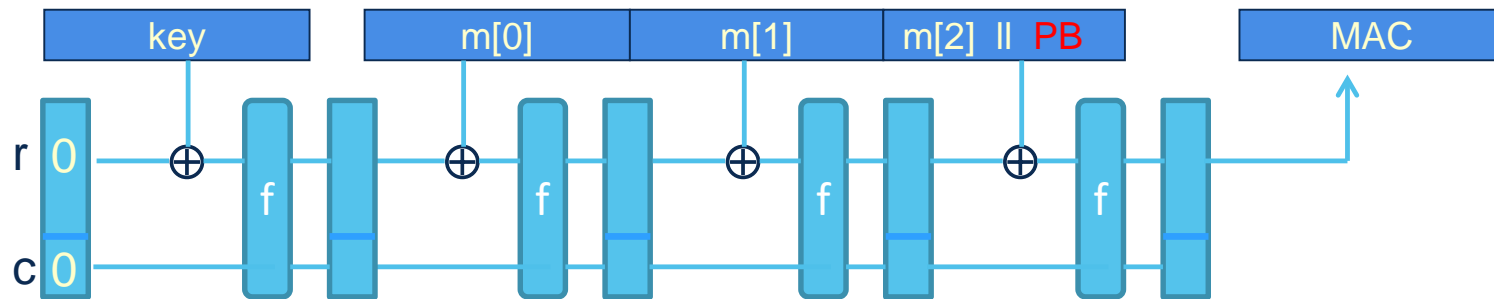
Algorithm	Output size	Block size	Bit security
SHA3-224	224	1152	112
SHA3-256	256	1088	128
SHA3-384	384	832	192
SHA3-512	512	576	256
SHAKE128	d	1344	128*
SHAKE256	d	1088	256*

* or d/2, if smaller

Can we implement a secure MAC as:

SHA-3($k \parallel m$)?

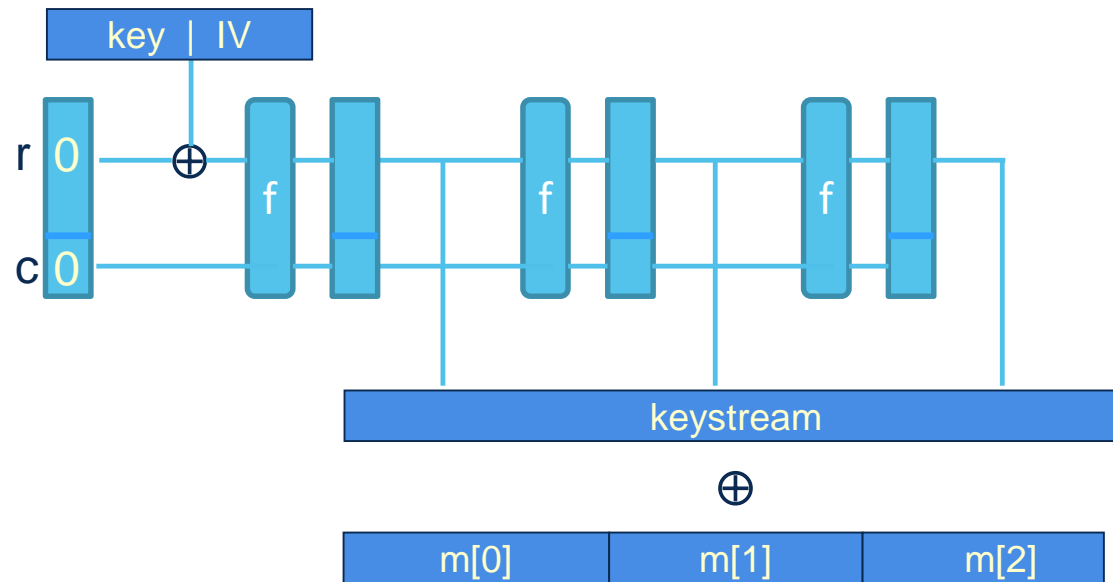
How?



Why?

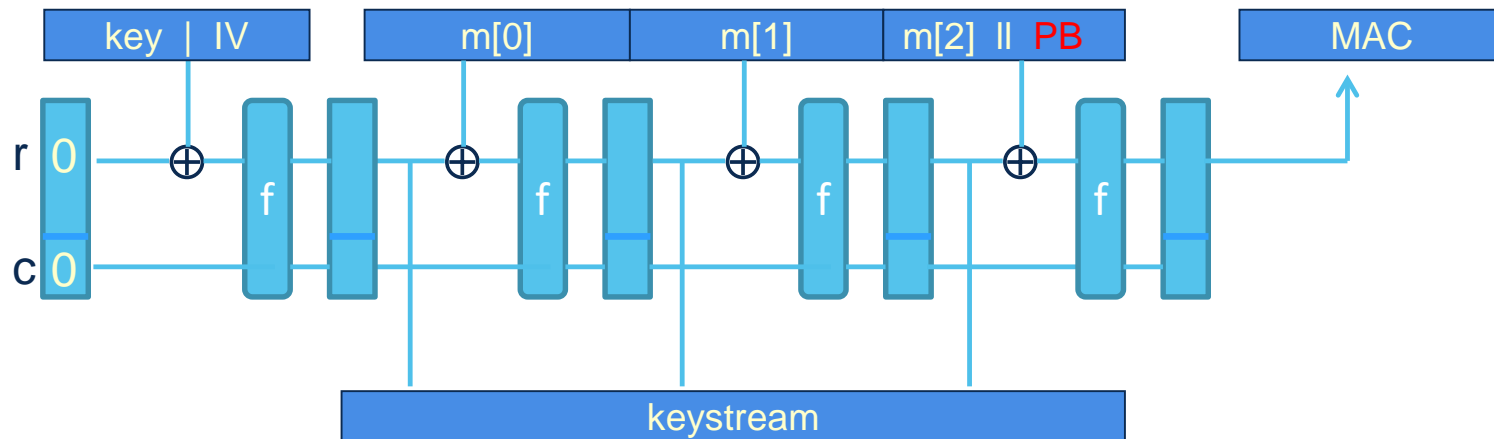
Can we implement a stream cipher with SHA3?

How?



Stream ciphers are malleable... Can we achieve authenticated encryption with SHA3?

How?



Can be secure against all adversaries and faster than previous MACs

Let q be a large prime (e.g. $q = 2^{128} + 51$) (or: 130-5)

key = $(a, b) \in \{1, \dots, q\}^2$ (two random ints. in $[1, q]$)

msg = $(m[1], \dots, m[L])$ where each block is 128 bit int.

$$S(\text{key}, \text{msg}) = P_{\text{msg}}(a) + b \pmod{q}$$

$$\text{where } P_{\text{msg}}(x) = \sum_{i=1}^L x^i \cdot m[i]$$

$$= m[L] \cdot x^L + \dots + m[1] \cdot x \text{ is a poly. of deg. } L$$

Let (S,V) be a secure one-time MAC over $(K_1, M, \{0,1\}^n)$.

Let $F: K_F \times \{0,1\}^n \rightarrow \{0,1\}^n$ be a secure PRF.

Carter-Wegman MAC: $CW((k_1, k_2), m) = (r, \underbrace{F(k_1, r)}_{\text{slow but short inp}} \oplus \underbrace{S(k_2, m)}_{\text{fast long inp}})$

for random $r \leftarrow \{0,1\}^n$.

Thm: If (S,V) is a secure **one-time** MAC and F a secure PRF then CW is a secure MAC outputting tags in $\{0,1\}^{2n}$.



A (*badly implemented*) verification oracle

- Compares output of MAC byte by byte
- (*and returns as soon as a byte differs*)

Mallory has access to verification oracle, aims at universal forgery



(Given arbitrary message (*pirated code*), guess correct MAC)

1. For B_i try all 256 possible values
2. Measure time until rejection (comparison per byte 2.2ms)

For 16 byte MAC (2^{128}), how many attempts do you need?

MACs verify integrity of messages

$S(k,m)$; $V(k,m,t)$ \rightarrow secret key must be used, known to verifier

MAC hard to forge without secret key, but *integrity purely mutual*:

- Once key is disclosed, receiver can create arbitrary new tags!
- \Rightarrow Proof of origin not towards third parties (no non-repudiation!)

But to achieve this, we first have to understand keys better...

You can explain the goals and ideas of message integrity

You know different adversary and security models for MACs

You have seen different constructions of hash functions

You specifically can explain the Merkle-Damgard construction, MD5, SHA-1

You know how to create collisions (and why that's bad)

You can explain how to create secure MACs from hash functions and PRFs

You can construct and explain the details of CBC, and HMAC