

	$0_{16}$	$1_{16}$	$2_{16}$	$3_{16}$	$4_{16}$	$5_{16}$	$6_{16}$	$7_{16}$	$8_{16}$	$9_{16}$	$a_{16}$	$b_{16}$	$c_{16}$	$d_{16}$	$e_{16}$	$f_{16}$
$0_{16}$	$0_{16}$	$1_{16}$	$2_{16}$	$3_{16}$	$4_{16}$	$5_{16}$	$6_{16}$	$7_{16}$	$8_{16}$	$9_{16}$	$a_{16}$	$b_{16}$	$c_{16}$	$d_{16}$	$e_{16}$	$f_{16}$
$1_{16}$	$1_{16}$	$0_{16}$	$3_{16}$	$2_{16}$	$5_{16}$	$4_{16}$	$7_{16}$	$6_{16}$	$9_{16}$	$8_{16}$	$b_{16}$	$a_{16}$	$d_{16}$	$c_{16}$	$f_{16}$	$e_{16}$
$2_{16}$	$2_{16}$	$3_{16}$	$0_{16}$	$1_{16}$	$6_{16}$	$7_{16}$	$4_{16}$	$5_{16}$	$a_{16}$	$b_{16}$	$8_{16}$	$9_{16}$	$e_{16}$	$f_{16}$	$c_{16}$	$d_{16}$
$3_{16}$	$3_{16}$	$2_{16}$	$1_{16}$	$0_{16}$	$7_{16}$	$6_{16}$	$5_{16}$	$4_{16}$	$b_{16}$	$a_{16}$	$9_{16}$	$8_{16}$	$f_{16}$	$e_{16}$	$d_{16}$	$c_{16}$
$4_{16}$	$4_{16}$	$5_{16}$	$6_{16}$	$7_{16}$	$0_{16}$	$1_{16}$	$2_{16}$	$3_{16}$	$c_{16}$	$d_{16}$	$e_{16}$	$f_{16}$	$8_{16}$	$9_{16}$	$a_{16}$	$b_{16}$
$5_{16}$	$5_{16}$	$4_{16}$	$7_{16}$	$6_{16}$	$1_{16}$	$0_{16}$	$3_{16}$	$2_{16}$	$d_{16}$	$c_{16}$	$f_{16}$	$e_{16}$	$9_{16}$	$8_{16}$	$b_{16}$	$a_{16}$
$6_{16}$	$6_{16}$	$7_{16}$	$4_{16}$	$5_{16}$	$2_{16}$	$3_{16}$	$0_{16}$	$1_{16}$	$e_{16}$	$f_{16}$	$c_{16}$	$d_{16}$	$a_{16}$	$b_{16}$	$8_{16}$	$9_{16}$
$7_{16}$	$7_{16}$	$6_{16}$	$5_{16}$	$4_{16}$	$3_{16}$	$2_{16}$	$1_{16}$	$0_{16}$	$f_{16}$	$e_{16}$	$d_{16}$	$c_{16}$	$b_{16}$	$a_{16}$	$9_{16}$	$8_{16}$
$8_{16}$	$8_{16}$	$9_{16}$	$a_{16}$	$b_{16}$	$c_{16}$	$d_{16}$	$e_{16}$	$f_{16}$	$0_{16}$	$1_{16}$	$2_{16}$	$3_{16}$	$4_{16}$	$5_{16}$	$6_{16}$	$7_{16}$
$9_{16}$	$9_{16}$	$8_{16}$	$b_{16}$	$a_{16}$	$d_{16}$	$c_{16}$	$f_{16}$	$e_{16}$	$1_{16}$	$0_{16}$	$3_{16}$	$2_{16}$	$5_{16}$	$4_{16}$	$7_{16}$	$6_{16}$
$a_{16}$	$a_{16}$	$b_{16}$	$8_{16}$	$9_{16}$	$e_{16}$	$f_{16}$	$c_{16}$	$d_{16}$	$2_{16}$	$3_{16}$	$0_{16}$	$1_{16}$	$6_{16}$	$7_{16}$	$4_{16}$	$5_{16}$
$b_{16}$	$b_{16}$	$a_{16}$	$9_{16}$	$8_{16}$	$f_{16}$	$e_{16}$	$d_{16}$	$c_{16}$	$3_{16}$	$2_{16}$	$1_{16}$	$0_{16}$	$7_{16}$	$6_{16}$	$5_{16}$	$4_{16}$
$c_{16}$	$c_{16}$	$d_{16}$	$e_{16}$	$f_{16}$	$8_{16}$	$9_{16}$	$a_{16}$	$b_{16}$	$4_{16}$	$5_{16}$	$6_{16}$	$7_{16}$	$0_{16}$	$1_{16}$	$2_{16}$	$3_{16}$
$d_{16}$	$d_{16}$	$c_{16}$	$f_{16}$	$e_{16}$	$9_{16}$	$8_{16}$	$b_{16}$	$a_{16}$	$5_{16}$	$4_{16}$	$7_{16}$	$6_{16}$	$1_{16}$	$0_{16}$	$3_{16}$	$2_{16}$
$e_{16}$	$e_{16}$	$f_{16}$	$c_{16}$	$d_{16}$	$a_{16}$	$b_{16}$	$8_{16}$	$9_{16}$	$6_{16}$	$7_{16}$	$4_{16}$	$5_{16}$	$2_{16}$	$3_{16}$	$0_{16}$	$1_{16}$
$f_{16}$	$f_{16}$	$e_{16}$	$d_{16}$	$c_{16}$	$b_{16}$	$a_{16}$	$9_{16}$	$8_{16}$	$7_{16}$	$6_{16}$	$5_{16}$	$4_{16}$	$3_{16}$	$2_{16}$	$1_{16}$	$0_{16}$

Tabelle 1: Ergebnis der XOR-Verknüpfung hexadezimal.

## Beispiele für die grundlegenden Rechenoperationen im AES

a) Verknüpfung von Bytes:

$$a(x) = x^6 + x^4 + x^2 + x + 1 = 0101\ 0111 = 57,$$

$$b(x) = x^7 + x + 1 = 1000\ 0011 = 83$$

$$c(x) = a(x) \oplus b(x) = x^7 + x^6 + x^4 + x^2 = 1101\ 0100 = D4.$$

$$\begin{aligned} d(x) &= a(x) \odot b(x) = (x^6 + x^4 + x^2 + x + 1)(x^7 + x + 1) \\ &= x^{13} + x^7 + x^6 + x^{11} + x^5 + x^4 + x^9 + x^3 + x^2 + x^8 + x^2 + x + x^7 + x + 1 \\ &= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \end{aligned}$$

$d(x)$  muss noch modulo  $m(x)$  reduziert werden; dazu wird der Rest von  $d(x)$  bei Division durch das Polynom  $m(x) = x^8 + x^4 + x^3 + x + 1$  bestimmt:

$$\begin{array}{r} (x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1) : (x^8 + x^4 + x^3 + x + 1) = x^5 + x^3 \\ \underline{x^{13} \quad + x^9 + x^8 + x^6 + x^5} \\ x^{11} \quad \quad \quad + x^4 + x^3 + 1 \\ \underline{x^{11} \quad + x^7 + x^6} \quad \quad + x^4 + x^3 \\ x^7 + x^6 \quad \quad \quad + 1 = r(x) \end{array}$$

$$d(x) = x^7 + x^6 + 1 = 1100\ 0001 = C1.$$

b) Verknüpfung von Polynomen mit Koeffizienten aus GF(2<sup>8</sup>):

$$a(x) = 01 \cdot x^3 + 03 \cdot x^2 + A1 \cdot x + 02,$$

$$b(x) = 02 \cdot x^3 + 01 \cdot x + FF$$

$$\begin{aligned} c(x) &= a(x) + b(x) \\ &= (01 \oplus 02) \cdot x^3 + (03 \oplus 00) \cdot x^2 + (A1 \oplus 01) \cdot x + (02 \oplus FF) \\ &= 03 \cdot x^3 + 03 \cdot x^2 + A0 \cdot x + FD \end{aligned}$$